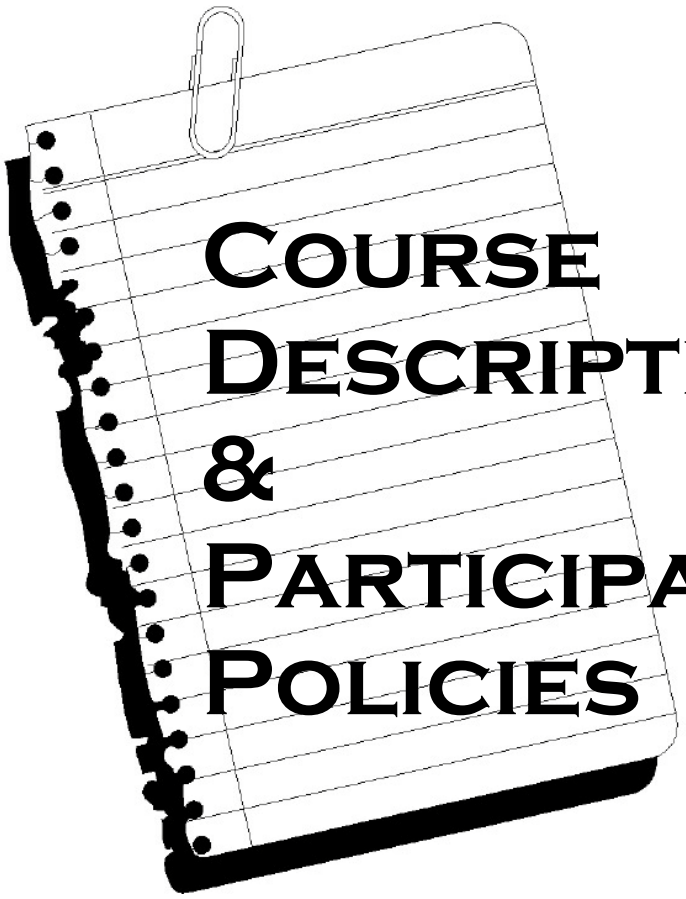

Course Descriptions & Participation Policies

Research Theme:
The Arithmetic of *L*-Functions

Education Theme:
Knowledge for Teaching Mathematics

IAS/Park City Mathematics Institute (PCMI)
2009 Summer Session

PCMI is a program of the **Institute *for* Advanced Study**

General Information:

Welcome to the 19th Annual PCMI Summer Session!

This booklet contains course descriptions, organized alphabetically by program, for each group of participants that will be meeting during the PCMI Summer Session. Be sure to read your program's section before you arrive at PCMI. Also, take a look at the information about the other PCMI programs; courses and seminars are open to all participants, no matter which program. Interaction is a key component of the PCMI Summer Session programming, and there will be many opportunities for you to interact with participants from all the programs.

Cross Program Activities are offered two or three times each week in addition to the regular course schedule. Although your main focus will be your own program, you are strongly encouraged to take part in the Cross Program Activities. Many of these activities deal with educational issues (teaching plays a large part in all our careers); others deal with general mathematical content and are extremely enjoyable.

A sample daily schedule can be found at the back of this booklet. The actual schedule for Monday, June 29h, will be available at PCMI Registration on June 28th.

In addition to attending the courses, seminars, and activities at PCMI, you should plan to take advantage of the Park City area and its diverse recreational and cultural opportunities. Wednesday afternoons and weekends are typically a time for recreational activities. Keep in mind that there is excellent hiking and bicycling in and near Park City, and several national parks are within driving distance. More information about the Summer Session and the Park City area can be found in the "Information and Policies" booklet distributed by PCMI.

Designing and Delivering Professional Development Program (DDPD)

This one-week seminar meets Sunday, July 5th through Friday, July 10th.

Facilitated by Johnny Lott, Director of the Center of Excellence in Teaching and Learning, University of Mississippi.

Designing and Delivering Professional Development consists of a one-week summer session experience for those who are responsible for professional development activities for secondary school mathematics teachers. This would include, among others,

- mathematicians;
- university mathematics educators;
- mathematics supervisors;
- leaders from the PCMI Professional Development and Outreach (PDO) groups.

This session is connected to and framed around the second week of PCMI's three-week Secondary School Teachers Program.

Partnerships among higher education mathematicians, curriculum coordinators/supervisors and classroom teachers have the potential to make a positive impact on student achievement.

The DDPD sessions will begin with a dinner meeting at 5;30 p.m. on Sunday, July 5th. Please be sure to have arrived in Park City by that time so that you are in attendance at the first meeting. Participants should meet in the Prospector Square front lobby at 5:20 p.m.

Monday, July 6th, through Friday, July 10th, the DDPD Participants will attend the morning sessions of the Secondary School Teachers Program (SSTP), which start at 8:20 a.m. The first morning session engages participants in doing mathematics collaboratively, and the second morning session is focused on reflecting on practice. Working groups meet in the afternoon. (See the SSTP section for more detailed descriptions of the SSTP courses and activities, plus any tools that might be needed for the SSTP activities.)

A final agenda for the week will be distributed at the dinner meeting on Sunday, July 5th.

All DDPD participants are asked to bring a laptop computer if at all possible.

From the organizers:

Dear Colleagues,

The summer session of the IAS/Park City Math Institute begins soon. We're looking forward to a week of intense and rewarding intellectual and social experiences. The 2009 summer program for the Designing and Delivering Professional Development (DDPD) program will begin on Sunday, July 5 with an organizational meeting and dinner at 5:30 p.m., and continue through Friday, July 10. The DDPD group consists of mathematicians and mathematics educators involved in professional development activities with teachers, including representatives from the PCMI Math Science Partnership Grant (PD^3) representatives from PCMI Professional Development and Outreach (PDO) Groups, and educators at large. We will provide a roster of attendees when you arrive.

Our vision of your experience at PCMI has multiple perspectives: first to allow newcomers to the group to take part in the activities and sessions of the Secondary School Teachers Program as well as other facets of PCMI and second, to allow the experienced DDPDers to work on other items during this same period and finally third, to concentrate as a group on issues relative to your work in professional development. To this end, some choices will be announced at our Sunday session. A draft agenda is included, and we will provide more details on July 5th.

Your work together will begin with the Sunday session and continue in the noted time slots Monday,

Tuesday, Wednesday with a half day off, Thursday, and Friday. The focus of the discussion will be twofold; the first is to share the activities some of our group does with teachers during the academic year. Several of you have already agreed to do this and if others of you would like to share as well, please let us know and we will adjust the schedule. The second is to consider some aspects of functions and how they relate to the work of professional development. How can we design and implement professional development experiences that make use of important aspects of mathematics and how might we approach mathematics through writing? We have included two readings. Johnny Lott will be the chair for our sessions.

Please read the *Participant Guidelines & Information* booklet and the other information included in the general packet sent to you about PCMI in May. If you did not receive that mailing, please go to http://pcmi.ias.edu/current/SummerSessionFormsandInformation.php and download the documents from that page.

To register for the conference, you will need to check in at Prospector Square Lodge and Conference Center located at the corner of Sidewinder Drive and Gold Dust Drive between 3:00 p.m. and 5:00 p.m. on Sunday, July 5. Our initial meeting is at 5:30 p.m. in the Grub Steak Restaurant across the street from the Center. Breakfast Monday morning will be in the Moose and Miner Room at the Grub Steak restaurant (across from the front of Prospector Square Conference Center, using the corner entry door).

Yours truly,

Johnny Lott	Gail Burrill	Jim King
Director	Coordinator	Coordinator

DDPD Participation Policy: participants who are funded by PCMI are expected to attend all scheduled DDPD sessions from Sunday, July 5th to Friday, July 10th.

Graduate Summer School (GSS)

The Graduate Summer School offers an intense introduction to problems and techniques in an active field of research. We expect about 70 graduate students to attend the Summer Session this year. Course descriptions and a schedule for the lectures are included in this booklet. It promises to be an exciting – and intense – three weeks!

As you can see from the summaries, the courses cover a lot of ground quickly. Because many post-docs and senior mathematicians will attend the lectures, there is great pressure on the speakers to go even faster and to raise the level of the lectures to suit the experts. It is your job to slow down the lecturers! Please feel free to interrupt the lecturers with questions to make sure they are proceeding at an appropriate speed.

There are other features of the Graduate Summer School designed to help you get the most out of the courses. The lecturers and their course assistants will run daily problem sessions related to the course material and will be available to answer questions. The lecturers will produce notes for the lectures, and the course assistants will distribute these notes during the Summer Session.

There will be many opportunities for mentoring relationships to develop, and you are encouraged to take the initiative in establishing these relationships. You and your mentor will have plenty of time to discuss mathematics in the relaxed atmosphere of Park City and away from teaching and the telephone. You are also encouraged to form study groups with other PCMI students. We think you will find meeting and working with other students to be an important part of the program.

There are three Summer School lectures each weekday: two in the morning and one in the early afternoon (except Wednesdays, as noted on the schedule).

Many of you will be tempted to attend the sessions of the Research Program as well. While we encourage you to sample them, we caution you not to burn out on an overdose! Three lectures a day for three weeks is more information than most people can absorb, so please don't try to do everything on the schedule.

Course Descriptions and Suggestions for Preparation:

Introduction to Stark's Conjectures; *John Tate, University of Texas, Austin*
These conjectures concern the leading term coefficient c(X) of the Taylor expansion at s=0 of the Artin L-function L(s.X) attached to the character of a representation of a Galois group G of a Galois extension K/k of number fields. In the case of the zeta function (K=k, X=1)the leading coefficient is -hR/w. Stark's great achievement in the 1970's was to suggest an analog for arbitrary characters, for which there is by now overwhelming evidence, both theoretical and computational, but no proof. After a brief introduction to L-functions, I will explain Stark's basic idea, and how it leads conjecturally to ``Stark units'', a vast generalization of cyclotomic units.

The lectures will assume a basic knowledge of Algebraic number theory: splitting of places in extensions, decomposition group, inertia group, Frobenius automorphism, S-class number, S-unit theorem, Regulator... . Especially important is to understand the statement of the unit theorem as the existence of an isomorphism of the group of S-units, mod roots of unity, onto a lattice spanning a hyperplane in the real vector space with basis the places in S.

A knowledge of the basics of the theory of linear representations of finite groups will also be assumed: semisimplicity, characters, orthogonality, induced representations, Frobenius reciprocity... .

Root Numbers; *David Rohrlich, Boston University*
We will be concerned with two issues: (i) How does one compute the factors in the functional equation (known or conjectured) of various kinds of L-functions (Hecke *L*-functions, Artin *L*-functions, and *L*-functions of elliptic curves, for example)? (ii) To what extent, or under what circumstances, should one expect the order of vanishing of an *L*-function at the center of its critical strip to be the smallest value permitted by its functional equation? The main prerequisites for the lectures are basic algebraic number theory and some familiarity with Dirichlet *L*-functions and with elliptic curves over the rationals.

Introduction to Elliptic Curves; *Alice Silverberg, University of California, Irvine*
This talk will be a brief introduction to elliptic curves. Topics will include: basic definitions; some discussion of elliptic curves over number fields, the complex numbers, and finite fields; the Mordell-Weil theorem; the Mazur/Merel torsion subgroup theorems; and ranks of elliptic curves over number fields.

Introduction to the Birch and Swinnerton-Dyer Conjecture; *Benedict Gross, Harvard University*
These lectures will give an overview of the Birch and Swinnerton-Dyer conjecture, which relates the *L*-function of an elliptic curve at s=1 to arithmetic information about the curve. We will formulate the conjecture, discuss the current state of progress on it, and describe some methods for attacking it.

The Equivariant Tamagawa Number Conjecture; *David Burns, King's College London and Guido Kings, UniversitätRegensburg*

This is a course on the Equivariant Tamagawa Number Conjecture (ETNC) for Dirichlet *L*-functions and *L*-functions associated to elliptic curves.The course will focus on:

1. stating the conjecture;
2. presenting evidence in support of the conjecture;
3. proving that the integral and p-adic refinements of Stark's Conjecture (a la Rubin and Gross) are consequences of the ETNC for Dirichlet *L*-functions and that the Birch and Swinnerton-Dyer Conjecture is a

consequence of the ETNC for *L*-functions associated to elliptic curves.

Integral Abelian Stark-type Conjectures; *Manfred Kolster, McMaster University and Cristian Popescu, University of California-San Diego*
Topics:

1. Integral refinements of Stark's Conjecture for abelian *L*-functions of arbitrary order of vanishing at s=0 and consequences.
2. p-adic refinements of Stark's Conjecture for abelian *L*-functions and consequences.
3. The conjectures of Lichtenbaum and Coates-Sinnott on special values of abelian *L*-functions at negative integers.
4. An equivariant main conjecture in Iwasawa theory and consequences.

Euler Systems; *Karl Rubin, University of California-Irvine*
Euler systems were introduced by Kolyvagin as a new tool for bounding the size of ideal class groups and Selmer groups, and for relating the sizes of those groups to special values of *L*-functions. In this course we will describe the basic Euler system machinery, and apply it in the fundamental cases of cyclotomic fields (class number formulas) and elliptic curves (the Birch and Swinnerton-Dyer conjecture).

The Birch and Swinnerton-Dyer Conjecture over Function Fields; *Douglas Ulmer, University of Arizona*
We know a lot more about ranks of elliptic curves and the conjecture of Birch and Swinnerton-Dyer over function fields than we do over number fields. I plan to discuss how one can prove special cases of the the BSD conjecture over function fields as well as how one can construct elliptic curves with large Mordell-Weil groups. Much of this also applies to higher dimensional Jacobians. The lectures should be accessible to anyone with a first course in algebraic geometry and some acquaintance with elliptic curves.

Complex Multiplication and Heegner Points; *Vinayak Vatsal, University of British Columbia*
We will start by discussing the Kronecker-Weber theorem, which gives a description of the abelian extensions of the rational numbers in terms of points of finite order on the circle group. We then move to the theory of complex multiplication, which gives an analogous description of the abelian extensions of imaginary quadratic fields in terms of point of finite order on certain special elliptic curves, the so called CM elliptic curves. From this we move on to Heegner points, namely, the points on modular curves associated to these CM elliptic curves. We will discuss their basic properties, and some of their surprising applications to number fields and the arithmetic of all elliptic curves. If time permits, we will discuss some of the many generalizations of CM points to higher dimensions, other number fields, and p-adic settings.

Graduate Summer School Lecture Schedule 2009

	Monday	Tuesday	Wednesday	Thursday	Friday
WEEK 1					
8:30 a.m.	Tate	Tate	Tate	Tate or Popescu	Popescu
11:00 a.m.	Silverberg	Rohrlich	Rohrlich	Rohrlich	Rohrlich
2:00 p.m.	Birch	Gross	(none)	Gross	Gross
WEEK 2					
8:30 a.m.	Ulmer	Ulmer	Ulmer	Ulmer	Ulmer
11:00 a.m.	Rohrlich	Rubin	Rubin	Rubin	Rubin
2:00 p.m.	Popescu	Popescu	(none)	Popescu or	Kolster
WEEK 3					
8:30 a.m.	Vatsal	Vatsal	Vatsal	Vatsal	Vatsal
11:00 a.m.	Rubin	Kings	Kings	Kings	Grad student research talks
2:00 p.m.	Burns	Burn	(none)	Burns	(thru remainder of day)

Research Program (RP)

The Research Program offers researchers a stimulating environment for discussion, collaboration, and individual work. This year the research program has about 50 members, covering a broad range from recent PhD's to senior researchers.

There will be two research seminars on most days, depending on the preferences of the organizers and participants. The schedule will be established as the Summer Session progresses. Topics for additional informal sessions and working groups may be arranged as well, if desired. It is expected that researchers will also be active in other aspects of the PCMI Summer Session by interacting with participants in the other groups and/or attending their sessions. The rest of the time will be free for work and informal discussions.

This year's program will focus on recent developments in the study of special values of L-functions. The central problems are Stark's conjectures, the Birch and Swinnerton-Dyer conjecture, and generalizations such as the equivariant Tamagawa number conjecture. A substantial amount of background on these questions will be covered in the graduate courses, while the research seminars are intended to present recent results and work in progress. Researchers will discuss their own work in a leisurely environment, allowing considerable time for informal discussion.

A primary goal of the research program is to foster the collaboration of a diverse group of participants. The program is designed to provide an opportunity for researchers to meet and talk together, interact with students entering the field, and hopefully even get some work done.

Please take the opportunity to interact with participants from other programs. You are encouraged to seek out students whom you can mentor.

Your responsibility towards these students can be as little as "lunch once a week" or as much as you and the student want. While this is a great opportunity to meet and interact with graduate students and other research mathematicians, the same is also true with regard to undergraduate students. In past years, volunteers who wished to work with undergraduates have been very welcome. Contact Aaron Bertram, Undergraduate Summer School Organizer, if you would like to help.

One popular feature of the Summer Session has been the Cross Program Activities. These activities are designed to introduce basic topics in, and explore aspects of, the area of concentration of the Summer Session and to provide a forum for interaction and discussion among participants from different programs.

The mathematicians who designed this integrated summer institute, and obtained the funding for it, have a serious commitment to working with secondary school teachers and learning from them. They have found it to be personally rewarding and believe it to be important for the future of our profession. You are invited and encouraged to take part.

Whether you have a particular interest in education or simply want to expand your horizons, the PCMI Summer Session offers an easy and interesting opportunity to interact with secondary school teachers and learn what mathematicians and mathematics educators are doing in the arena of education reform. For example, it is possible to arrange to attend some of the sessions in which teachers present their materials.

We also suggest sharing lunch tables or organizing joint leisure activities with participants from other programs.

Secondary School Teachers Program (SSTP)

The Secondary School Teacher Program regards the teacher as the primary agent for promoting and implementing classroom reform. This year's program will have about 55 participants from the United States and Canada.

The SSTP is designed to enable teachers to make informed decisions and to implement change with confidence. Gail Burrill of Michigan State University, James King, University of Washington, and Carol Hattan of Skyview High School, Vancouver, Washington, organize the program.

The SSTP is a paradigm for the lifelong professional development of secondary level school teachers, just as PCMI's graduate summer school/research component is a paradigm for the lifelong professional development of a research mathematician. As such, the Secondary School Teachers Program includes the following three components:

- continued rigorous mathematical learning
- analysis of classroom practice
- research, production, and dissemination of materials for other teachers and their students.

Reflecting these three components, the PCMI summer session for secondary school teachers has three strands:

1.Questions and Problems in Arithmetic (2 hours per day, 5 days per week)
This course will investigate questions like these:

In how many ways can an integer be written as the sum of two squares?
In how many ways can an integer be written as the sum of four squares?
What's the probability that an integer picked at random has no perfect square factor?
What's the probability that two integers picked at random have no common factor?

Which linear functions $f(x) = ax+b$ (a and b integers) generate infinitely many prime numbers for integer values of x?
What is the probability that an integer picked at random between 1 and 1020 is a prime number?

The real goal of the course is to answer the following question:

How are all of the above questions related?

In this three week course, you will investigate questions like those above and develop underlying structural similarities among them. In particular, you will see how to use the algebra of sequences and series as a general- purpose tool for these investigations. No prior knowledge of number theory or sequences and series is assumed, and by the end of three weeks, you'll be astounded at what you've discovered about arithmetic and its applications to the 7-12 curriculum.

2. Reflecting on practice: Connections to Research (1 hour per day, 5 days per week, plus opportunities for informal sessions in late afternoon and evenings)
Participants will consider research related to teaching and learning mathematics with a particular focus on student thinking and questions designed to provoke student thinking. The discussion will be grounded in the study of discourse in lessons and classroom practice in both the United States and other countries. Participants will work collaboratively to consider different approached to questioning that better enable students to learn mathematics.

3. Working Groups (2 hours, 4 days a week)
As part of their summer activities, each participant in the 2009 Secondary School Teacher Summer Program will be assigned to a small, subject-specific working group, which will prepare an activity or resource for the profession (with the associated mathematics) for piloting during the following year.

Participants will collaborate with others in their working group to create a product that can be shared with others at PCMI, and across the larger mathematical education community. Each working group is composed of 4-6 teacher-participants and a resource person. Working collaboratively, the participants will research existing classroom materials and techniques, technologies, and other materials related to the topic, for dissemination and eventual publication by PCMI. The focus of the work will be on creating and elaborating meaningful tasks, designing activities to extend content knowledge, illustrating the use of technology and reflecting on ways to improve teaching and learning of mathematics.

The 2009 working groups are:
Exploring Discrete Mathematics: Members of this working group will explore topics from the rich field of discrete mathematics, which is increasingly important in both various applications and K-12 education. Possible topics include counting techniques, graph theory, logic games, and in the light of this year's research topic, coloring maps on bagels. Participants will develop resources to help their peers incorporate this material into their classrooms

Investigating Geometry: Participants will consider one or more rich geometry topics from multiple mathematical and pedagogical perspectives with special emphasis on "hands-on-investigations," model building, dynamic software, and other active approaches. From this work, the group will create lessons, activities, or discussion documents that relate to the classroom and geometrical investigations.

Learning from Teaching Cases: Participants will engage in rich mathematical tasks, watch video of classroom cases, read and discuss relevant articles, all of which will be developed in context of our larger goal of developing mathematical knowledge for teaching through instructional cases. We will co-construct ways of talking about classroom practices and student learning, which will help us collectively move forward on learning how to productively observe and interpret learning and teaching on video and in person. The group will create a product that reflects the discussions and the big ideas we developed in our group.

Teacher Professional Continuum Writing Group: As a part of the Teacher Professional Continuum (TPC) program the National Science Foundation has funded PCMI to create and disseminate materials based on the Developing Mathematics course. The TPC

working group will make decisions about what support facilitators would need to teach these courses, and then work on creating that facilitator support.

Implementing Lesson Study: Lesson study, Jugyuu Kenkyuu, is a professional development activity that is at the heart of much of the professional development for teachers in Japan. In this group, participants will select a daily goal for a "study lesson," spend an extended period of time collaboratively creating a detailed lesson plan, observe a participant teach the lesson to students, participate in a colloquium to discuss the lesson, and revise and refine the lesson to share with others.

Visualizing Functions: Participants will explore selected functions from multiple perspectives. Functions of interest may include polynomials and trigonometric functions, but also matrix functions and geometric transformations. One possible topic would be the use of computer algebra systems to work with functions, and how these can support or hinder learning. For this topic teachers could study relevant research, design and evaluate lessons based on this study, or discuss implications for their own work with students.

Analytic Number Theory: This working group will take part in an undergraduate faculty mathematics course related to the Institute theme — and then, facilitated by an SSTP member, will consider the implication of mathematics for the high school and their work as teachers.

Reasoning from Data and Chance: Participants will explore ideas relating to data and chance using the Navigation Series from the National Council of Teachers of Mathematics as a resource. Technology, such as the statistical software FATHOM, will be used as a tool for learning. The goal is to produce lessons, describe teaching strategies, or present an issue related to statistics or probability. Content related to the study of data in beginning secondary courses as well as in AP Statistics will be explored

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A special letter from the organizers of the SSTP:

The summer session of the IAS/Park City Math Institute is fast approaching. We're looking forward to meeting you and to three weeks of intense and rewarding intellectual and social experiences. While some of you will be returning to Park City, many will be attending for the first time. The 2008 summer program is of three weeks' duration, from Mondays through Fridays June 29–July 17.

All SSTP participants will attend a Number Theory course called "Some Questions and Problems in Arithmetic " and a course "Reflecting on Practice: Connections to Research" in the mornings. The afternoons will be devoted to the various Working Groups (Reasoning, Data and Chance; Exploring Discrete Mathematics; Investigating Geometry; Learning from Teaching Cases; Visualizing Functions; or Analytic Number Theory.) You have received a separate email assigning you to a working group. Your group may do some work ahead of time so that you will be able to know what supplies to bring. The last part of the afternoon will be taken up with a cross program activity that involves everyone at PCMI or a focus group to be defined after you arrive. The last hour of the day and the evening will be scheduled with a variety of activities including group presentations, joint meetings with other groups, technology training, or session where you can share activities with other teachers. No sessions or activities will be scheduled on Wednesday afternoons, Wednesday and Friday evenings, or the weekends.

On several evenings we will work with hands-on activities that will allow us to share and get acquainted with each other. In addition the returning participants will share special activities they learned at PCMI. During subsequent weeks the leadership group will schedule various sessions. If you have interesting activities that would enrich these evenings please bring them along. If there are special supply needs, please let Carol Hattan know before you come.

Some basic supplies to bring:

Math tools

Compass, ruler, protractor, and calculator (we will have TI 84s, and TI NSPIRE CAS available)

Laptop

All SSTP participants are encouraged to bring a laptop computer. A wireless network is available.

Ideas

Do you have any "gem" problems of the day/week, that are accessible to high school students, but interesting enough to challenge teachers and mathematicians? Do you have an idea for a collaborative project that could involve participants from other programs? Do you have an idea that you would really like to share during a hands-on evening meeting? Even though we may have limited time to share these with the whole group, you will have plenty of informal time to share with your colleagues.

Read

Please read the "Information and Policies" booklet and the other information distributed by PCMI.

Registration will take place in the Grand Theater foyer of the Prospector Square Conference Center between 1:00 p.m. and 8:00 p.m. See the "Information and Policies" booklet for instructions on what to do if you arrive later than 8:00 p.m. on Sunday.

An Opening Social will be held at 7:00 p.m., Sunday, June 28th, in the SSTP meeting room at the Conference Center building. Light refreshments will be served.

Classes in the Secondary School Teacher Program begin at 8:15 a.m., so we'll see you bright and early on June29th!

Yours truly,

Gail Burrill, Carol Hattan, Jim King
Organizers,
PCMI Secondary School Teacher Program

SSTP Participation Policy: Secondary School Teachers Program participants who are funded by the IAS/Park City Mathematics Institute are expected to participate in all three components of the program (Problem Solving Course, Reflecting on Practice, and Working Groups) from the beginning of the summer session on the morning of June 29thh to the end of the summer session on the afternoon of July 17th.

Evenings and Wednesday afternoons may be used for study or teacher-organized mathematical gatherings or as free time. There will be late afternoon (4:30) and evening sessions (7:30) that are optional – these are scheduled to take advantage of the rich source of expertise available from PCMI participants.

If permission has been granted for a less than three-week participation, stipend and meal allowance checks will be pro-rated accordingly.

Stipend checks will be issued on Friday, July 17th.

Undergraduate Faculty Program (UFP)

Building an Undergraduate Course in Algebraic Number Theory; *David Pollack, Wesleyan University*.

When studying the integer solutions to a polynomial equation one is often led to work with more general numbers than just the integers. For example, to understand the solutions to the equation $x^2 + y^2 = p$ for a prime p it is natural to attempt to factor the left hand side as $(x + iy)(x - iy)$ and then to search for ways to factor p in the ring $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}\}$ of *Gaussian integers*. Factorization in $\mathbb{Z}[i]$ behaves very much like factorization of ordinary integers, but if we venture even a little further striking new phenomina pop up. For example, in the ring $\mathbb{Z}[\sqrt{-5}]$ we can no longer factor all numbers uniquely as a product of primes: the equation $(1 + \sqrt{-5})(1 - \sqrt{-5}) = (2)(3)$ gives two different factorizations of 6, and none of the four factors appearing can be factored any further!

Here $\mathbb{Z}[i]$ and $\mathbb{Z}[\sqrt{-5}]$ are examples of *number rings*, which are the central objects of study in algebraic number theory. Algebraic number theory is an excellent next step for undergraduates who have already studied abstract algebra. A course in algebraic number theory can provide an opportunity to reinforce much of the material from abstract algebra, and can give students a deeper understanding of how some of those ideas are used in higher mathematics. For example, to resolve the lack of uniqueness of factorization in $\mathbb{Z}[\sqrt{-5}]$ we are forced to work with factorization of *ideals* rather than of elements, and to understand the prime ideals of more general number rings we need to use ideas from Galois theory.

Our goal in the Undergraduate Faculty Program will be to develop such an undergraduate algebraic number theory course, with an emphasis on exploration and problem solving. We'll begin by exploring concrete examples, including the rings $\mathbb{Z}[\sqrt{d}]$ as above, and then introduce the required machinery to put all of our examples into a common framework. The concepts we'll require from abstract algebra will be reviewed along the way.

UFP Participation Policy: Undergraduate Faculty Program participants who are funded by the IAS/Park City Mathematics Institute are expected to participate from the beginning of the summer session on the morning of June 29th to the end of the summer session on the afternoon of July 17th.

Undergraduate Summer School (USS)

The Undergraduate Summer School offers a combination of lectures, research projects and computer experimentation to help the students gain an understanding of topics related to *The Arithmetic of L-functions*. There will be about 45 undergraduate students attending the Summer Session this year. It promises to be an exciting – and intense – three weeks!

Dirichlet's *L*-functions, Generalizations, and Applications; *Keith Conrad, University of Connecticut*: The first use of *L*-functions, by Dirichlet (1837), was in his proof that there are infinitely many primes in any arithmetic progression $a, a+m, a+2m, \ldots$, where a and m are relatively prime. This course will begin by proving Dirichlet's theorem and developing some basic properties of Dirichlet *L*-functions: analytic continuation, functional equation, and special values. Analogies between $\mathbf{Z}$ and $\mathbf{F}[T]$,

where **F** is a finite field, are an important theme in number theory, and we will see the analogue of Dirichlet's theorem and Dirichlet *L*-functions for **F**[*T*]. Moreover, the Riemann hypothesis can be proved in this setting, which leads to bounds on character sums of classical interest (with applications to counting points mod *p* on elliptic curves). Returning to **Z**, we will develop algebraic number theory for cyclotomic fields in order to rewrite Dirichlet *L*-functions in terms of characters on abelian Galois groups. Then we will see how to do this for the analogous *L*-functions on **F**[*T*]. At the end we will introduce Artin *L*-functions, which are a generalization of Dirichlet *L*-functions that are associated to possibly non-abelian Galois groups.

Please access for more detailed information:
http://www.math.uconn.edu/~kconrad/pcmi2009/

Please access for preliminary reading recommendations:
http://www.math.uconn.edu/~kconrad/pcmi2009/

Elliptic Curves, Modular Forms, and L-functions;
Alvaro Lozano-Robledo, University of Connecticut.
This course will be an introduction to elliptic curves and modular forms, with an emphasis on examples. We will begin with some motivating problems, such as the congruent number problem, and the definitions, and then explain how a link between elliptic curves and modular forms is suggested through *L*-functions. Students will learn how to manipulate elliptic curves, modular forms and *L*-functions to extract interesting arithmetic information such as rational points, the rank, or congruences (using the free software SAGE). We will discuss some of the big theorems and conjectures - such as the Mordell-Weil theorem and Birch and Swinnerton-Dyer conjecture -, and their consequences. For example, we will sketch how the modularity of elliptic curves is used to prove Fermat's Last Theorem.

The prerequisites for this course are elementary number theory, linear algebra and group theory.

More extensive information is found at
http://www.math.uconn.edu/~alozano/PCMI2009/index.html

Course assistants will run problem sessions and will be available to answer questions. The lecturers will produce notes for distribution.

Undergraduates are also welcome to attend the morning problem solving course of the Secondary School Teachers Program, the mathematics course of the Undergraduate Faculty Program, or the more introductory lectures of the Graduate Summer School. However, please don't overdo it! It is too easy to become overwhelmed by the vast amount of mathematics available at PCMI. Concentrate on what is the most valuable to you at your current stage of preparation.

USS Participation Policy: Undergraduate Summer School participants who are funded in any way by the IAS/Park City Mathematics Institute are expected to participate from the beginning of the summer session on the morning of June 29th to the end of the summer session on the afternoon of July 17th.

Stipend checks will be issued on Friday, July 17th.

Sample 2009 Daily Schedule

	Graduate Summer School	Secondary School Teachers Program	Research Program	Undergraduate Faculty Program	Undergraduate summer school
6:45-8:15	Breakfast (Moose and Miner Room at the Grub Steak Restaurant)				
8:30-9:30	Lecture: (Grand theater)	Developing Mathematics (begins at **8:20** each day) (Silver King 2-4)			
9:30-9:40	Break				
9:40-10:40	Problem Session (PCMI tent)	Developing Mathematics continued (Silver King 2-4)	Seminar: (Coalition 3 or Grand Theater)		Advanced course (Coalition 1&2)
10:40-11:00	Break				
11:00-12:00	Lecture: (Grand Theater)	Reflection on Teaching Practice (Silver King 2-4)			Intro course (Coalition 1&2)
12:00-1:15	Lunch (PCMI Tent)				
1:00-2:00	Problem Session (Grand Theater)	Working groups (meeting rooms to be announced)		Course: (Coalition 1-2)	
2:00-3:00	Lecture: (Grand Theater)	Working groups continued			
3:00-3:15	Break				
3:15-4:15	Cross Program Activity:				
4:15-4:30	Afternoon Tea (Foyer outside Grand Theater)				
4:30-6:30		Session: *TBA* (Silver King 2-4)	Seminar: (Grand Theater)	Seminar (Coalition 3)	4:30-5:30 Problem Session (Coalition 1&2) 5:30-6:30 Problem Session (Coalition 1&2)

The final schedule for Monday, June 29th, will be available at PCMI Registration in Park City.