PITP 2014
String Compactification

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Recap (I)

Heterotic compactification to 4d with N=1 SUSY:

⟶ Calabi-Yau three-fold, holomorphic poly-stable bundle

Type IIB compactification to 4d with N=2 SUSY:

⟶ Calabi-Yau three-fold, RR fields with vanishing flux

Break to N=1 ⟶ Fluxes

⟶ D-branes and O-planes

U(N), SO(N) or USp(N)

Recap (II)

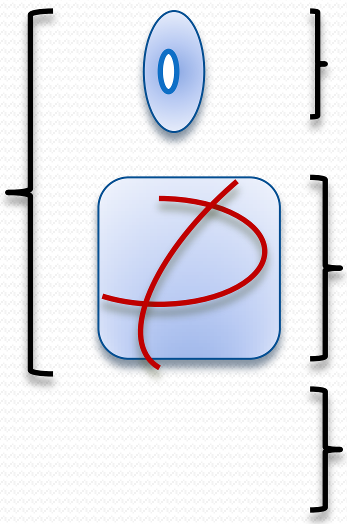
F-theory

encodes IIB with varying axio-dilaton by elliptic fibration over IIB space-time

Weierstrass: $y^2 = x^3 + f\,x + g$

F-theory compactification on Y to four dimensions with $N=1$ SUSY:

Y



E F-theory torus

B_3 Compactification manifold

$R^{1,3}$ Minkowski space

Y = elliptic Calabi-Yau four-fold

Singular elliptic fibers over discriminant locus: 7-branes

Questions

F-theory somehow encodes 7-branes in the geometry of elliptic fibrations

In type IIB, the world-volume Yang-Mills theory was obtained by quantizing open strings.

Quantizing open strings connecting parallel branes gave us non-abelian gauge symmetry.

Although we didn't discuss it, quantizing open strings on intersecting branes gives us charged matter.

Now we would like to understand the F-theory analogues:

How do we see the worldvolume gauge fields come out?

How do we see the gauge symmetry enhancement when D-branes collide?

Abelian gauge fields

M-theory perspective:

Bosonic fields: g_{MN}, $C_{(3)MNP}$

Abelian gauge fields are obtained by expanding C_3 in harmonic forms:

$$C_{(3)} = A^I_\mu dx^\mu \wedge \omega_{(2),I} \qquad \omega_{(2),I} \in H^2(Y, \mathbf{R})$$

It turns out that to survive in the F-theory limit, $\omega_{(2),I}$ must have one index on T^2 and one index on B_3.

This is precisely what we need to get a IIB interpretation:

$$dC_{(3)} = -\frac{i}{\mathrm{Im}(\tau)} \underbrace{\left(dC_{(2)} - \tau dB_2\right)}_{B_3} \wedge \underbrace{d\bar{z}}_{T^2} + c.c.$$

dz is (1,0) form on elliptic fiber

ADE singularities

To understand how non-abelian gauge symmetries arise, first need to discuss ADE singularities. Simplest example:

$$xy = z^2 \quad \subset \quad \boldsymbol{C}^3$$

Two ways to desingularize.

<u>Blow-up</u>:

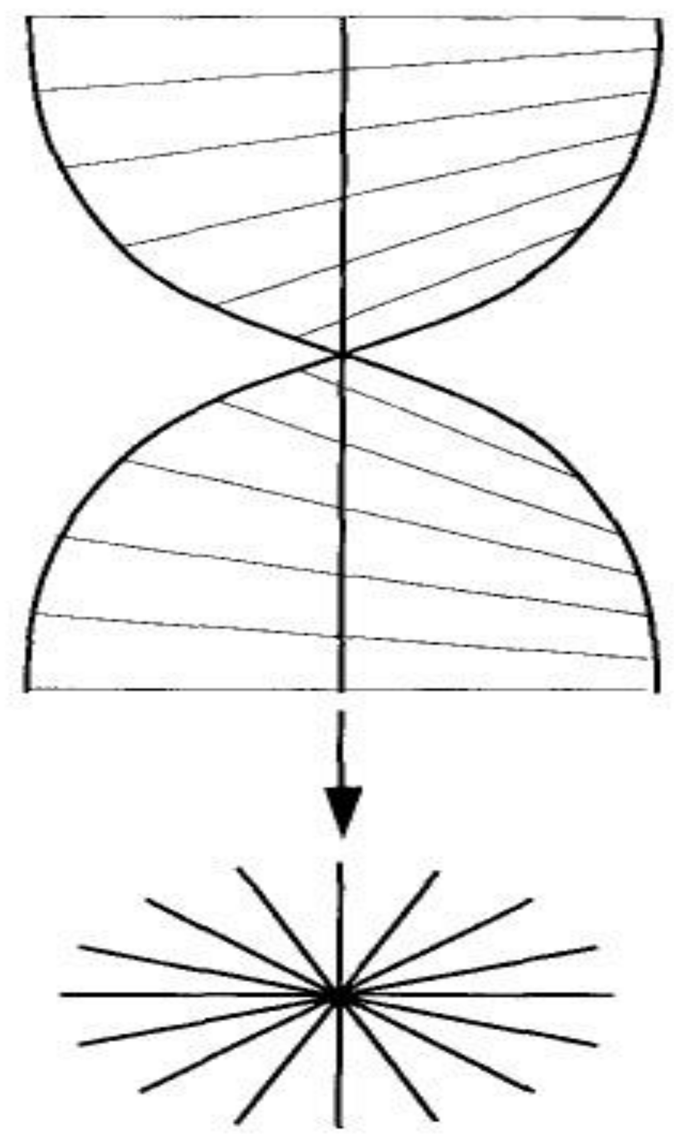
Replace *xy*=*z*^2 by pair of equations $\begin{pmatrix} x & z \\ z & y \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \quad \subset \quad \boldsymbol{C}^3 \times \boldsymbol{CP}^1$

Linear equations ⟶ non-singular

- when $(x, y, z) \neq (0,0,0)$ ⟶ solve for $(\lambda_1, \lambda_2) \in \boldsymbol{CP}^1$
  ⟶ isomorphic to *xy* = *z*^2

- when $(x, y, z) = (0,0,0)$ ⟶ $(\lambda_1, \lambda_2) \in \boldsymbol{CP}^1$ unconstrained
  ⟶ singular point replaced by $\boldsymbol{CP}^1$

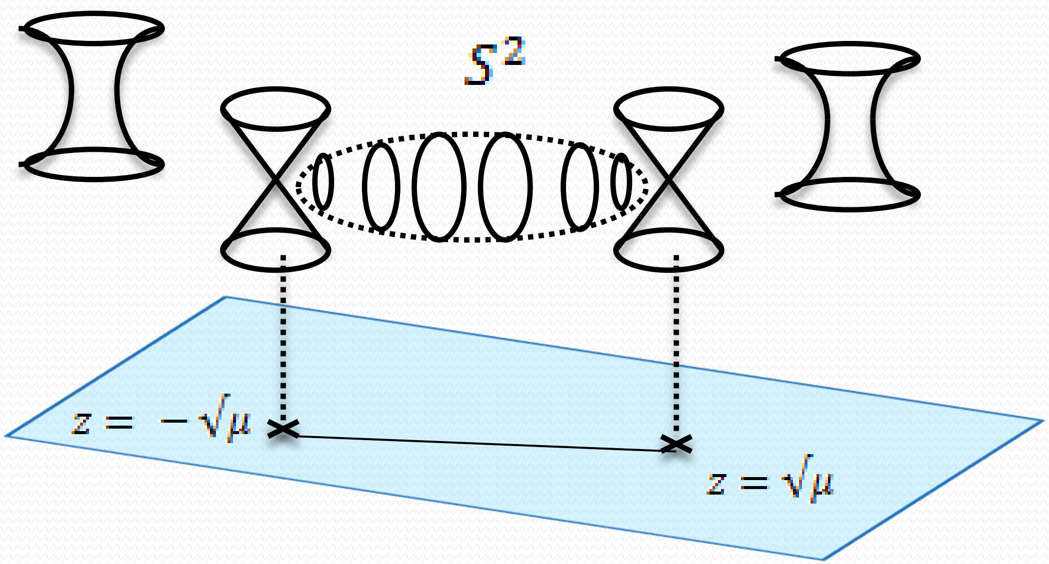
Picture of blow-up



ADE singularities

Second way to desingularize:

<u>Deformation</u>: $xy = P_2(z) = z^2 - \mu$

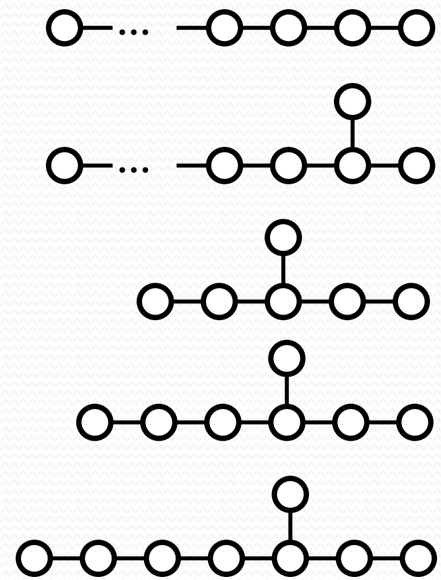


In both cases we find a finite size S². ⟶ $H_2(S_{A_1}, \mathbf{Z}) = \mathbf{Z}$

ADE singularities

Part of a general class:

Dynkin type	equation	restrictions
A_n	$xy + z^{n+1} = 0$	$n \geq 1$
D_n	$x^2 + y^2 z + z^{n-1} = 0$	$n \geq 4$
E_6	$y^2 + x^3 + z^4 = 0$	
E_7	$y^2 + x^3 + x z^3 = 0$	
E_8	$y^2 + x^3 + z^5 = 0$	



Many nice properties, eg. admit (non-compact) Calabi-Yau metric

After deformation or resolution:

$$H_2(S_{ADE}, \mathbf{Z}) \cong H_2(S_{ADE}, \mathbf{Z}) \cong \Gamma_{ADE}$$

ADE root lattice

M-theory on ADE singularities

Main claim:

M-theory on S_{ADE} gives non-abelian gauge theory with ADE gauge group

- Abelian gauge fields from $C_{(3)} = A^I_\mu dx^\mu \wedge \omega_{(2),I}$

$$\omega_{(2),I} \in H^2(S_{ADE}, \boldsymbol{R})$$

(& corresponding adjoint fields from metric moduli)

- Non-abelian W-bosons from quantizing M2-branes wrapped on $\alpha \in H_2(S_{ADE}, \boldsymbol{Z})$

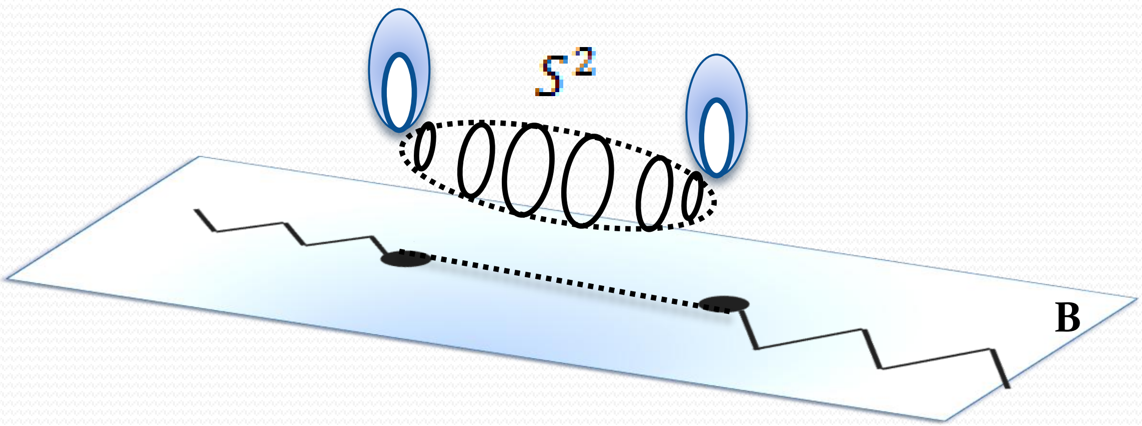
$$\alpha \cdot \alpha = -2$$

This should carry over to F-theory.

IIB perspective

Back to IIB/F-theory. From IIB we expect to get enhanced gauge symmetry when 7-branes collide.

What does this look like from the point of view of the elliptic fibration?



M2 –brane wrapped on S^2 maps to string stretched between D7-branes

As 7-branes collide, get enhanced *SU(2)* gauge symmetry from A_1 singularity.

Note that only the elliptic fibers are singular, IIB space-time is still smooth.

Kodaira classification

Kodaira classified all the singular elliptic fibers you can get (in complex codim 1).

From our perspective, these are all the singular fibers you get by colliding multiple 7-branes

$ord(f)$	$ord(g)$	$ord(\Delta)$	fiber type	sing.type
≥ 0	≥ 0	0	$smooth$	$-$
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	$-$
1	≥ 2	3	II	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8

$$y^2 = x^3 + f\,x + g$$

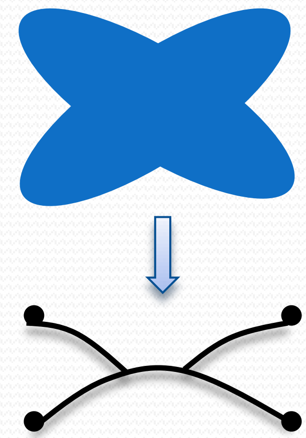
$$\Delta \equiv 4\,f(z)^3 + 27\,g(z)^2$$

As you can see, the singularity type (and hence the enhanced gauge symmetry) is of type ADE.

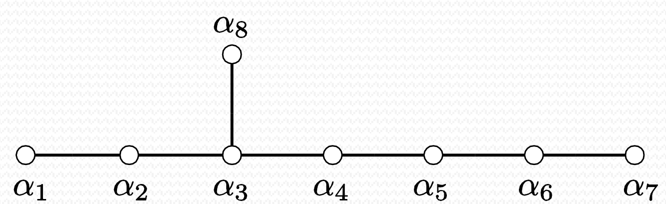
Multi-pronged strings

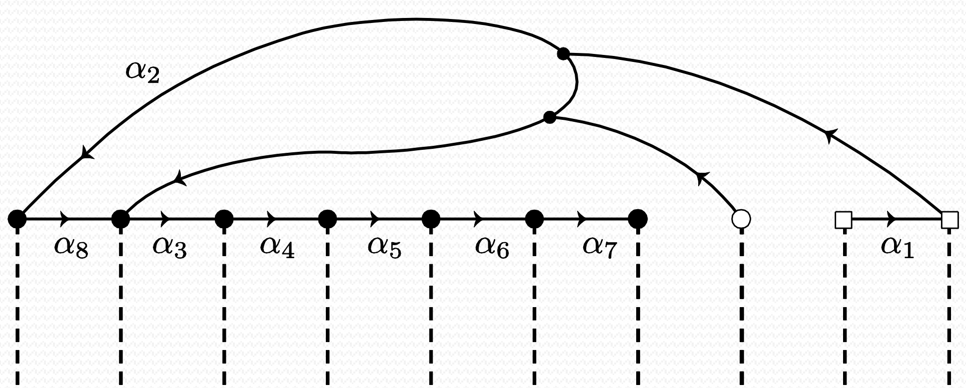
In F-theory we also got exceptional gauge symmetries. How do we get these from open strings?

In general M2-brane maps to
multi-pronged (p,q)-string
in IIB space-time



Open strings for E8:





Dot = (1,0)-brane

Circle = (1,-1)-brane

Box = (1,1)-brane

(DeWolfe et al.)

Splitting O-planes

In fact the (p,q) 7-branes we discussed so far did not include the O7-planes of perturbative IIB. What happened with these?

$SL(2,Z)$ monodromy for (p,q) 7-brane:

$$\begin{pmatrix} 1+pq & p^2 \\ -q^2 & 1-pq \end{pmatrix}$$

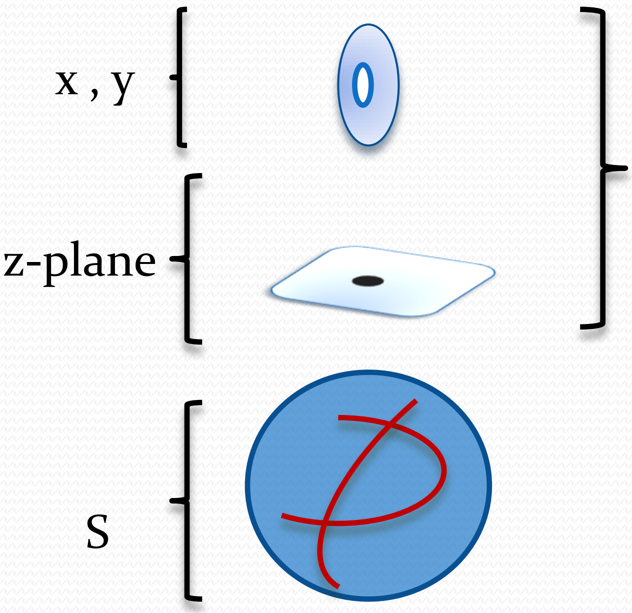
$SL(2,Z)$ monodromy for O7-plane:

$$\begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$$

The O7-plane splits into a (1,1) 7-brane and a (1,-1) 7-brane at finite string coupling.

Application

As an application. Let us consider the following (non-compact) four-fold:



$$y^2 = x^3 + a_0 z^5 + a_2 x z^4 + a_3 y z^3 + a_4 x^2 z + a_5 xy$$

Copy of S at x = y = z = 0.

With help from Kodaira:

$z = 0 \longrightarrow$ SU(5) singularity

$z = a_5 = 0 \longrightarrow$ SO(10) singularity

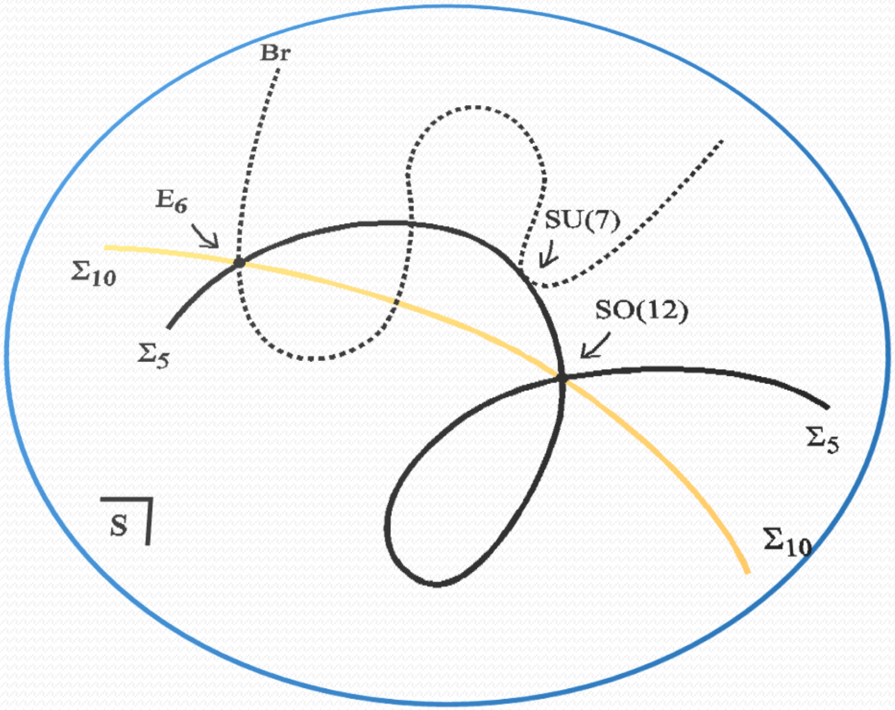
$z = a_0 a_5^2 - a_2 a_3 a_5 + a_3^2 a_4 = 0 \longrightarrow$ SU(6) singularity

$z = a_5 = a_4 = 0 \longrightarrow$ E_6 singularity

$z = a_5 = a_3 = 0 \longrightarrow$ SO(12) singularity

What does this mean??

Application



F-theory picture of SU(5) Grand Unified Theories

$SU(6)$ enhancement $\Rightarrow$ charged matter in $\mathbf{5} \oplus \overline{\mathbf{5}}$ of $SU(5)$

$$\mathbf{35} = \mathbf{24} \oplus \mathbf{5} \oplus \overline{\mathbf{5}} \oplus \mathbf{1}$$

$SO(10)$ enhancement $\Rightarrow$ charged matter in $\mathbf{10} \oplus \overline{\mathbf{10}}$ of $SU(5)$

$$\mathbf{45} = \mathbf{24} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}$$

E_6 enhancement $\Rightarrow$ $\mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}$ Yukawa coupling

$SO(12)$ enhancement $\Rightarrow$ $\mathbf{10} \cdot \overline{\mathbf{5}} \cdot \overline{\mathbf{5}}$ Yukawa coupling

IIB limit

F-theory can reduce to perturbative IIB and heterotic in suitable limits. Briefly discuss IIB limit, also known as the Sen limit.

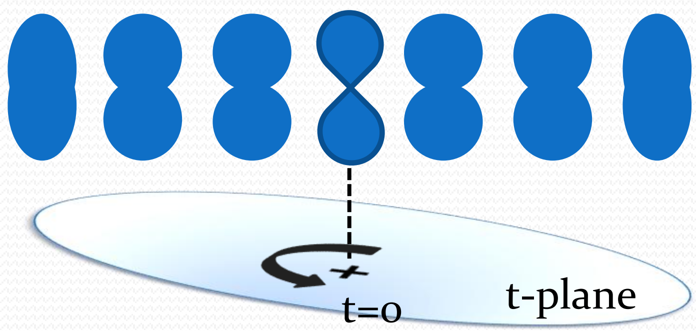
Instead of Weierstrass form, write elliptic fibers as:

$$y^2 = \frac{1}{3}s^3 + b_2 s^2 + 2\,b_4 s + b_6$$

Now add a parameter t:

$$y^2 = \frac{1}{3}s^3 + b_2 s^2 + 2\,b_4 st + b_6 t^2$$

The IIB limit corresponds to $t \to 0$

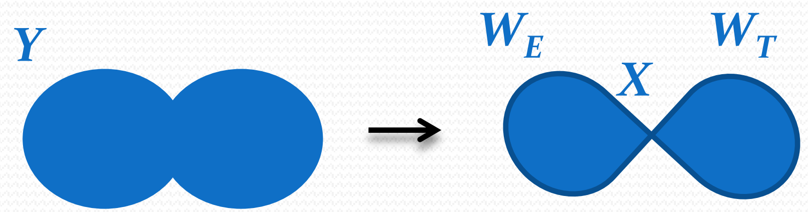


Calabi-Yau 4-fold Y splits into two components

More precisely, the elliptic fibers split into two components

IIB limit

$$y^2 = \frac{1}{3}s^3 + b_2 s^2 + 2\, b_4 s t + b_6 t^2$$



- Naive $t \to 0$

$$\longrightarrow \quad (y/s)^2 = \frac{1}{3}s + b_2 \qquad W_T$$

- Define $s = t\tilde{s}, \quad y = t\tilde{y}$. Alternative $t \to 0$

$$\longrightarrow \quad \tilde{y}^2 = b_2 \tilde{s}^2 + 2\, b_4 \tilde{s} + b_6 \qquad W_E$$

Intersection:

$W_T \cap W_E = X: \quad \{\xi^2 = b_2\}$ $\longrightarrow$ IIB compactification manifold with O7-plane

Singular fibers of W_E:

$\{b_2 b_6 - b_4^2 = 0\}$ $\longrightarrow$ D7-branes

$\longrightarrow$ Exactly the generic D7/O7 system that we saw in perturbative IIB

$\longrightarrow$ Turning on IIB string coupling corresponds to smoothing out Y

F-theory summary

Bookkeeping device for IIB vacua with varying axio-dilaton/finite g_s

For *4d N=1* SUSY, require elliptically fibered Calabi-Yau four-fold

7-branes geometrized in terms off elliptic fibration

Colliding 7-branes ⟶ ADE singularities ⟶ enhanced gauge symmetries

Perturbative IIB (and heterotic string) as special limits