#MCF: The physics of magnetic confinement in 180 minutes

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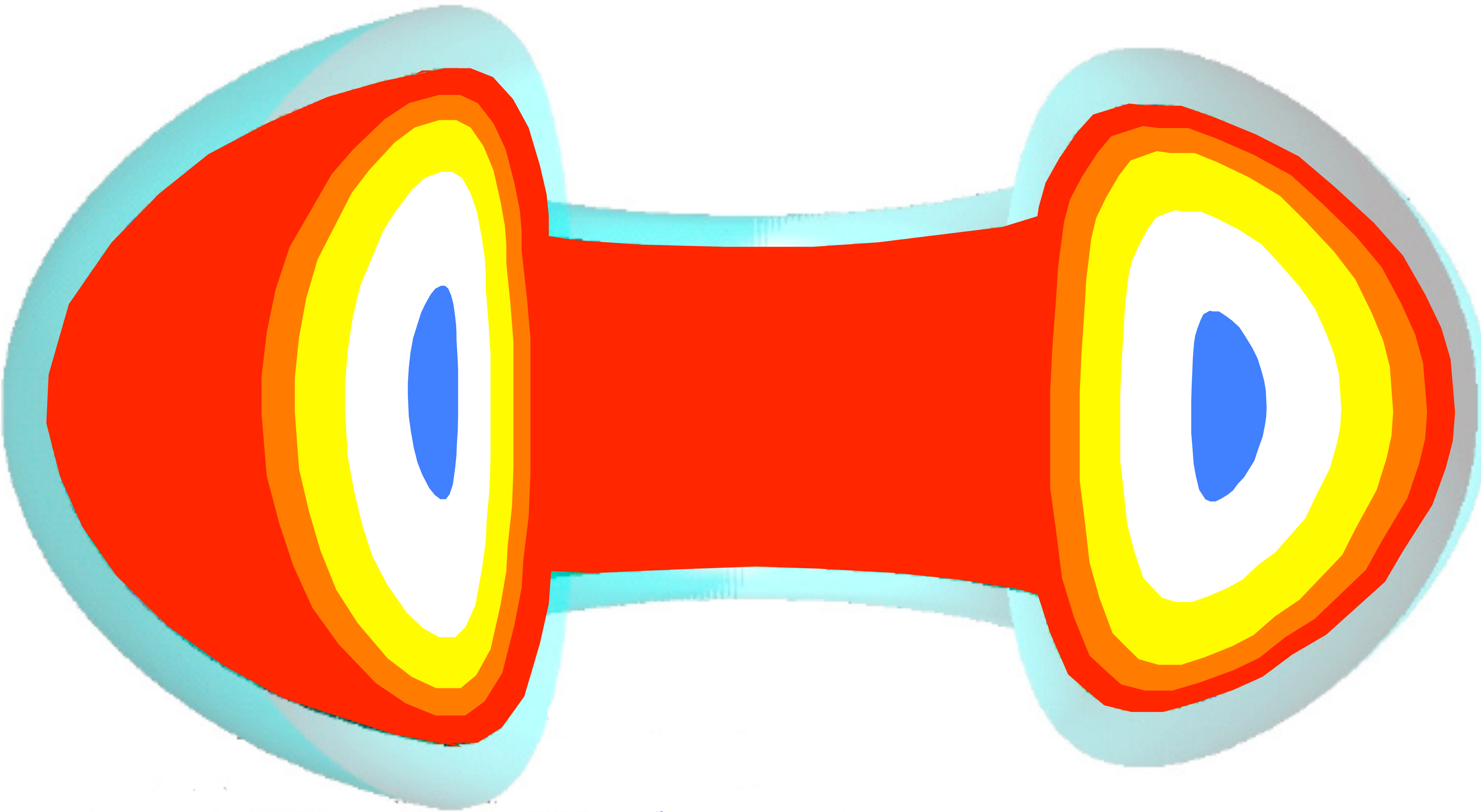
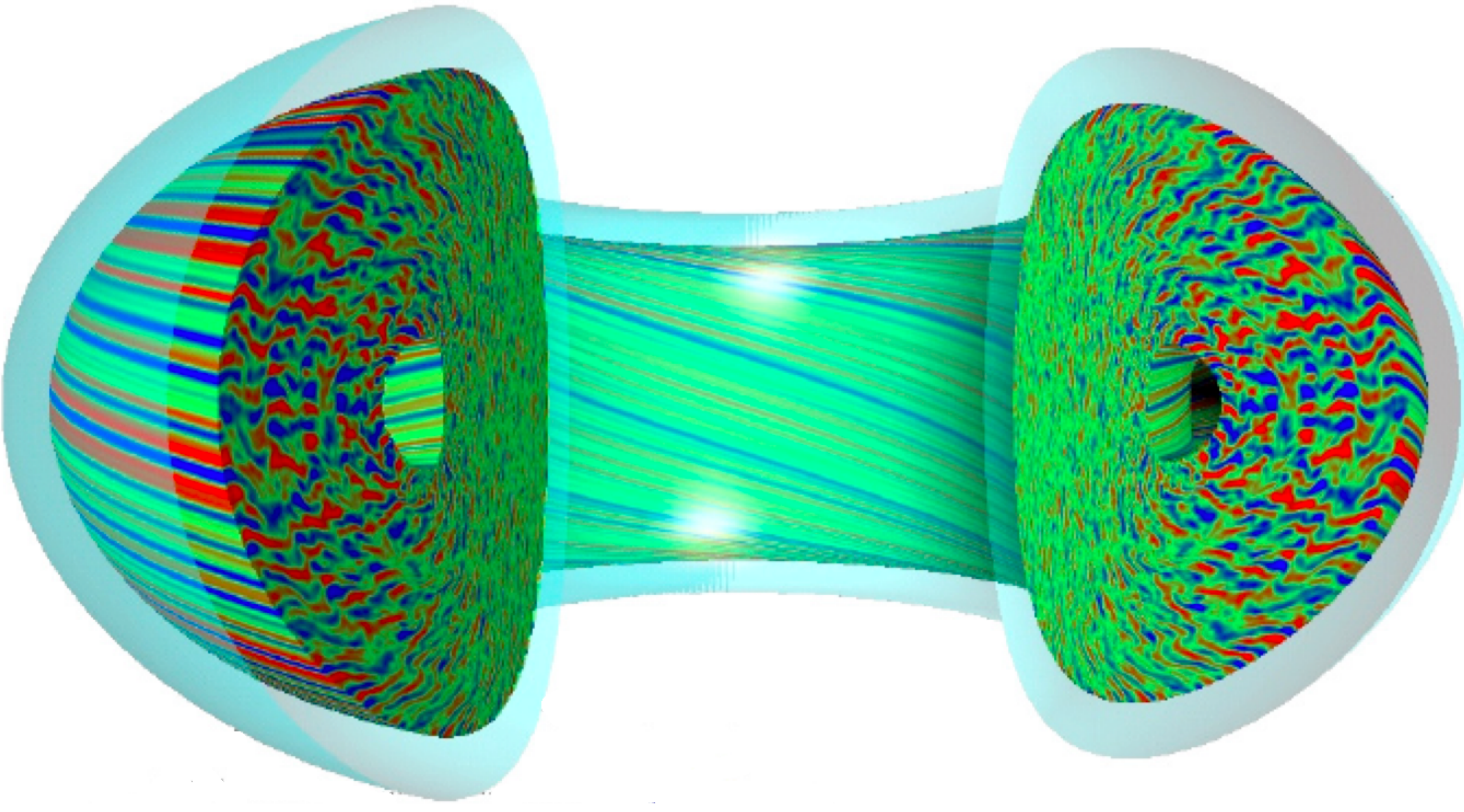
Turbulence and confinement

- What determines particle, momentum, and energy transport in magnetically confined plasmas? (spoiler: it's microturbulence)
- How big is turbulent transport and can we reduce it? (practical MCF question)
- What are the properties of microturbulence in (nearly) collisionless, magnetized plasma? (fundamental physics question, interesting in laboratory/space/astro plasmas)

Turbulence and transport in hot, magnetized plasmas



Turbulence and transport in hot, magnetized plasmas



How good must the confinement be?

$$\frac{p}{\tau_E} = \left(\frac{\partial p}{\partial t}\right)_{\text{transport}} = \mathcal{S}_{\text{fusion}}$$

$$\mathcal{S}_{\text{fusion}} = n^2 \langle \sigma v \rangle E_{\text{fusion}}$$

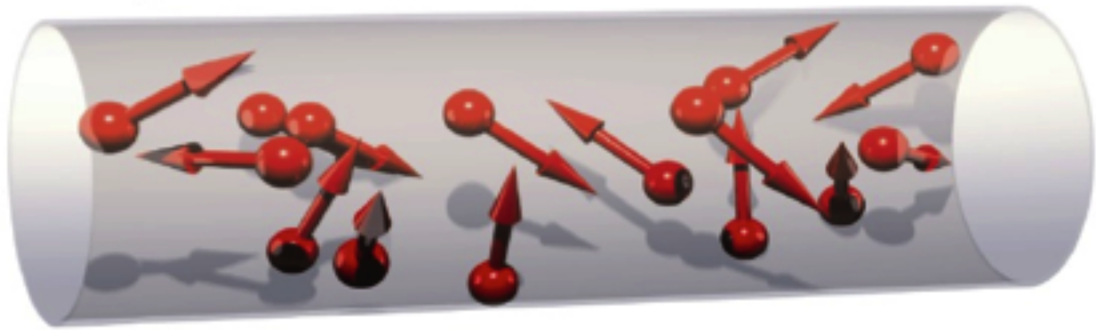
$$n \sim 10^{20}/m^3,\ T \sim 10keV,$$

$$\langle \sigma v \rangle \sim 10^{-22} m^3/s,\ E_{\text{fusion}} = 3.5MeV$$

$$\tau_E \sim 1s$$

Confinement time without magnetic field

without magnetic field

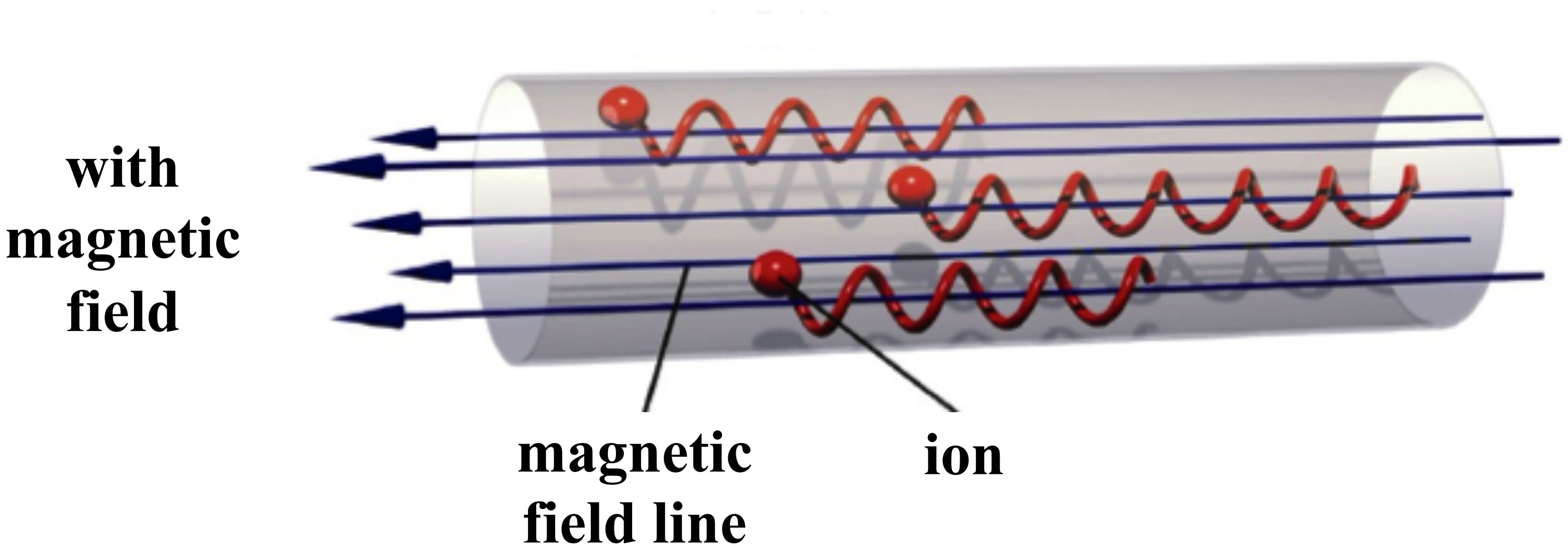


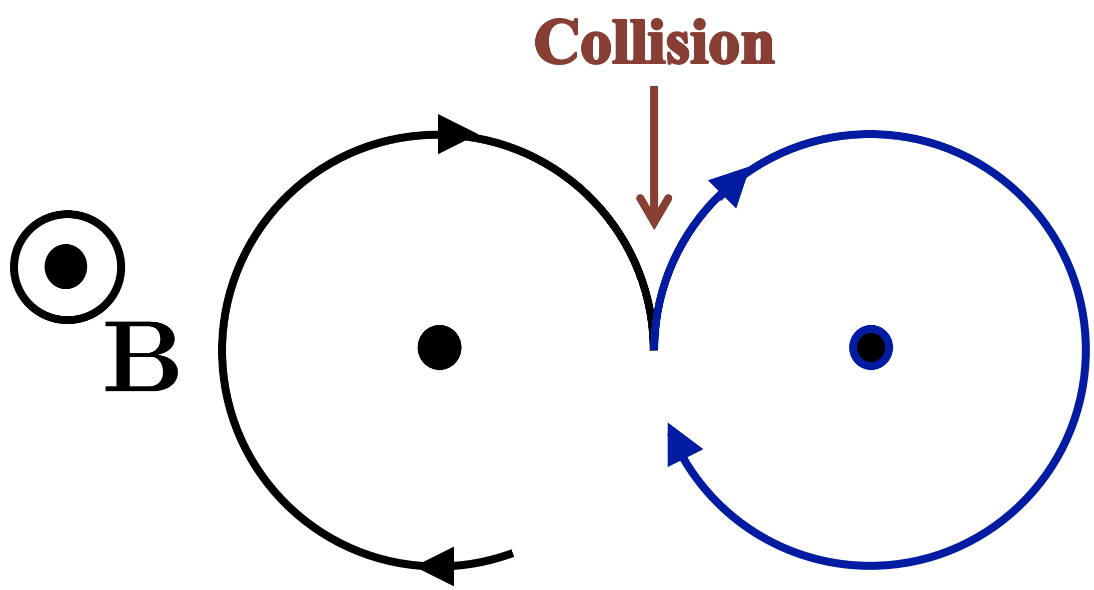
Distance from center of toroidal volume to wall = a ~ 1 m

Average speed of particles is thermal speed, $v_{th} \sim 10^5$ m/s

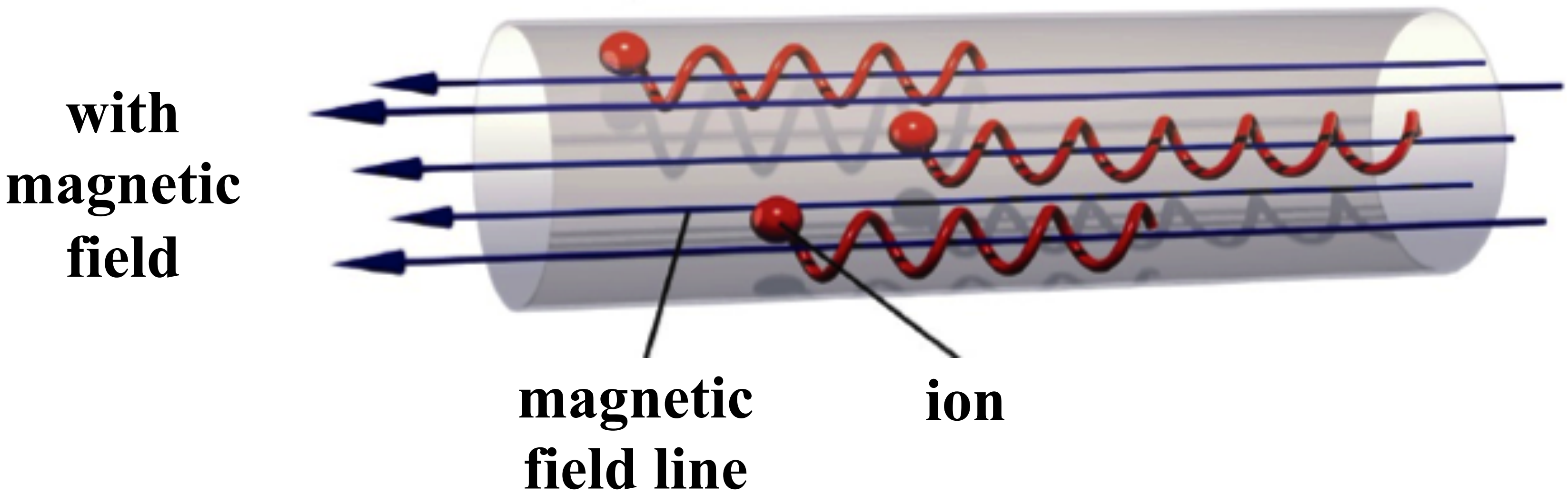
=> Energy confinement time = $\tau_E \sim a/v_{th} \sim 10^{-5}$ s

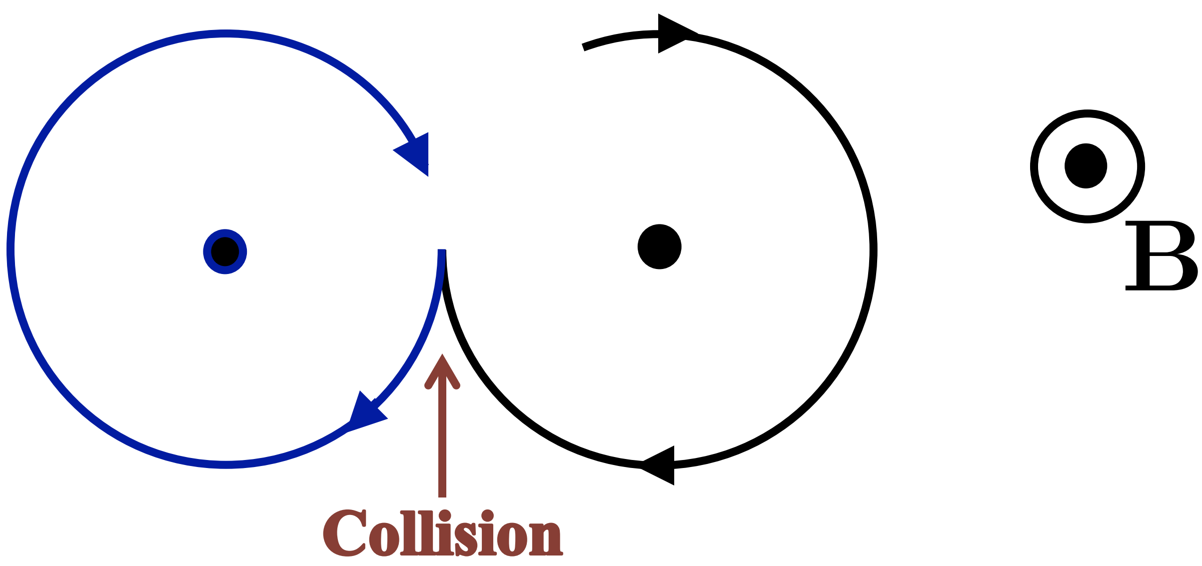
Collisional transport in magnetized plasma





Collisional transport in magnetized plasma

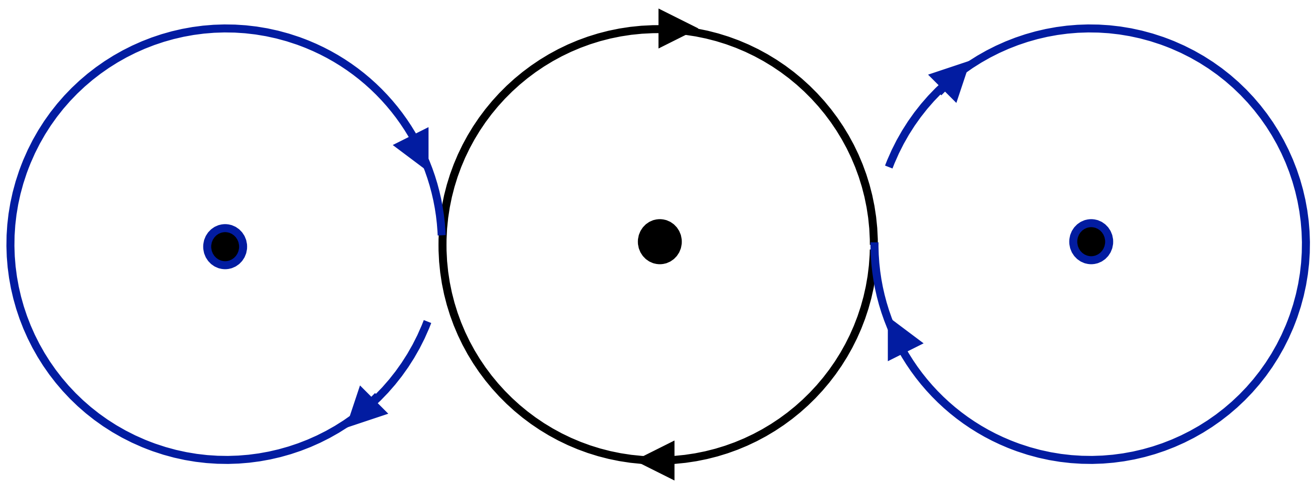




Random walks and diffusion

Think of the process as a random walk. Particles and energy at given space-time point determined by particles and energy at neighboring space-time points:

$$p(x,t) = \frac{p(x+\ell, t-\tau) + p(x-\ell, t-\tau)}{2}$$



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$$p(x,t) = \frac{p(x+\ell, t-\tau) + p(x-\ell, t-\tau)}{2}$$

Taylor expand about the space-time point (x,t), i.e. ℓ, τ small, and use definition of derivative to get:

$$\frac{\partial p(x,t)}{\partial t} \approx \frac{\ell^2}{2\tau} \frac{\partial^2 p(x,t)}{\partial x^2}$$

This is a diffusion equation, with diffusion coefficient D:

$$D \equiv \frac{\ell^2}{2\tau}$$

Estimate for (classical) collisional transport

Random walk step size ~ gyration radius $\longrightarrow$ $\ell \sim \rho$

Average time between steps ~ collision time

$$\longrightarrow \tau \sim \tau_C \sim \frac{\lambda_{\mathrm{mfp}}}{v_{th}}$$

Combine with diffusion equation to get estimate for energy confinement time:

$$\frac{\partial p(x,t)}{\partial t} \approx \frac{\ell^2}{2\tau} \frac{\partial^2 p(x,t)}{\partial x^2}$$

$$\longrightarrow \frac{p}{\tau_E} \sim v_{th} \frac{\rho^2}{\lambda_{\mathrm{mfp}}} \frac{p}{a^2} \longrightarrow \tau_E \sim \underbrace{\left(\frac{a}{\rho}\right)^2}_{10^5} \underbrace{\left(\frac{a}{v_{th}}\right)}_{10^{-5}\ \mathrm{s}} \left(\frac{\lambda_{\mathrm{mfp}}}{a}\right)$$

Much better confinement than observed

Random walk step size ~ gyration radius $\longrightarrow \ell \sim \rho$

Average time between steps ~ collision time

$$\longrightarrow \tau \sim \tau_C \sim \frac{\lambda_{\mathrm{mfp}}}{v_{th}}$$

Combine with diffusion equation to get estimate for energy confinement time:

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$$\longrightarrow \frac{p}{\tau_E} \sim v_{th} \frac{\rho^2}{\lambda_{\mathrm{mfp}}} \frac{p}{a^2} \longrightarrow \tau_E \sim \left(\frac{\lambda_{\mathrm{mfp}}}{a}\right) \mathrm{s} \gg 1 \mathrm{~s}$$

Tamm's Theorem revisited

If confining field axisymmetric, canonical angular momentum conserved:

$$p_\zeta = Rmv_\zeta + \frac{ZeRA_\zeta}{c} = const$$

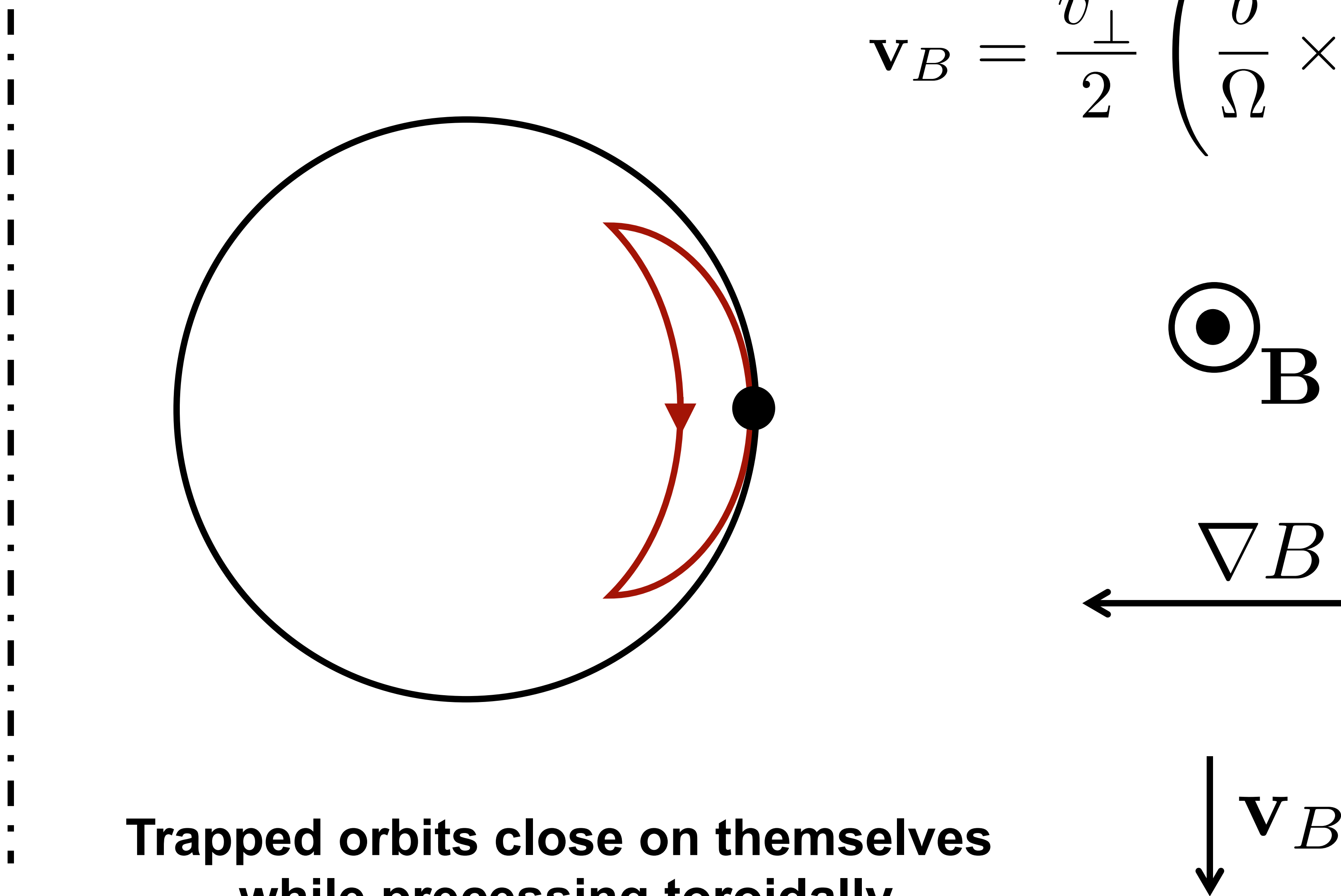
$$\psi_* = \psi_p - \frac{mcRv_\zeta}{Ze} = const$$

$$\psi_p = const \times \left(1 + O\left[\frac{\rho}{a}\frac{B}{B_p}\right]\right)$$

$$\rho_p \equiv \frac{B}{B_p}\rho \sim 10\rho$$

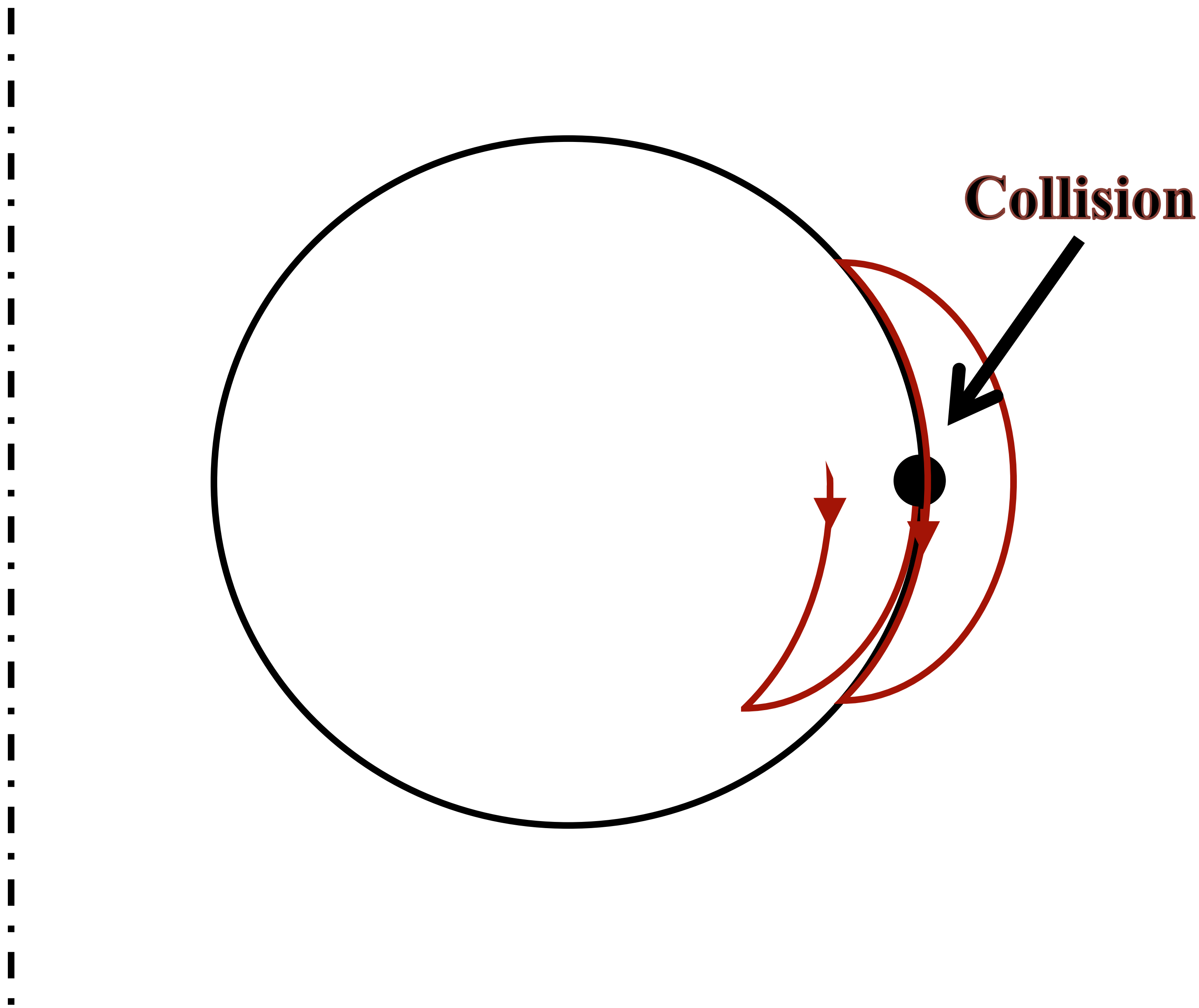
(Neoclassical) collisional transport

$$\mathbf{v}_B = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega} \times \frac{\nabla B}{B}\right)$$



Trapped orbits close on themselves while precessing toroidally

(Neoclassical) collisional transport



Still better confinement than observed

Random walk step size ~ poloidal gyroradius $\longrightarrow$ $\ell \sim \rho_p$

Average time between steps ~ collision time

$$\longrightarrow \tau \sim \tau_C \sim \frac{\lambda_{\mathrm{mfp}}}{v_{th}}$$

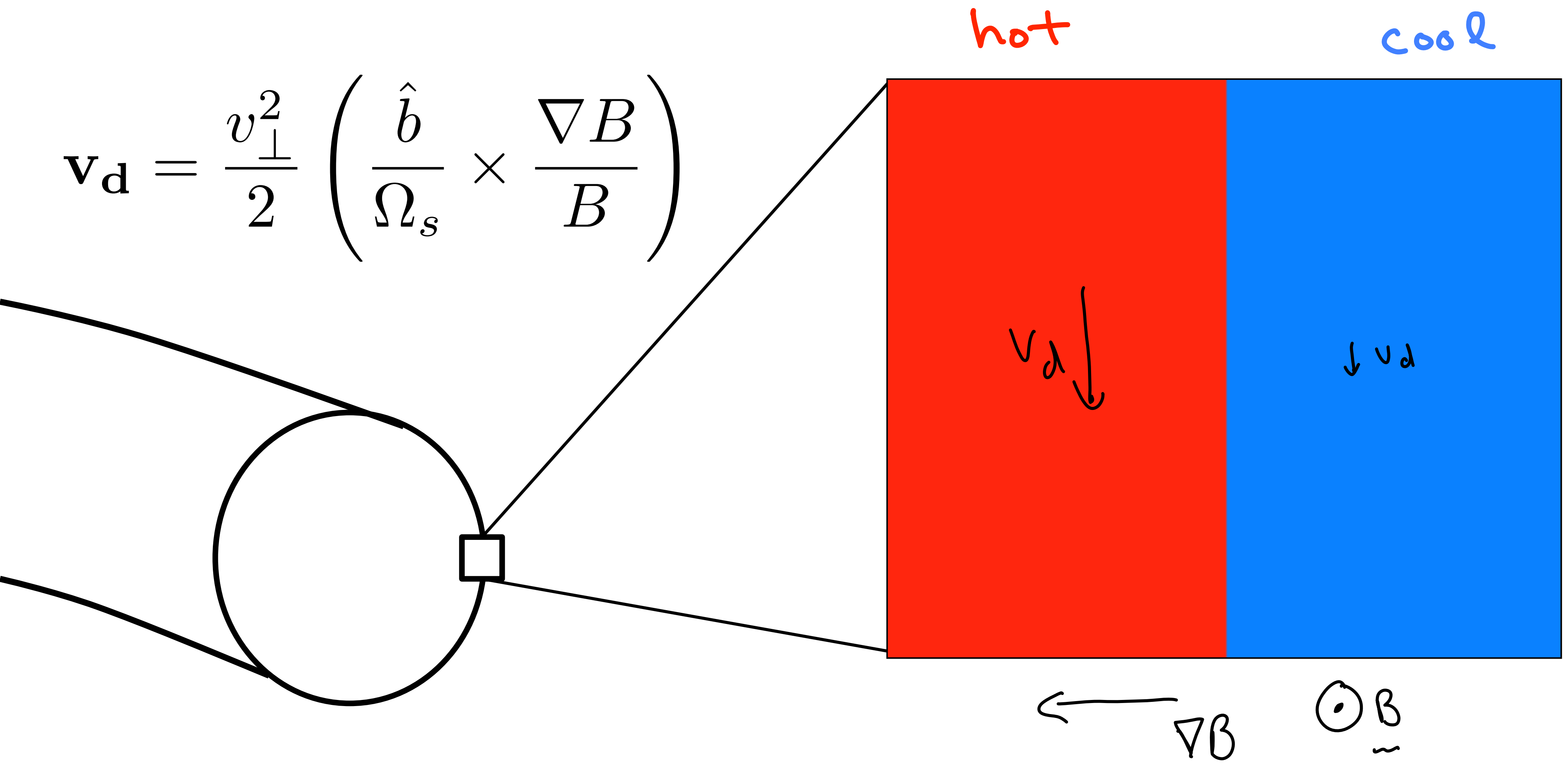
Combine with diffusion equation to get estimate for energy confinement time:

$$\frac{\partial p(x,t)}{\partial t} \approx \frac{\ell^2}{2\tau} \frac{\partial^2 p(x,t)}{\partial x^2}$$

$$\longrightarrow \frac{p}{\tau_E} \sim v_{th} \frac{\rho_p^2}{\lambda_{\mathrm{mfp}}} \frac{p}{a^2} \longrightarrow \tau_E \sim \frac{1}{100} \left(\frac{\lambda_{\mathrm{mfp}}}{a} \right) \mathrm{s} \gg 1\mathrm{s}$$

Toroidal drift instability (ITG/ETG)

$$\mathbf{v_d} = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega_s} \times \frac{\nabla B}{B}\right)$$



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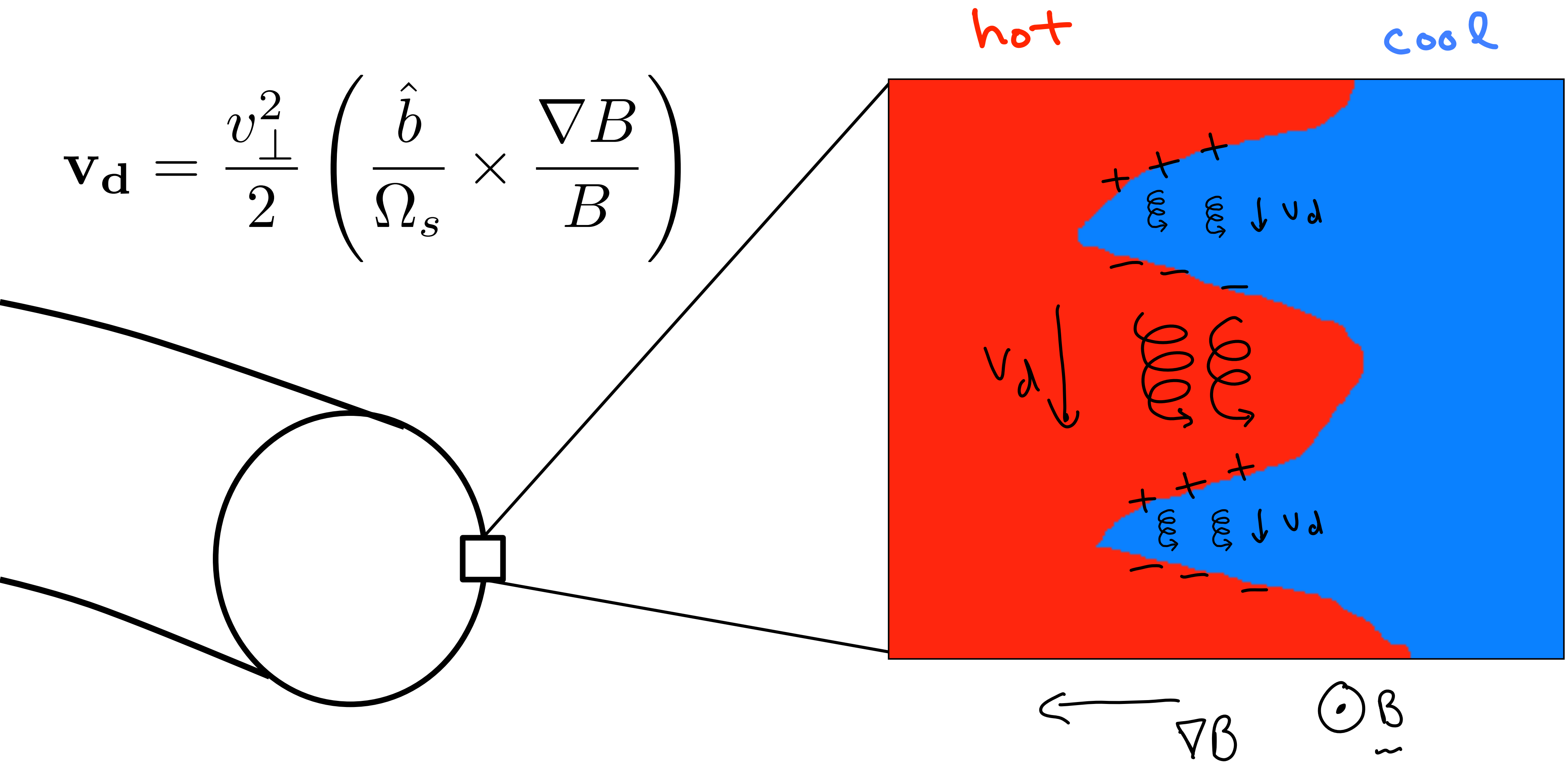
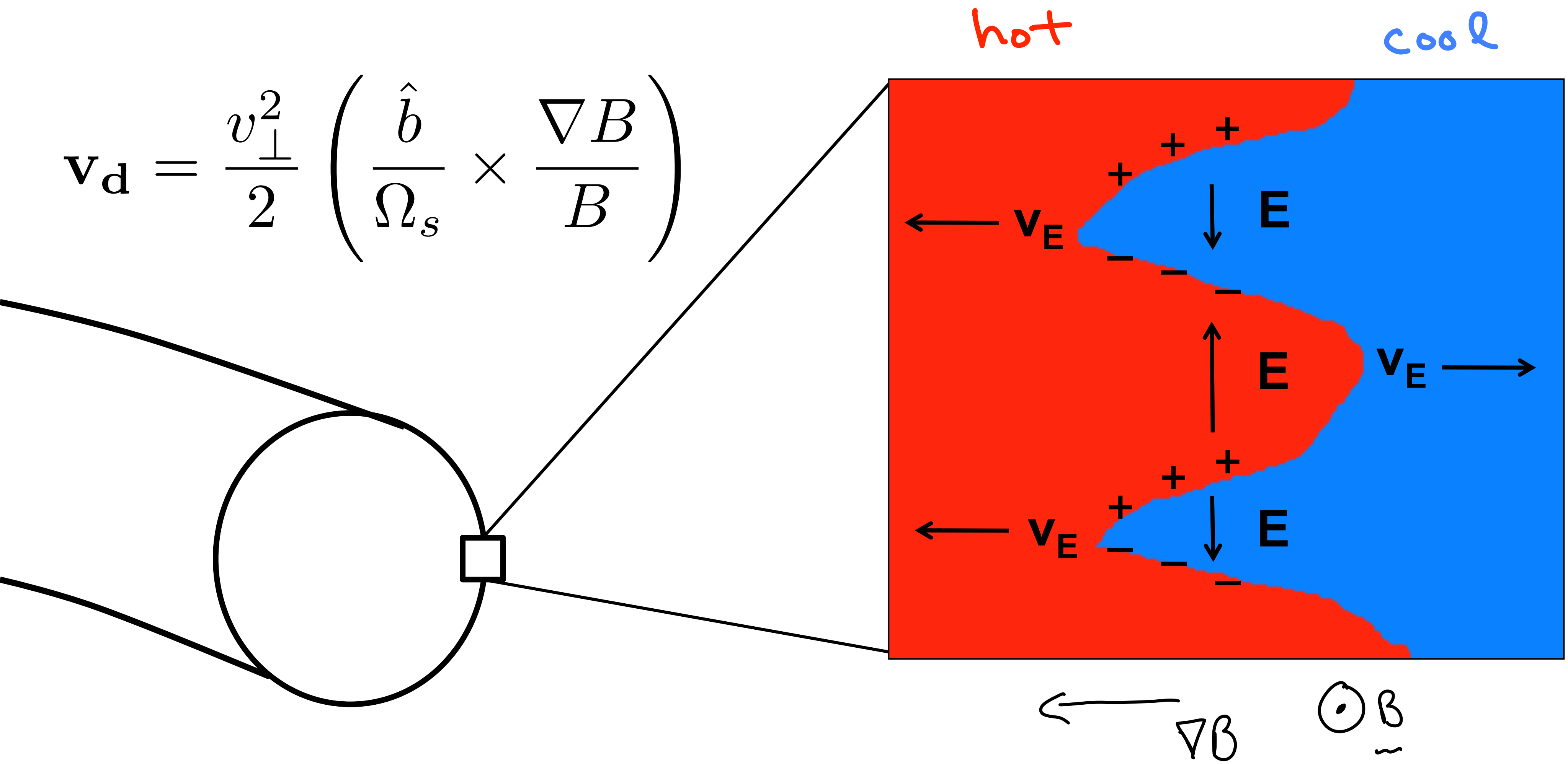


Toroidal drift instability (ITG/ETG)

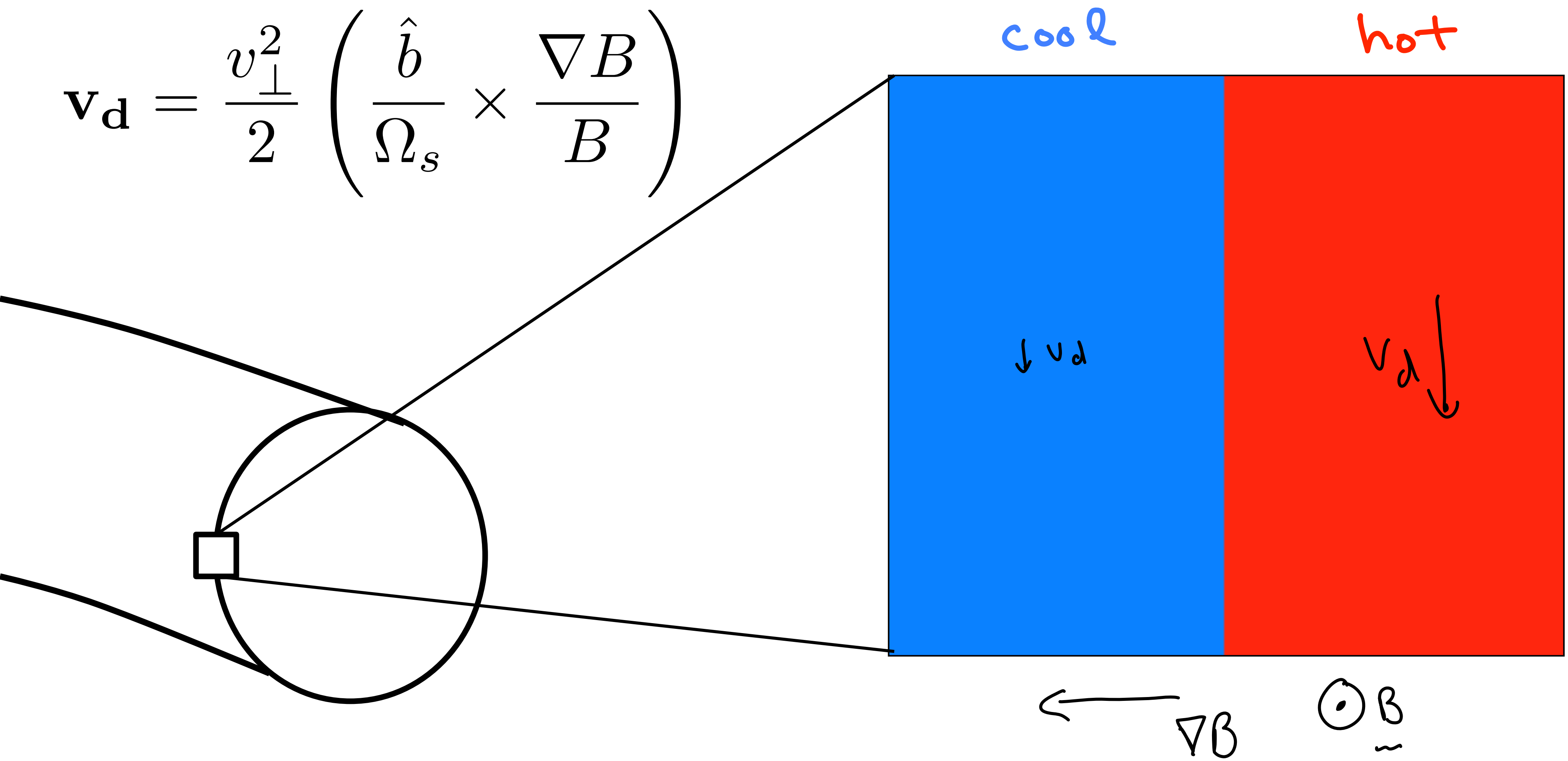
$$\mathbf{v_E} = \frac{c}{B^2}\mathbf{E} \times \mathbf{B}$$

$$\mathbf{v_d} = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega_s} \times \frac{\nabla B}{B}\right)$$



Good curvature vs. bad curvature

$$\mathbf{v_d} = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega_s} \times \frac{\nabla B}{B}\right)$$



Good curvature vs. bad curvature

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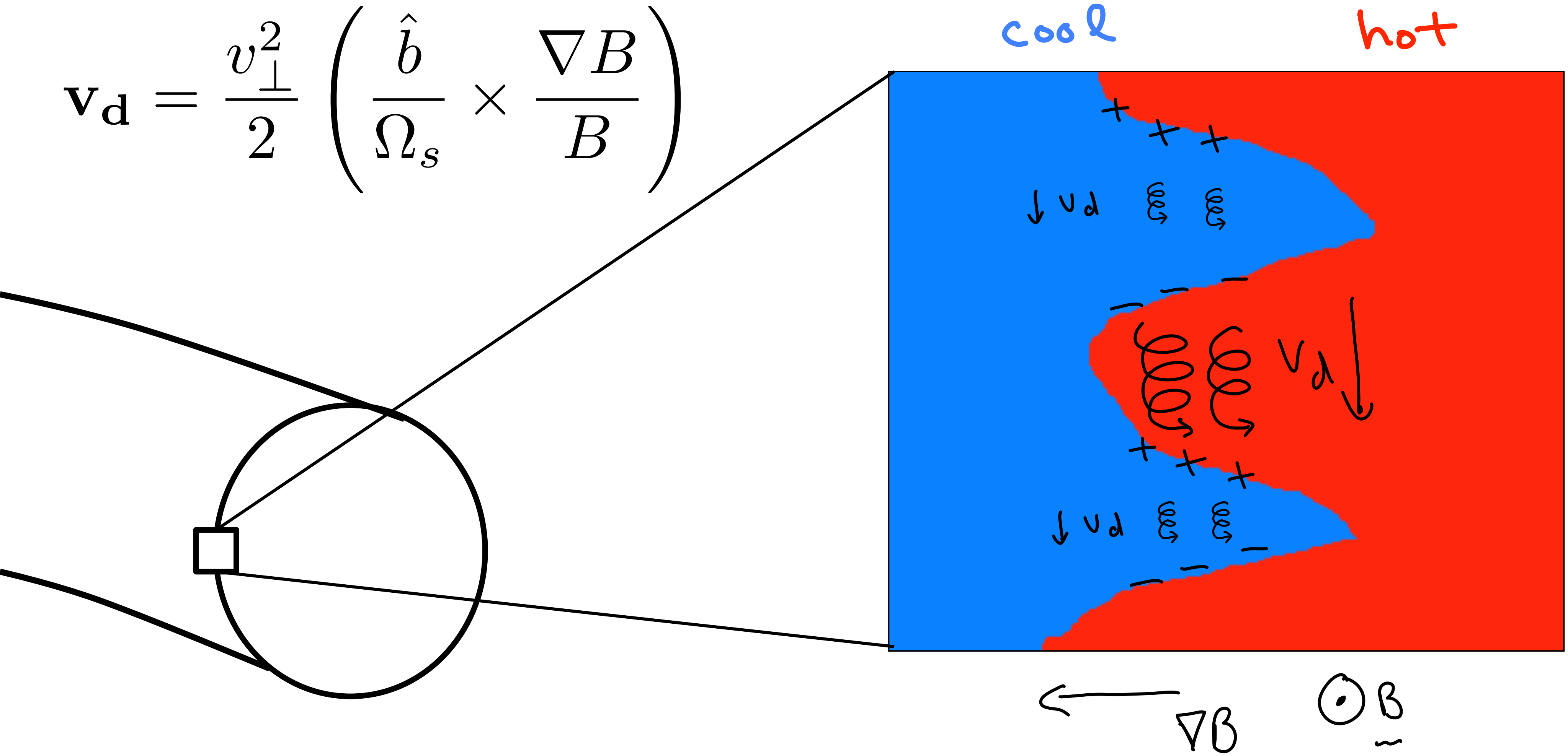
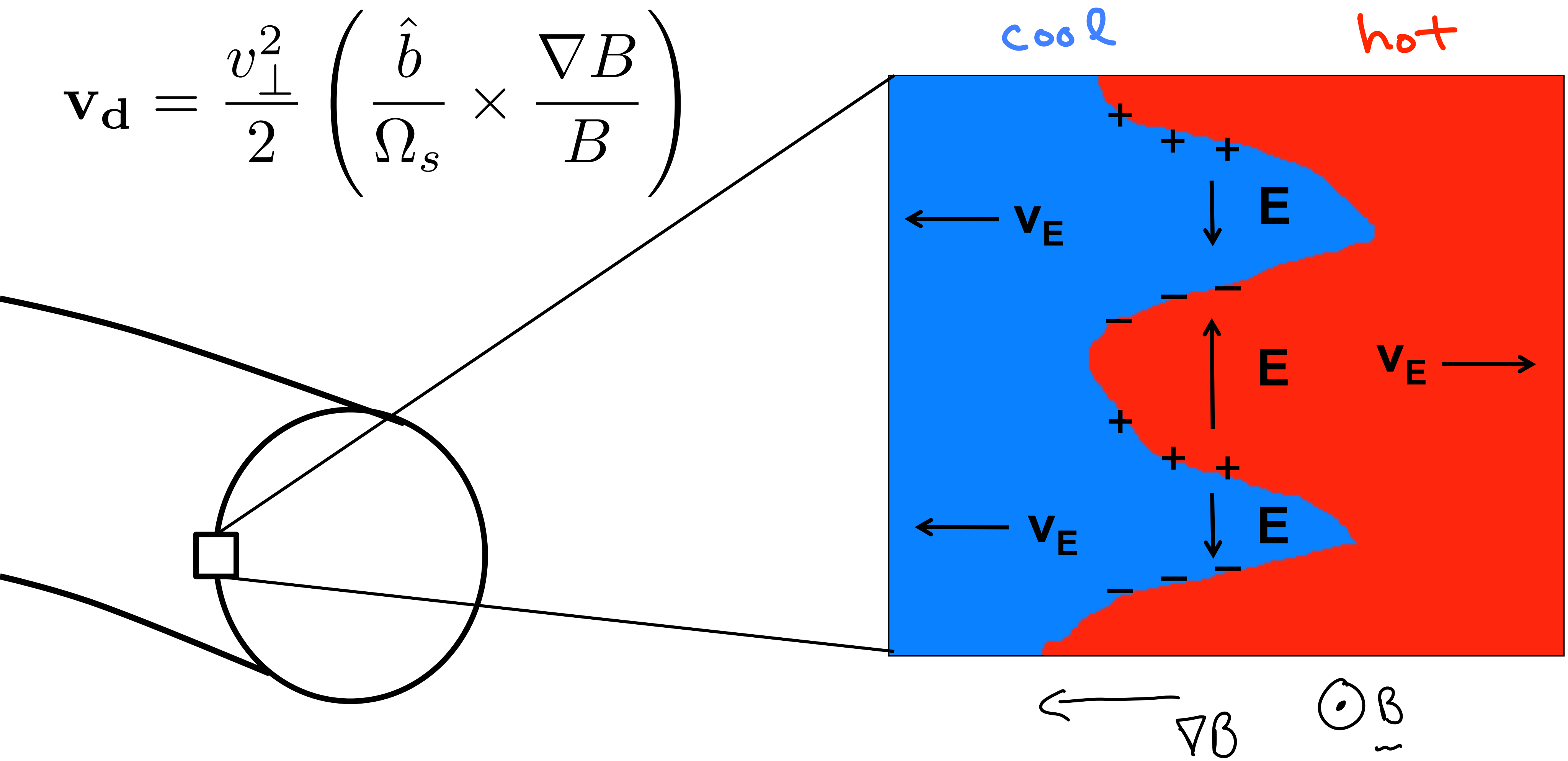


Turbulence stable on inside

$$\mathbf{v_E} = \frac{c}{B^2}\mathbf{E} \times \mathbf{B}$$

$$\mathbf{v_d} = \frac{v_\perp^2}{2}\left(\frac{\hat{b}}{\Omega_s} \times \frac{\nabla B}{B}\right)$$



Instability has a critical gradient

Growth rate of instability in bad curvature region increases with magnitude of temperature gradient:

$$\gamma = \gamma(T')$$

Plasma in bad curvature region is swept along field lines into good curvature region, making a competition between stabilization and destabilization. Expect net instability when:

$$\gamma(T') \gtrsim \frac{v_{th}}{qR}$$

This implies a critical temperature gradient necessary to sustain instability:

$$T' > T'_c$$

What wavelengths are unstable?

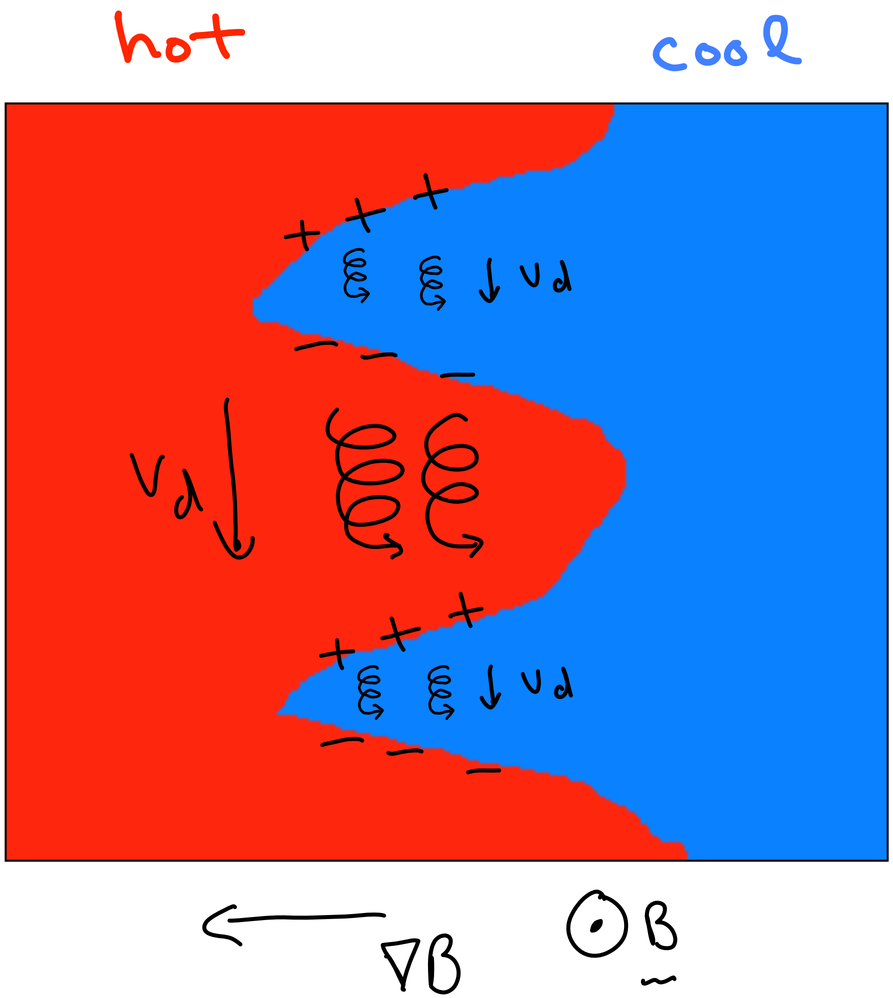
Instability depends on charge separation via magnetic drifts

$$\mathbf{v}_d \sim \frac{\rho}{R} v_{th}$$

Growth rate of instability related to rate at which charge separation occurs:

$$\gamma_{k_\perp} \sim k_\perp v_d \sim \frac{v_{th}}{R} k_\perp \rho$$

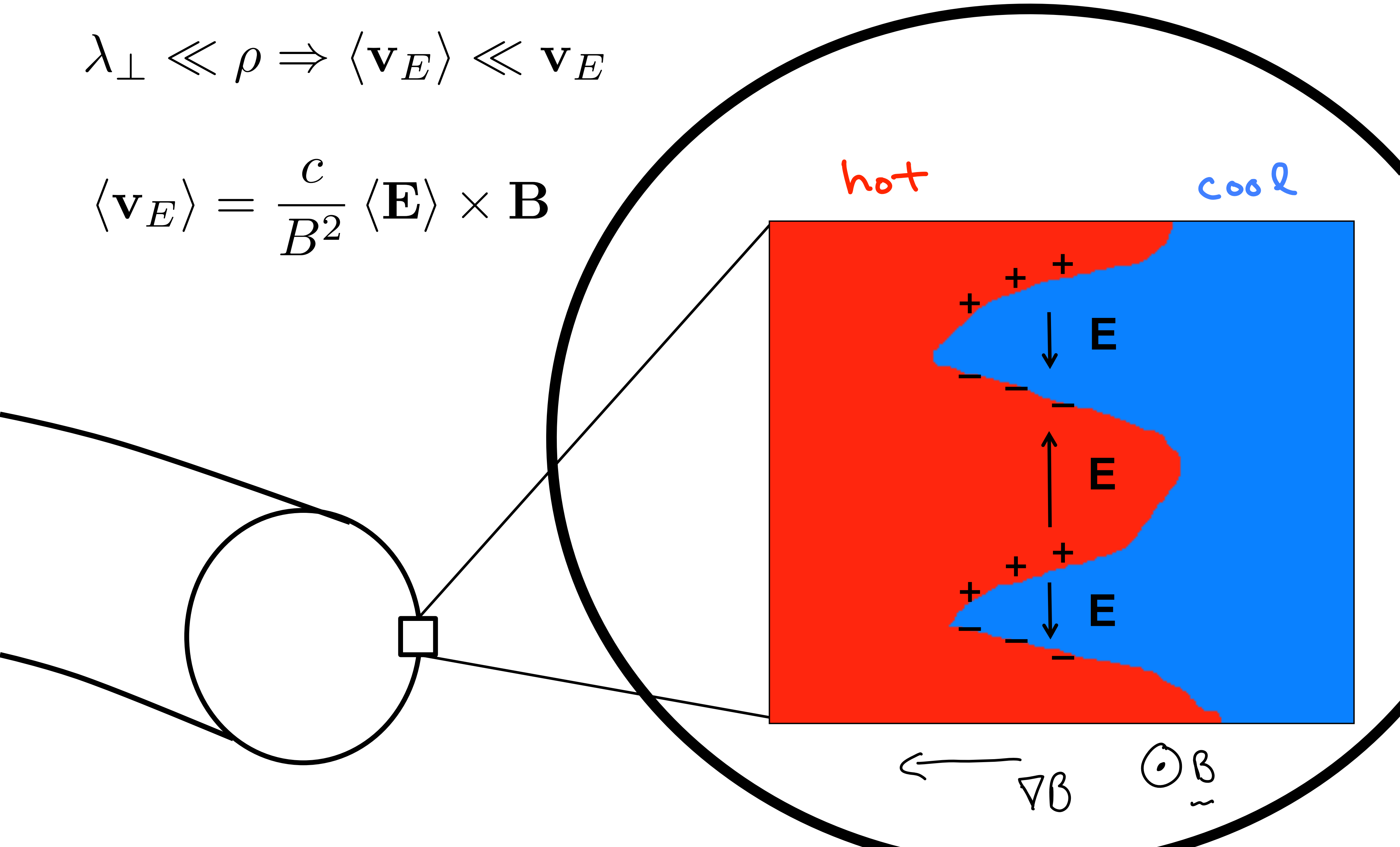
If gyroradius large compared to perturbation scale, reduces instability drive



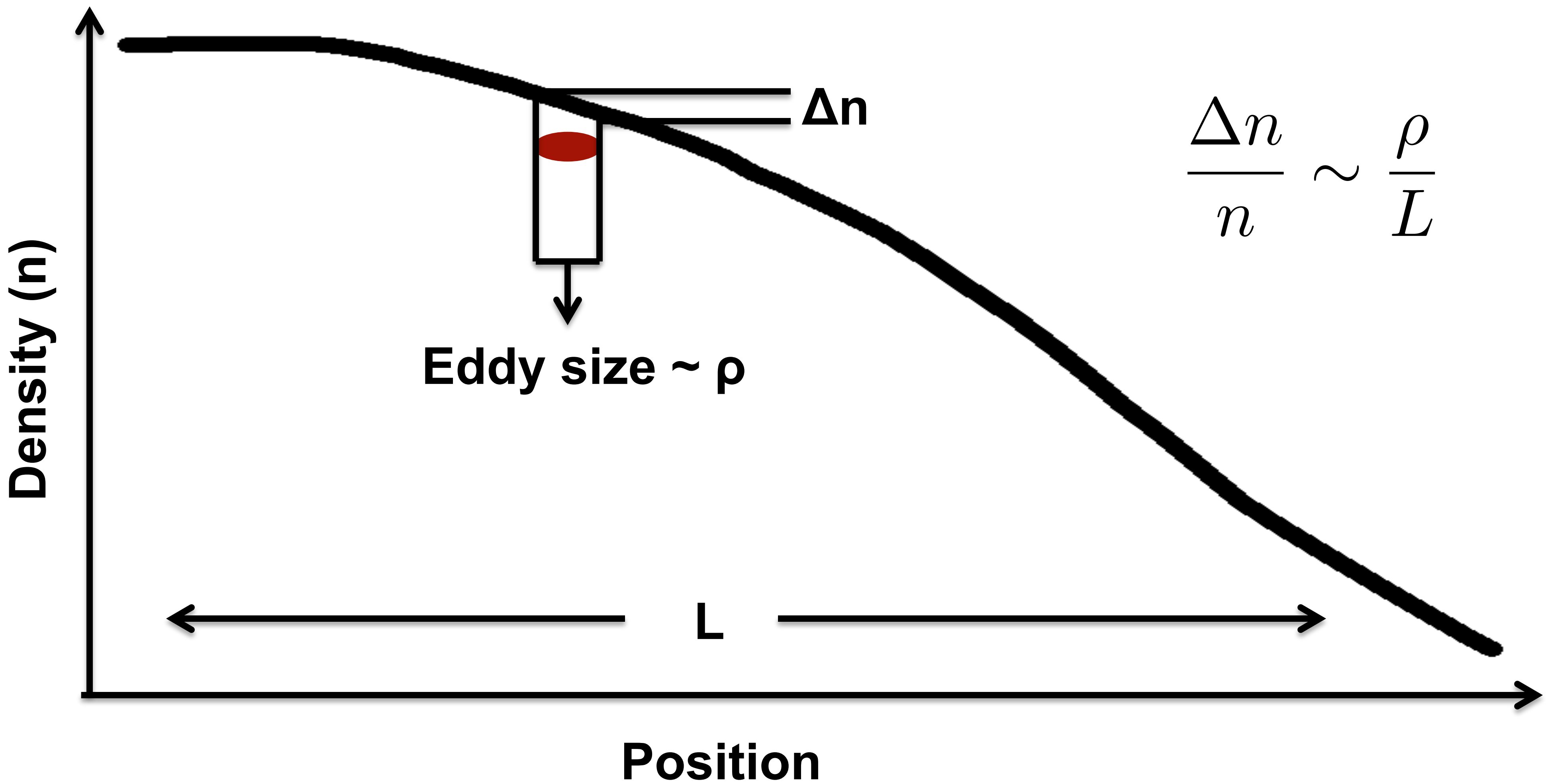
What wavelengths are unstable?

$$\lambda_\perp \ll \rho \Rightarrow \langle \mathbf{v}_E \rangle \ll \mathbf{v}_E$$

$$\langle \mathbf{v}_E \rangle = \frac{c}{B^2} \langle \mathbf{E} \rangle \times \mathbf{B}$$



Turbulent fluctuation amplitude small



$$\frac{\Delta n}{n} \sim \frac{\rho}{L}$$

Estimate for turbulent transport

Random walk step size ~ eddy size ~ gyration radius $\longrightarrow \ell \sim \rho$

Average time between steps ~ eddy turnover time

$$\longrightarrow \tau \sim \tau_t \sim \frac{\rho}{\mathrm{v_E}} \sim \frac{a}{v_{th}} \qquad \left(\mathrm{v_E} \sim v_{th}\frac{\rho}{a}\right)$$

Combine with diffusion equation to get estimate for energy confinement time:

$$\frac{\partial p(x,t)}{\partial t} \approx \frac{\ell^2}{2\tau}\frac{\partial^2 p(x,t)}{\partial x^2}$$

$$\longrightarrow \frac{p}{\tau_E} \sim v_{th}\frac{\rho^2}{a}\frac{p}{a^2} \longrightarrow \tau_E \sim \underbrace{\left(\frac{a}{\rho}\right)^2}_{10^5} \underbrace{\left(\frac{a}{v_{th}}\right)}_{10^{-5}\ \mathrm{s}}$$

Confinement time just right!

Random walk step size ~ eddy size ~ gyration radius $\longrightarrow \ell \sim \rho$

Average time between steps ~ eddy turnover time

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$$\longrightarrow \frac{p}{\tau_E} \sim v_{th}\frac{\rho^2}{a}\frac{p}{a^2} \longrightarrow \tau_E \sim 1 \text{ s}$$

Resultant turbulence

DIII-D Shot 121717

GYRO Simulation

Cray X1E, 256 MSPs

Kinetic corrections

- So far, picture has been essentially fluid, even though collisional mean free path is long; what about kinetic effects?
- Not all particles travel at sound speed along field
  - Significant number of particles travel slower
  - Some particles are trapped in bad curvature region due to magnetic mirroring

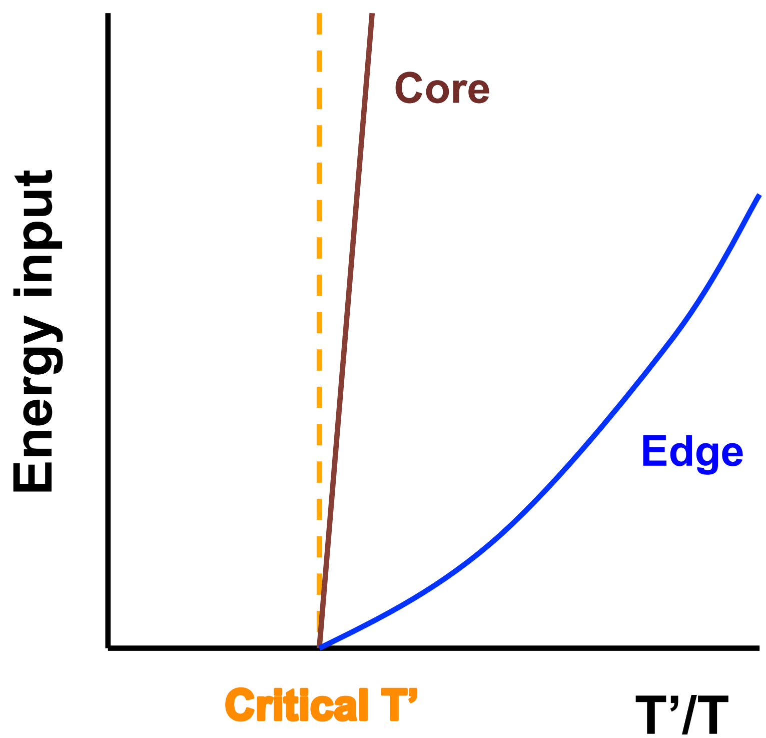
$$(T_c')_{\rm kinetic} < (T_c')_{\rm fluid}$$

Turbulence determines plasma profiles

Turbulence diffusion coefficient depends strongly on temperature:

$$\frac{\partial p(x,t)}{\partial t} \approx \frac{\ell^2}{2\tau} \frac{\partial^2 p(x,t)}{\partial x^2}$$

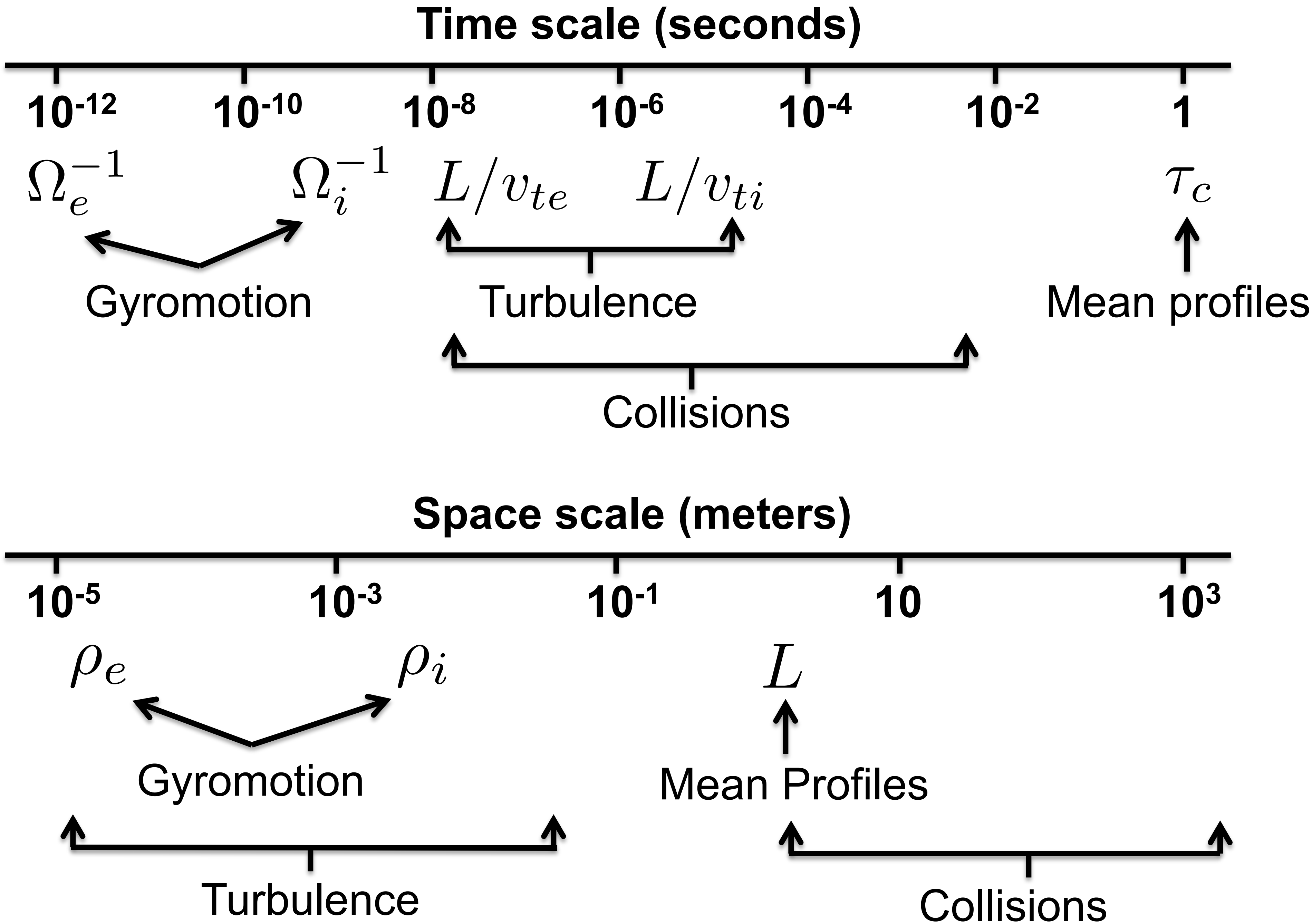
$$\rightarrow D_{\text{eff}} \sim \frac{v_{th}}{a} \left(\frac{\rho}{a}\right)^2 p \propto T^{5/2}$$



Diffusion coefficient large in core where temperature is high, making it difficult to go above critical temperature gradient

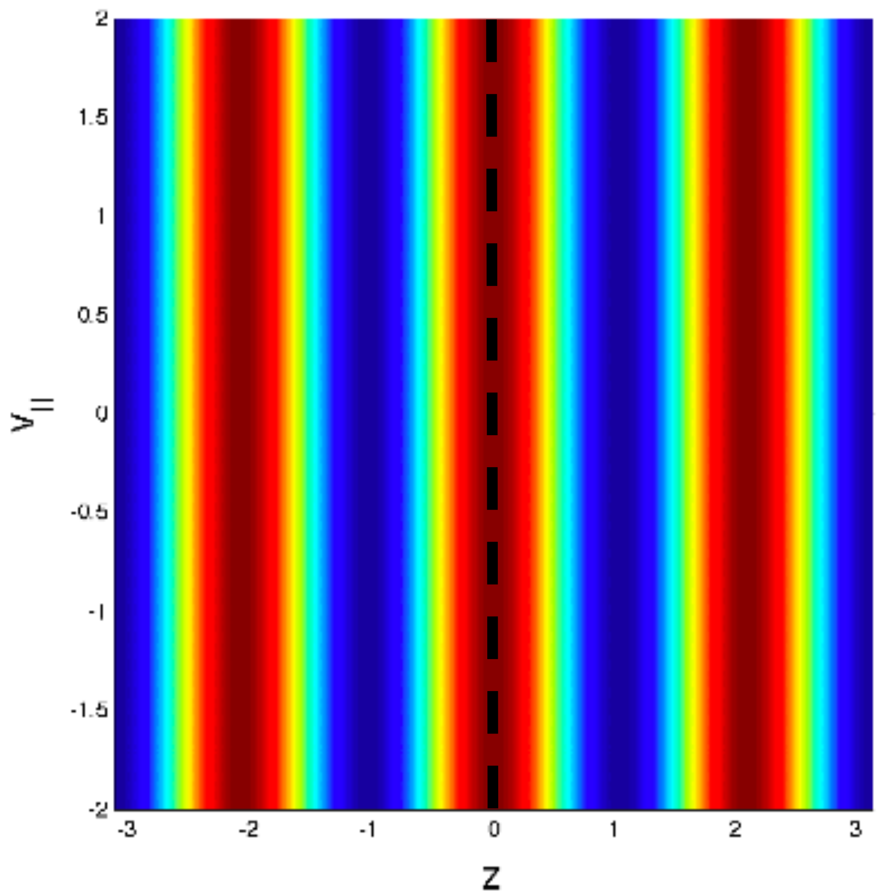
Edge temperature is very important!

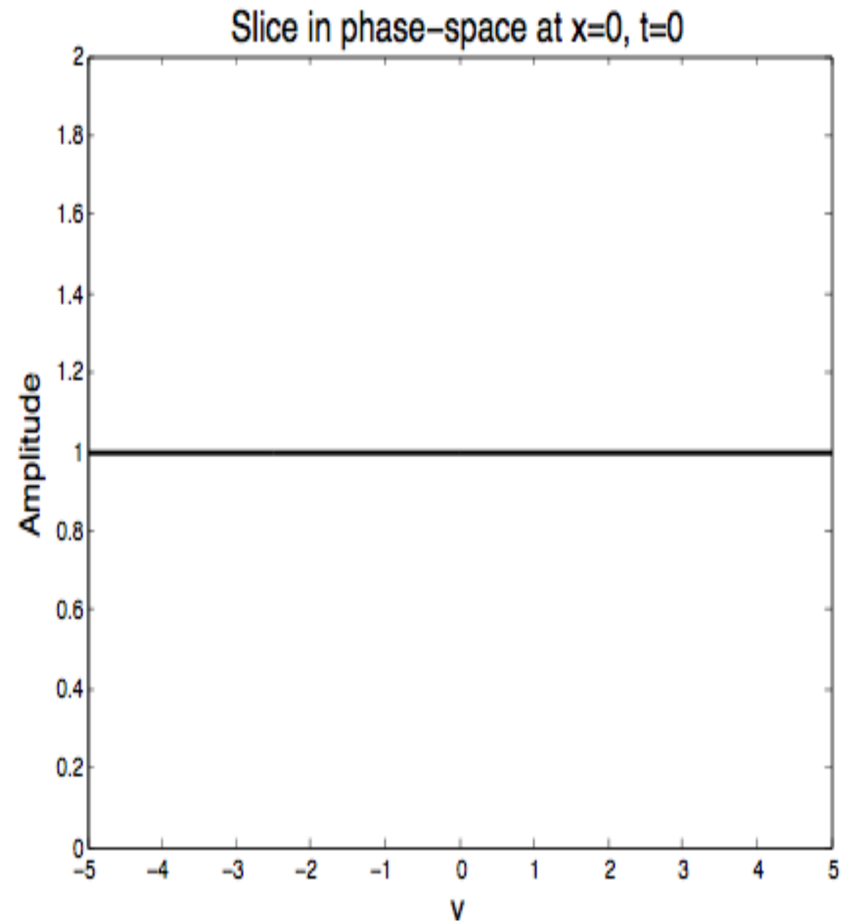
Range of scales



Formation of small scales in v-space

$$f = f(\mathbf{r}, \mathbf{v}, t)$$





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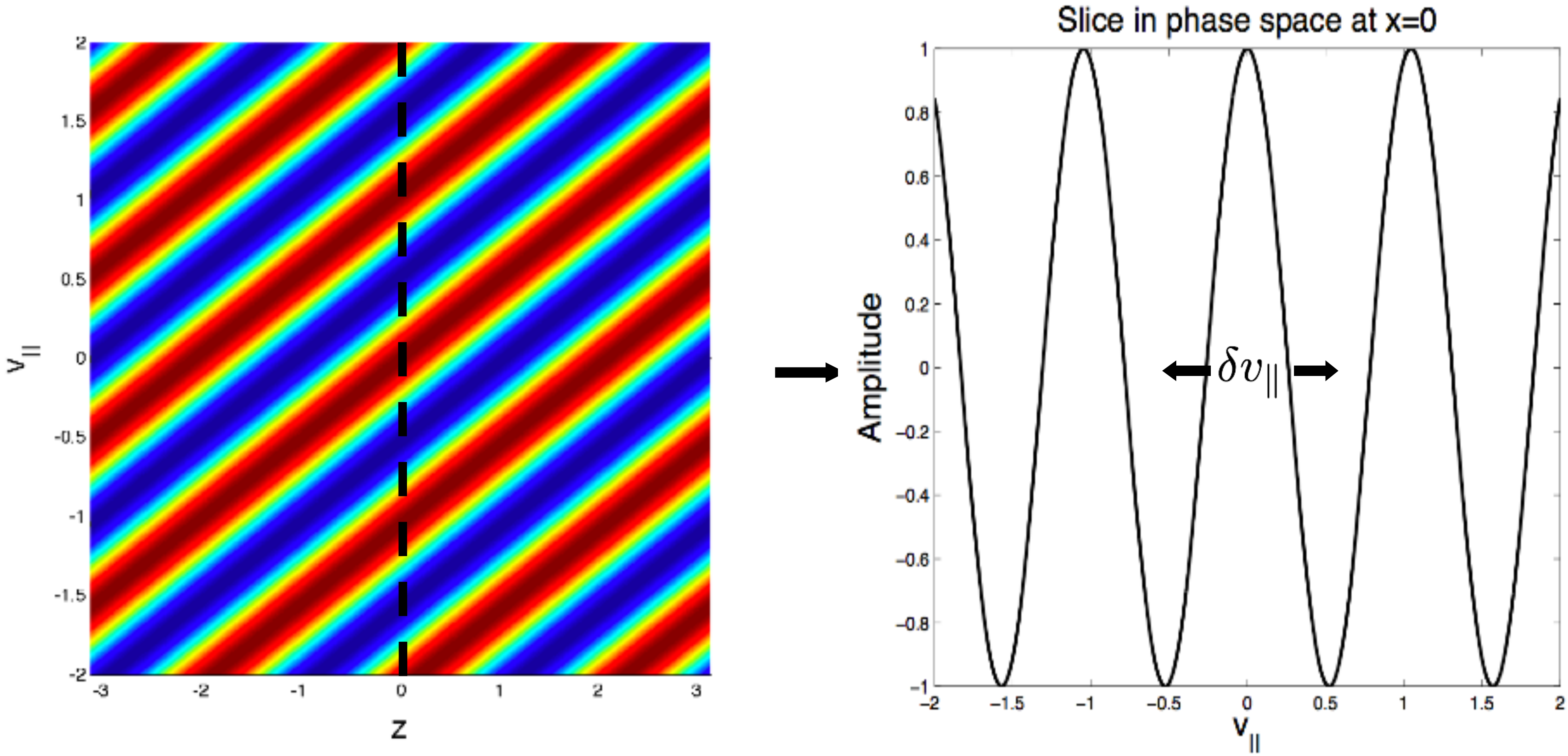

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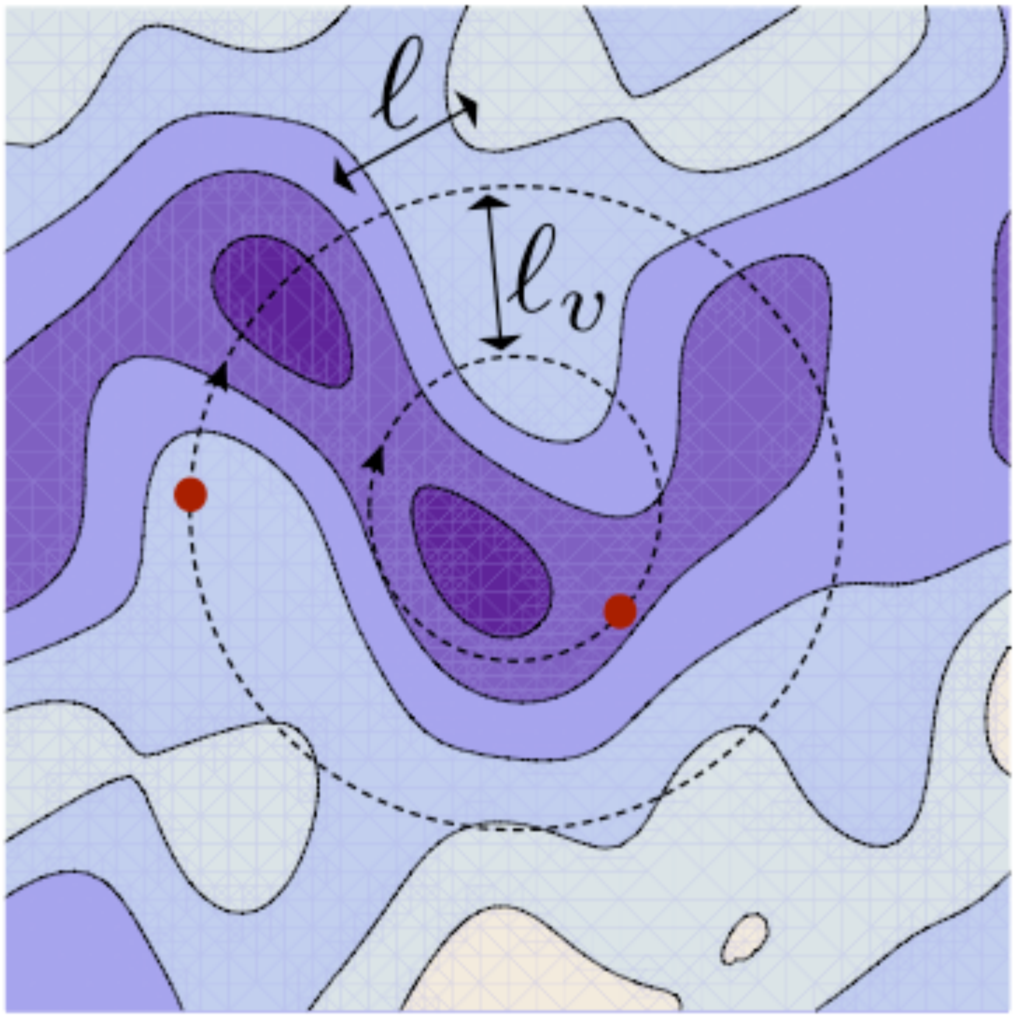


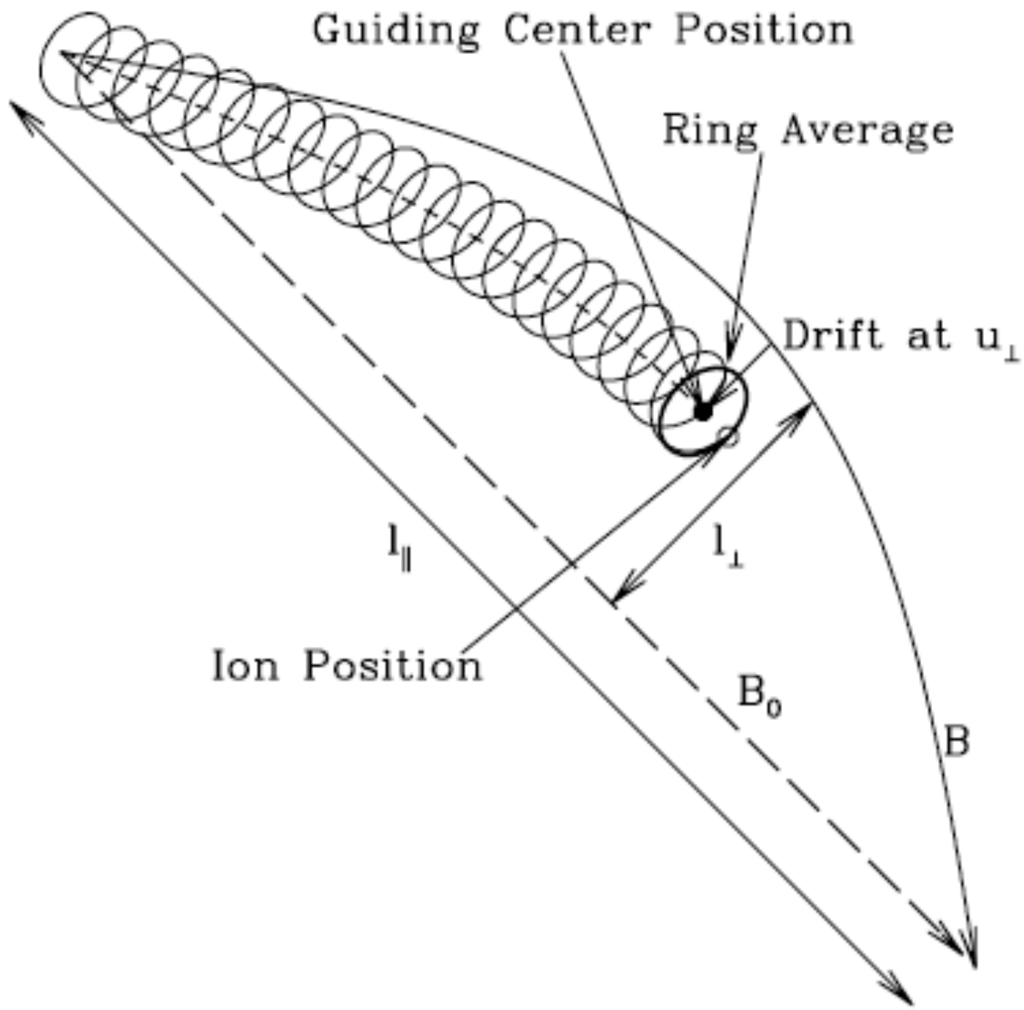
Schekochihin et al., PPCF 2008

- Drift velocity = $F[\langle\Phi\rangle]$
- Particles with Larmor orbits separated by turbulence wavelength 'see' different averaged potential
- Drift velocities decorrelated, thus phase mixing

$$\Rightarrow \frac{\delta v_\perp}{v_{th}} \sim (k_\perp \rho_i)^{-1}$$

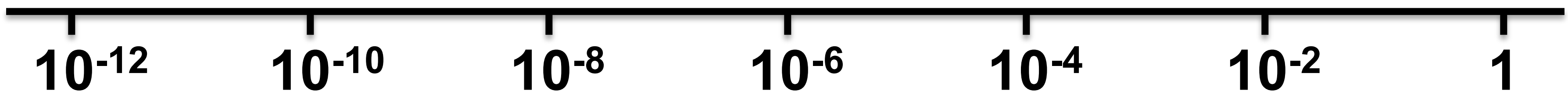
Kinetic description of dynamics

$$\frac{\partial f_s}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{d\mathbf{v}}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = C[f_s]$$

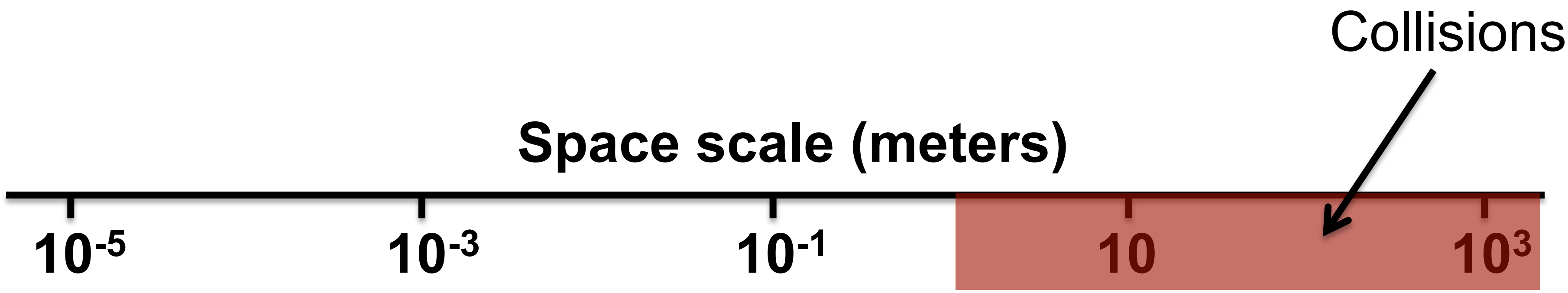


Numerical expense (brute force)

Time scale (seconds)



Temporal grid: ~10^{13} time steps



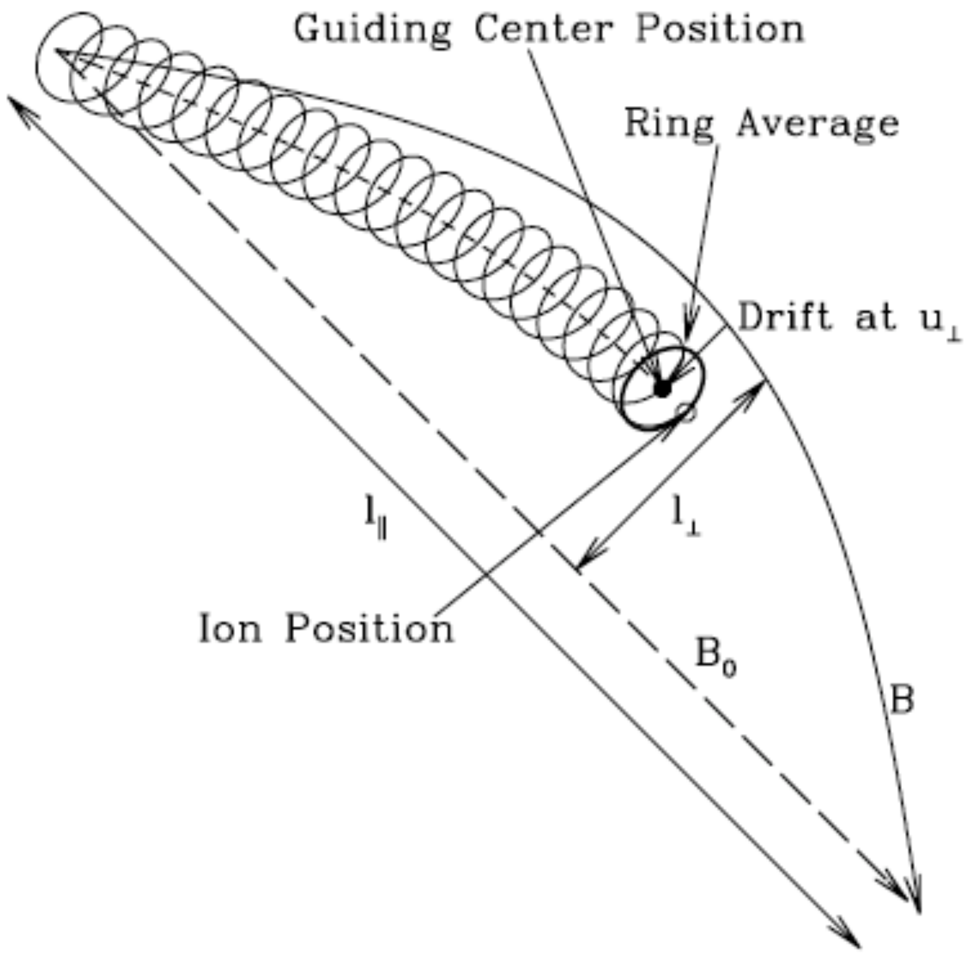
Spatial grid: ~10^6 grid points x 3-D = 10^{18} grid points

Velocity grid: ~10 grid points x 3-D = 10^3 grid points

Total: ~10^{34} total grid points

Gyrokinetic description of dynamics

$$\frac{\partial f_s}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{d\mathbf{v}}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = C[f_s]$$

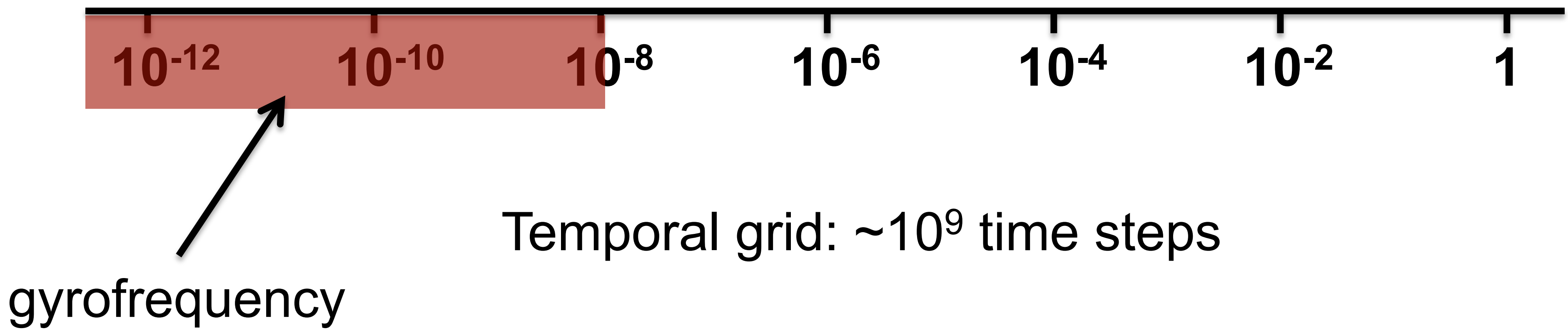


$$f = f(\mathbf{r}, \mathbf{v}, t)$$

- Average over fast gyro-motion and follow 'guiding center' position
- Eliminates fast time scale and gyro-angle variable (6-D → 5-D)

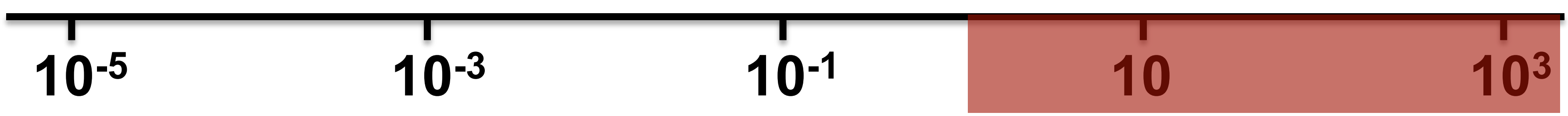
Numerical expense (gyrokinetics)

Time scale (seconds)



Temporal grid: ~10^9 time steps

Space scale (meters)



Perpendicular spatial grid: ~10^6 grid points x 2-D = 10^{12} grid points

Parallel spatial grid: ~10 grid points x 1-D = 10 grid points

Velocity grid: ~10 grid points x 2-D v-space = 10^2 grid points

Total: ~10^{24} total grid points (10^{10} savings)

Multi-scale gyrokinetics

Decompose f into mean and fluctuating components:

$$f = F + \delta f$$

Mean varies perpendicular to mean field on system size while fluctuations vary on scale of gyro-radius:

$$\nabla_\perp \ln F \sim L^{-1} \qquad \nabla_\perp \ln \delta f \sim \rho^{-1}$$

Fluctuations are anisotropic with respect to the mean field:

$$\nabla_\| \ln \delta f \sim L^{-1}$$

=> Turbulent fluctuations are low amplitude: $\delta f \sim \epsilon f$

=> Mean profile evolution slow compared to turbulence:

$$\frac{\partial \ln F}{\partial t} \sim \epsilon^2 \omega \sim \epsilon^3 \Omega$$

$$\boxed{\epsilon \equiv \frac{\rho}{L} \ll 1}$$

Gyrokinetic-Poisson system

$$\delta f_s = h_s - \frac{e_s \varphi}{T_s} F_{M,s}$$

Gyrokinetic equation:

$$\frac{\partial}{\partial t}\left(h_s - \frac{e\langle\varphi\rangle_{\mathbf{R}}}{T_s} F_{M,s}\right) + \left(\mathbf{v}_{\|} + \mathbf{v}_{M,s} + \langle\mathbf{v}_E\rangle_{\mathbf{R}}\right)\cdot\nabla h_s$$
$$= \langle C[h_s]\rangle_{\mathbf{R}} - \langle\mathbf{v}_E\rangle_{\mathbf{R}}\cdot\nabla F_{M,s}$$

Quasineutrality: $$\sum_s e_s\left(\int d^3v\ h_s - \frac{e_s\varphi}{T_s} n_s\right) = 0$$

Equilibrium: $$\frac{3}{2}\frac{\partial p_s}{\partial t} + \frac{1}{\mathcal{J}}\frac{\partial}{\partial r}\mathcal{J} Q_r = \mathcal{S}$$

Gyrokinetic-Poisson system

NB: GK ordering implies $\mathbf{E} = -\nabla\varphi$. $\beta \ll 1$ implies fluctuations approximately electrostatic

Gyrokinetic equation:

$$\frac{\partial}{\partial t}\left(h_s - \frac{e\left\langle\varphi\right\rangle_{\mathbf{R}}}{T_s}F_{M,s}\right) + \left(\mathbf{v}_\parallel + \mathbf{v}_{M,s} + \left\langle\mathbf{v}_E\right\rangle_{\mathbf{R}}\right)\cdot\nabla h_s$$

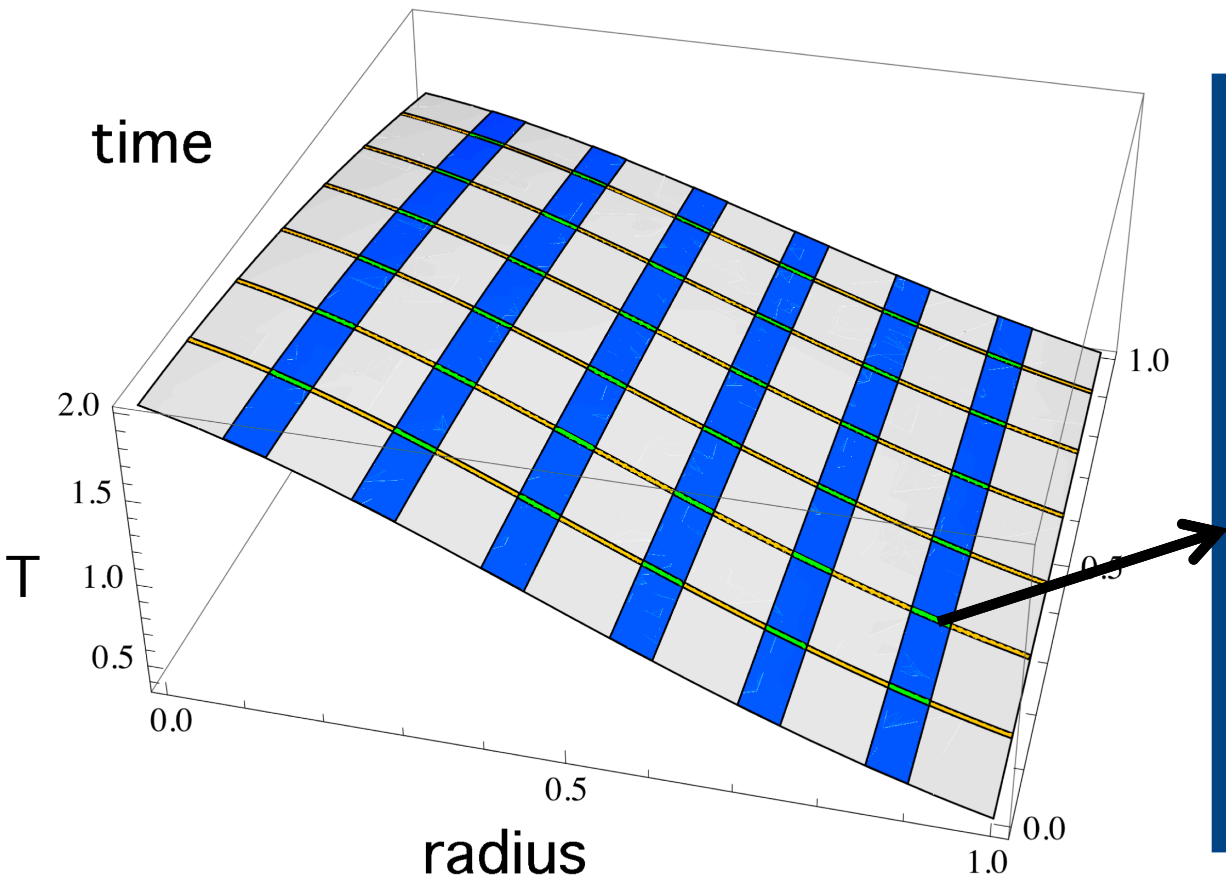
$$= \left\langle C[h_s]\right\rangle_{\mathbf{R}} - \left\langle\mathbf{v}_E\right\rangle_{\mathbf{R}}\cdot\nabla F_{M,s}$$

Quasineutrality: $$\sum_s e_s\left(\int d^3v\ h_s - \frac{e_s\varphi}{T_s}n_s\right) = 0$$

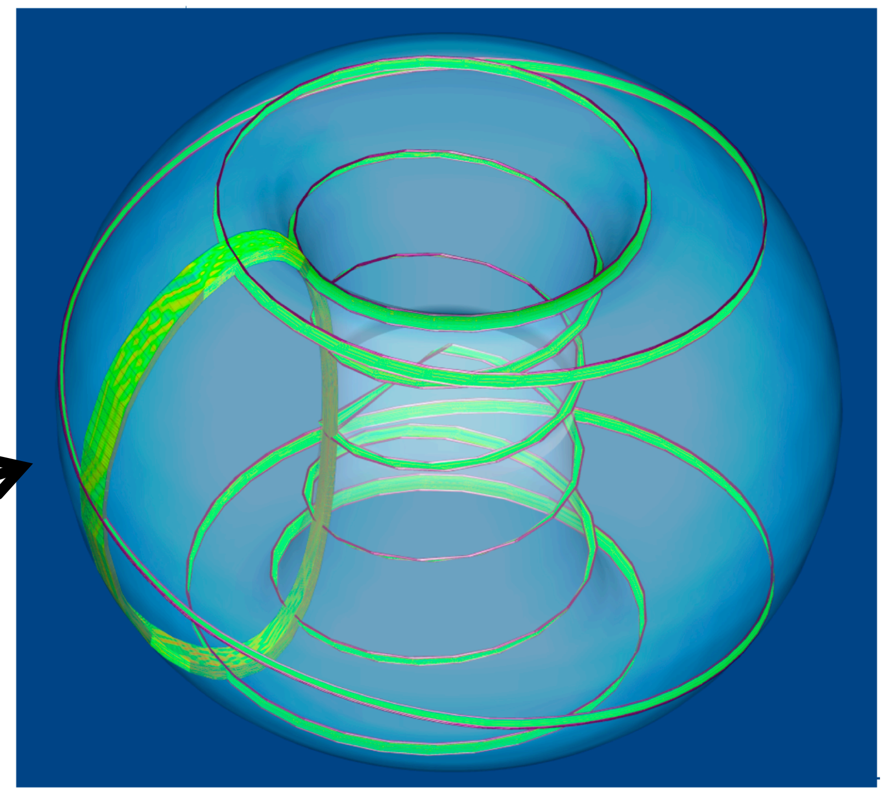
Equilibrium: $$\frac{3}{2}\frac{\partial p_s}{\partial t} + \frac{1}{\mathcal{J}}\frac{\partial}{\partial r}\mathcal{J}Q_r = \mathcal{S}$$

Multi-scale gyrokinetics

Turbulent fluctuations calculated in small regions of fine space-time grid embedded in coarse grid for mean quantities

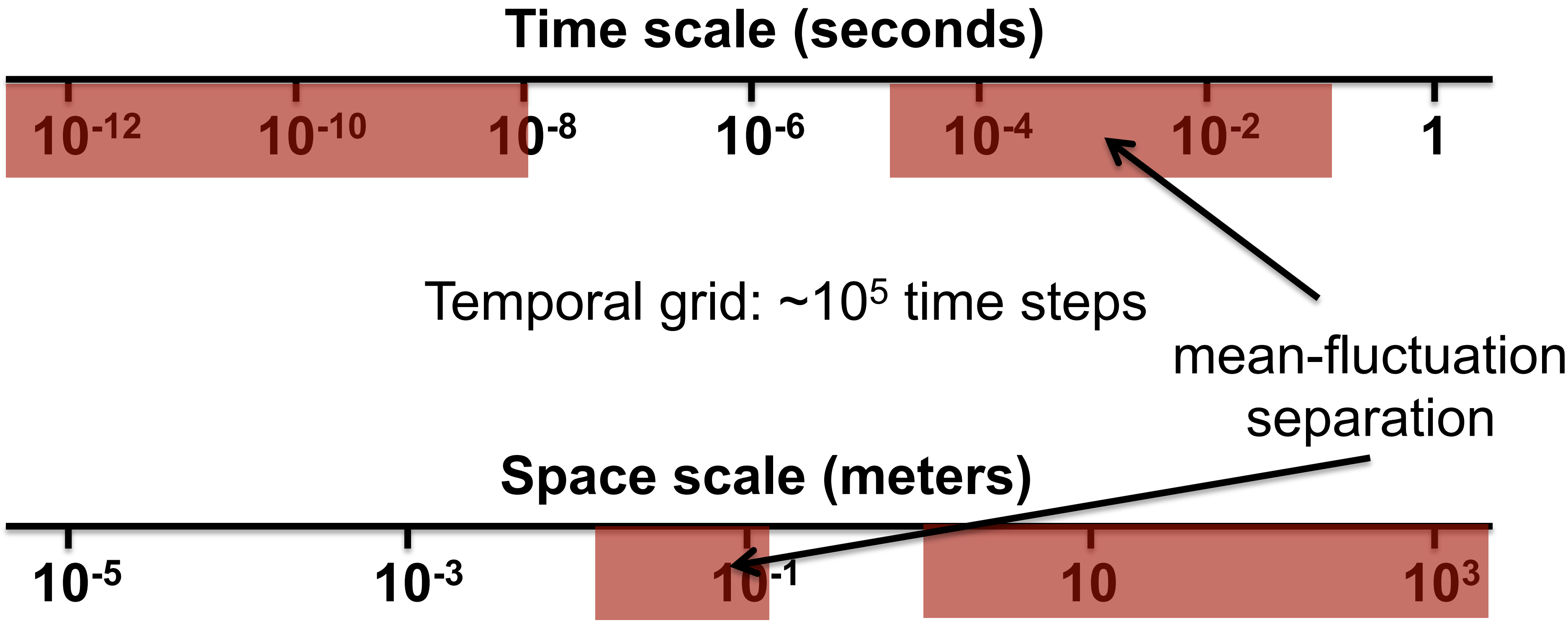


Trinity



GS2

Numerical expense (multi-scale GK)



Temporal grid: ~10^5 time steps

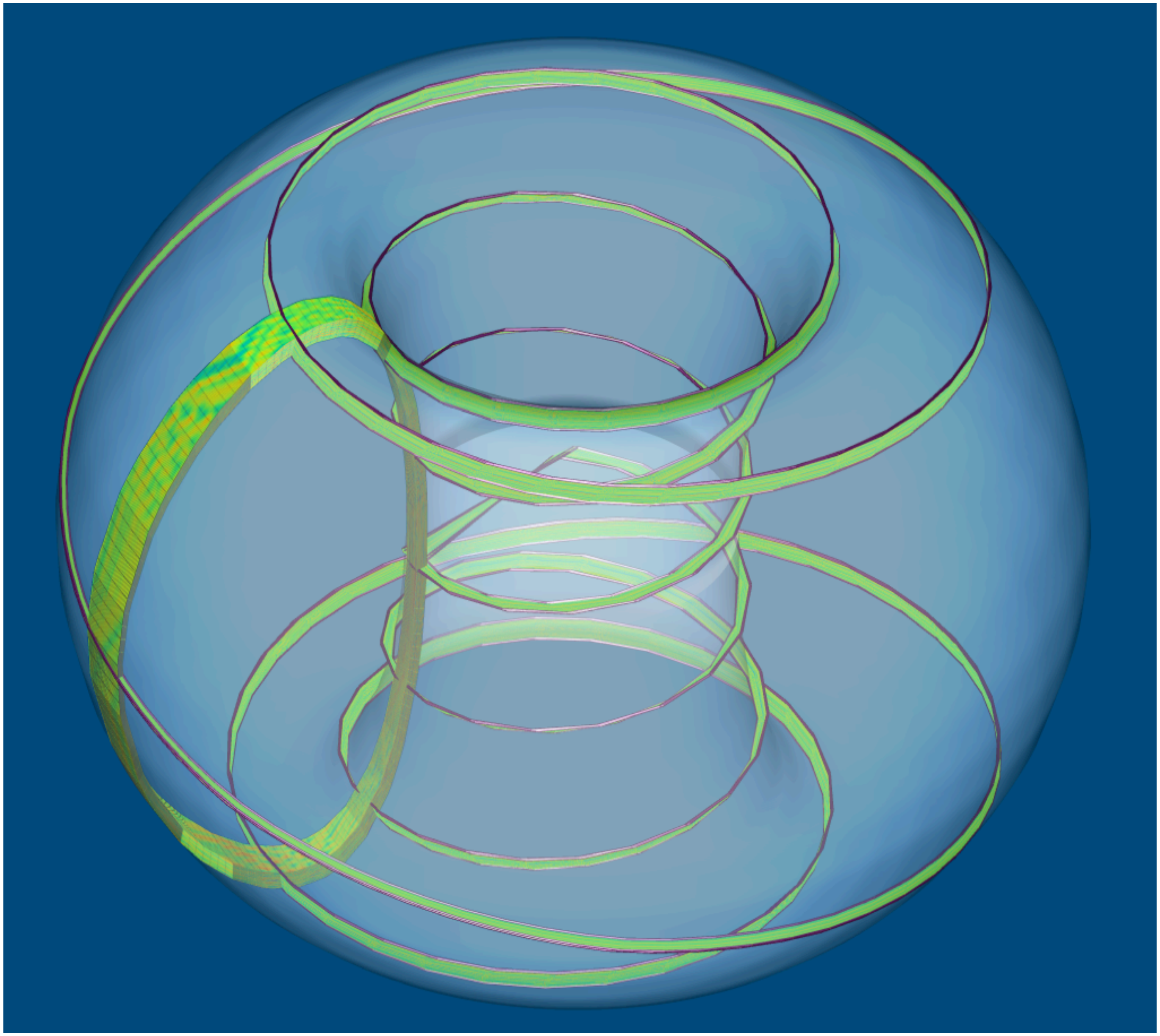
Perpendicular spatial grid: ~10^5 grid points x 2-D = 10^{10} grid points

Parallel spatial grid: ~10 grid points x 1-D = 10 grid points

Velocity grid: ~10 grid points x 2-D v-space = 10^2 grid points

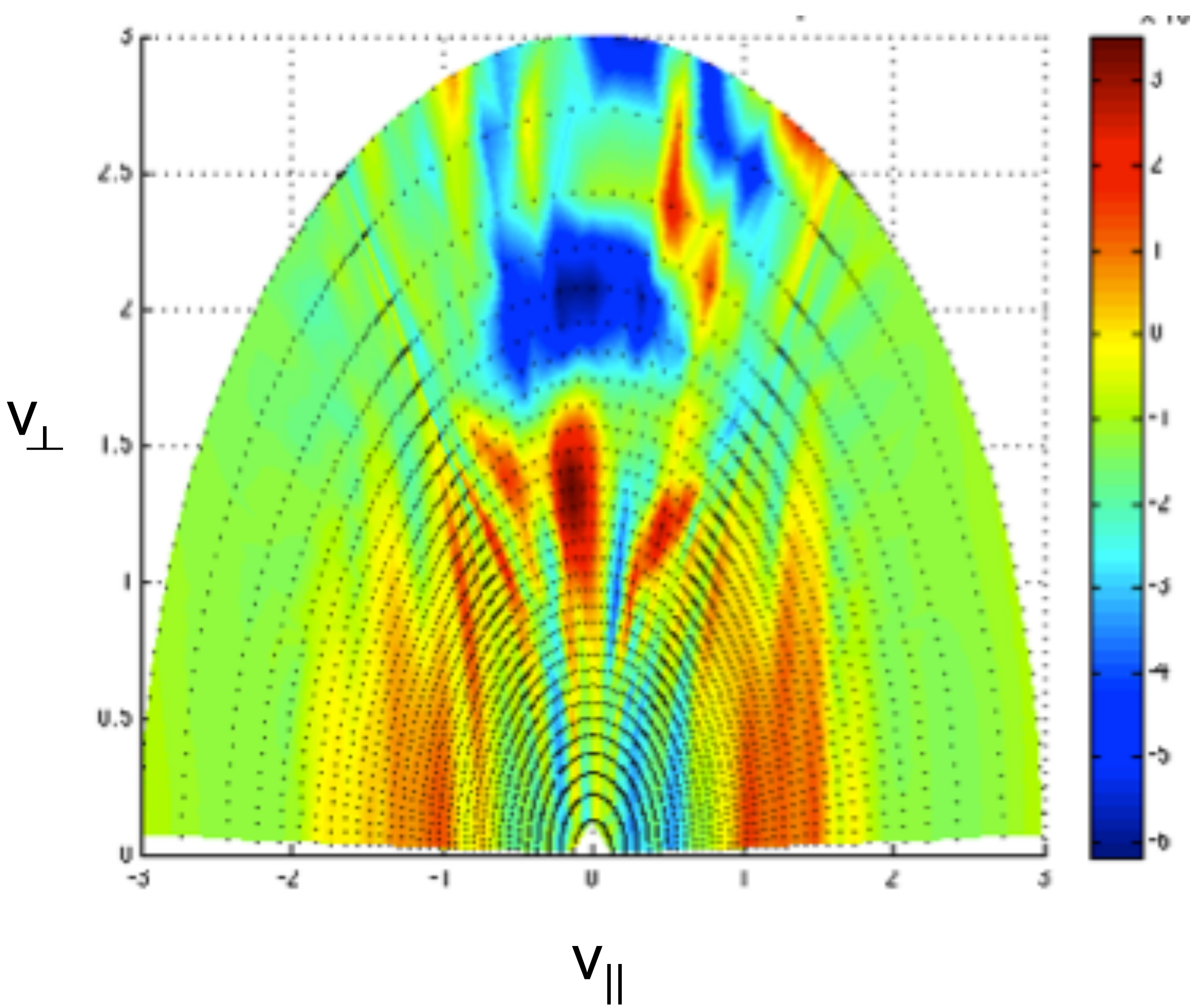
Total: ~10^{18} total grid points (10^6 savings)

Simulating gyrokinetic turbulence

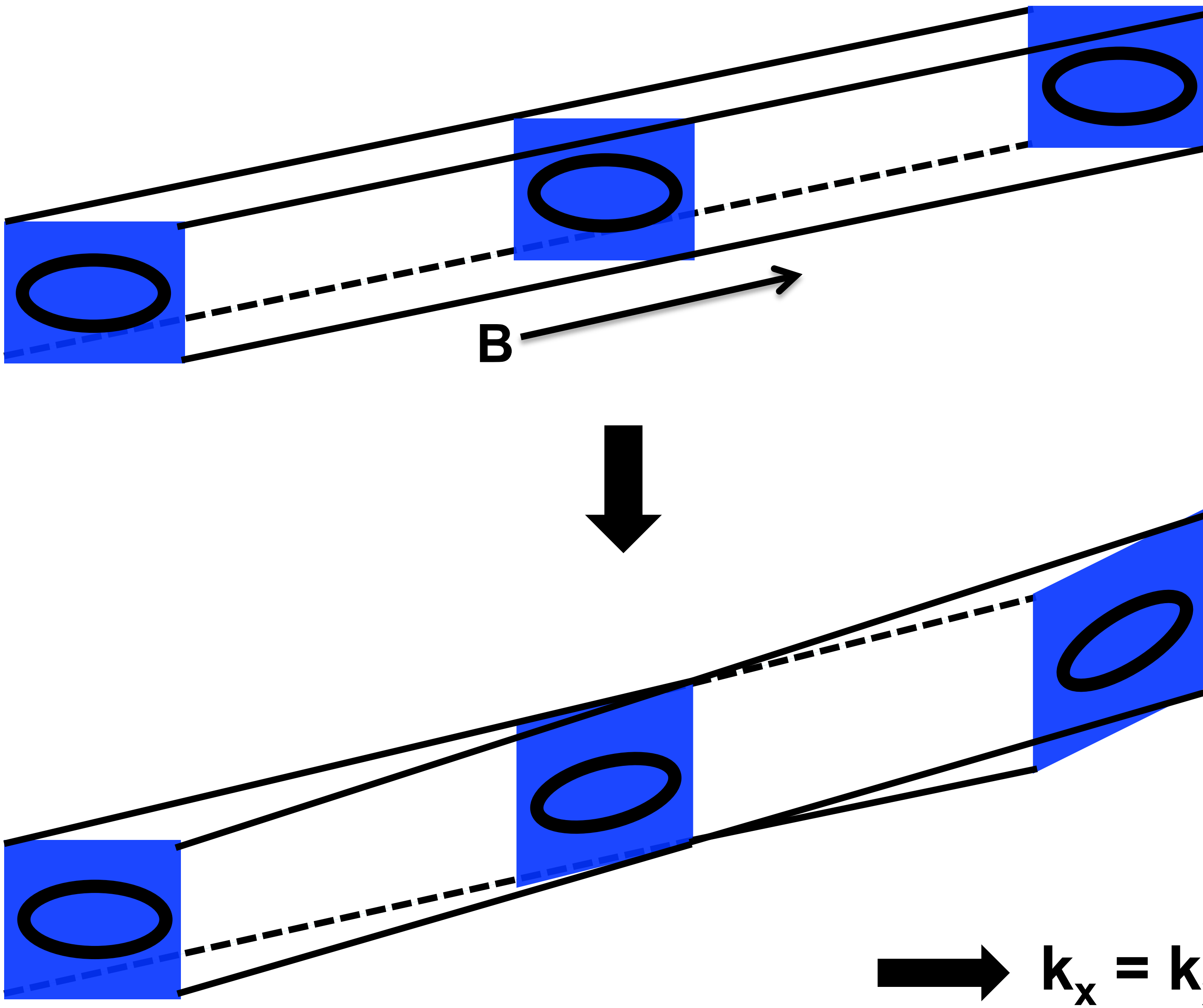


Simulating gyrokinetic turbulence

$$f = f(\mathbf{r}, \mathbf{v}, t)$$

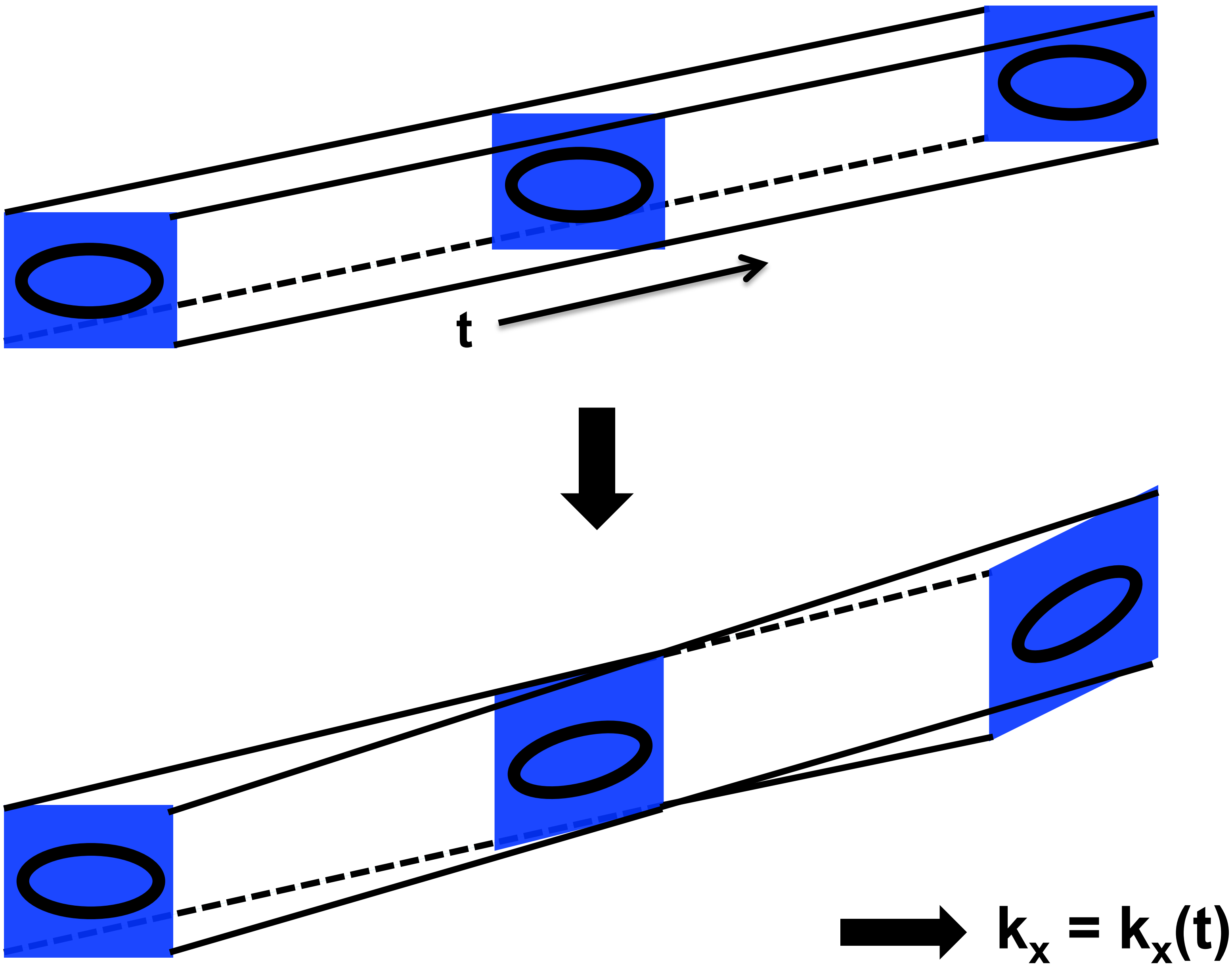


(Magnetically) sheared box

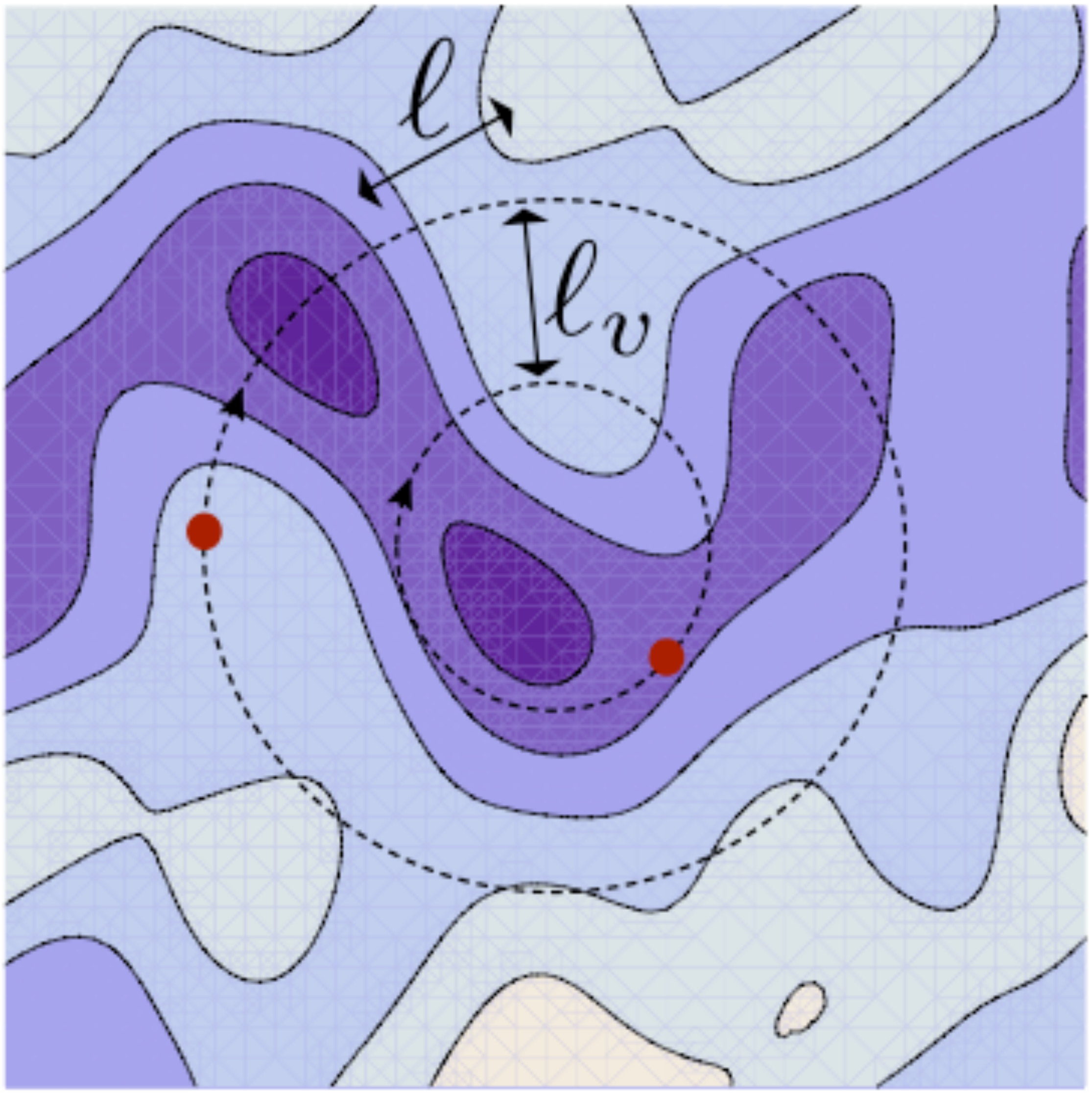


$\Rightarrow k_x = k_x(z)$

(Flow) sheared box



Numerical gyroaveraging



- Gyro-average is non-local in physical space
- Local in Fourier space

Numerical gyroaveraging

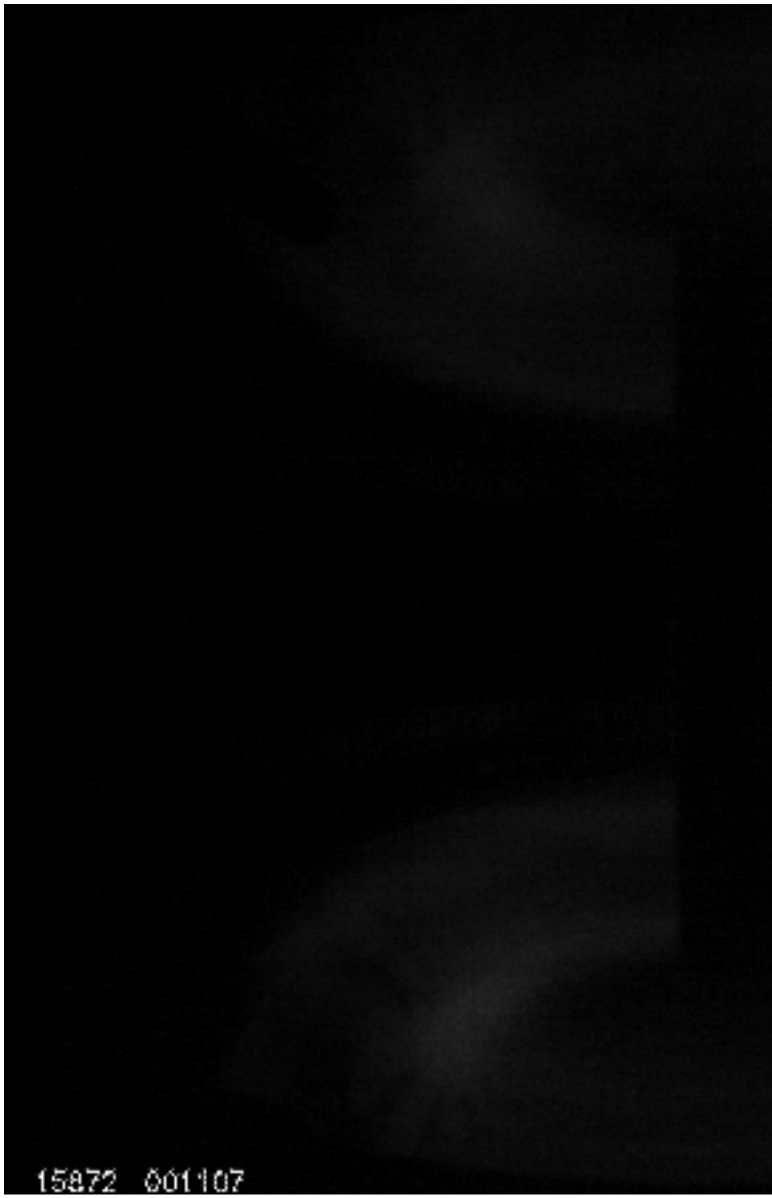
- Gyro-average is non-local in physical space
- Local in Fourier space:

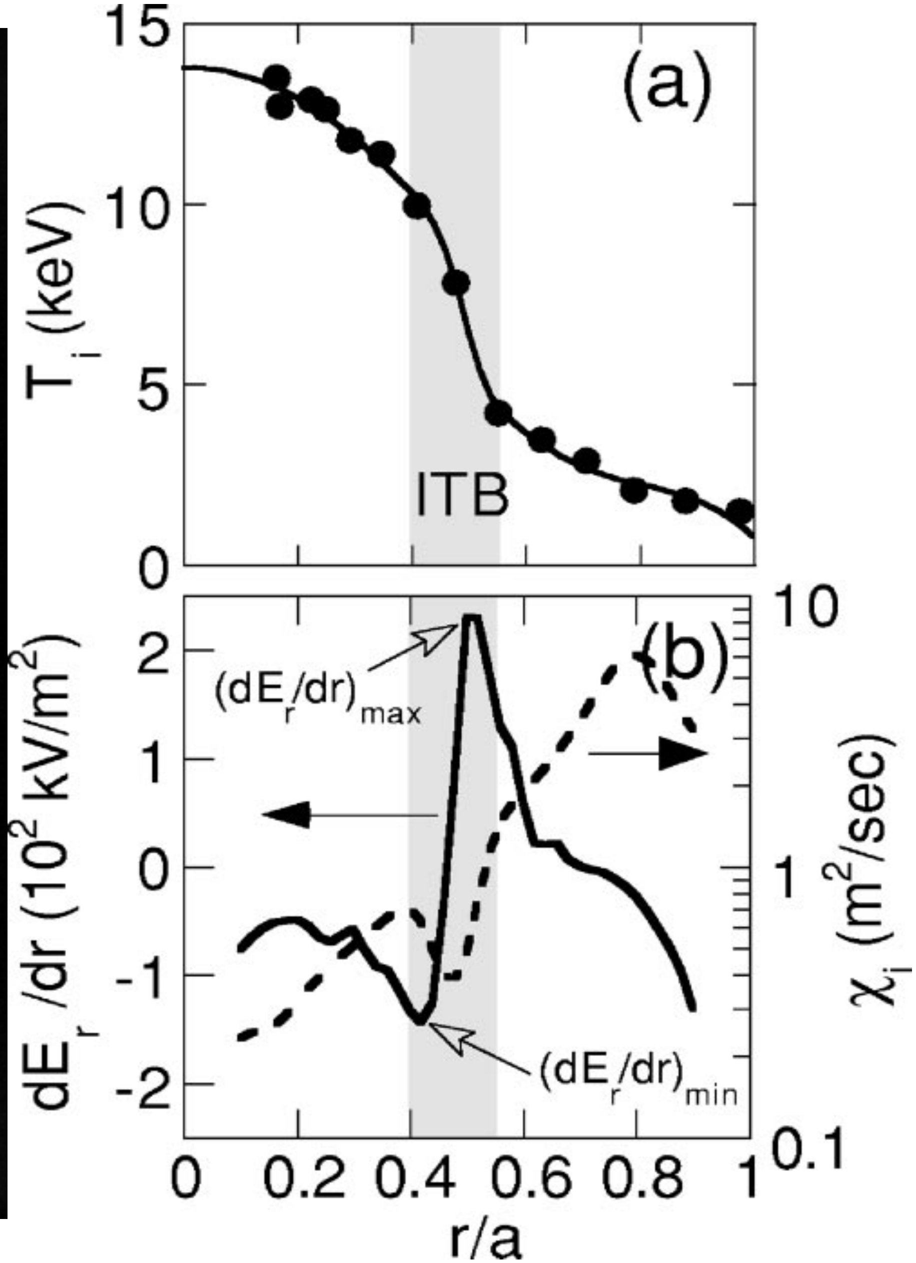
$$\langle \varphi(\mathbf{r}) \rangle = \left\langle \sum_{\mathbf{k}_\perp} \hat{\varphi}_{\mathbf{k}_\perp}(z) e^{i\mathbf{k}_\perp \cdot \mathbf{r}} \right\rangle$$

$$= \frac{1}{2\pi} \oint d\vartheta \sum_{\mathbf{k}_\perp} \hat{\varphi}_{\mathbf{k}}(z) e^{i\mathbf{k}_\perp \cdot \mathbf{R}_s} e^{i\mathbf{k}_\perp \cdot \boldsymbol{\rho}_s(\vartheta)}$$

$$= \sum_{\mathbf{k}_\perp} J_0\left(\frac{k_\perp v_\perp}{\Omega_s}\right) \hat{\varphi}_{\mathbf{k}_\perp}(z) e^{i\mathbf{k}_\perp \cdot \mathbf{R}_s}$$

Turbulence suppression

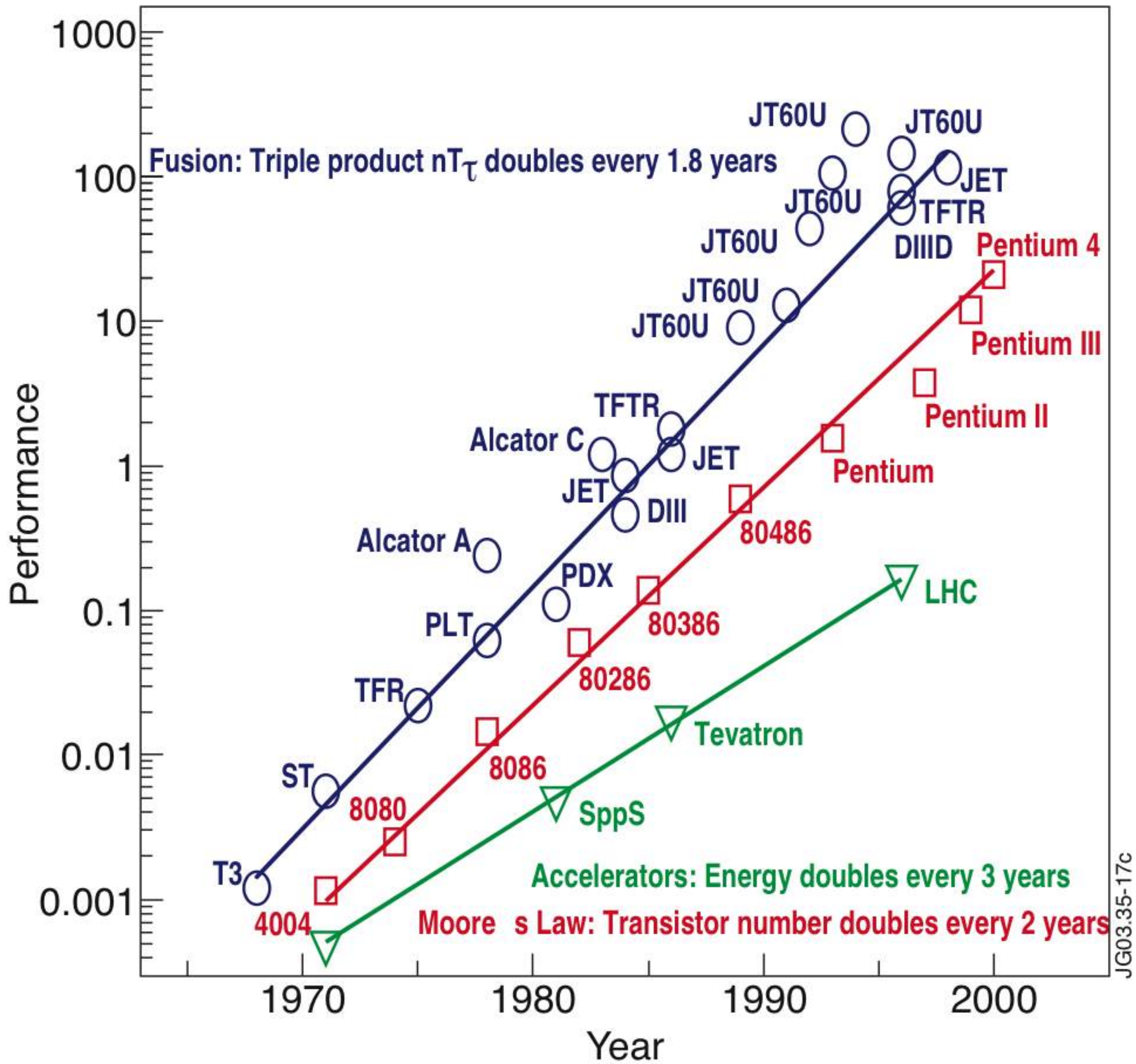




Fast camera image of MAST plasma

JT-60U data, Y. Miura et al.

Progress in magnetic confinement fusion

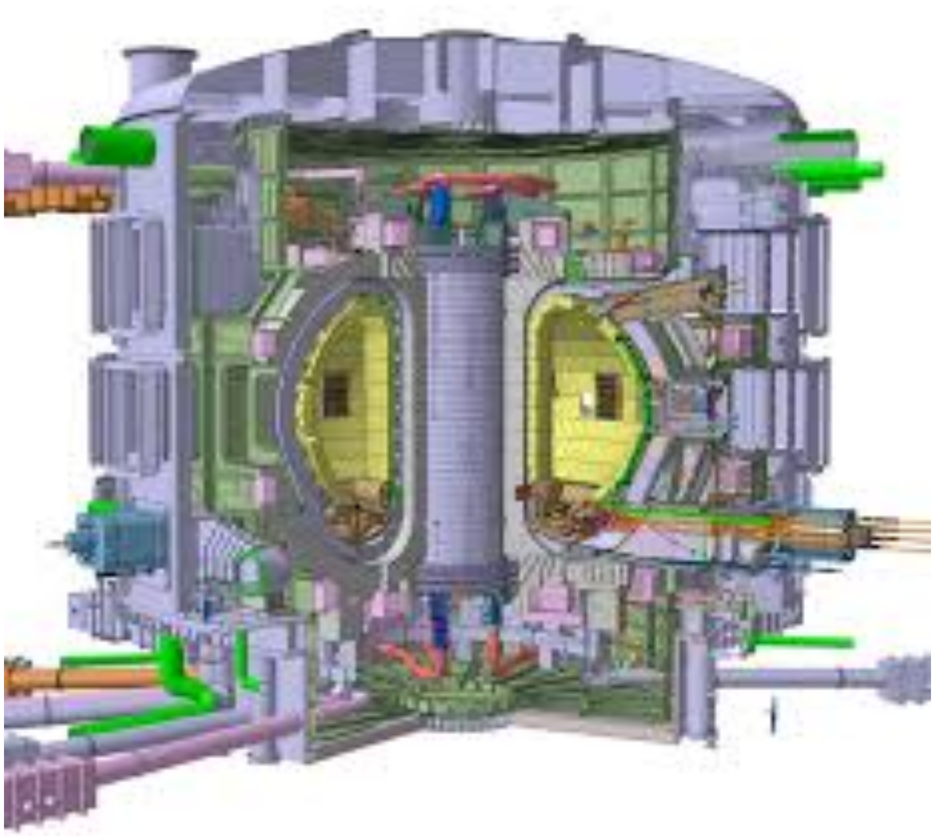


Challenges ahead

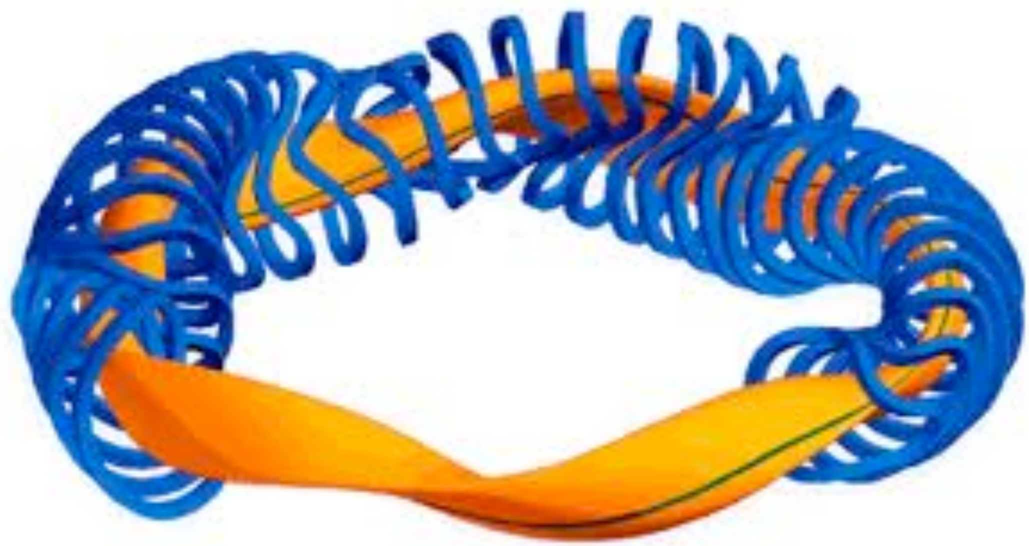
- Technological: plasma-wall interaction, superconducting magnets, etc.
- Controlling macroscopic instabilities
- Reducing turbulent transport
- Steady state operation

Moving forward

ITER



Wendelstein 7-X



And now for something completely different…

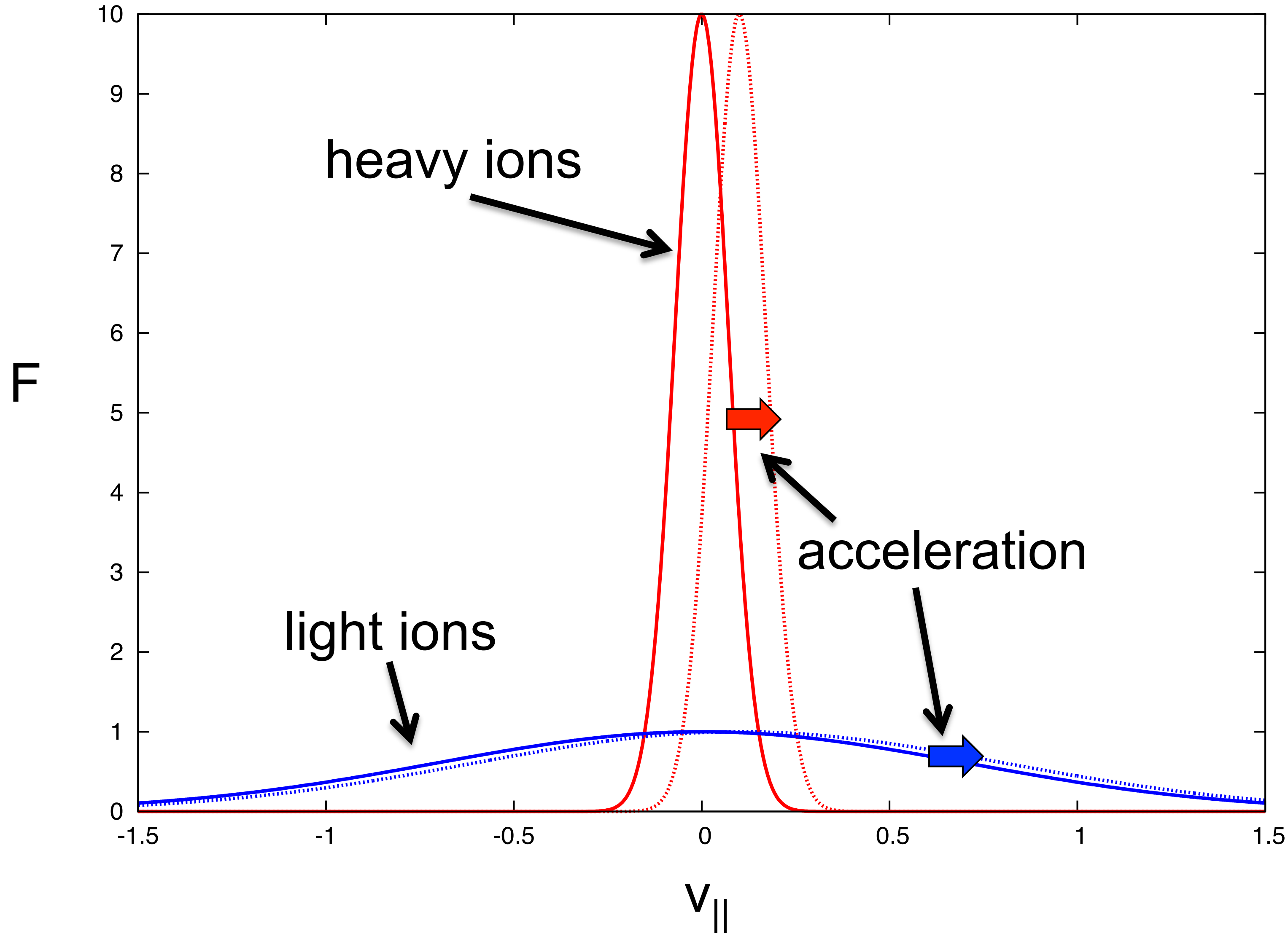
Turbulent heating

- Assume turbulent plasma dominated by electrons and hydrogenic ions
- How much are passive minority ions heated by turbulence?
- Gyrokinetics restricts us to isotropic heating because magnetic moment is conserved
- Turbulent heating mechanism is Joule heating:

$$H_s = \delta \boldsymbol{J}_{\|,s} \cdot \delta \mathbf{E}_{\|} \propto e_s v_{th,s} \delta n_s \propto m_s^{1/2} \delta n_s$$

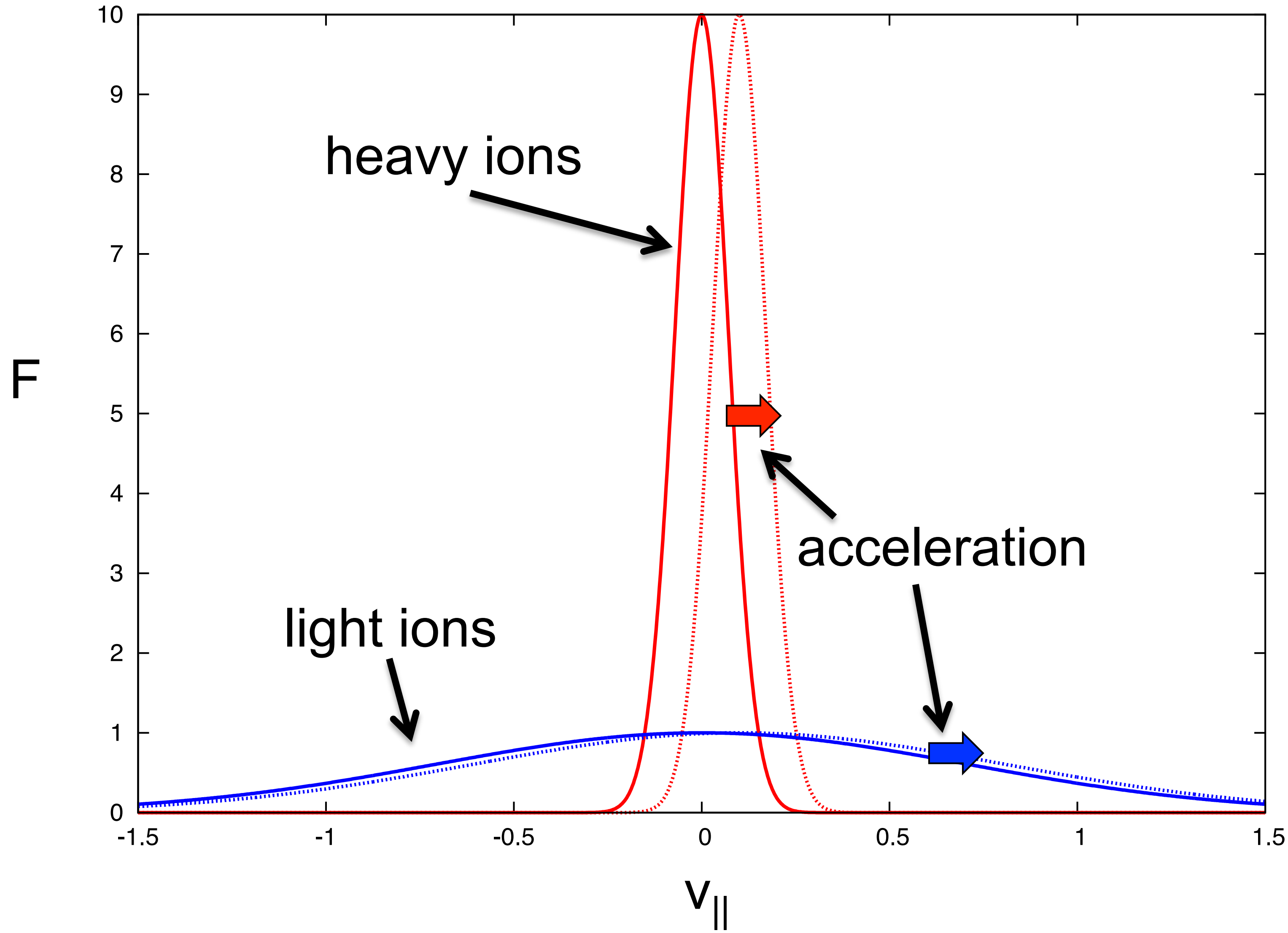
Cartoon of mass-dependent fluctuations

Acceleration = $(Ze/m)\delta E_{||}$



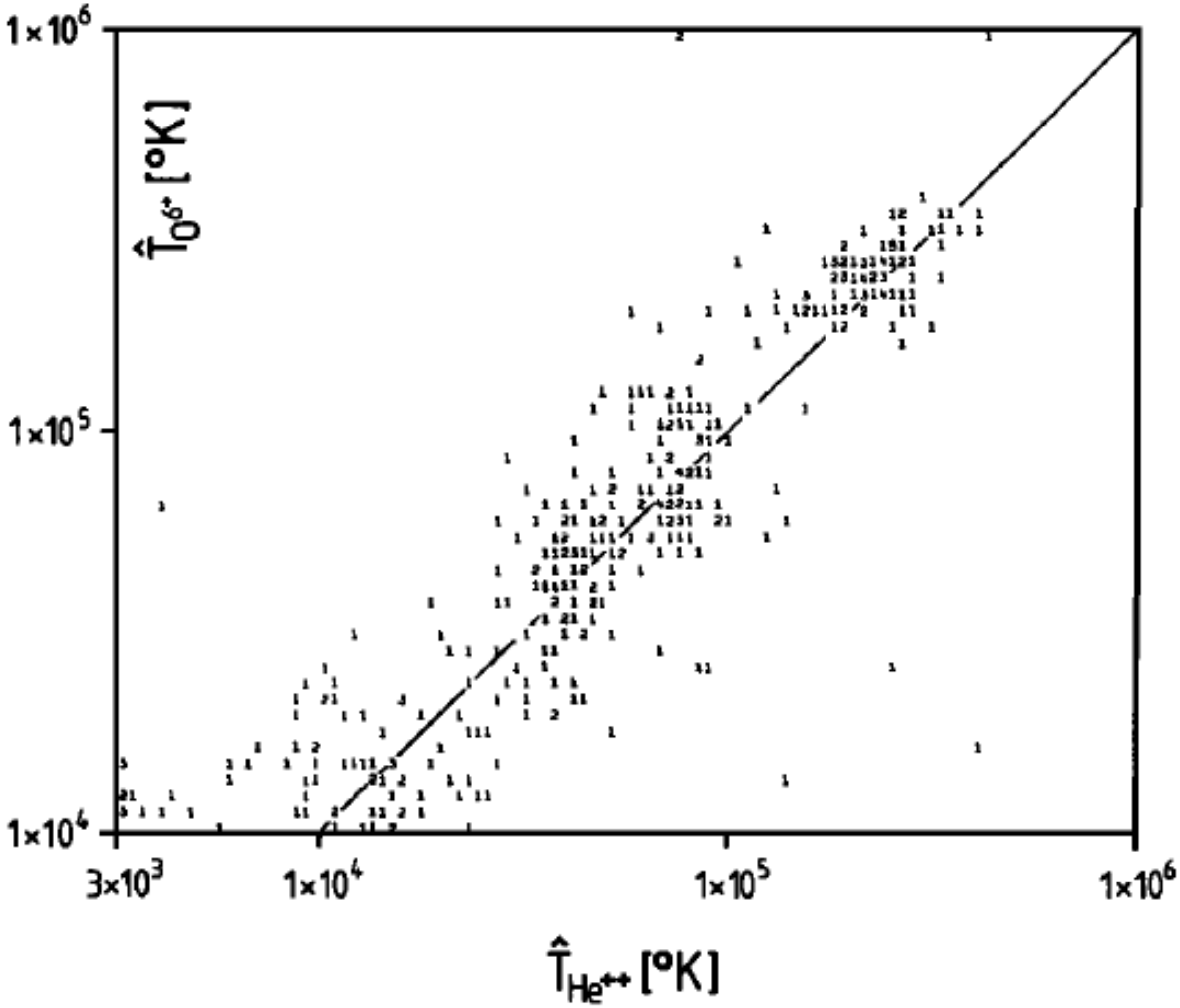
Cartoon of mass-dependent fluctuations

$$\frac{\delta f_s}{f_s} \sim \frac{\Delta v_{||}}{v_{th,s}} \propto m_s^{1/2} \Rightarrow H_s \propto m_s$$



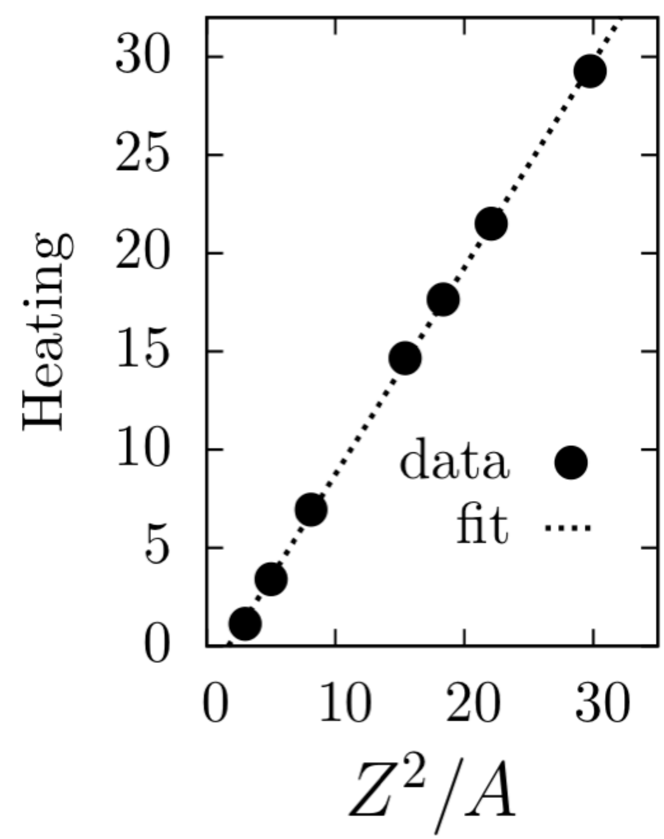
Simple picture consistent with GK simulations and solar wind observations

Minority ion temperatures in solar wind



Schmidt et al., Geophys. Res. Lett. (1980).

Heating from GK simulation



Barnes et al. PRL (2012)

Fundamental properties of GK turbulence

Take GK and make a few simple conjectures:

- Isotropy in plane perpendicular to B-field
- Position and velocity space scales linked
- Parallel streaming time and nonlinear turnover time comparable at all scales (critical balance)
- Parallel length at outer scale set by system size (connection length)

Smooth, isotropic v-space (kρ < 1)

Assume $$\frac{\partial h}{\partial v_\parallel} \sim \frac{\partial h}{\partial v_\perp} \sim \frac{h}{v_{th}}$$

Quasineutrality: $$\sum_s e_s \left(\int d^3v\ h_s - \frac{e_s \varphi}{T_s} n_s \right) = 0$$

$$\int d^3v\ h \sim v_{th}^3 h \Rightarrow \frac{h}{F_M} \sim \frac{e\varphi}{T}$$

Critical balance

- Physical idea: two points along field correlated only if information propagates between them before turbulence decorrelated in perpendicular plane

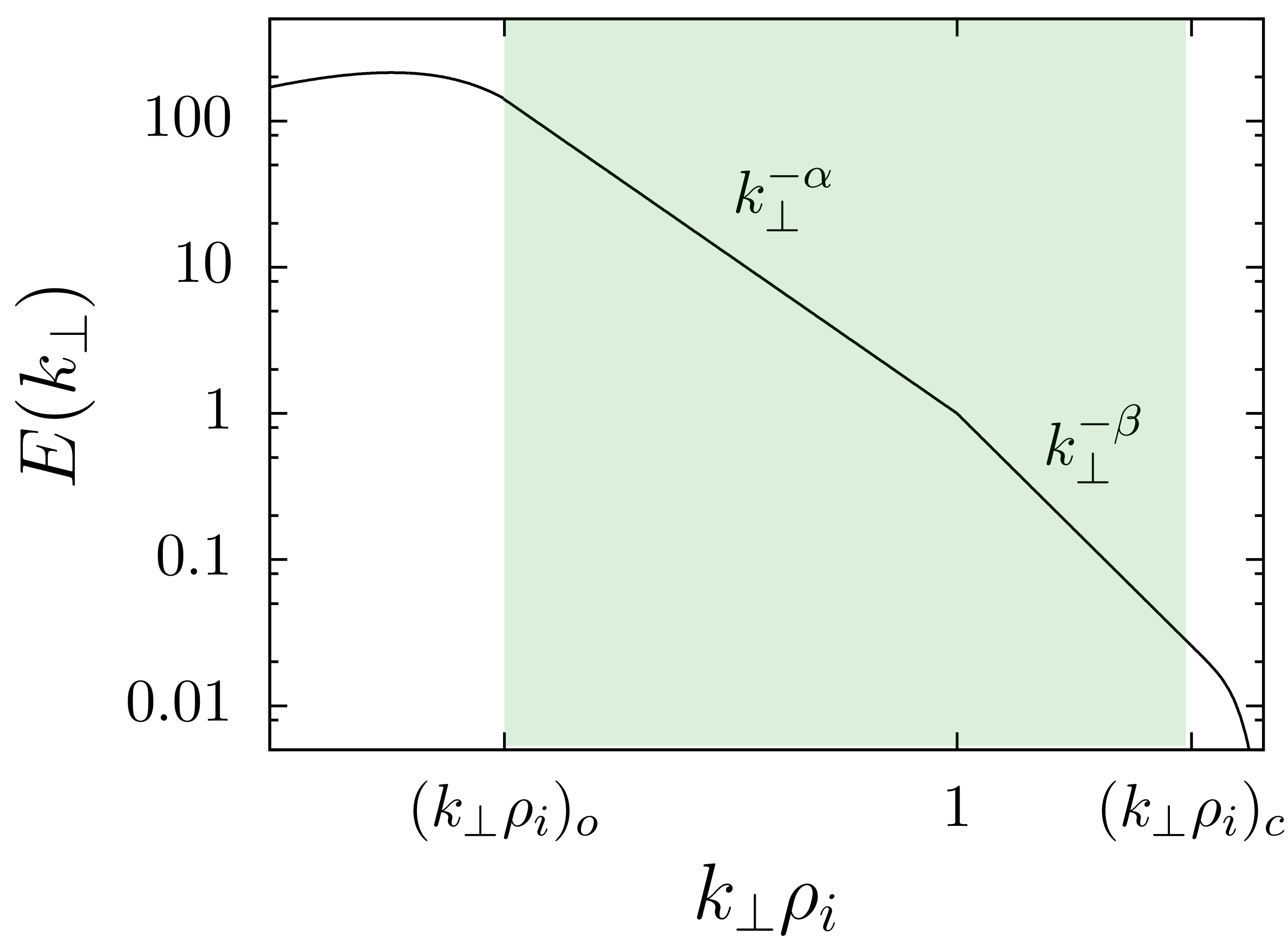
Definition of nonlinear time scale:

$$\frac{h_k}{\tau_k} \sim \left(\langle \mathbf{v}_E \rangle_{\mathbf{R}} \cdot \nabla\right)_k$$

Critical balance:

$$k_{\|} v_{th} \sim \tau_k^{-1}$$

Inertial range



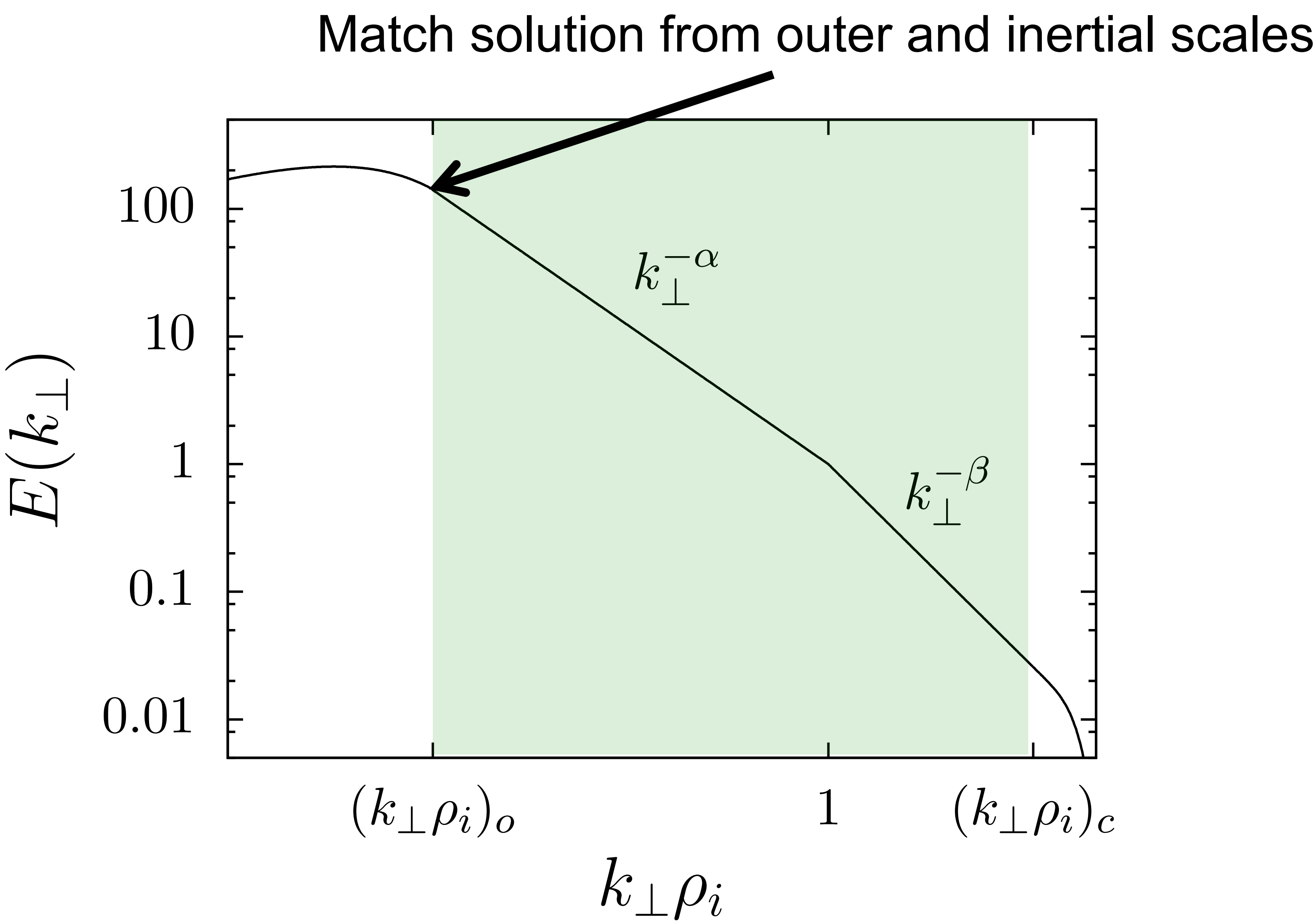
Inertial range (kρ < 1)

Free energy: $W = V^{-1} \sum_s \int d^3r \int d^3v \left(\frac{T_s \delta f_s^2}{F_{M,s}} \right)$

- Flux of free energy (nonlinear invariant) scale-independent in inertial range:

$$\frac{W_k}{\tau_k} \sim (k_\perp \rho_i)^2 \, \frac{v_{th}}{R} \left(\frac{\rho_i}{R} \right)^2 \Phi_k^3 \sim \text{constant}$$

Matching



Match solution from outer and inertial scales
$k_\perp^{-\alpha}$
$k_\perp^{-\beta}$
100
10
1
0.1
0.01
$E(k_\perp)$
$(k_\perp \rho_i)_o$
1
$(k_\perp \rho_i)_c$
$k_\perp \rho_i$

Inertial range (kρ << 1)

Free energy: $W = V^{-1} \sum_s \int d^3r \int d^3v \left(\frac{T_s \delta f_s^2}{F_{M,s}} \right)$

- Flux of free energy (nonlinear invariant) scale-independent in inertial range:

$$\frac{W_k}{\tau_k} \sim (k_\perp \rho_i)^2 \frac{v_{th}}{R} \left(\frac{\rho_i}{R} \right)^2 \Phi_k^3 \sim \text{constant}$$

$$\Phi_k \sim q^{1/3} \left(\frac{R}{L_T} \right)^{4/3} (k_\perp \rho_i)^{-2/3}$$

Inertial range (kρ << 1)

- Convert expression for Φ_k into 1D spectrum

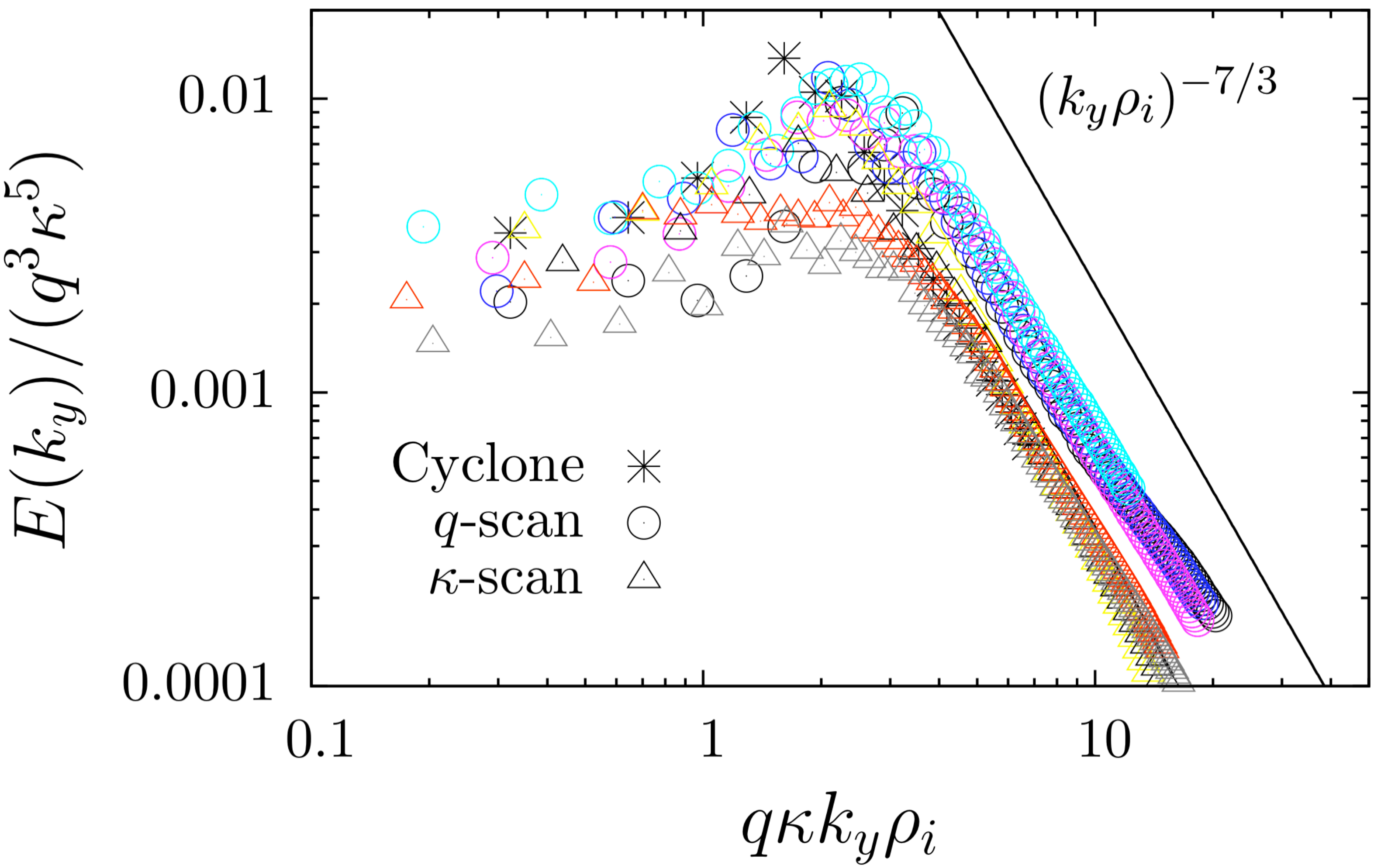
$$\int dk_y \ \rho_i E(k_y) = V^{-1} \int d^3 r \ \Phi^2$$

$$E(k_y) \sim q^{2/3} \left(\frac{R}{L_T}\right)^{8/3} (k_y \rho_i)^{-7/3}$$

- Use critical balance with Φ_k to relate $k_\parallel$ and $k_\perp$

$$k_\parallel q R \sim \left(k_\perp \rho_i \frac{qR}{L_T}\right)^{4/3}$$

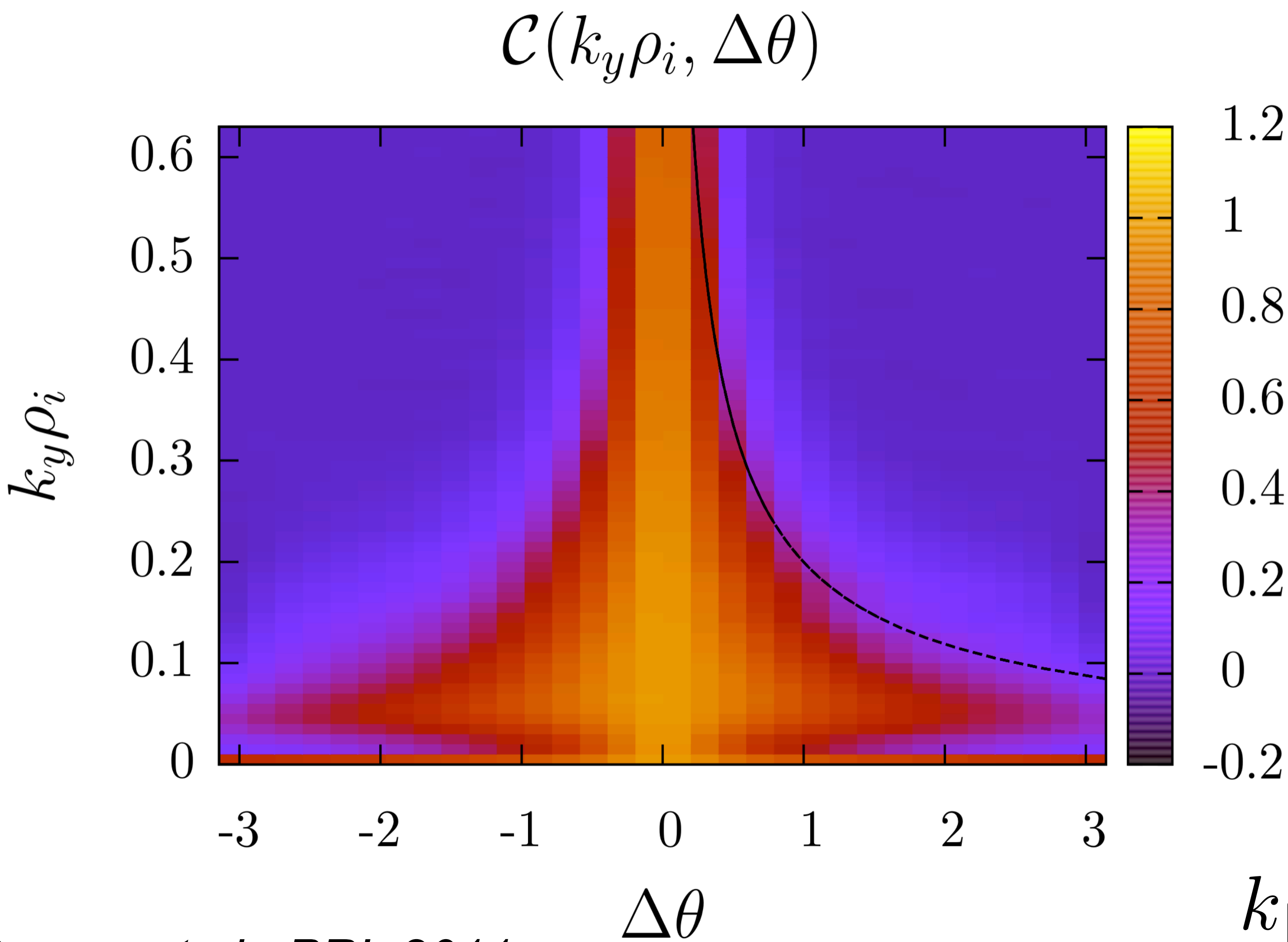
Inertial range spectra



Barnes et al., PRL 2011

Critical balance test

Correlation function: $$\mathcal{C}(k_y, \Delta\theta) = \frac{\sum_{k_x} \Phi(\theta=0)\Phi^*(\theta=\Delta\theta)}{\sum_{k_x} \left|\Phi(\theta=0)\right|^2}$$



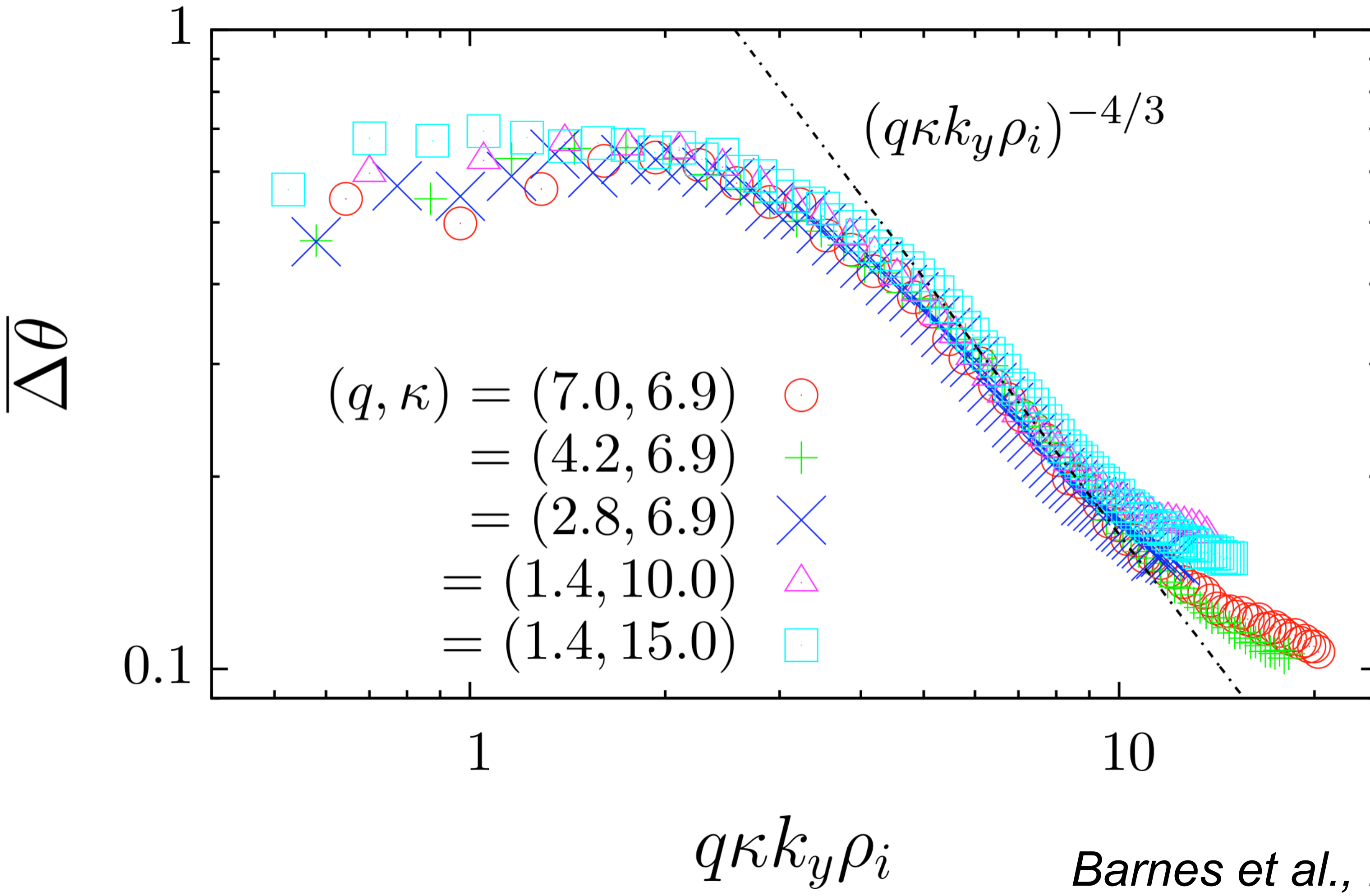
$$k_\parallel qR \sim (k_\perp \rho_i)^{4/3}$$

Barnes et al., PRL 2011

Inertial range critical balance

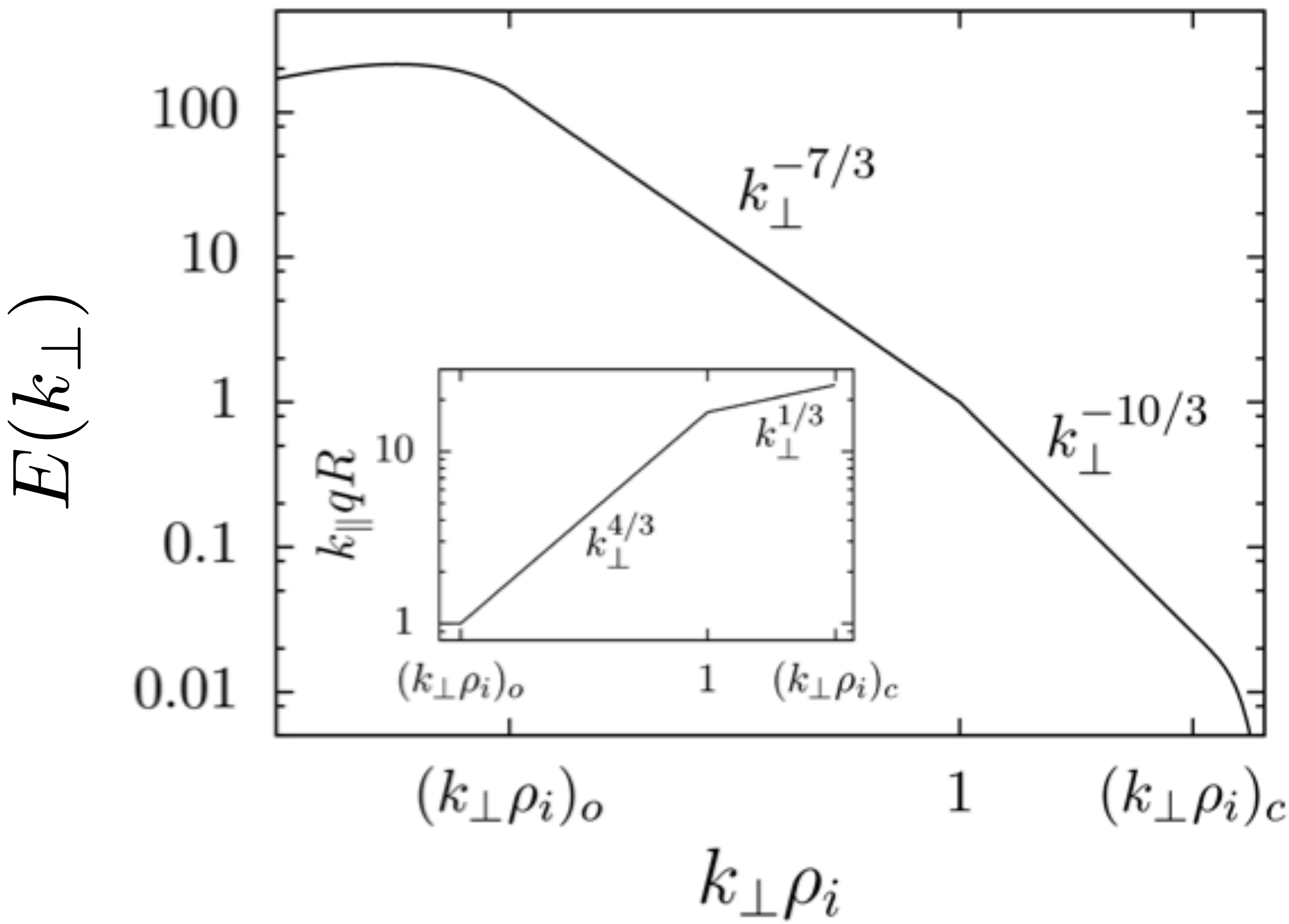
$$k_\parallel qR \sim \left(k_\perp \rho_i \frac{qR}{L_T}\right)^{4/3}$$

$$\overline{\Delta\theta}(k_y) = \int d(\Delta\theta)\mathcal{C}(k_y, \Delta\theta)$$



Barnes et al., PRL 2011

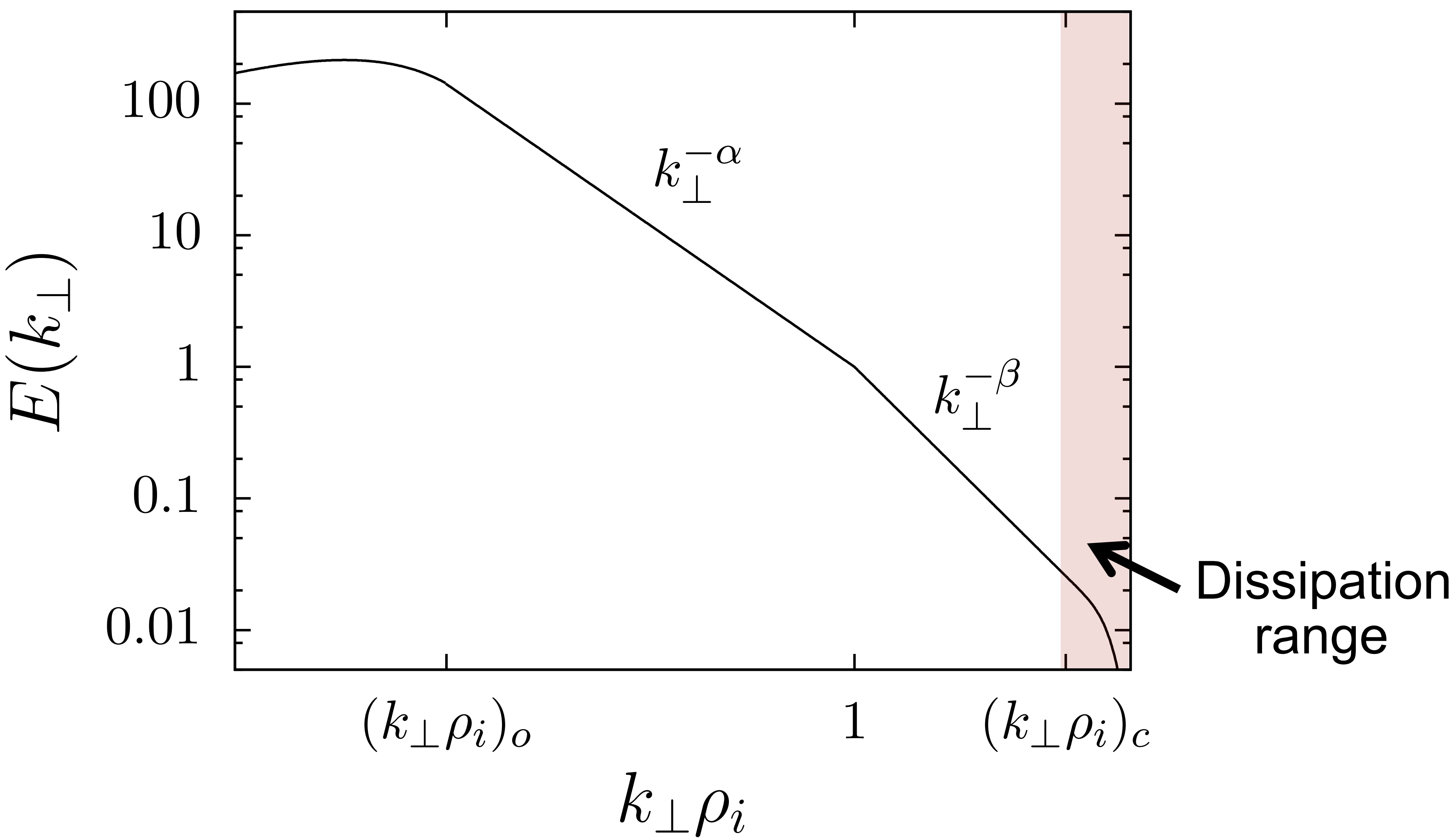
Turbulence properties



Barnes et al., PRL 2011

Schekochihin et al., ApJ 2009

Dissipation scale



Dissipation scale

- At dissipation scale, dissipation rate comparable to nonlinear decorrelation rate

$$C \sim \nu v_{th}^2 \frac{\partial^2}{\partial v^2} \sim \nu \left(\frac{v_{th}}{\delta v}\right)^2 \sim \frac{1}{\tau_{\ell_c}}$$

- Decorrelation in v-space due to finite Larmor radius

$$\frac{\delta v_c}{v_{th}} \sim \frac{\ell_c}{\rho_i}$$

$$\Rightarrow \quad \frac{\ell_o}{\ell_c} \sim \left(q^2 \left(\frac{R}{L_T}\right)^3 \frac{v_{th}}{\nu R}\right)^{3/5} \doteq \mathrm{Do}^{3/5}$$