Relativistic Magnetohydrodynamics

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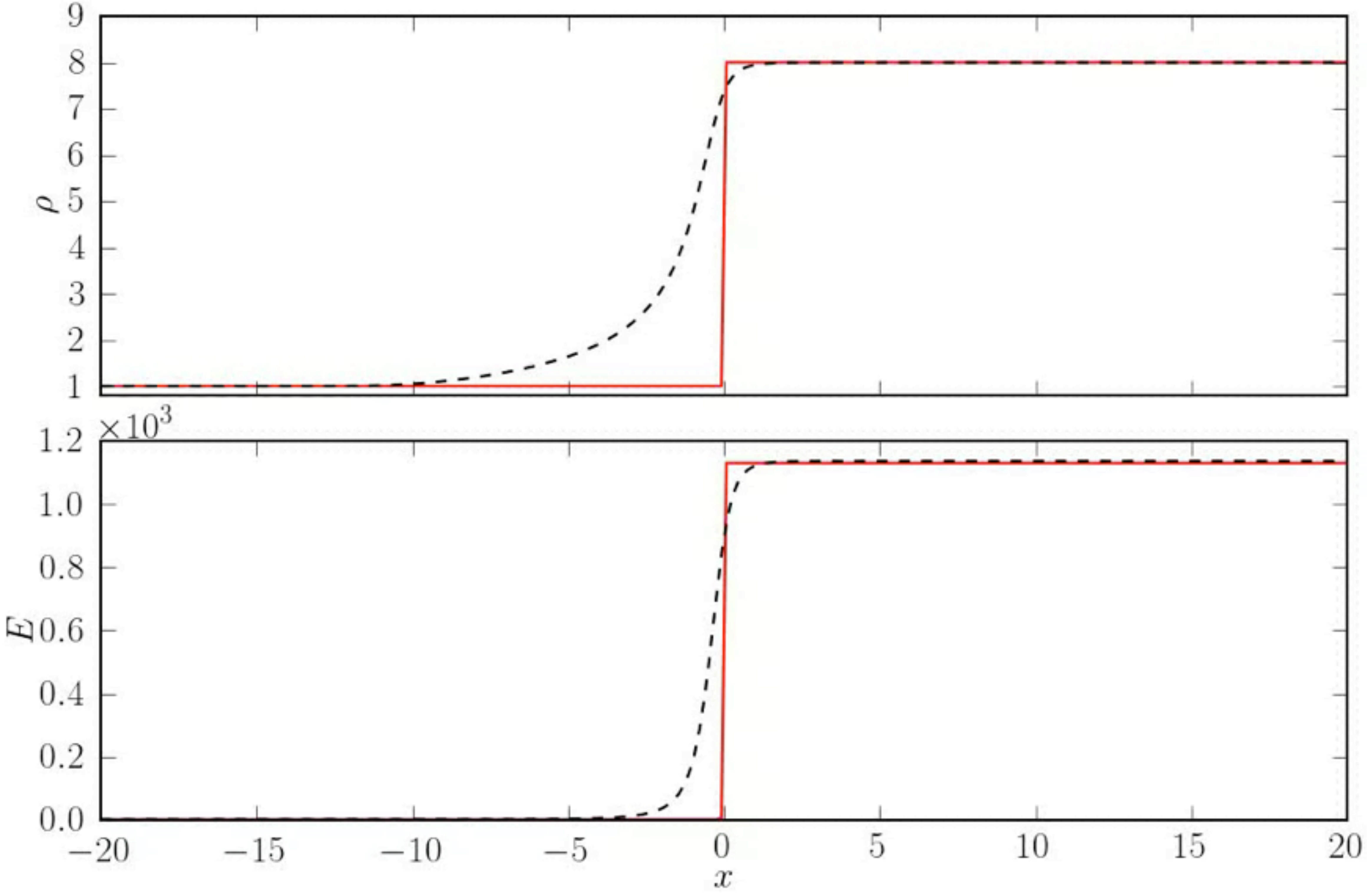
PiTP, July 2016

Farris Shock 3: Γ ~ 10 shock, downstream Pgas/Prad ~ 1



Relativistic MHD

- When is relativistic MHD required?
- Basic equations; conserved currents
- Numerical techniques; dirty secrets
- Beyond ideal MHD

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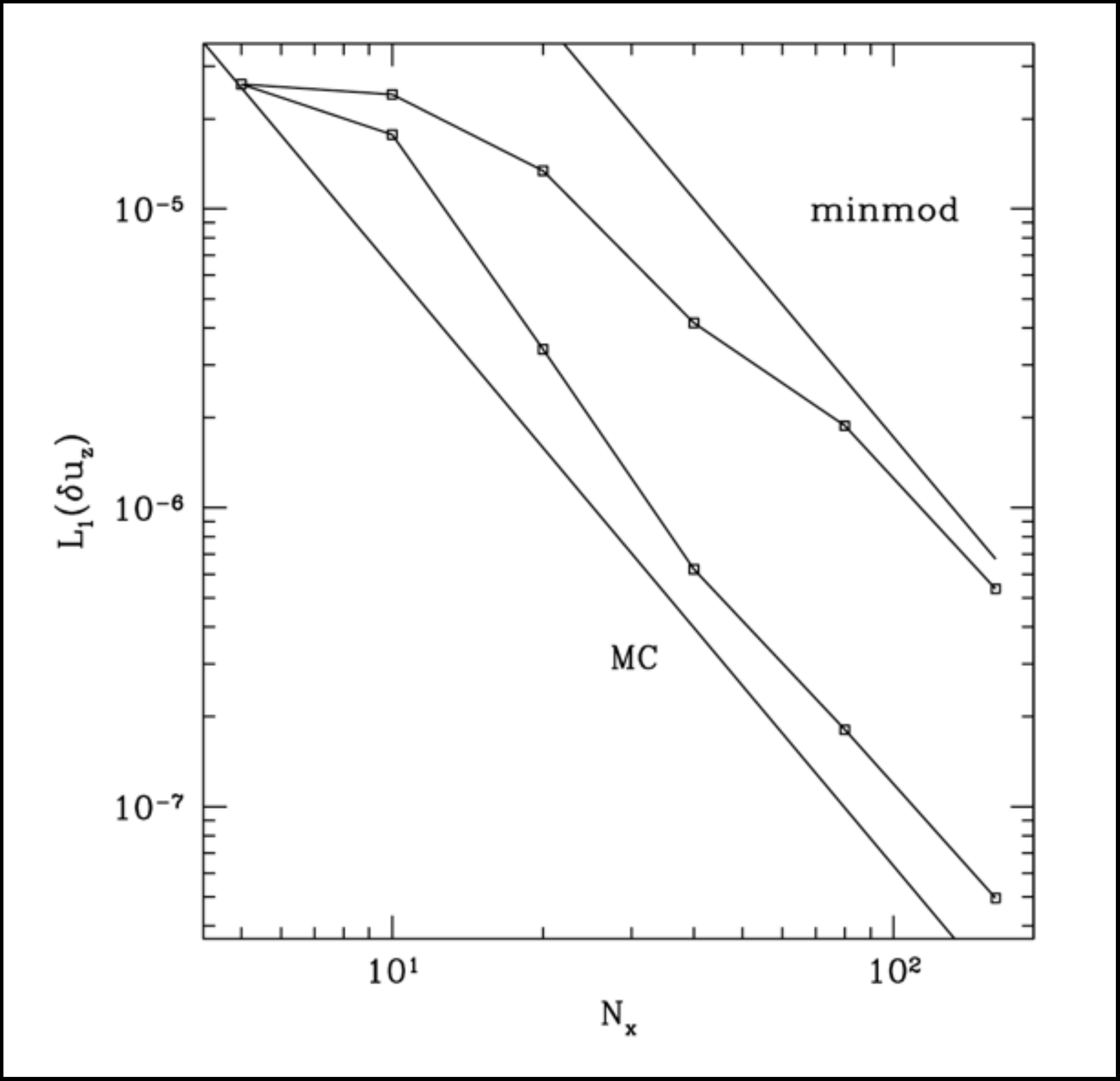
Feynman:

“the first principle is that you must not fool yourself, and you are the easiest person to fool.”

“I'm talking about a specific, extra type of integrity that is not lying, but bending over backwards to show how you're maybe wrong”

In computational astrophysics:

- test your code
- expose failure modes.

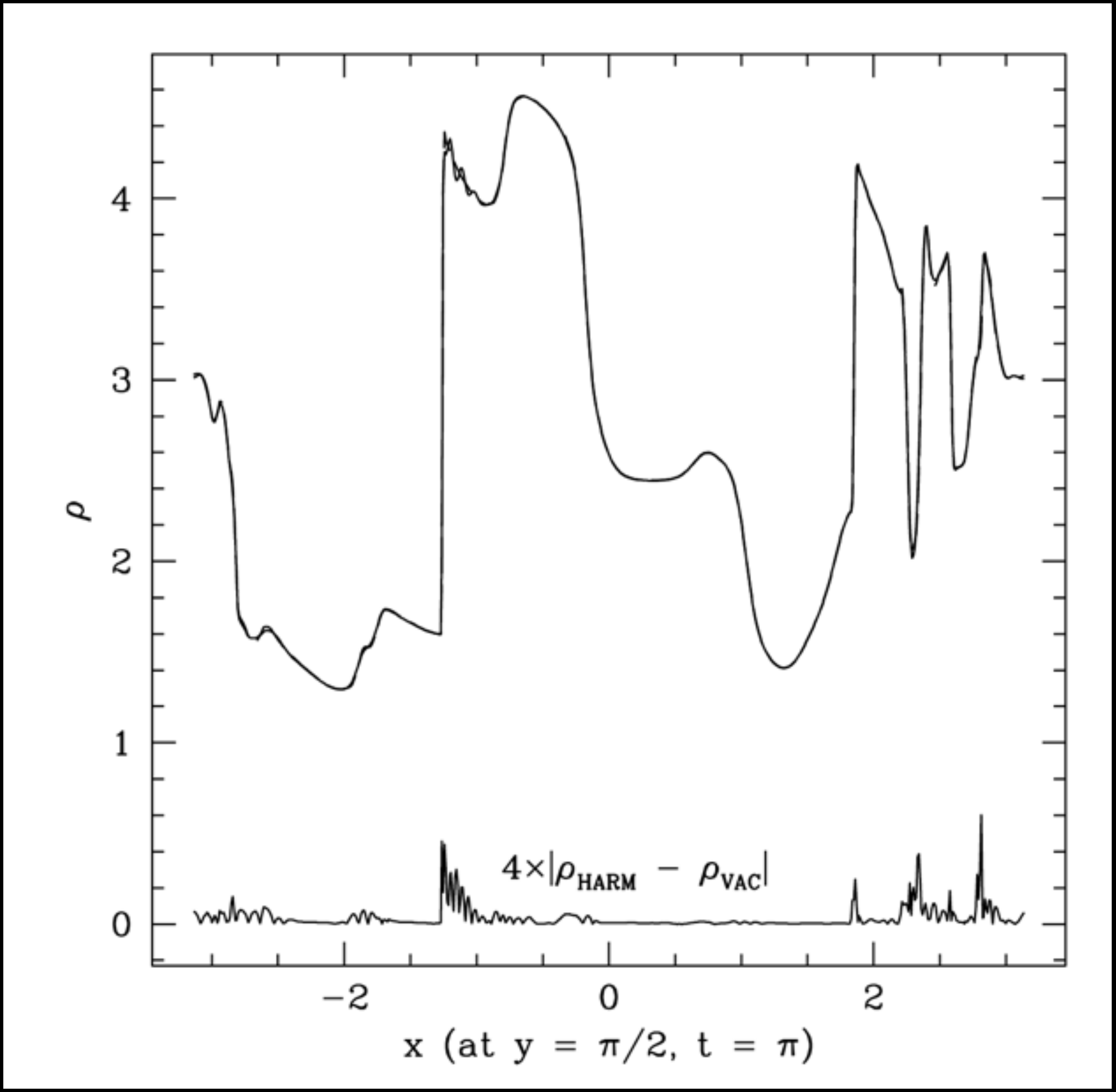


Alfven wave
test problem

Gammie+ 2003

convergence test
vs.
linear theory

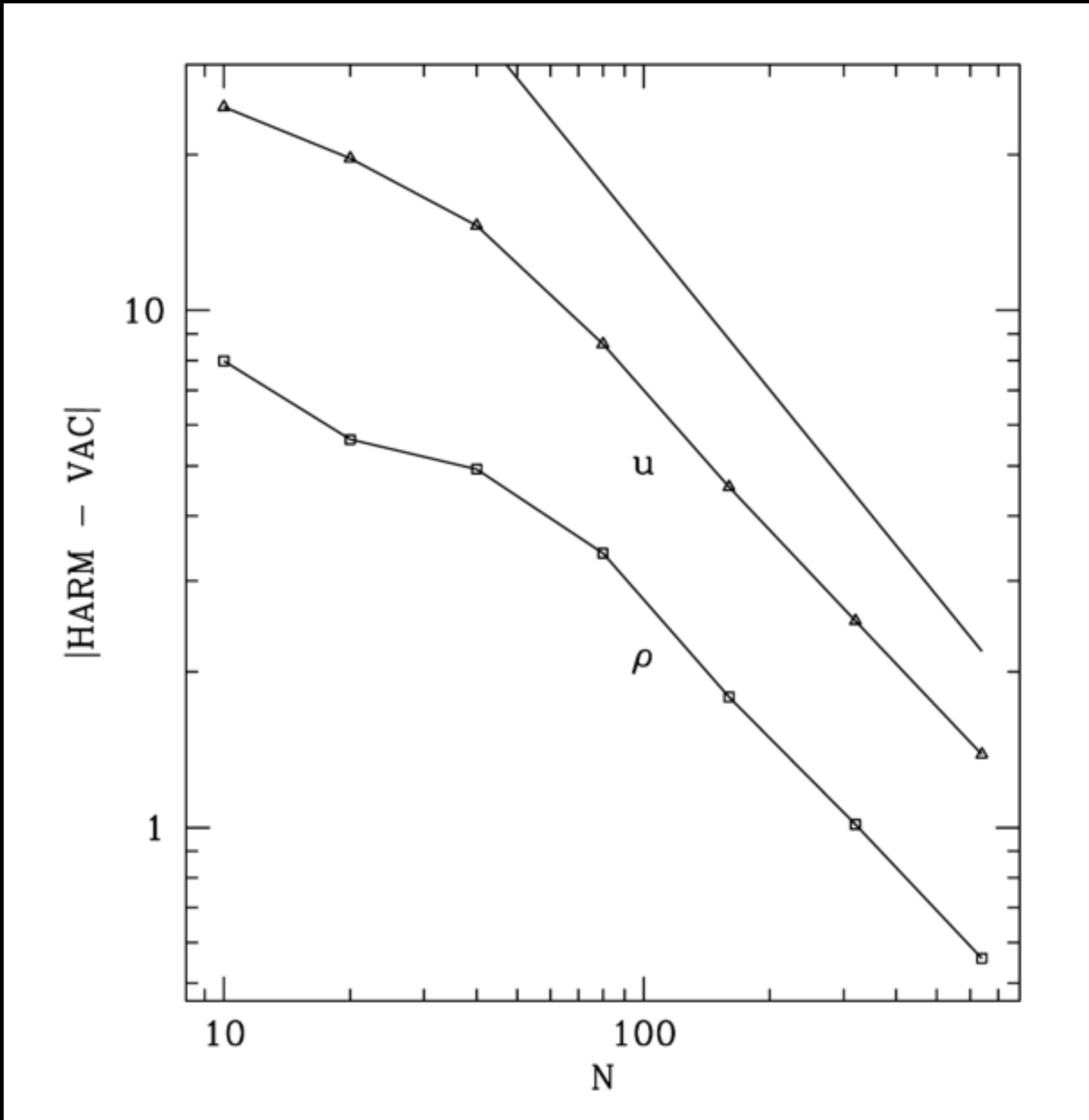
$\mathscr{L}_1(f) \equiv \int |f| \, d^2x$



Orszag-Tang
Vortex

Gammie+ 2003

nonlinear
test
vs.
VAC



Orszag-Tang Vortex

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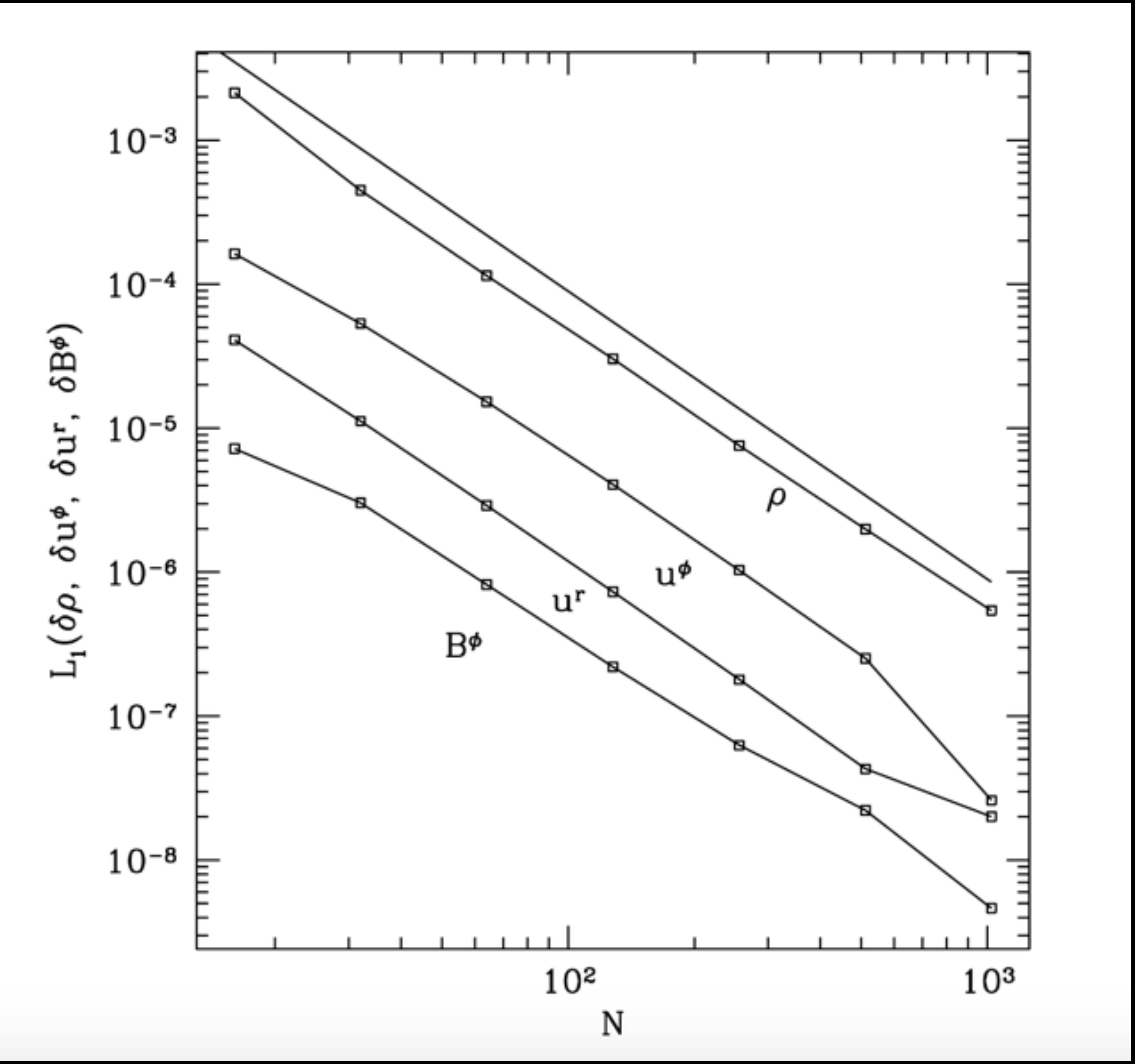
convergence test vs. VAC

$\mathscr{L}_1(f) \equiv \int |f| \, d^2x$



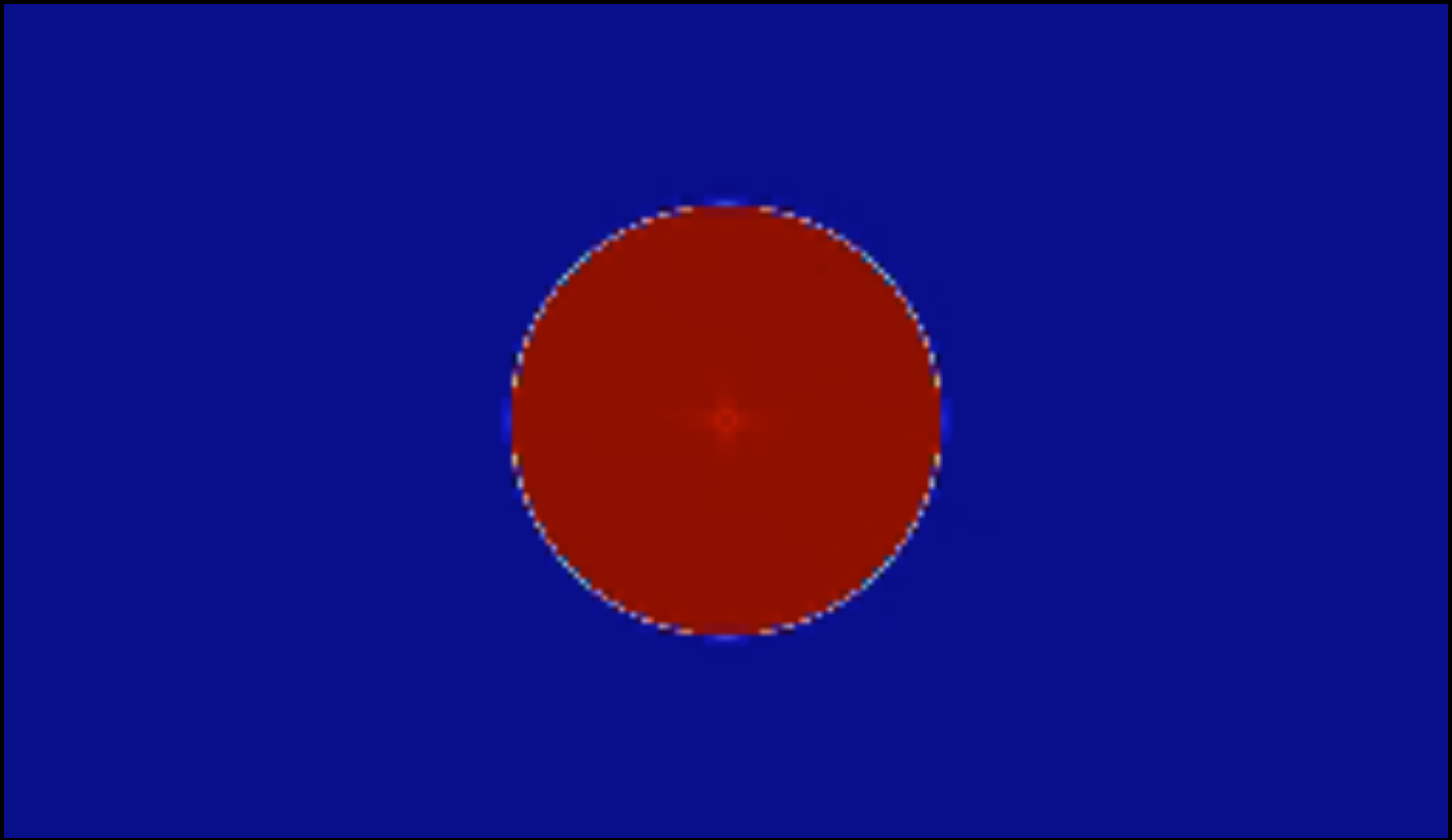
Kerr inflow
(inside-out
Parker wind)

Gammie+ 2003

convergence test
vs.
"exact" solution

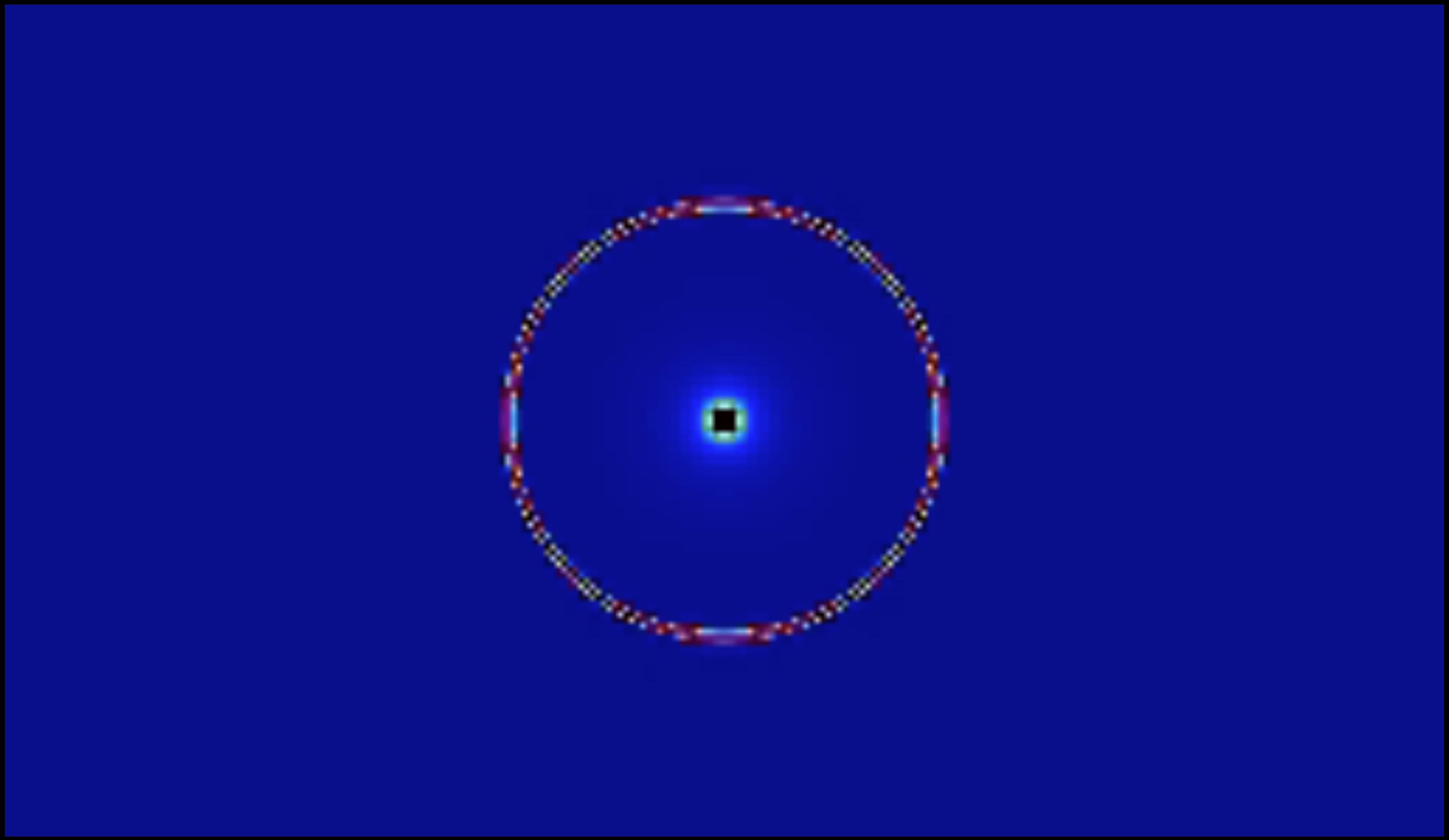
$\mathscr{L}_1(f) \equiv \int |f|\, d^2x$

Field loop advection test



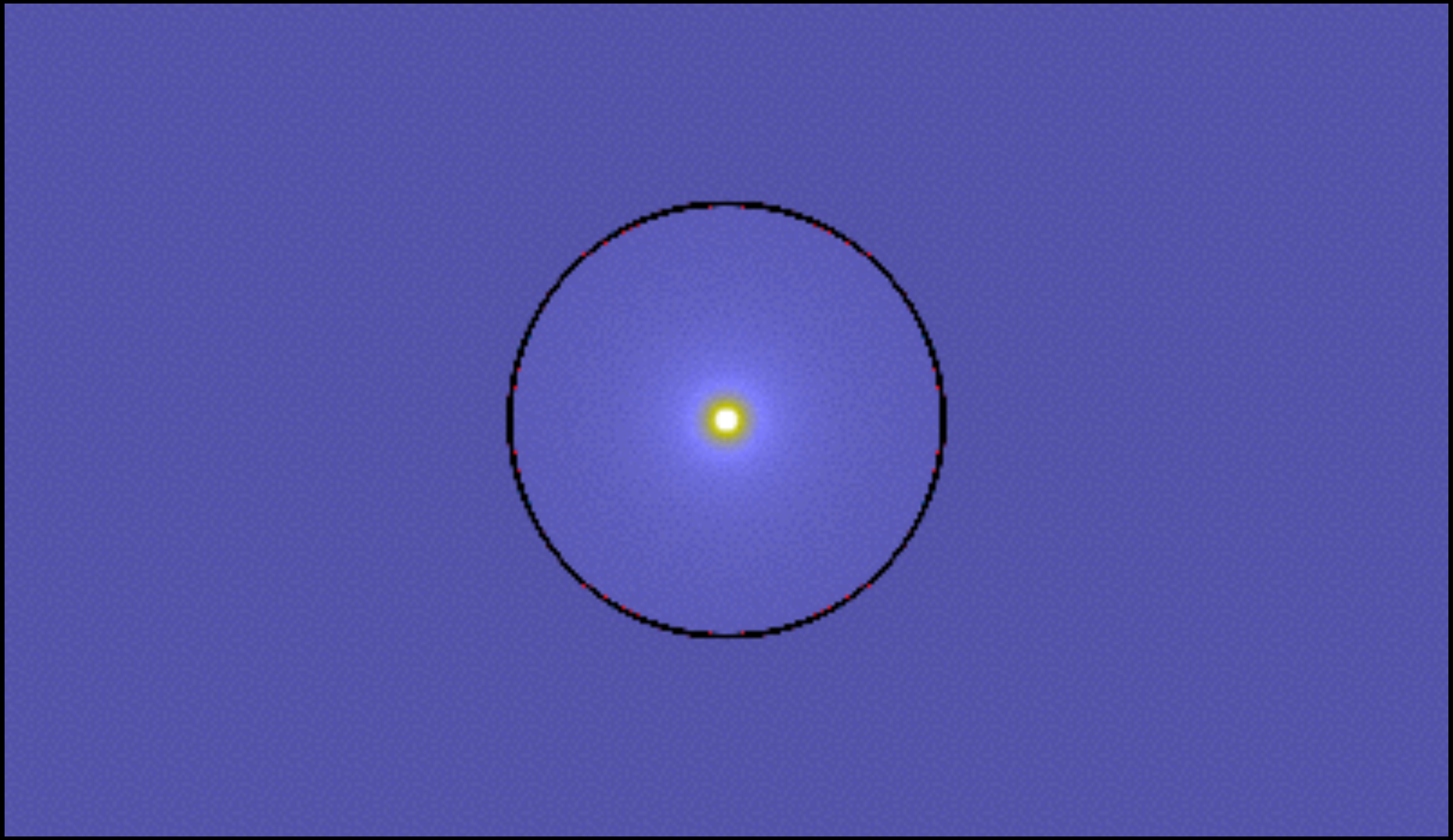
`harm` color shows b^2 $A_z \sim MAX(r_0 - r, 0)$

Field loop advection test



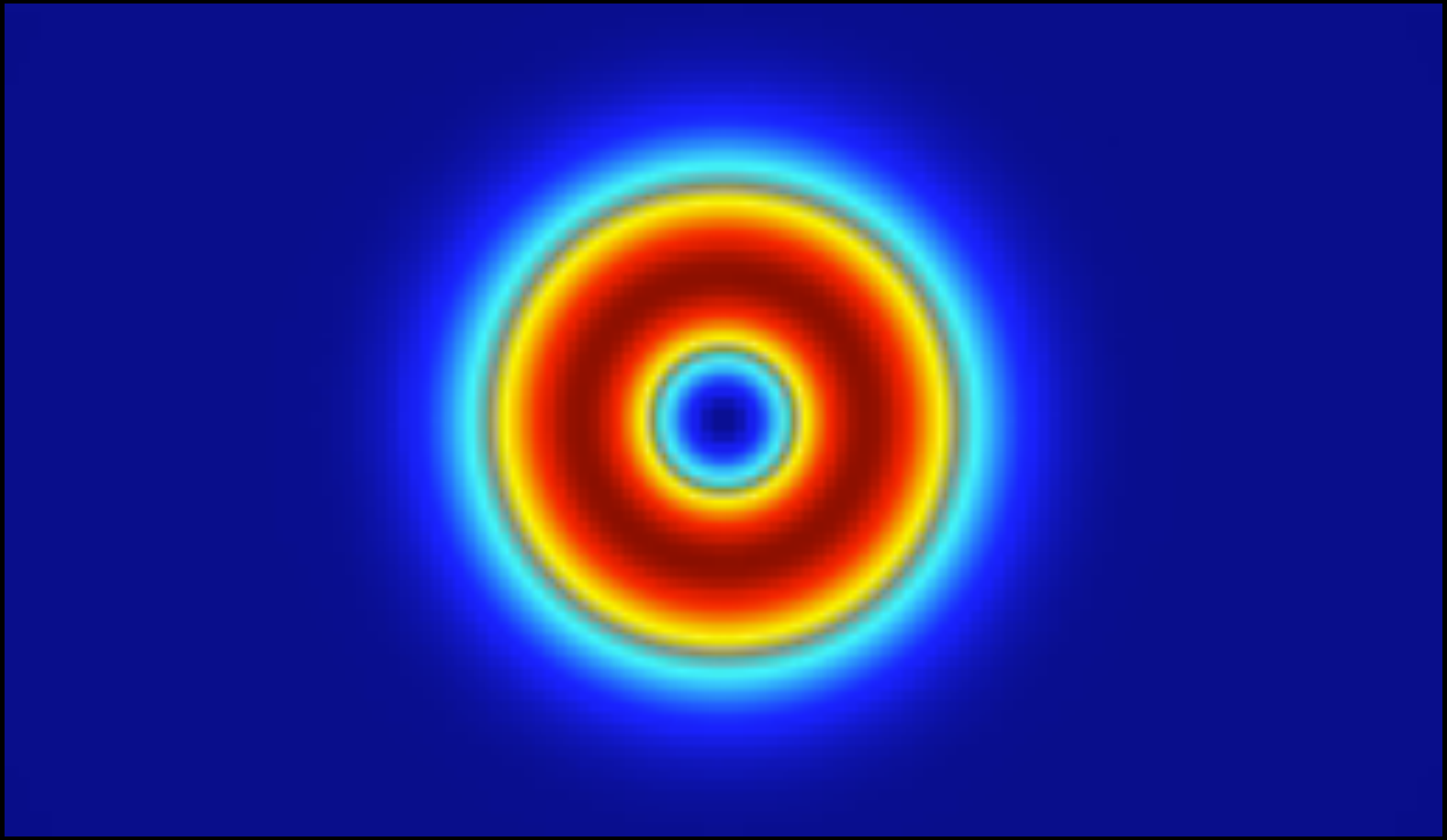
`harm` color shows j^2 $A_z \sim MAX(r_0 - r, 0)$

Field loop advection test



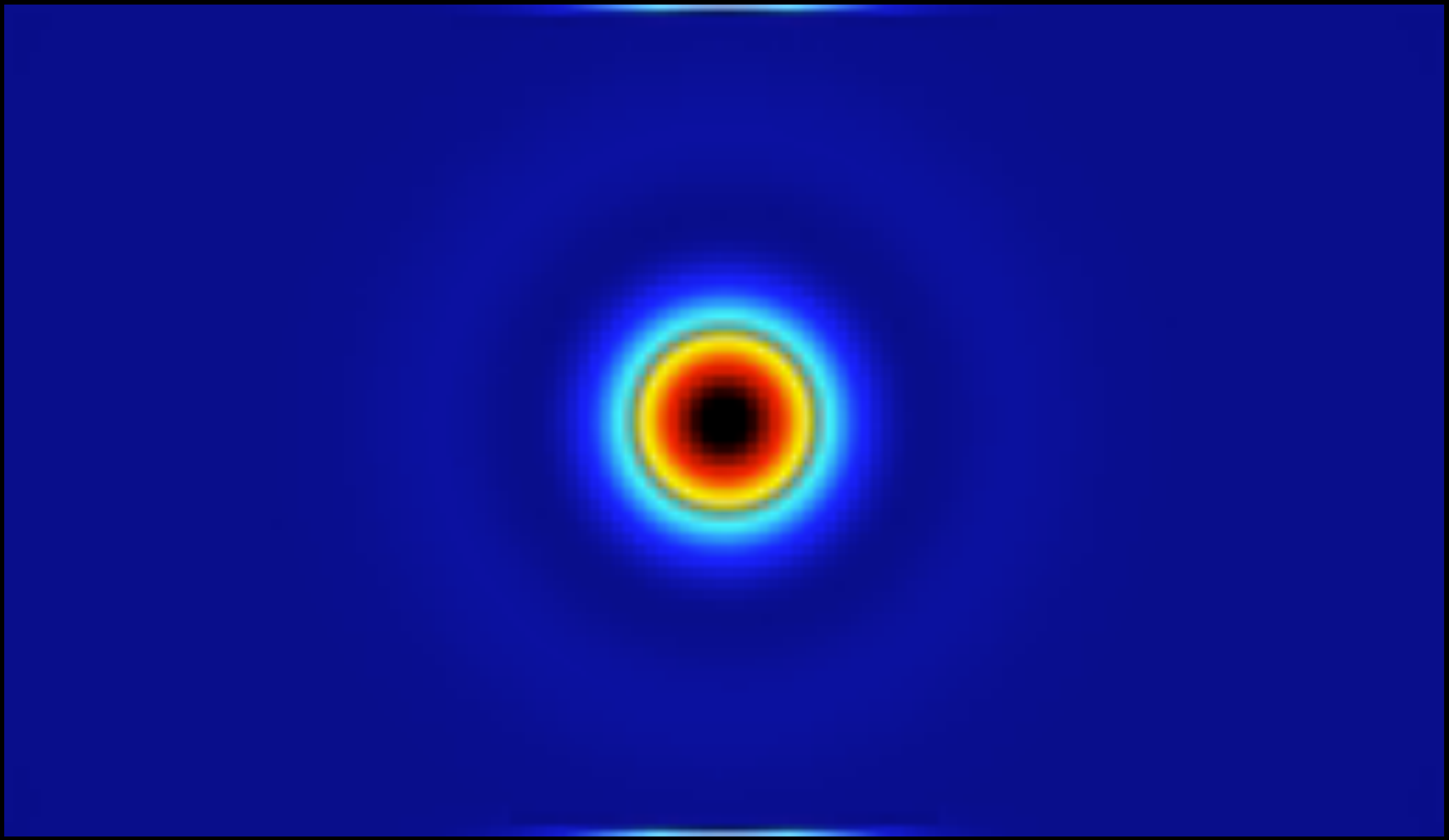
athena color shows j^2 $A_z \sim \mathrm{MAX}(r_0 - r, 0)$

Field loop advection test



`harm` color shows b^2 $A_z \sim \exp(-r^2/w^2)$

Field loop advection test



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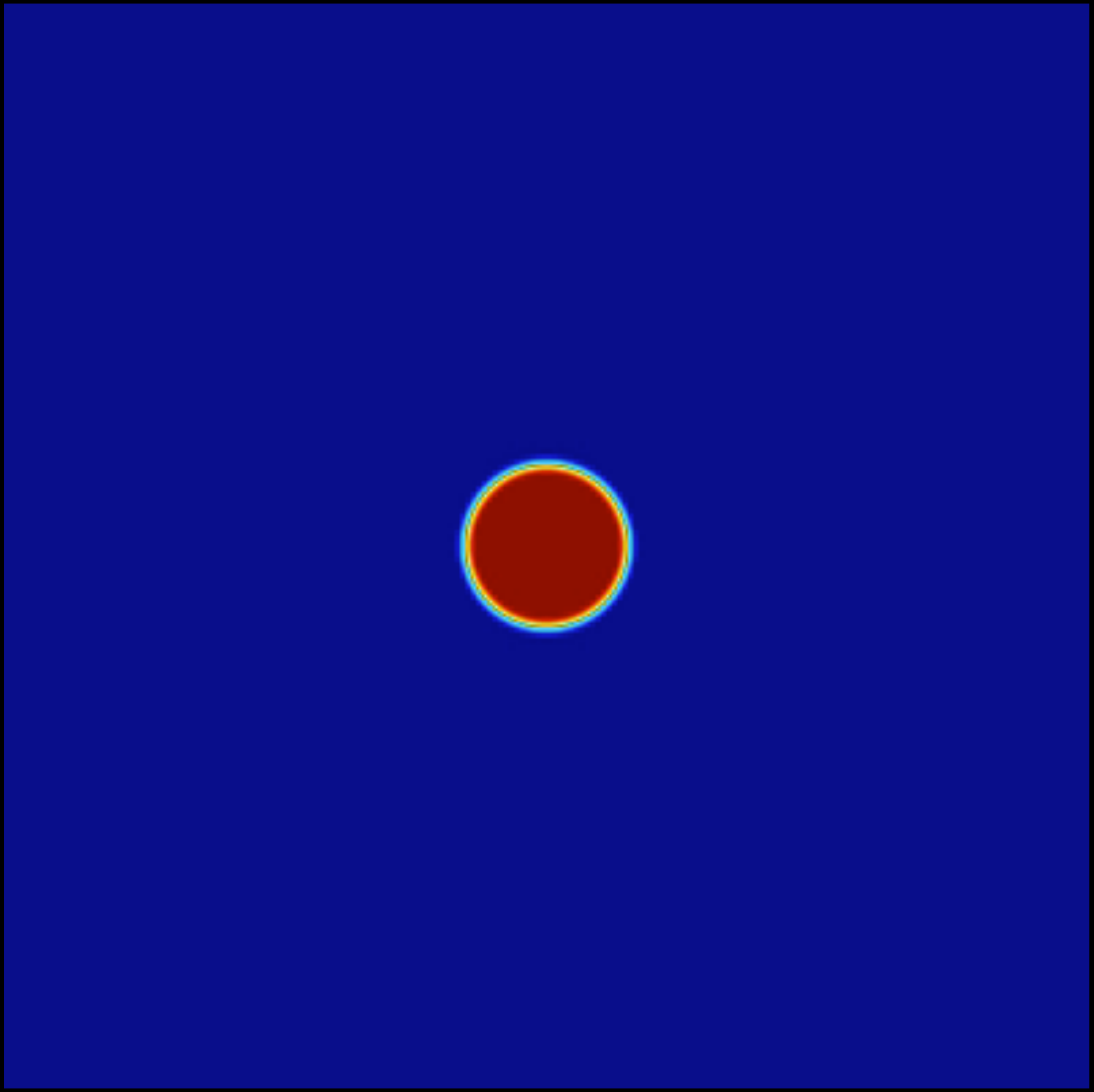
Komissarov's sadistic explosion problem



color shows log density

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Beyond Ideal MHD

In low $\dot{M}$ black hole accretion flows:

$\lambda_{mfp,\parallel}$ for Coulomb scattering by ions, electrons $\gg GM/c^2$

$\lambda_{mfp,\perp}$ for ions, electrons $\ll GM/c^2$

viscosity $\boldsymbol{\nu} \sim v_{th}\, \lambda_{mfp}$

⇒ anisotropic viscosity

⇒ anisotropic conduction

⇒ electrons and ions decouple, distinct temperatures

covariant extended MHD (Chandra+ 2015)
electron thermodynamics (Ressler+ 2015)

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Covariant EMHD Model

Heat flux parallel to field

$q^\mu = q\, b^\mu$

$b^\mu \equiv$ unit spacelike four-vector || b-field

Momentum transport parallel to field

$\tau^{\mu\nu} = -\Delta P\, [b^\mu b^\nu - (1/3)\, h^{\mu\nu}]$

$h^{\mu\nu} \equiv$ projection tensor, $\perp u^\mu$

$u^\mu \equiv$ four-velocity

Chandra+ 2015

Covariant EMHD Model

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Momentum transport parallel to field

$\tau^{\mu\nu} = -\Delta P\, [b^{\mu}\, b^{\nu} - (1/3)\, h^{\mu\nu}]$

Naive theory ($q \sim \nabla T + T\, a$) unstable:
promote q, ΔP to dependent variables and evolve

Chandra+ 2015

Covariant EMHD Model

Covariant, causal, stable model.

Governing equations:

Conservation of rest-mass, energy, momentum (5)

Induction equation (ideal) (3)

Relaxation equations for q, ΔP (2)

$q_0 = -\rho\, \chi\, b^\mu\, [\nabla_\mu \Theta + a_\mu \Theta]$

$\Delta P_0 = 3\, \rho\, \nu\, [b^\mu b^\nu\, \nabla_\mu u_\nu - (1/3)\, \nabla_\mu u^\mu]$

Closure relation for χ,ν

Chandra+ 2015

`grim` code

Ideal GRMHD codes ~ solved.

New problem:

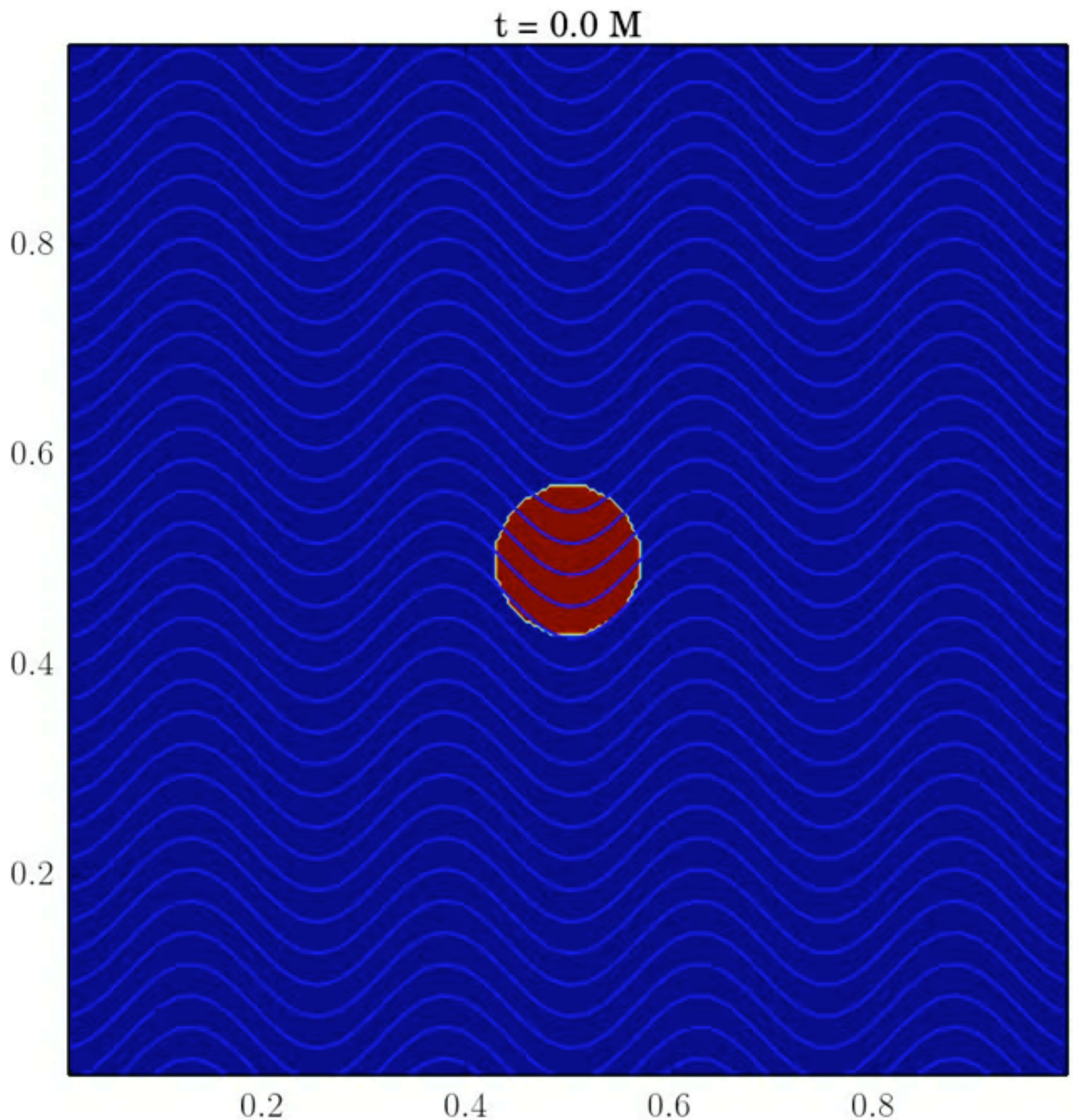
$$dq/d\tau = -(q - q_0)/\tau_R + \ldots$$

$$d\Delta P/d\tau = -(\Delta P - \Delta P_0)/\tau_R + \ldots$$

$q_0, \Delta P_0$ contain both space *and* time derivatives

⇒ new algorithm, implicit/explicit evolution

Chandra, Foucart, Gammie, in prep



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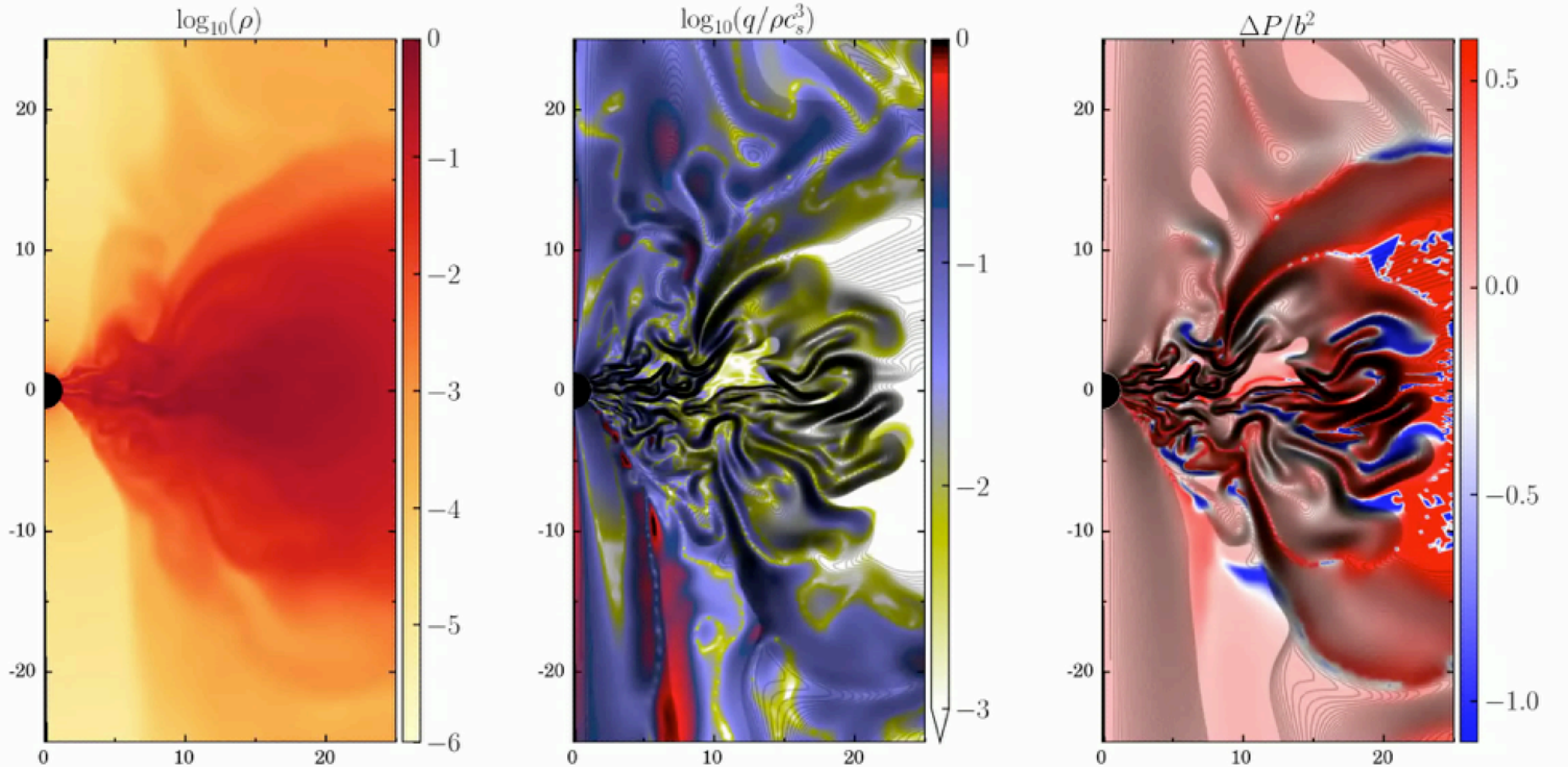

Foucart+ 2015

Further Reading

- Komissarov 1999, A Godunov-type scheme for relativistic magnetohydrodynamics, MNRAS, 303, 343-366.
- Gammie et al. 2003, HARM: A Numerical Scheme for General Relativistic Magnetohydrodynamics, ApJ, 589, pp. 444-457.
- Anile, 1990, Relativistic Fluids and Magneto-fluids, Cambridge.
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- Rezzolla & Zanotti 2013, Relativistic Hydrodynamics, Oxford.
- White et al. 2016, An Extension of the Athena++ Code Framework for GRMHD Based on Advanced Riemann Solvers and Staggered-Mesh Constrained Transport, ApJ in press.