Resetting Reentrant Excitation Oscillations

Nessy Tania, Tongli Zhang
Steve Cantrell, Eric Cytrynbaum
PCMI 2005
Reentry on cardiac tissue

- Normal heart rhythm is set by the sinoatrial node.
- Cardiac arrhythmia arises from abnormal propagation of cardiac impulses.
- Reentry causes an abnormally rapid heart beat which may be fatal:
  - Anatomical block
  - Premature stimulus applied to a partially refractory region (functional block)
- Treatment by application of periodic electrical stimuli through an electrode on a surface of cardiac tissue.
Consider the cell as a long cable with a number of short pieces of isopotential membrane. In any piece all currents must balance

- **Transmembrane Current** - membrane as a capacitor and ion channels
- **Axial Current**

$$C_m \frac{\partial V}{\partial t} = D \frac{\partial^2 V}{\partial x^2} - I_{ion}(V, h, w)$$

$$\frac{\partial w}{\partial t} = G(V, w)$$

where \( V = V_i - V_e \) is the transmembrane potential and \( w \) represents gating/recovery variable.

\( D \) is the conductivity tensor.
Fitzhugh-Nagumo Equation

\[ C_m \frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} - w - v(v - a)(v - 1) + I(x, t) \]
\[ \frac{\partial h}{\partial t} = \epsilon(v - bw) \]

\[ 0 \leq x < L \]
\[ a = 0.125, \ b = 0.139, \ \epsilon = 0.008 \]
\[ D = 0.001, \ L = 2\sqrt{5} \]

- Stable oscillation (pulse traveling on the ring) exists with period of \( T_o = 349.4 \)
- Supratreshold stimulus either leads to resetting or annihilation
Resetting and Annihilation of Wave on a 1D Ring
Resetting and Annihilation of Wave on a 1D Ring

- Topology of defibrillation (Keener)
- Movies?
Assume that the cycle length of the stably circulating wave is $T_o$.

Phase definition is arbitrary - designate a particular event as phase 0. For example take $\phi = 0$ corresponding to the time when $v$ hits its maximum at halfway around the ring.

After a stimulus is introduced, find the next time $\phi$ reaches 0 again.
Phase Resetting on a 1D Periodic Ring

- **Continuity Rule**: Resetting curve is continuous provided that stimulus leaves system in the basin of attraction (i.e. subthreshold stimulus)
  - Gedeon and Glass
- There exists a range of phase leading to annihilation of reentry pulse
- Discontinuity in phase resetting curve
  - Glass and Josephson
Resetting on a Different Geometry

- Location of reentry circuit is typically unknown - What’s the dynamics for off-circuit pacing?

- Termination from off-circuit pacing seems impossible in homogenous media (Sinha and Christini)
  - Once a branch of the stimulus hits the circuit, it must be blocked by the refractory tail of reentrant wave.
  - Otherwise, resetting will just occur.
Resetting on a Different Geometry

- What's the effect of having an off-circuit path on resetting curve?
- Is it possible to get discontinuity without annihilating the reentry rhythm?
- Will periodic pacing from an off-circuit path work?
- Heterogeneities in reentry circuit
Phase Resetting on a 1D Ring with a Tail

Larger range of phase for which annihilation will occur

Off-circuit pacing however is not likely to lead to annihilation

Periodic pacing doesn’t seem to help either
References

- T Gedeon and L Glass, Continuity of resetting curves for FHN eqn on the circle (1999)
- L Glass and ME Josephson, resetting and annihilation of reentrant abnormally rapid heartbeat (1995)
- S Sinha and DJ Christini, Termination of reentry in an inhomogenous ring of model cardiac cells (2002)