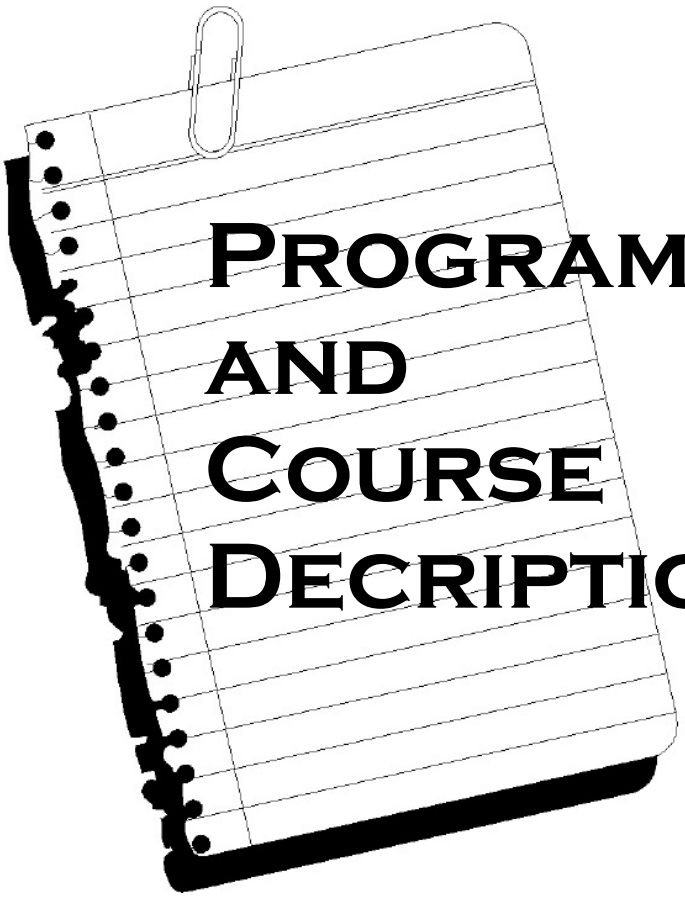

Program and Course Decriptions

Research Theme:
Mathematical Biology

Education Theme:
The Mathematics Education of Mathematics Teachers

IAS/Park City Mathematics Institute (PCMI) 2005 Summer Session

PCMI is a program of the **Institute *for* Advanced Study**

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The Graduate Summer School

The Graduate Summer School offers an intense introduction to problems and techniques in an active field of research. We expect 50-60 graduate students to attend the Summer Session this year. Course descriptions and a schedule for the lectures are included in this packet. It promises to be an exciting – and intense – three weeks!

As you can see from the summaries, the courses cover a lot of ground quickly. Because many post-docs and senior mathematicians will attend the lectures, there is great pressure on the speakers to go even faster and to raise the level of the lectures to suit the experts. It is your job to slow down the lecturers! Please feel free to interrupt the lecturers with questions to make sure they are proceeding at an appropriate speed.

There are other features of the Graduate Summer School designed to help you get the most out of the courses. The lecturers and their course assistants will run daily problem sessions related to the course material and will be available to answer questions. The lecturers will produce notes for the lectures, and the course assistants will distribute these notes during the Summer Session.

There will be many opportunities for mentoring relationships to develop, and you are encouraged to take the initiative in establishing these relationships. You and your mentor will have plenty of time to discuss mathematics in the relaxed atmosphere of Park City and away from teaching and the telephone. You are also encouraged to form study groups with other PCMI students. We think you will find meeting and working with other students to be an important part of the program.
There are three Summer School lectures each weekday: two in the morning and one in the early afternoon (except Wednesdays).

In addition, there are Cross Program Activities two or three times each week in the afternoon. Although your main focus will be the Graduate Summer School, you are strongly encouraged to take part in the Cross Program Activities. Teaching plays a large part in all our careers, and many of these activities deal with educational issues. Others deal with general mathematical content and are extremely enjoyable.

Please take a look at the information about the other PCMI programs as well. We are sure you will enjoy meeting and working with the other participants.

Many of you will be tempted to attend the sessions of the Research Program as well. While we encourage you to sample them, we caution you not to burn out on an overdose of *Mathematical Biology*. Three lectures a day for three weeks is more information than most people can absorb, so please don't try to do everything on the schedule.

Finally, you should plan to take advantage of the Park City area and its diverse recreational and cultural opportunities. Wednesday afternoons and weekends are typically a time for recreational activities.

Graduate students are strongly encouraged to bring a laptop to the summer session in order to participate in the GSS group projects being planned by the organizers.

Graduate Summer School Lectures

Introduction to Biological Dynamics
Lecturer: Mark Lewis

Description: A foundational module in which the basic tools of nonlinear systems, such as ordinary and partial differential equations will be addressed. Each lecture will focus on the interplay between mathematical tools and biological needs.

Introducation to Biological Dynamics
Lecturer: James Keener

Description: A foundational module in which the basic tools of nonlinear systems, such as ordinary and partial differential equations will be addressed. Each lecture will focus on the interplay between mathematical tools and biological needs.

Cell and Tissue Physiology
Lecturer: Alexander Mogilner

Description: Lecture module regarding the usage of mathematical models to deduce the structures and functions of cells (e.g. cytoskeleton) and tissue (e.g. myocardium.)

Epidemiology and Disease
Lecturer: David Earn

Description: Lecture module designed to discuss the interplay of mathematics and epidemiology to understand and propose controls for diseases such as measles, influenza and SARS.

Cancer
Lecturer: Helen Byrne

Description: Lecture module to discuss how cancer tumor formation and angiogenesis can be understood with complex spatial models. These models are now at a stage where they can describe the different stages of cancer progression. New mathematical approaches can be used to look for methods of controlling cancer.

Neurobiology
Lecturer: Paul Bressloff

Description: Lecture module concerned with the dynamical behavior of synaptically coupled spiking neurons. Topics will include stochastic dynamics, oscillations and synchrony, spatially-structured networks and synaptic plasticity. Applications to a variety of neurobiological systems will be described.

Ecological Dynamics
Lecturer: J.M. Cushing

Description: The broad themes of this module concern the temporal dynamics of biological populations and the interplay of mathematical models and experimental data. The lectures will include a synopsis of the modeling methodology and analysis for structured populations (in discrete time), with a focus on a bifurcation theory approach, and of methods for connecting the models with data. Applications will focus on laboratory experiments (with insects) conducted over the last fifteen years (by an interdisciplinary research team) that were designed on the basis of predictions of parameterized and validation models. The integrated model/experimental systems were used to investigate a variety of nonlinear phenomena in real biological populations under controlled, replicated conditions, including: equilibria and cycles, quasi-periodic dynamics, a bifurcation route to chaos, sensitivity to initial conditions, saddles and stable manifolds, state space lattice effects, environmental and demographic stochasticity, effects of periodically fluctuating habitats, and nonlinear resonance. If time permits, some discussion of current research topics under study will be included (such as multi-species competitive interactions, populations with synchronous life cycle stages, and the evolutionary of various life cycle traits).

Topological Approaches to Biological Dynamics
Lecturer: Leon Glass

Description: Lecture module on topical approaches to biolical dynamics.

For more information, please refer to the tentative lecture schedule that follows. A final schedule will be distributed in your Welcome Packet at Registration.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Week 1	8:30 Lewis/Keener 11:00 Lewis/Keener 2:00 Lewis/Keener	8:30 Lewis/Keener 11:00 Lewis/Keener 2:00 Mogilnor	8:30 Earn 11:00 Glass (2:00 no lecture)	8:30 Mogilner 11:00 Earn 2:00 Glass	8:30 Mogilnor 11:00 Earn 2:00 Glass
Week 2	PCMI is not in Session to celebrate Independence Day – July 4th	8:30 Mogilnor 11:00 Earn 2:00 Glass	8:30 Mogilnor 11:00 Earn 2:00 Glass	(8:30 no lecture) 11:00 Byrne 2:00 Bressloff	8:30 Cushing 11:00 Byrne 2:00 Bressloff
Week 3	8:30 Cushing 11:00 Byrne 2:00 Bressloff	8:30 Cushing 11:00 Byrne 2:00 Bressloff	8:30 Cushing 11:00 Byrne (2:00 no lecture)	8:30 Bressloff 11:00 Cushing 2:00 Wrap Up	8:30 Reports 11:00 Reports 2:00 Reports

Secondary School Teachers Program

The Secondary School Teacher Program regards the teacher as the primary agent for promoting and implementing classroom reform. This year's program will have a total of about 55 participants from the United States and Canada.

The Secondary School Teachers Program is designed to enable teachers to make informed decisions and to implement change with confidence. Gail Burrill of Michigan State University, and Carol Hattan of Skyview High School, Vancouver, Washington organize the program.

The PCMI Summer Session Secondary School Teachers Program is a paradigm for the lifelong professional development of high school teachers, just as PCMI's graduate summer school/research component is a paradigm for the lifelong professional development of a research mathematician. As such, the Secondary School Teachers Program includes the following three components:

- continued rigorous mathematical learning
- analysis of classroom practice
- research, production, and dissemination of materials for other teachers and their students.

Reflecting these three components, the PCMI summer session for secondary school teachers has three strands:

Developing Mathematics: Doing it with Differences
(2 hours per day, 5 days per week)

Focused on learning mathematics by working problems together, this course explores the fundamental mathematics on a topic that has its roots in secondary level, and is related to the mathematical theme of the Institute. Careful work on this topic allows teachers (and students) to understand exactly how elementary and more advanced procedures in the specific content area are derived and generalize. The course is structured so that each participant can work at his/her own level. Those who are more mathematically advanced may be asked to help those with less preparation. The course is conducted by teacher leaders from the PROMYS program at Boston University. The focus of this strand is entirely on mathematics, although opportunity is provided within the course for reflection on the approach used by the instructors and to consider the implications of such an approach for teaching in secondary classrooms. The topic for the summer is described below:

Many situations, from monthly payments on a car loan to the absorbtion of drugs into the bloodstream, to Fibonacci numbers, to models of population growth can be described with {\em difference equations \/}, equations that relate one term in a sequence to previous terms in a regular way. This course will develop several general-purpose methods for dealing with difference equations, including methods that use algebra, matrix algebra, combinatorics, and the theory of equations. Little by way of background is assumed, but we promise new and beautiful results by the end of week three.

Reflecting on Practice: Connections to Research
(1 hour per day, 5 days per week, plus opportunities for informal sessions in late afternoon and evenings)

After considering research related to teaching and learning mathematics, participants will reflect on the implications of this research for what takes place in classrooms. The discussion will be grounded in the development of lessons, student work, and videos of classroom practice. Participants will conduct small research investigations around some topic of interest in their secondary curriculum.

Working Groups
(2 hours, 4 days a week)
As part of their summer activities, each participant selected for the 2005 Secondary School Teachers Summer Program will be assigned to a small subject-specific working group, which will prepare an activity or resource for the profession (with the associated mathematics) for piloting during the following year.

Data Analysis, Statistics and Probability
This group will develop lessons that use technology, such as the statistical software FATHOM, as a tool for understanding the mathematics and the application of mathematics in contexts that require data analysis and statistical concepts. The goal is to produce lessons, describe teaching strategies, or present an issue related to the mathematics content at different mathematical levels from beginning algebra to Advanced Placement Statistics.

Geometry
Participants will consider one or more rich geometry topics from multiple mathematical and pedagogical perspectives with special emphasis on "hands-on investigations," model building, dynamic software, and other active approaches. From this work, the group will create lessons, activities, or discussion documents that relate to the classroom and geometrical investigations.

Lesson Study
Lesson study, Jugyuu Kenkyuu, is a professional development activity that is at the heart of much of the professional development for teachers in Japan. In this group, participants will select a daily goal for a "study lesson," spend an extended period of time collaboratively creating a detailed lesson plan, observe a participant teach the lesson to students, participate in a colloquium to discuss the lesson, and revise and refine the lesson to share with others.

Discrete Mathematics
Counting, networks and relations to discrete mathematics topics that have applications to mathematical biology, the research topic for PCMI this year. Participants in this working group will explore both the mathematics and biology in some of these applications, and work together to develop resource materials to help educators incorporate these interdisciplinary topics in their teaching.

Visualizing Functions
Participants will explore selected functions from multiple perspectives. Functions of interest may include polynomials and trigonometric functions, but also matrix functions and geometrical transformations. One possible topic would be the use of computer algebra systems to work with functions, and how these can support or hinder learning. For this topic teachers could study relevant research, design and evaluate lessons based on this study, or discuss implications for their own work with students.

Learning from Teaching Labs
For ten days during PCMI, Deborah Ball, University of Michigan, will be teaching a group of fifth grade students at PCMI, and reflecting on the student learning that takes place. This math education component of PCMI is the Teaching Lab. The lab will be open to some live observation, and it will also be videotaped extensively.

The Working Group will focus on learning from video of classroom practices. We will center our work on the Teaching Lab, which will be done in collaboration with Professor Ball and her team. Participants in the Working Group will attend the Lab in person one or two times, but most of the work will be done during Working Group time using video. We will construct ways of talking about classroom practices and student learning, as well as how we can continue our conversations beyond PCMI using tools like instant messaging and web chats.

Over a multi-year period, the working groups will:

- Review and critique existing materials and activities for the selected topic in the secondary curriculum
- Prepare and pilot 1 to 3 units or activities, together with the associated mathematics
- Prepare the activities for eventual publication in some form

[Note: The program for **Math Supervisors** will take place June 27-July 1. Information regarding that program will be sent directly to the participants of the group.]

[Note: The program for the **PD^3** leadership will take place the second week from July 5-July 9.]

Interaction is a key component of the PCMI Summer Session, and there will be many opportunities for you to interact with other mathematics educators, graduate students, researchers, and undergraduates.

In addition to the parts of the summer program specifically designed for the secondary school teacher participants, there are Cross Program Activities two or three times each week in the afternoon. Although your main focus will be the Secondary School Teachers Program and its activities, you are strongly encouraged to take part in the Cross Program Activities. Teaching plays a large part in all our careers, and many of these activities deal with educational issues. Others deal with general mathematical content and are extremely enjoyable.

Please take a look at the information about the other PCMI programs as well. We are sure you will enjoy meeting and working with the other participants.

Evenings and Wednesday afternoons may be used for study or teacher-organized mathematical gatherings or as free time. Weekends are free for adventures with other PCMI participants. Keep in mind that there is excellent hiking and bicycling in and near Park City, and several national parks are within driving distance.

Research Program (in Mathematics)

The Research Program offers researchers a stimulating environment for discussion, collaboration, and individual work. This year the research program has about 50 members. While no distinction is made between participants at different levels, most of the participants fall into the "postdoctoral" category, which is considered to be the mathematicians who are within five years of their PhD or do not yet have a tenured job. We report the distribution to funding agencies, and one of the strengths of this program is that so many younger mathematicians are funded.

15 research seminar meetings
Leaders of the field will discuss their own work in a leisurely environment, allowing considerable time for questions and comments. Participants will reflect on the current research agenda in the area of mathematical biology.

Topics for additional workshops and working groups will be chosen at the beginning of the Summer Session. It is expected that researchers will be active in other aspects of the PCMI Summer Session as well by offering talks or seminars to the other groups or attending their sessions. The rest of the time will be free for work and informal discussions.

The goal of the program is to provide an opportunity for researchers to meet and talk together, to contact the younger people entering the field, and hopefully even to get some work done. In the past, a few of the younger mathematicians have been asked to organize "working groups" in certain topics, just in case these groups do not form of their own accord. In general it is hoped that researchers will have time to pursue their own interests.

Please take the opportunity to interact with participants from other programs. You are encouraged to seek out students whom you can mentor. Your responsibility towards these students can be as little as "lunch once a week" or as much as you and the student want. This is a great opportunity to meet and interact with graduate students and other research mathematicians. The same goes for undergraduate students. In past years, volunteers who wished to work with undergraduates have been very welcome. Contact Roger Howe or Bill Barker, Undergraduate Program Organizers, if you would like to help.

We have left the most important part of the interaction to last. Whether you have a particular interest in education or simply want to expand your horizons, the PCMI Summer Session offers an easy and interesting opportunity to interact with secondary school teachers and learn what mathematicians and mathematics educators are doing in the arena of the education reform. In the past we have found research mathematicians to be very interested but not sure how to proceed. One popular feature of the Summer Session has been the Cross Program Activities. These activities are designed to introduce basic topics in, and explore aspects of, the area of concentration of the Summer Session and to provide a forum for interaction and discussion.

We also suggest sharing lunch tables or organizing joint leisure activities. And it is possible to arrange to attend some of the sessions in which teachers present their materials. The mathematicians who designed this integrated summer institute, and obtained the funding for it, have a serious commitment to working with secondary school teachers and learning from them. They have found it to be personally rewarding and believe it to be important for the future of our profession. You are invited and encouraged to take part.

Mathematics Education Research Program

The Mathematics Education Research Program offers an opportunity for intensive collaboration and individual work to researchers studying the teaching and learning of mathematics.

Elementary Mathematics Teaching Lab*
July 5-July 15

The goal of the Elementary Mathematics Lab is to investigate what is involved in teaching mathematics at the elementary level in ways that treat the mathematics with integrity and students' ideas with respect and mathematical sensitivity. Of particular interest is how essential ideas and ways of working that characterize mathematics at advanced levels might be made accessible to young students,

and how students might learn practices essential to mathematical work. A corollary problem is the mathematical knowledge needed for teaching: What do teachers have to do and "be" mathematically in order to engage students in such mathematical work?

The centerpiece of the program is a summer school course for approximately 20 fifth grade students that functions as a "laboratory" for the teaching of mathematics. During class, the EML participants observe and take notes on the lessons without interacting with the children while the classroom teacher teaches and the rest of the primary research team documents the class and assists as needed. The primary research team and the other participants make observations, form hypotheses, and "try out" mathematical representations, language, tasks, and ideas in the designs of the next lesson; they also carefully examine the collected documentation and discuss the results each day. For this reason, we often refer to the summer school course as a "lab class." It provides a site of study in which to examine "live" school mathematics as it is taught to young students. The EML provides a data-rich environment in which the disciplinary perspectives and expertise of mathematicians, mathematics educators, and K-12 teachers can be brought to bear on problems of teaching and learning mathematics. The lab class and the work of the participants are described in more detail below.

The real line will be used as the mathematical and representational context for students' work this summer, as they explore number concepts and operations. A mathematical emphasis will be placed on notation and representation, making generalizations, the use of definitions, and justification. The goal of this year's work is to explore how students of this age might learn about number and operations in the mathematical environment of the number line.

**Because of the necessity of limiting the number of classroom observers, participation in this program is by invitation only.*

Undergraduate Faculty Program

The Undergraduate Faculty Program offers mathematicians the opportunity to enhance the teaching of mathematics, especially at the collegiate level. We are expecting 17 participants in the program this year.

Introduction to Molecular Cell Biology for Mathematicians
John Tyson

Description: Topics addressed during the PCMI summer session will include:

- Cell structure: space and time scales in MCB
- Informational macromolecules of the cell: proteins and nucleic acids
- Bioenergetics: implications of the first and second laws of thermodynamics
- Enzyme catalysis, kenetics and regulation
- Metabolic pathways: glycolysis in detail, metabolic control theory
- Replication of gene expression
- Cell cycle of regulation and cancer
- Membrane structure, function and transport
- Membrane potential and electrical signaling in cells
- Membrane receptors, ligands and signal transduction pathways
- Calcium and cyclic AMP as second messengers
- Cytoskeleton, mobility and contractility

Recommended text(s): *The World of the Cell,* 4th Edition, Wayne M. Becker, et. Al.

Special Supplementary Course in conjunction with Undergraduate Summer School:
The Mathematics of Phylogenetic Trees
Elizabeth S. Allman, University of Southern Maine, and John A. Rhodes, Bates College

Description: Until recently, the inference of the evolutionary history of currently living species was based primarily on painstaking studies of their morphological similarities, together with comparison to the fossil record. Now a vast new source of evolutionary data is available through genetic sequencing. While similarities in DNA sequences among species suggest close ancestoral relationships and differences suggest greater evolutionary divergence, how to infer an entire evolutionary tree from biological sequences is a rich mathematical question.

This course begins with an overview of the sorts of biological questions of interest, and a look at thenature of biological sequence data. We then develop several of the modern approaches to

sequence-based phylogenetics, focusing on the modeling of the process of molecular evolution along a tree. Shortcomings of the various methods and models, both theoretical and practical, will be used to motivate new ones.

Necessary mathematical and biological background will be kept minimal: basic probability and linear algebra are helpful but can be picked up along the way. The course will also include elements of combinatorics, algorithmics, Markov models and statistics, as well as hands-on computer work with real and simulated data.

Undergraduate Summer School

The Undergraduate Summer School offers a combination of lectures, research projects and computer experimentation to help the students gain a deep understanding of *Mathematical Biology*. There will be about 40 undergraduate students attending the Summer Session this year. It promises to be an exciting – and intense – three weeks!

The lecturers for the Undergraduate Program will be Fred Adler, University of Utah, Lisette de Pillis, Harvey Mudd College, Elizabeth Allman, University of Southern Maine and John Rhodes, Bates College. Course assistants will run problem sessions and will be available to answer questions. The lecturers will produce notes for distribution.

Introductory Course: Dynamics, diseases and diversity
Lecturer: Fred Adler

Description: Mathematical biologists study a wide range of biological processes, including population growth, physiology and genetics. The underlying dynamics are mathematically interesting due to the strong positive and negative feedback among the many inter-connected components of complex living systems. This course will cover three broad areas. Firstly, we will use differential equations to study how diseases spread, do damage, and eventually die out. Secondly, we will use discrete-time dynamical systems to study how consumers, such as ourselves, interact with biological resources with potentially distressing consequences. And finally, we will use some elegant, but elementary, methods of probability theory to study how genetic systems maintain diversity, and show how modern genetic methods cast light on problems throughout biology.

Advanced Course: An Introduction to Cancer Modeling with Optimal Control
Lecturer: Lisette de Pillis

Description: The modeling of cancer growth and treatment is one that does not admit only narrow knowledge, but requires skills from multiple disciplines. This field of study lies at the intersection of biology and medicine, with mathematics at the core. Cancer development and the dynamics of the immune system have been a significant focus of mathematical modeling in recent decades. Immunotherapy, a treatment approach that enhances the body's natural ability to fight cancers, is becoming increasingly prevalent in many multi-stage treatment programs that also include chemotherapy, radiation, and surgery. The critical importance of the immune system in combating cancer has been verified both clinically and through mathematical models.

In this course, we will begin with an overview of the growing field of cancer modeling, surveying the broad number of mathematical techniques that have been taken to attacking this large and complex problem. We will then focus on specific models of cancer at the cellular level that include immune system responses, chemotherapy and immunotherapy. Model dynamics will be explored through bifurcation analysis techniques and numberical experiments. We will then introduce the calculus of variations and the mathematical theory of optimal control, which will be applied to these cancer models to determine theoretically improved treatment protocols.

Special Supplementary Course: The Mathematics of Phylogenetic Trees
Elizabeth S. Allman, University of Southern Maine, and John A. Rhodes, Bates College

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sequence-based phylogenetics, focusing on the modeling of the process of molecular evolution along a tree. Shortcomings of the various methods and models, both theoretical and practical, will be used to motivate new ones.
Necessary mathematical and biological background will be kept minimal: basic probability and linear algebra are helpful but can be picked up along the way. The course will also include elements of combinatorics, algorithmics, Markov models and statistics, as well as hands-on computer work with real and simulated data.

Most PCMI undergraduates will regularly attend two or three of the above sessions and possibly sample some of the others, such as the problem solving course of the Secondary School Teachers Program, or the more introductory lectures of the Graduate Summer School. However, please don't overdo it! It is too easy to become overwhelmed by the vast amount of mathematics available at the Institute. Concentrate on what is the most valuable to you at your current stage of preparation.

Most PCMI undergraduates will regularly attend two or three of the above sessions and possibly sample some of the others, such as the problem solving course of the Secondary School Teachers Program or the introductory lecture of the Graduate Summer School.

In addition to these sessions you are strongly encouraged to take part in the Cross Program Activities. We are sure you will enjoy meeting and working with participants in the other PCMI programs.

Finally, you should plan to take advantage of the Park City area and its diverse recreational and cultural opportunities. Wednesday afternoons and weekends are typically a time for recreational activities.