Resetting Reentrant Excitation Oscillations

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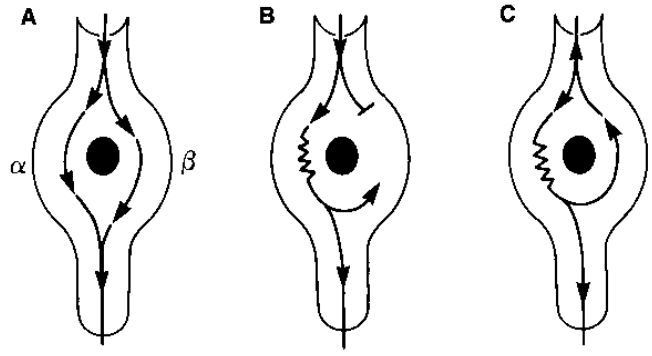
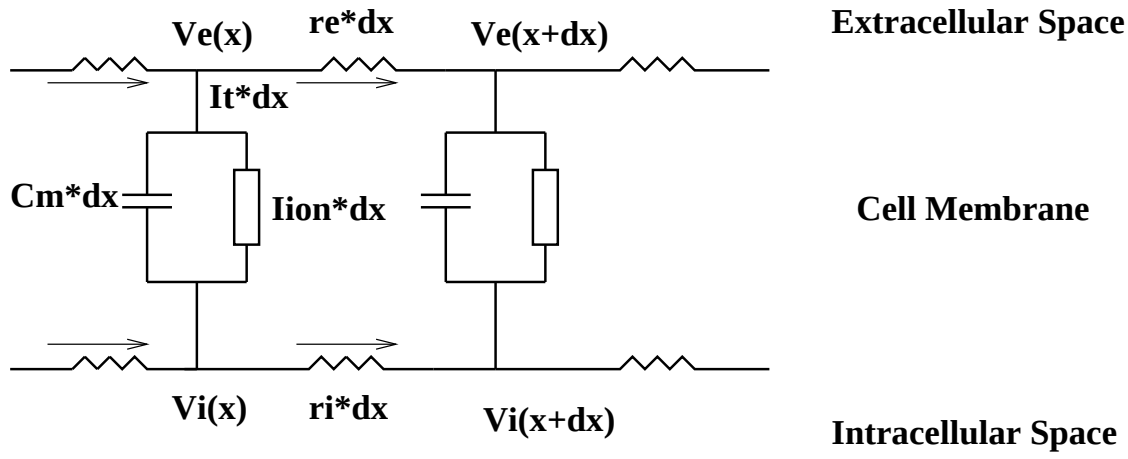
Reentry on cardiac tissue



- Normal heart rhythm is set by the the sinoatrial node
- Cardiac arrhythmia arises from abnormal propagation of cardiac impulses
- Reentry causes an abnormally rapid heart beat which may be fatal
  - Anatomical block
  - Premature stimulus applied to a partially refractory region (functional block)
- Treatment by application of periodic electrical stimuli through on an electrode on a surface of cardiac tissue

Modeling Cardiac Electrical Activity



Consider the cell as a long cable with a number of short pieces of isopotential membrane. In any piece all currents must balance

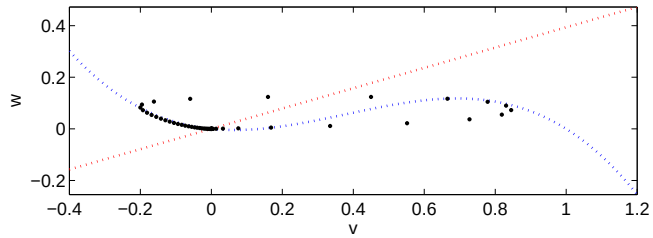
- *Transmembrane Current* - membrane as a capacitor and ion channels
- *Axial Current*

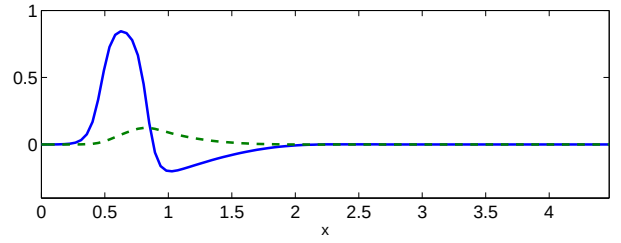
$$C_m \frac{\partial V}{\partial t} = D \frac{\partial^2 V}{\partial x^2} - I_{ion}(V, h, w)$$
$$\frac{\partial w}{\partial t} = G(V, w)$$

where $V = V_i - V_e$ is the transmembrane potential and w represents gating/recovery variable.

D is the conductivity tensor

Fitzhugh-Nagumo Equation





$$C_m \frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} - w - v(v-a)(v-1) + I(x,t)$$

$$\frac{\partial h}{\partial t} = \epsilon(v - bw)$$

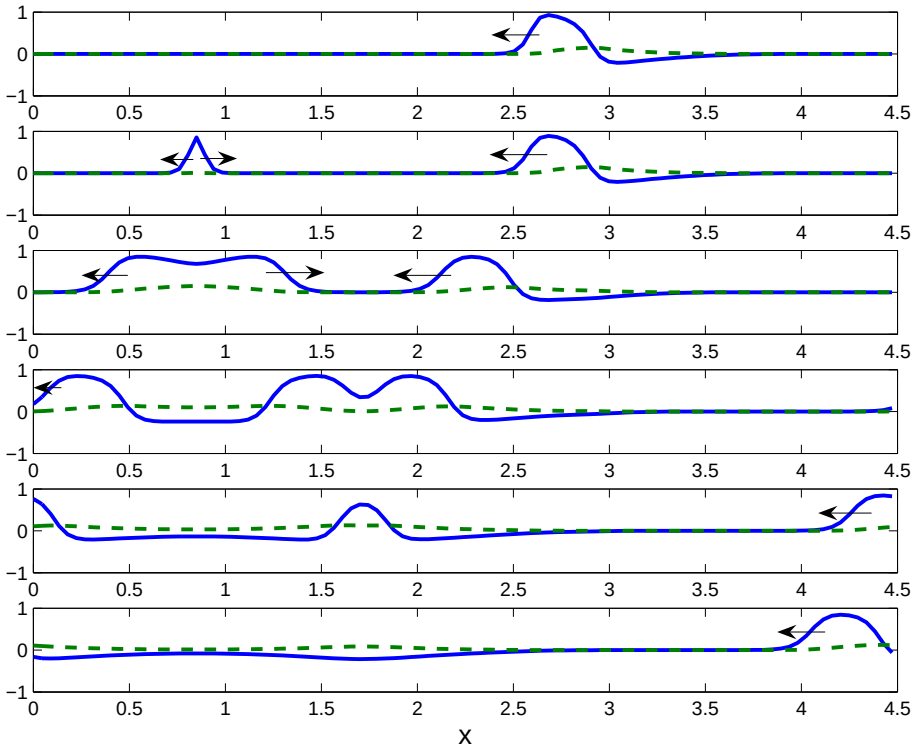
$0 \leq x < L$

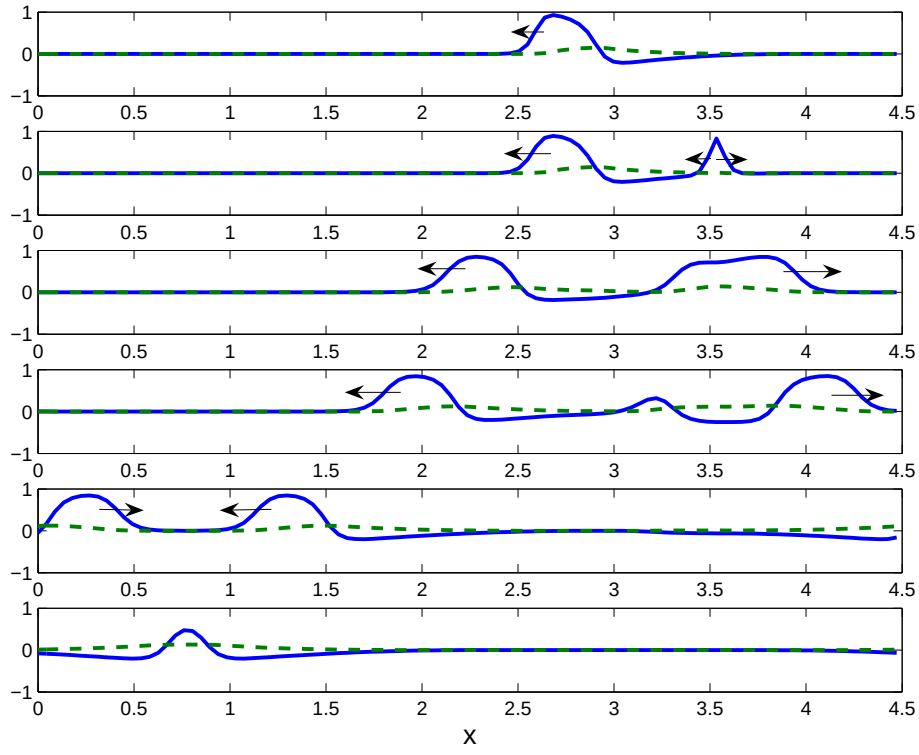
$a = 0.125,\ b = 0.139,\ \epsilon = 0.008$

$D = 0.001,\ L = 2\sqrt{5}$

- Stable oscillation (pulse traveling on the ring) exists with period of $T_o = 349.4$
- Supratreshold stimulus either leads to resetting or annihilation

Resetting and Annihilation of Wave on a 1D Ring

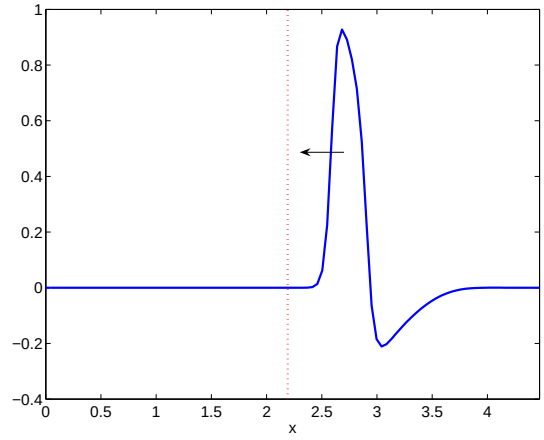




Resetting and Annihilation of Wave on a 1D Ring

- Topology of defibrillation (Keener)
- Movies?

Phase Resetting



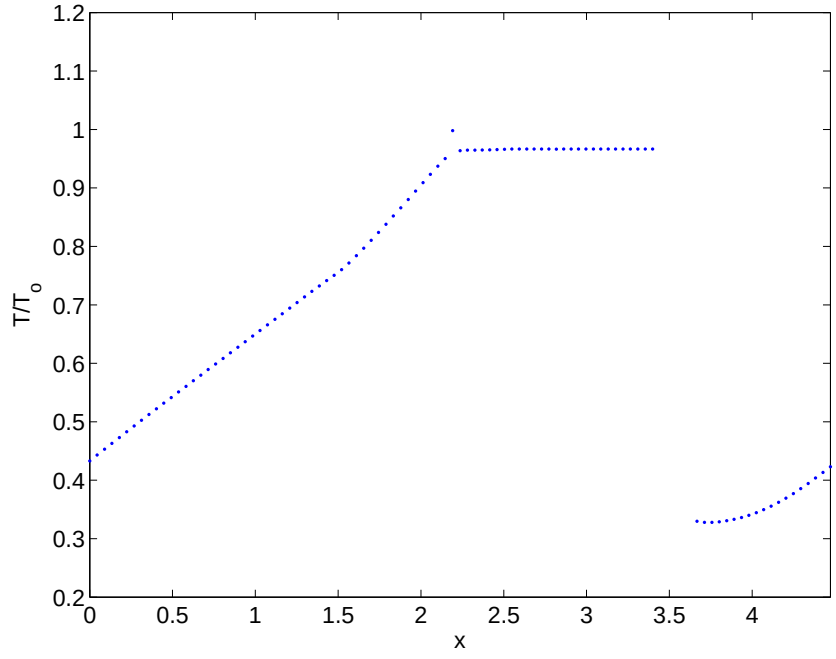
Assume that the cycle length of the stably circulating wave is T_o.

Phase definition is arbitrary - designate a particular event as phase 0. For example take $\phi = 0$

corresponding to the time when v hits its maximum at halfway around the ring .

After a stimulus is introduced, find the next time ϕ reaches 0 again.

Phase Resetting on a 1D Periodic Ring



- *Continuity Rule:* Resetting curve is continuous provided that stimulus leaves system in the basin of attraction (i.e. subtreshold stimulus)
  - Gedeon and Glass
- There exists a range of phase leading to annihilation of reentry pulse
- Discontinuity in phase resetting curve
  - Glass and Josephson

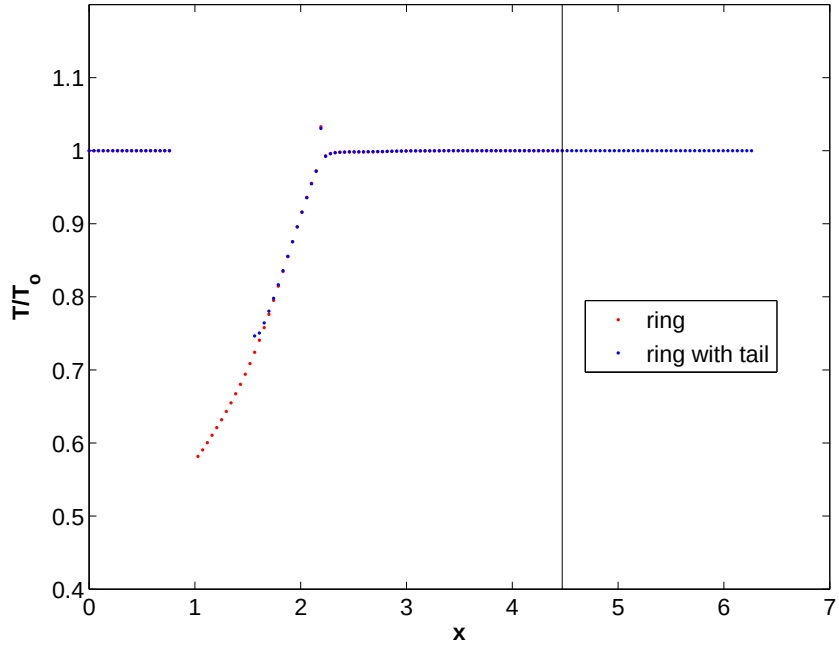
Resetting on a Different Geometry

- Location of reentry circuit is typically unknown - What's the dynamics for off-circuit pacing?
- Termination from off-circuit pacing seems impossible in homogenous media (Sinha and Christini)
  - Once a branch of the stimulus hits the circuit, it must be blocked by the refractory tail of reentrant wave.
  - Otherwise, resetting will just occur.

Resetting on a Different Geometry

- What's the effect of having an off-circuit path on resetting curve?
- Is it possible to get discontinuity without annihilating the reentry rhythm?
- Will periodic pacing from an off-circuit path work?
- Heterogeneities in reentry circuit

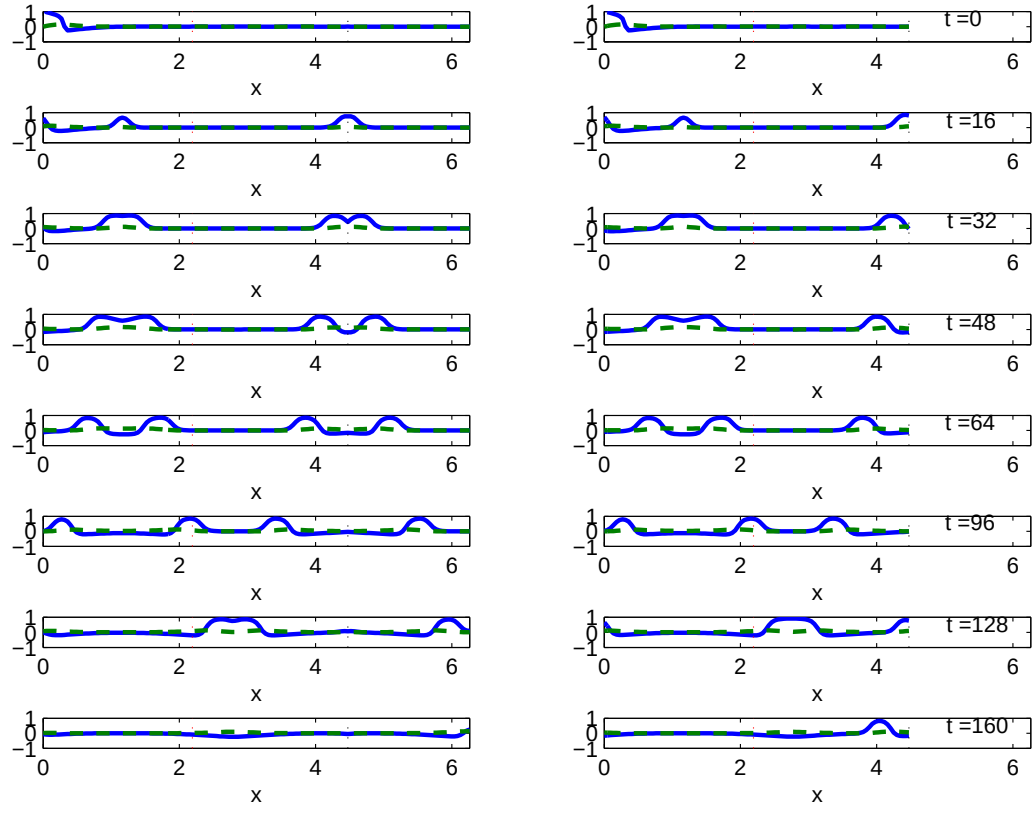
Phase Resetting on a 1D Ring with a Tail



- Larger range of phase for which annihilation will occur
- Off-circuit pacing however is not likely to lead to annihilation
- Periodic pacing doesn't seem to help either

References

- JP Keener, Topology of defibrillation (2003)
- T Gedeon and L Glass, Continuity of resetting curves for FHN eqn on the circle (1999)
- L Glass and ME Josephson, resetting and annihilation of reentrant abnormally rapid heartbeat (1995)
- S Sinha and DJ Christini, Termination of reentry in an inhomogenous ring of model cardiac cells (2002)


t =0
t =16
t =32
t =48
t =64
t =96
t =128
t =160
x