Astrophysics of Accretion Disks

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PiTP, July 2016

M66 / NASA, ESA, Hubble Heritage team, and S.Van Dyk+


Urbana, Illinois
United States
MONTANA
NORTH DAKOTA
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Disk Astrophysics

- Part 1: Child’s Garden of Astrophysical Disks
- Part 2: Disk Evolution
- Part 3: Turbulence in Disks
- Part 4: Current Problems in Disk Theory

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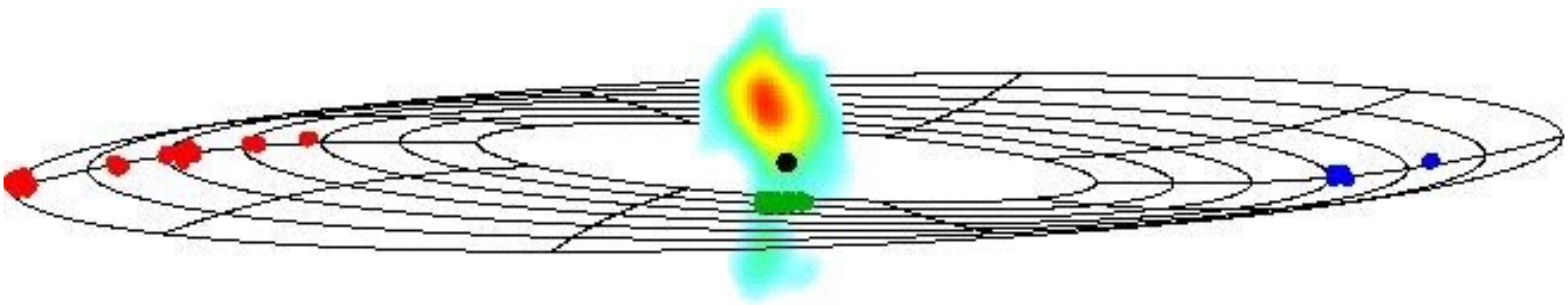
Disks: the incomplete list

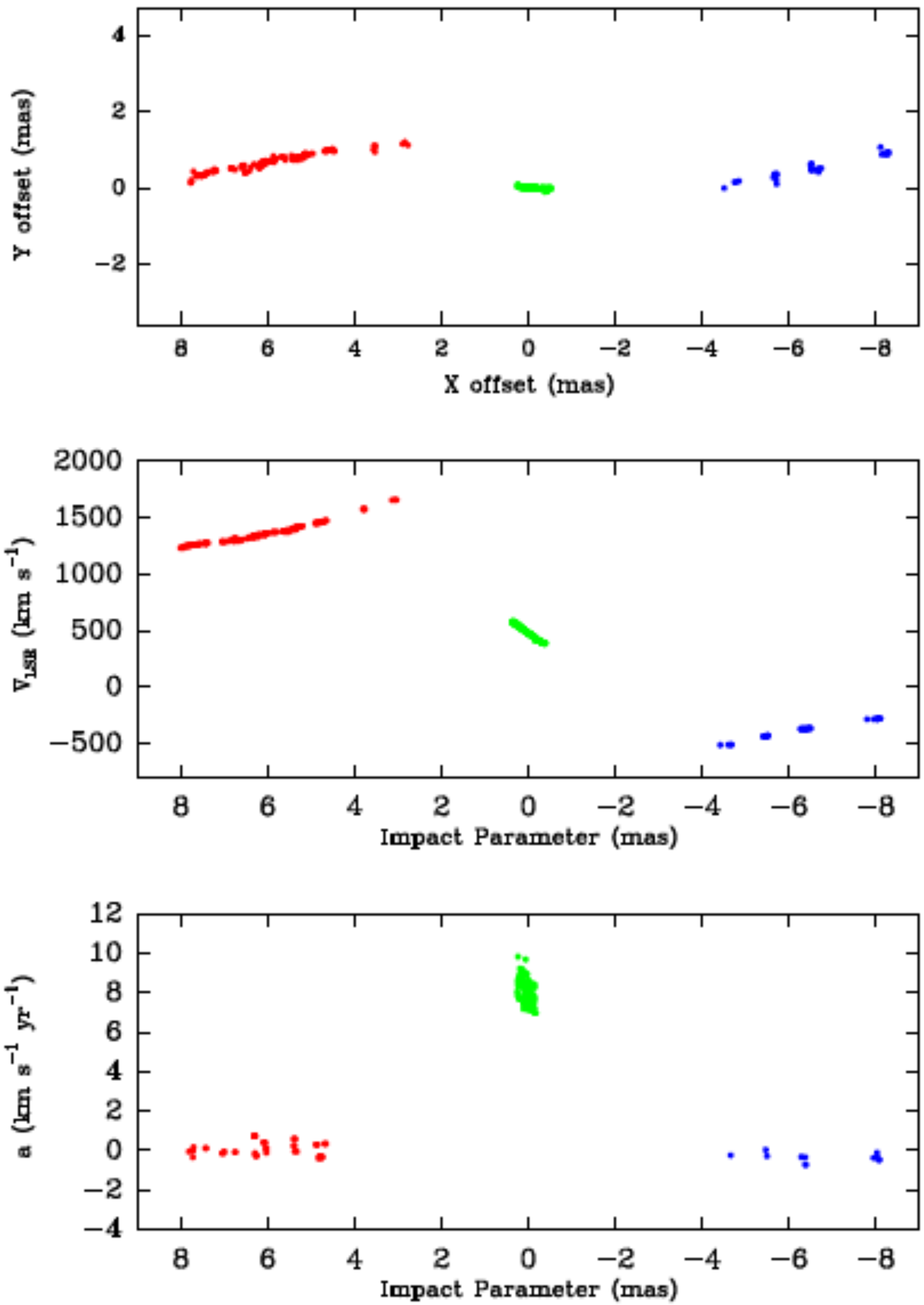
galactic disk	spiral	
	elliptical	NGC 4278
supermassive BH	Quasar	3C 273
	Seyfert	MCG -6-30-15
	LINER	NGC 4258
	LLAGN	Sgr A*
	TDE	Swift J1644+57
stellar mass BH	microquasar	GRS 1915+105
	gamma-ray burst	long bursts?
neutron star	LMXB	Aql X-1
	HMXB	Cyg X-1
	gamma-ray burst	short bursts?
white dwarf	dwarf nova	SS Cyg
	nova	RS Oph
protostar	protoplanetary	HL Tau
	debris	Fomalhaut
planet	protolunar disk	Earth/Moon
	planetary rings	Saturn

NGC 4258 (M106), NASA, ESA, Hubble Heritage, R. Gendler

NGC 4258 maser spots



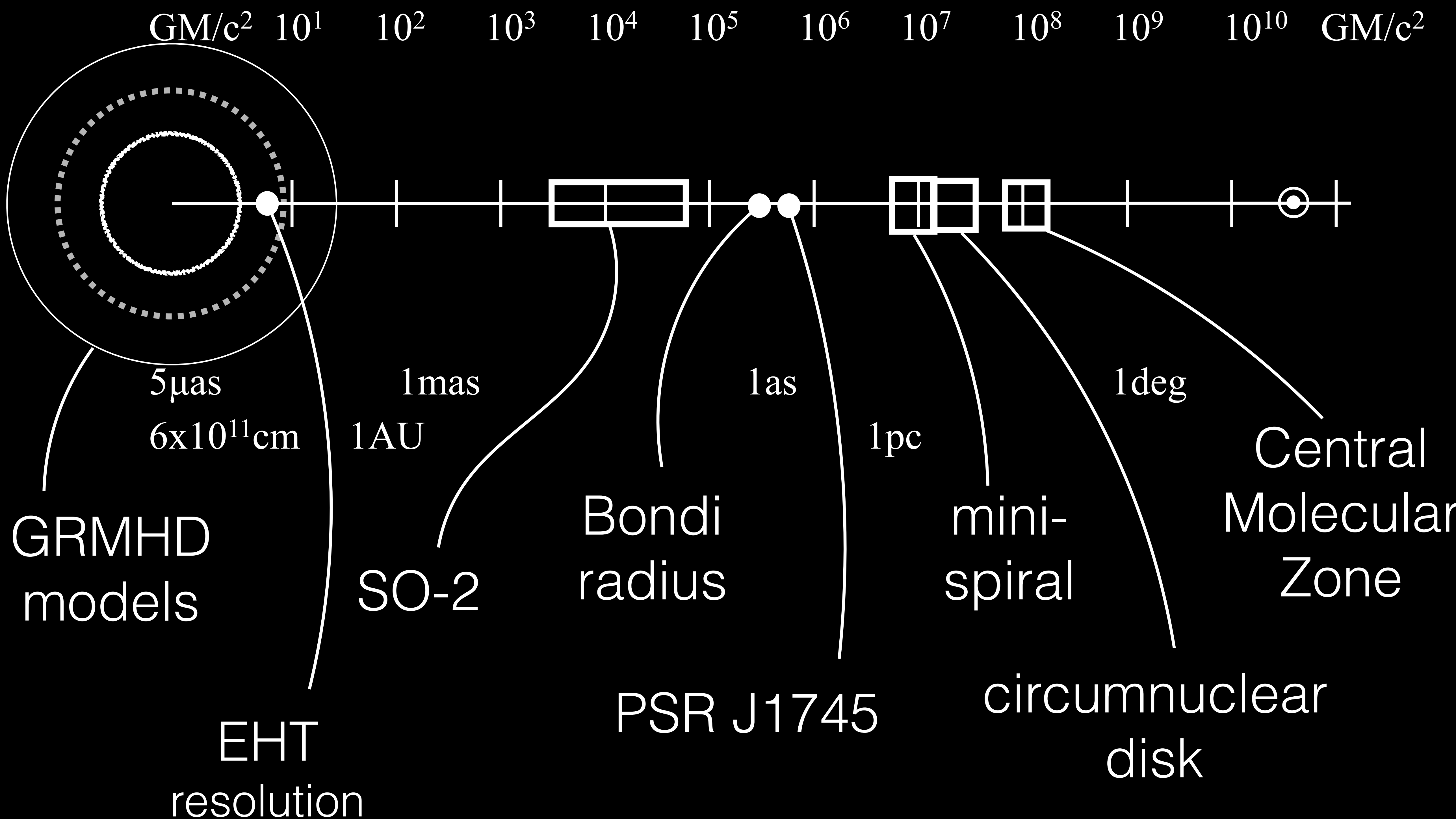
$0.1 < r < 0.3 pc$



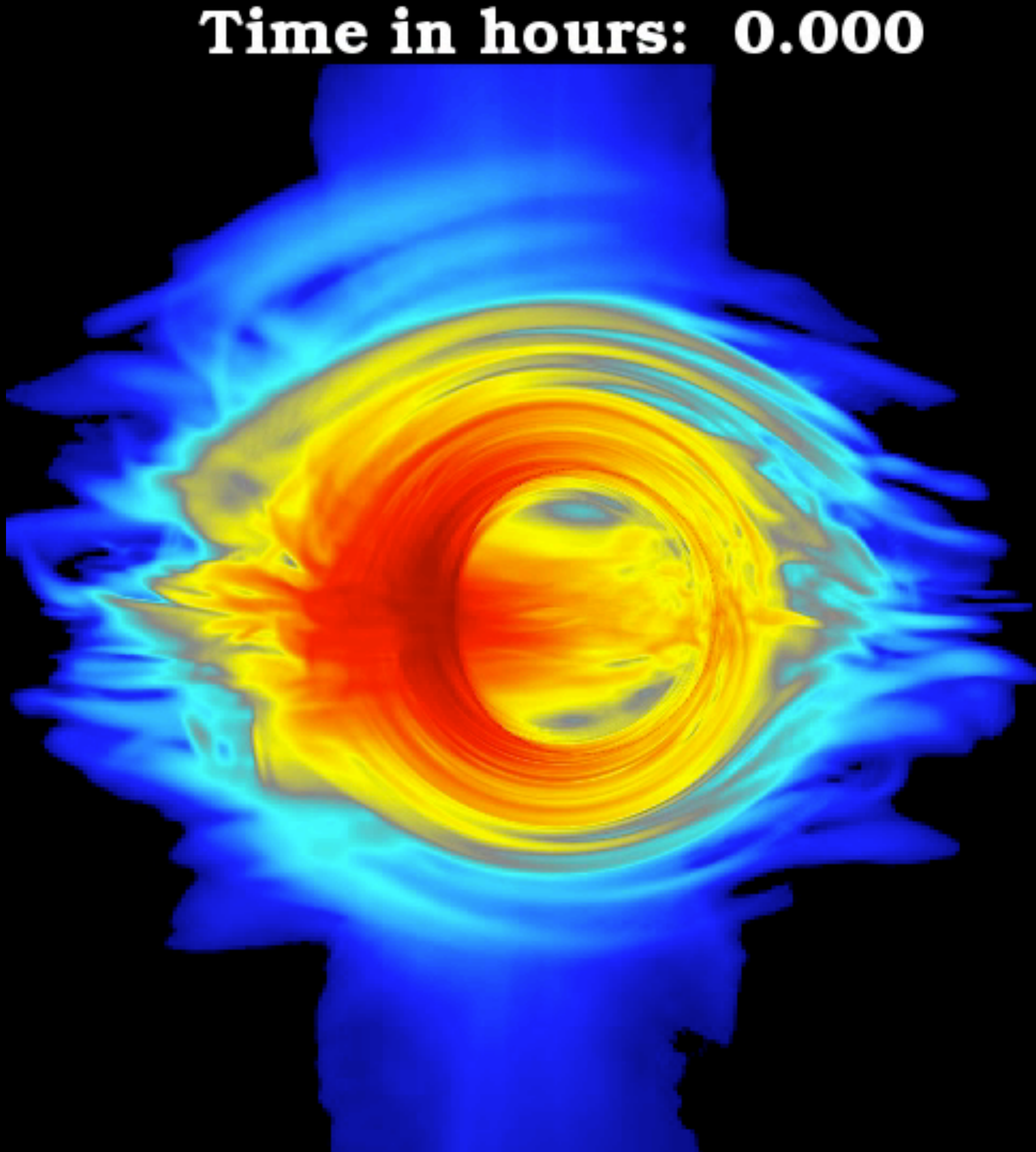
Humphreys+ 2013


NASA / JPL-Caltech / S. Stolovy

Galactic Center Overview

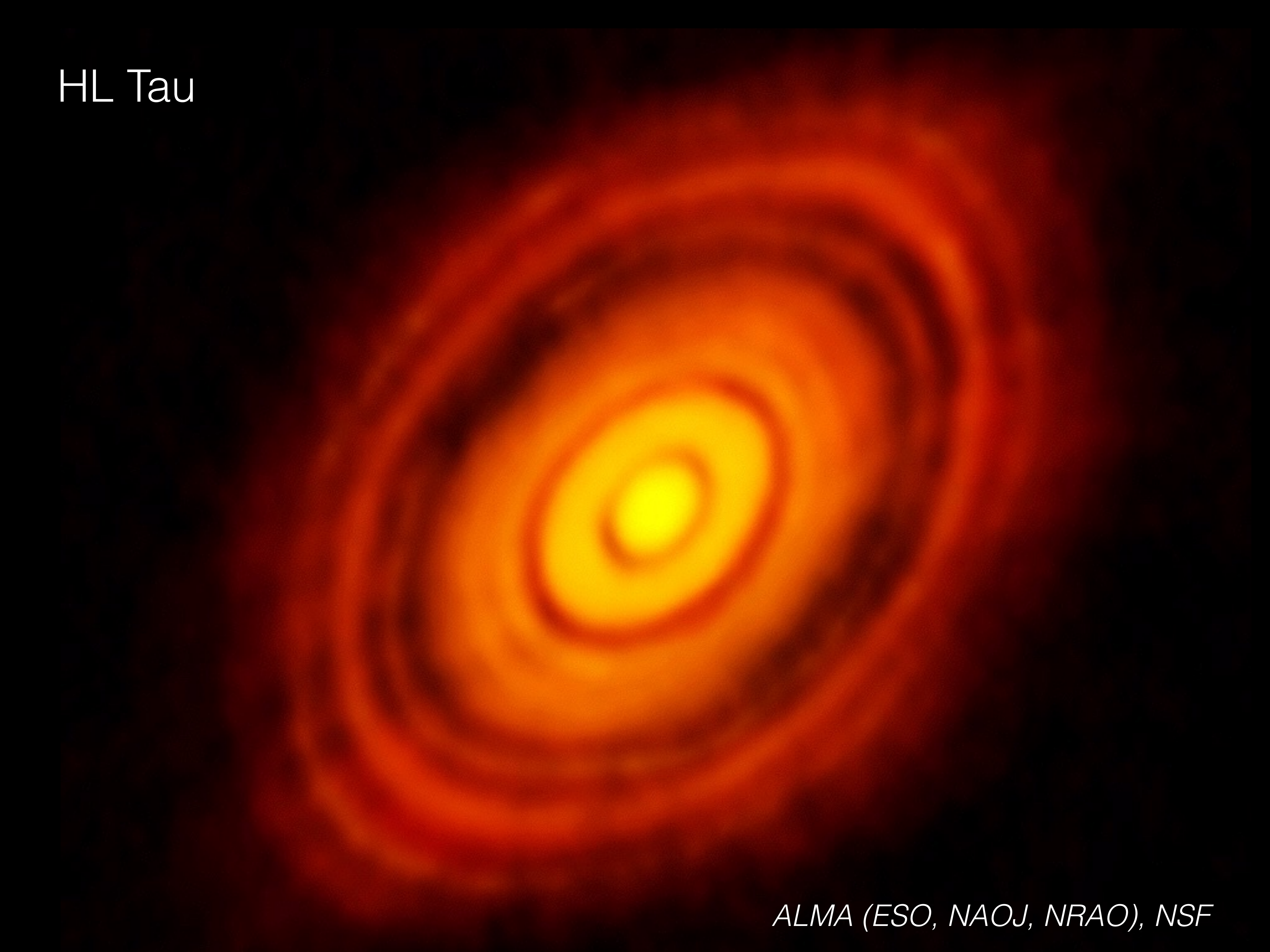


Sgr A*



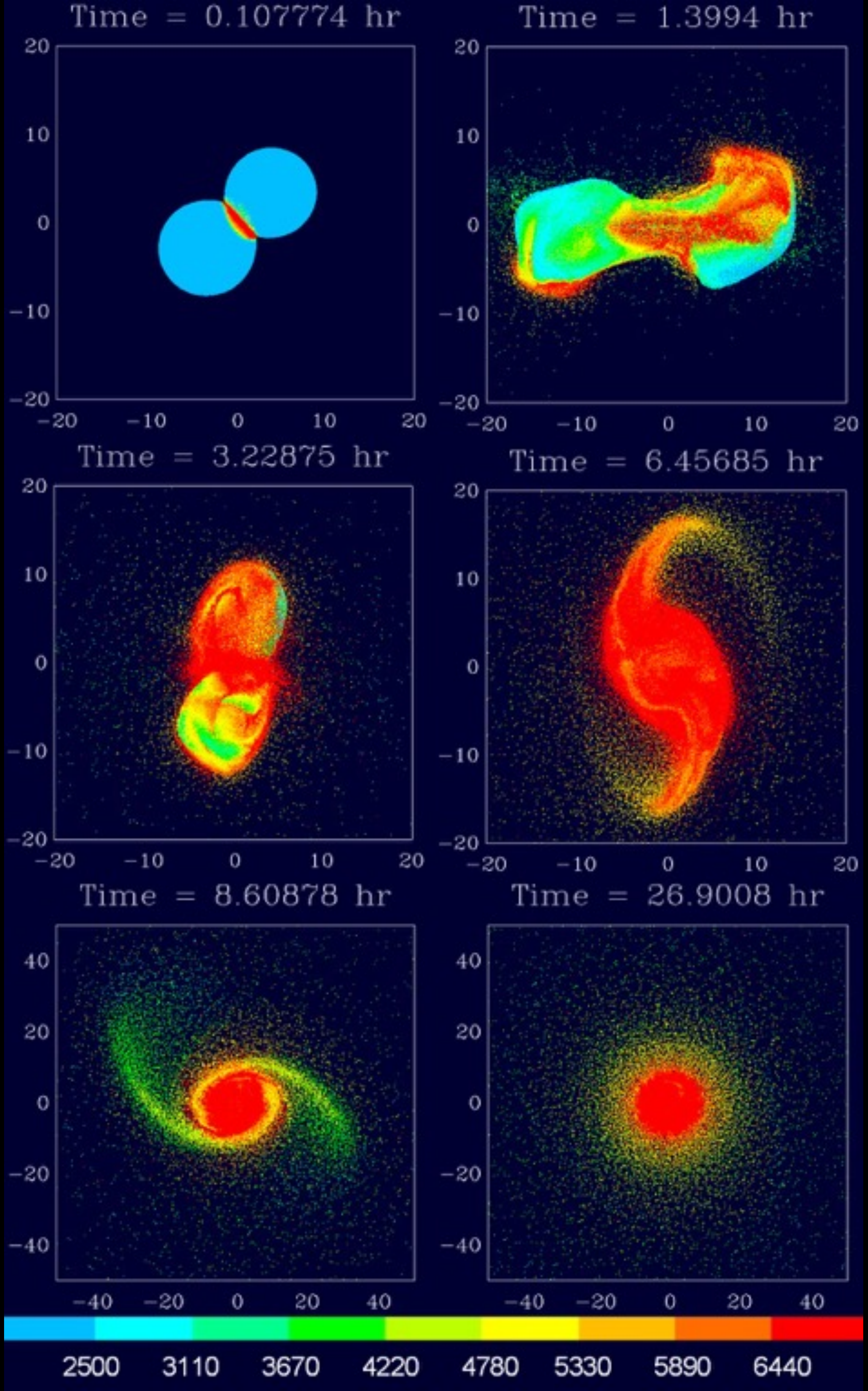
J. Dolence

HL Tau

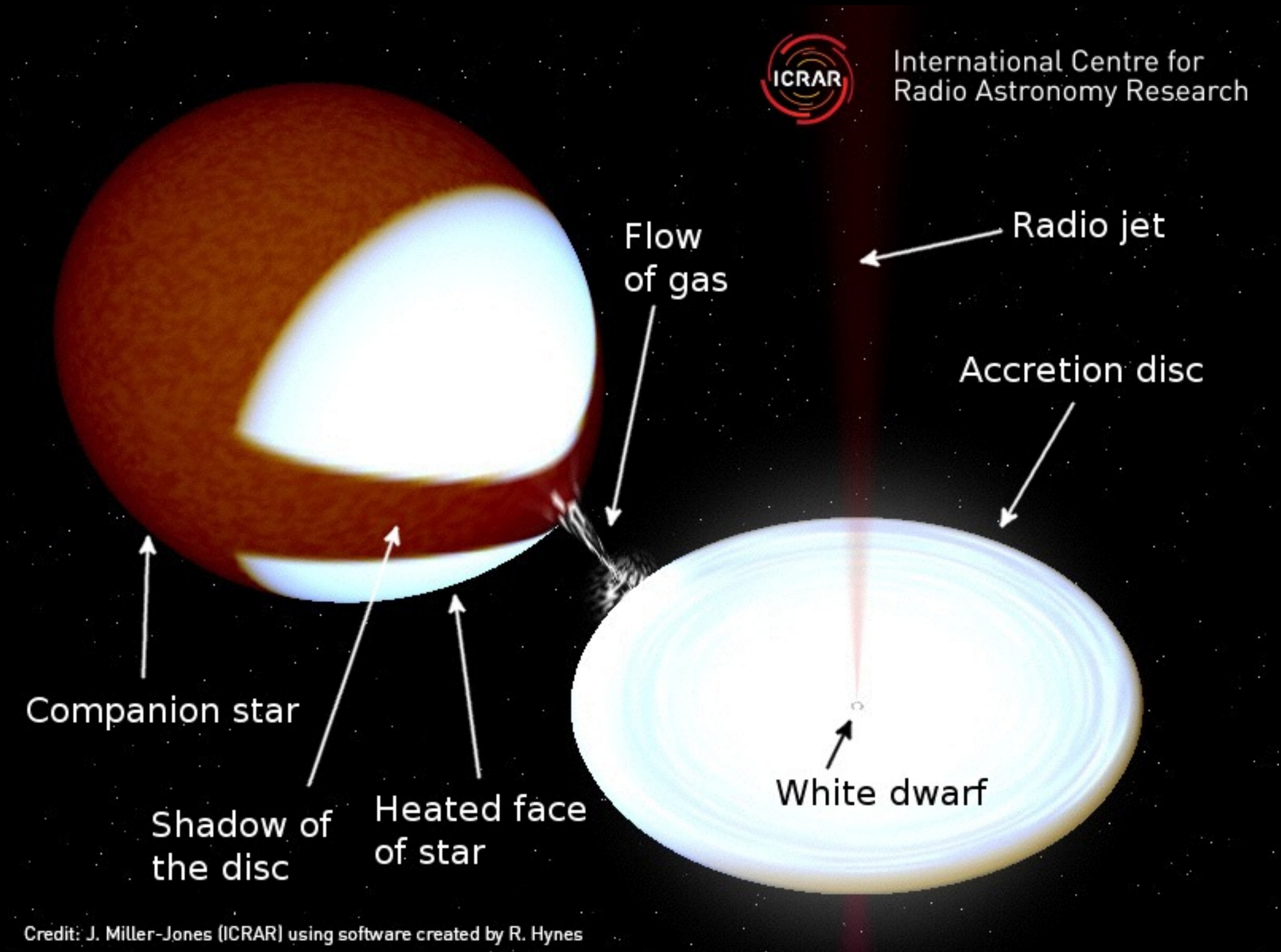


ALMA (ESO, NAOJ, NRAO), NSF

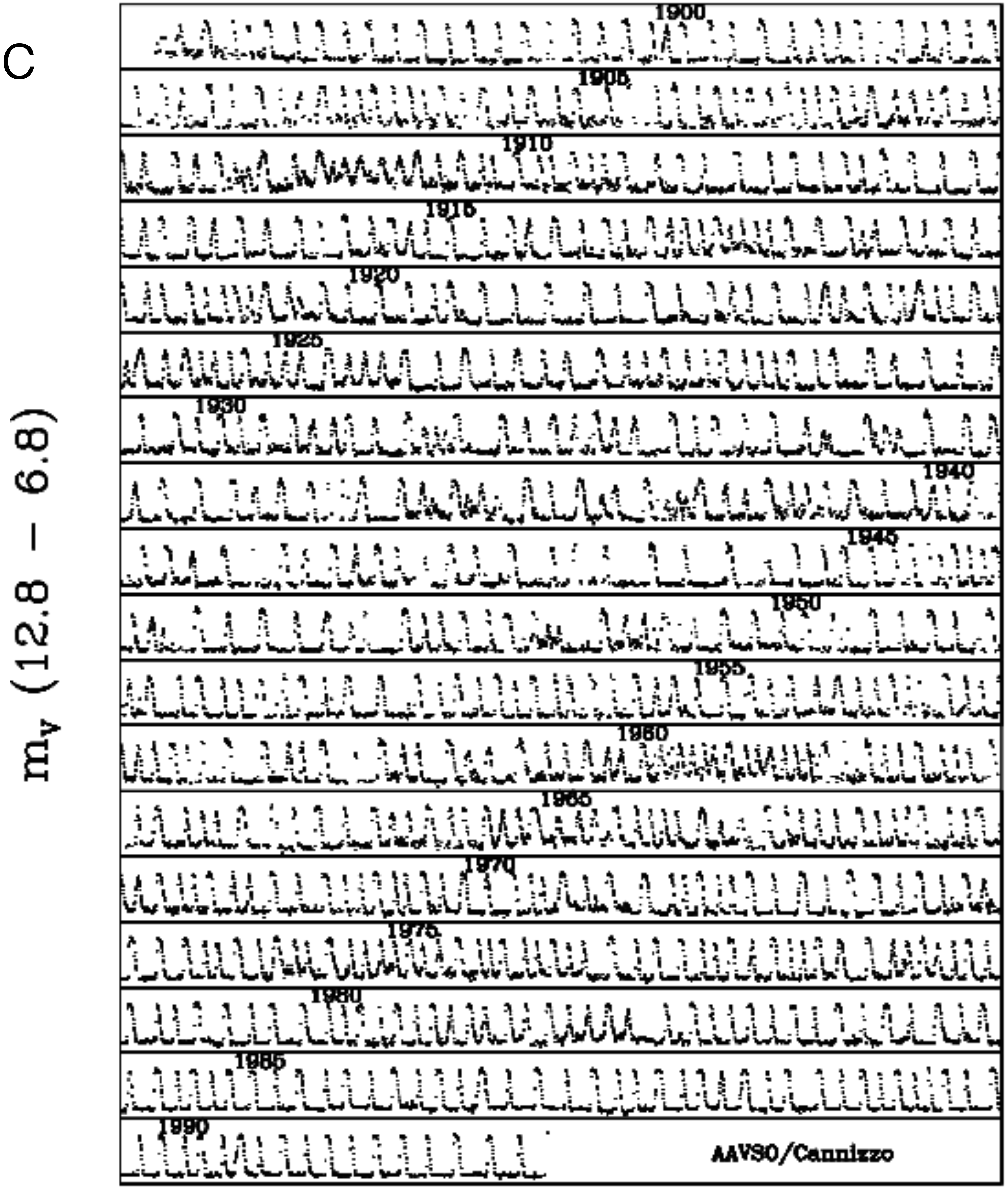
Moon Formation



Cúk & Stewart 2012


International Centre for
Radio Astronomy Research
ICRAR
Flow
of gas
Radio jet
Accretion disc
Companion star
Shadow of
the disc
Heated face
of star
White dwarf
Credit: J. Miller-Jones (ICRAR) using software created by R. Hynes

Cataclysmic
Variable



SS Cyg (27 Sep 1896–7 Apr 1992)

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All About L

L E_{kin} E_{th}

All About L

L E_{kin} E_{th}

All About L

L E_{kin} E_{th}

All About L

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All About L

L E_{kin} E_{th}

thin disk!

Disk Equilibrium

- dynamical equilibrium:
  $\Omega = (GM/R^3)^{1/2} + O(H/R)^2$ $\qquad$ $v_z = 0$
  $\Delta t \sim \Omega^{-1}$

- thermal equilibrium: $Q^+ \simeq Q^-$
  $\Delta t \sim \Sigma\, c_s^2/Q^+ \sim (\alpha\Omega)^{-1}$

- inflow equilibrium: $\dot{M} \simeq \text{const.}$
  $\Delta t \sim M_{disk}/\dot{M} \sim (\alpha\Omega)^{-1}\,(R/H)^2$

Disk Evolution

Disk evolution equation:

$$\partial_t \Sigma = \frac{2}{r}\partial_r\left(\frac{\Omega}{r\kappa^2}\partial_r(r^2 W_{r\phi}) - \frac{\Omega}{\kappa^2}\tau\right) + \dot{\Sigma}_{ext}$$

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$$\Sigma \equiv \int \rho \, dz \; = \text{surface density}$$

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$\tau \propto$ external torque/area

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full derivation: exercise for student.

hint: d(specific angular mom.)/dr = $r\kappa^2/(2\Omega)$

hint: vertical, azimuthal integration of mass and angular momentum conservation equation.

α Disks

Shakura & Sunyaev 1973
Lynden-Bell & Pringle 1974

adopt simple scaling argument for diffusion of angular moment by turbulence.

model: turbulent viscosity $\boldsymbol{\nu} \simeq \alpha\, c_s\, H$

ignore external torques, infall/winds, variation in α.

Disk Evolution

Disk evolution equation becomes:

$$\partial_t \Sigma = \frac{2}{r}\partial_r\left(\frac{\Omega}{r\kappa^2}\partial_r(r^2 W_{r\phi})\right) = 0$$

α Disks

thin, Keplerian disk: $\Omega \cong (G\,M/r^3)^{1/2}$

vertical equilibrium: $H \simeq c_s/\Omega$

opacity: $\kappa \simeq \kappa_0\,\rho^a\,T^b$

turbulent viscosity: $\boldsymbol{\nu} \simeq \alpha\,c_s\,H$

vertical integration: $\Sigma \simeq 2\,\rho\,H$

optical depth: $\tau \simeq \Sigma\,\kappa\,/\,2$

surface temperature: $F = \sigma\,T_{eff}^{4} \simeq (9/8)\,\Sigma\,\boldsymbol{\nu}\,\Omega^2$

radiative equilibrium: $T_{eff}^{4} \simeq (8/3)\,\sigma\,T^4/\tau$

steady state: $\dot{M} = 3\,\pi\,\Sigma\,\boldsymbol{\nu}$

α Disks

Example: steady state disk, stellar mass black hole

inner zone: radiation pressure ≫ gas pressure
electron scattering opacity dominates

find:

$T \simeq 4.3 \text{ x } 10^7 \, \alpha^{-1/4} \, m^{-1/4} \, x^{-3/8}$ [K]

$\Sigma \simeq 0.4 \, x^{3/2} \, \alpha^{-1} \, \dot{m}^{-1}$ [g cm^{-2}]

$H/r \simeq 10 \, \dot{m} \, x^{-1}$ etc.

see `alpha_disk.ma` mathematica script for details

Disk Evolution

Disk evolution equation:

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What are $W_{r\varphi}$, τ, Σ_{ext}?

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Turbulence in Disks

α disk model posits turbulent diffusion of AM

what generates turbulence?

possibilities:

- magnetorotational instability
- gravitational instability
- zombie vortex instability
- subcritical baroclinic instability
- vertical shear instability

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Turbulence in Disks

magnetorotational instability (MRI)

Balbus & Hawley (1991)

local, linear instability of weakly magnetized disks driven by exchange of angular momentum.

start with mechanical analogy:
two masses in orbit on a spring

MRI linear theory facts

- Ideal fluid instability requires $d\Omega^2/dr < 0$
- Maximum growth rate (Keplerian): $(3/4)\ \Omega$
- Fastest growing mode: $(k \cdot V_A)^2 = (15/16)\Omega^2$
- Local instability for vertical field
- Local instability for azimuthal field

MRI simulations

- local or global
- stratified or unstratified
- explicit dissipation or ILES
- isothermal or energetically self-consistent

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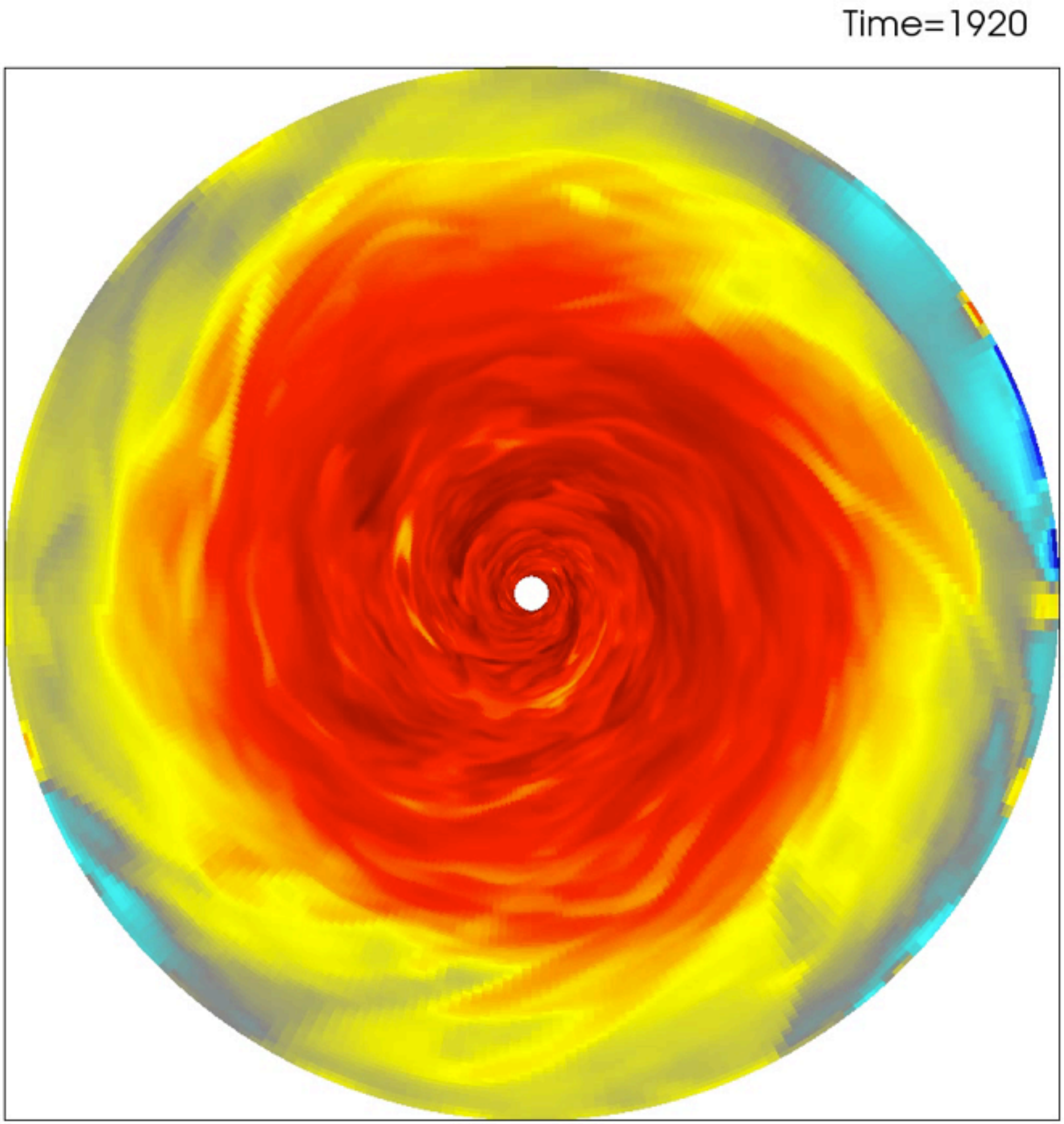

H. Shiokawa

MRI simulations

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- explicit dissipation or ILES
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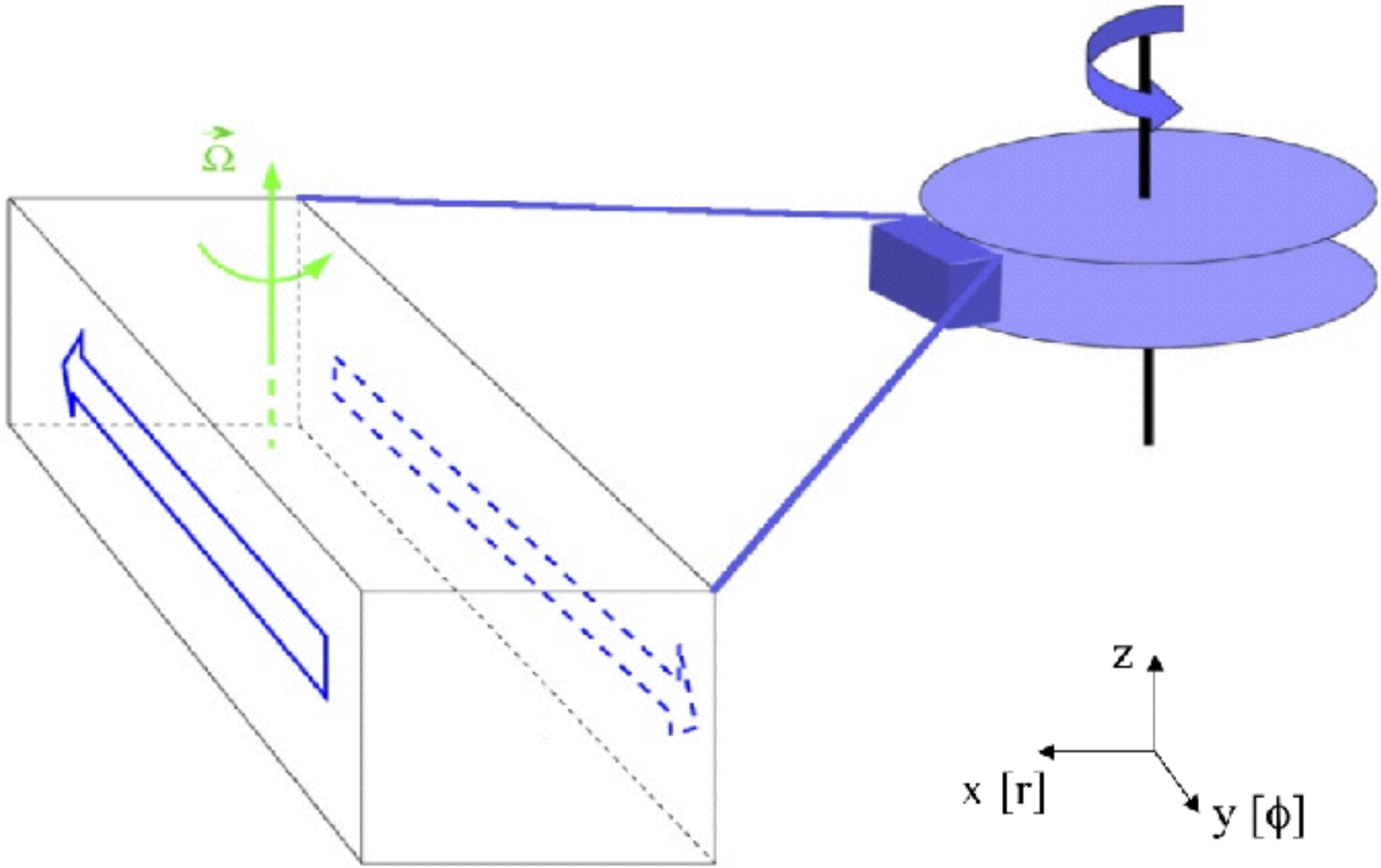
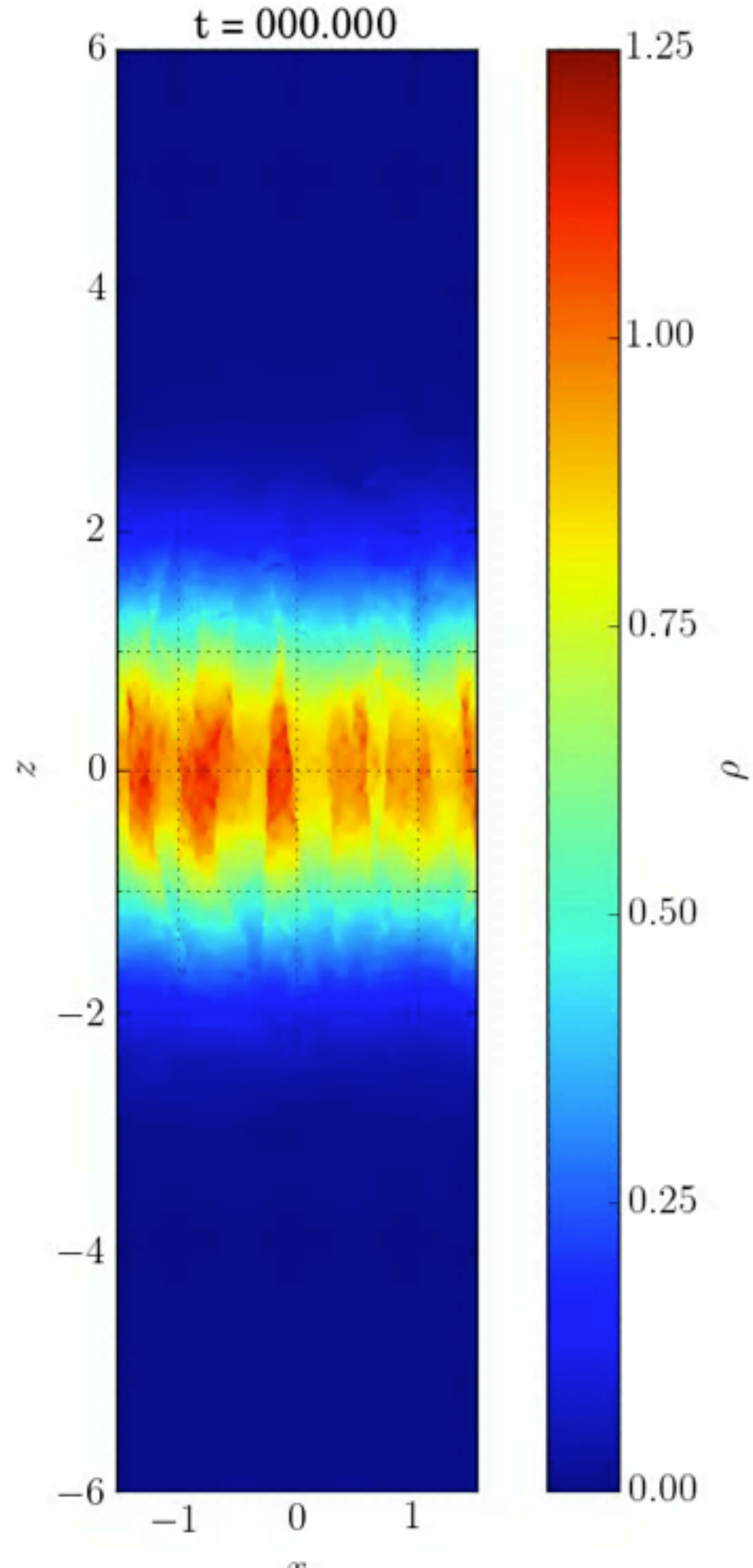


G. Lesur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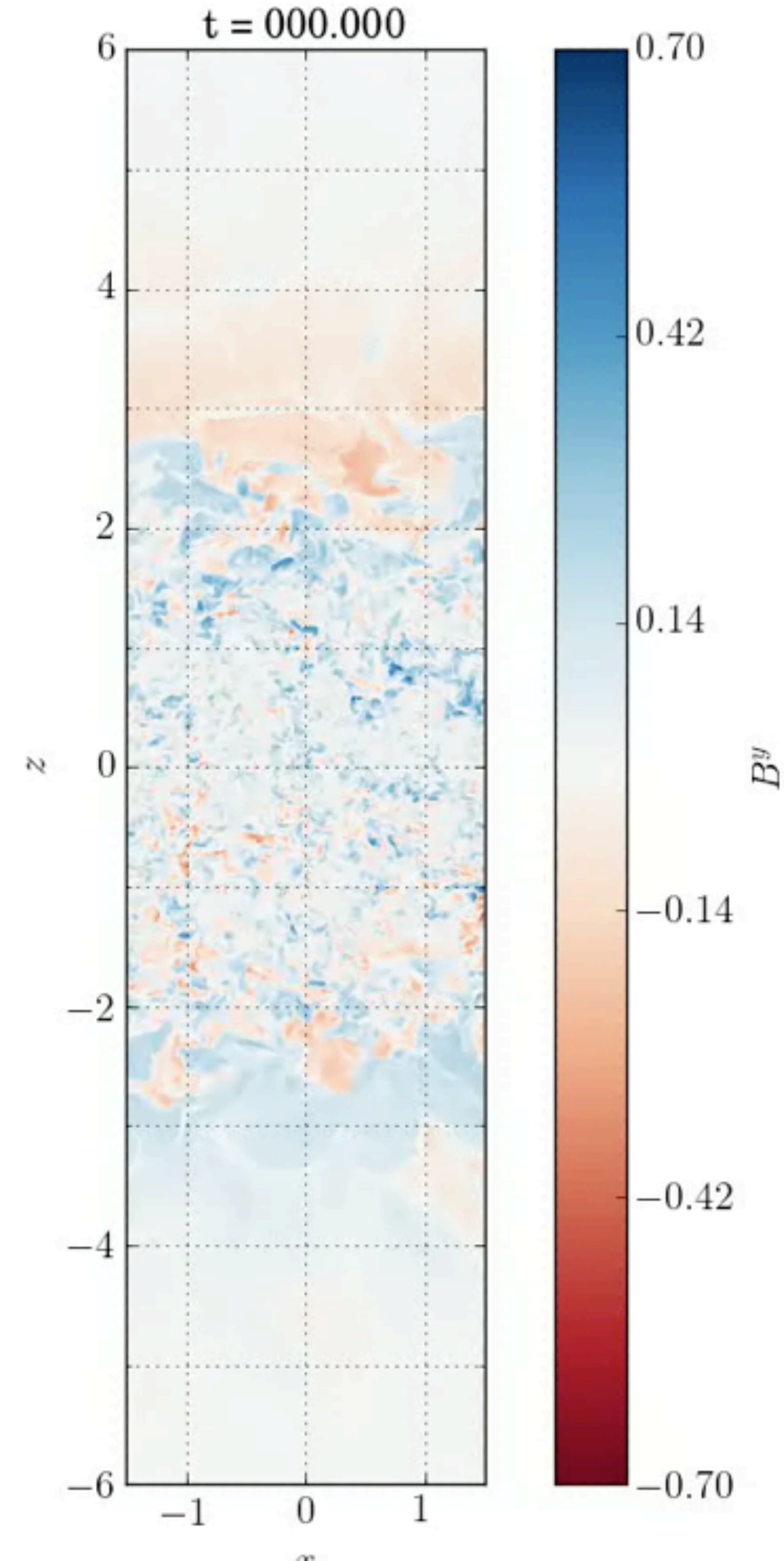
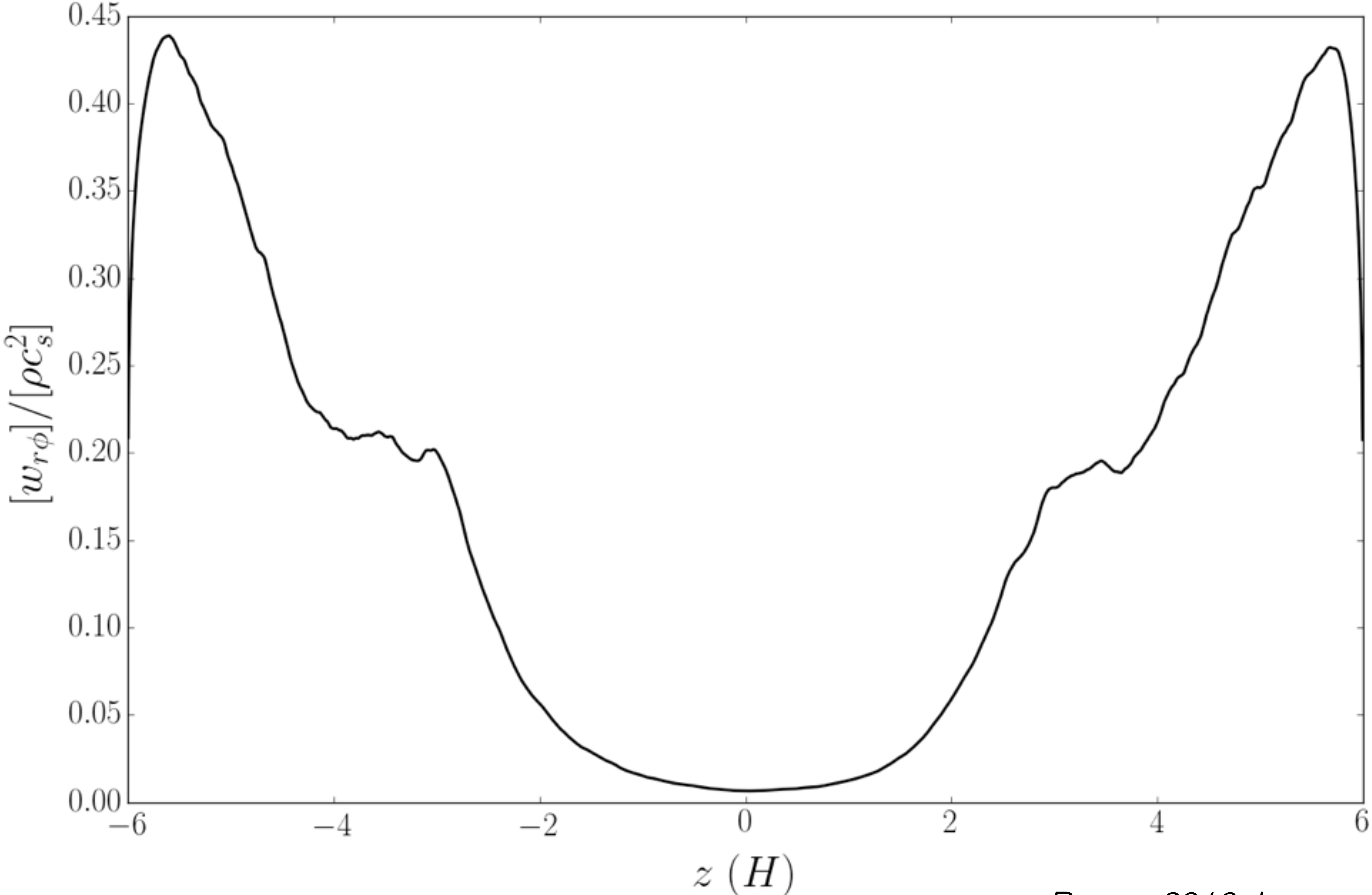
6
4
2
0
−2
−4
−6
z
−1
0
1
1.25
1.00
0.75
0.50
0.25
0.00
ρ


t = 000.000
6
4
2
0
−2
−4
−6
z
−1
0
1
0.70
0.42
0.14
−0.14
−0.42
−0.70
B^y

B. Ryan+

MRI simulation facts

- In 2D MRI leads to turbulence and α
- In 2D MRI does not converge
- In 3D MRI leads to turbulence and α
- Sometimes, 3D MRI simulations converge
- $\alpha = \alpha(z, \langle B_z \rangle, Re_M, Pr_M, \ldots)$



Ryan+ 2016, in prep

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Problems in Disk Theory

- structure of radiation dominated disks
- structure of low collisionality, low $\dot{M}$ flows
- magnetic flux problem
- *ab initio* dwarf nova evolution
- ILES problem

Homework

- Derive the disk evolution equation
- Download, compile, and run iharm2d
  https://github.com/AFD-Illinois/iharm2d_v3
  on a torus problem. See README in the home directory for a typical workflow.
- Run current loop advection problem with a sharp edge and with a smooth distribution. What can you conclude about harm's performance?
- Download, compile, and run ibothros2d
  https://github.com/AFD-Illinois/ibothros2d
  and make 1mm images of your torus run.