Selected Topics in Plasma Astrophysics

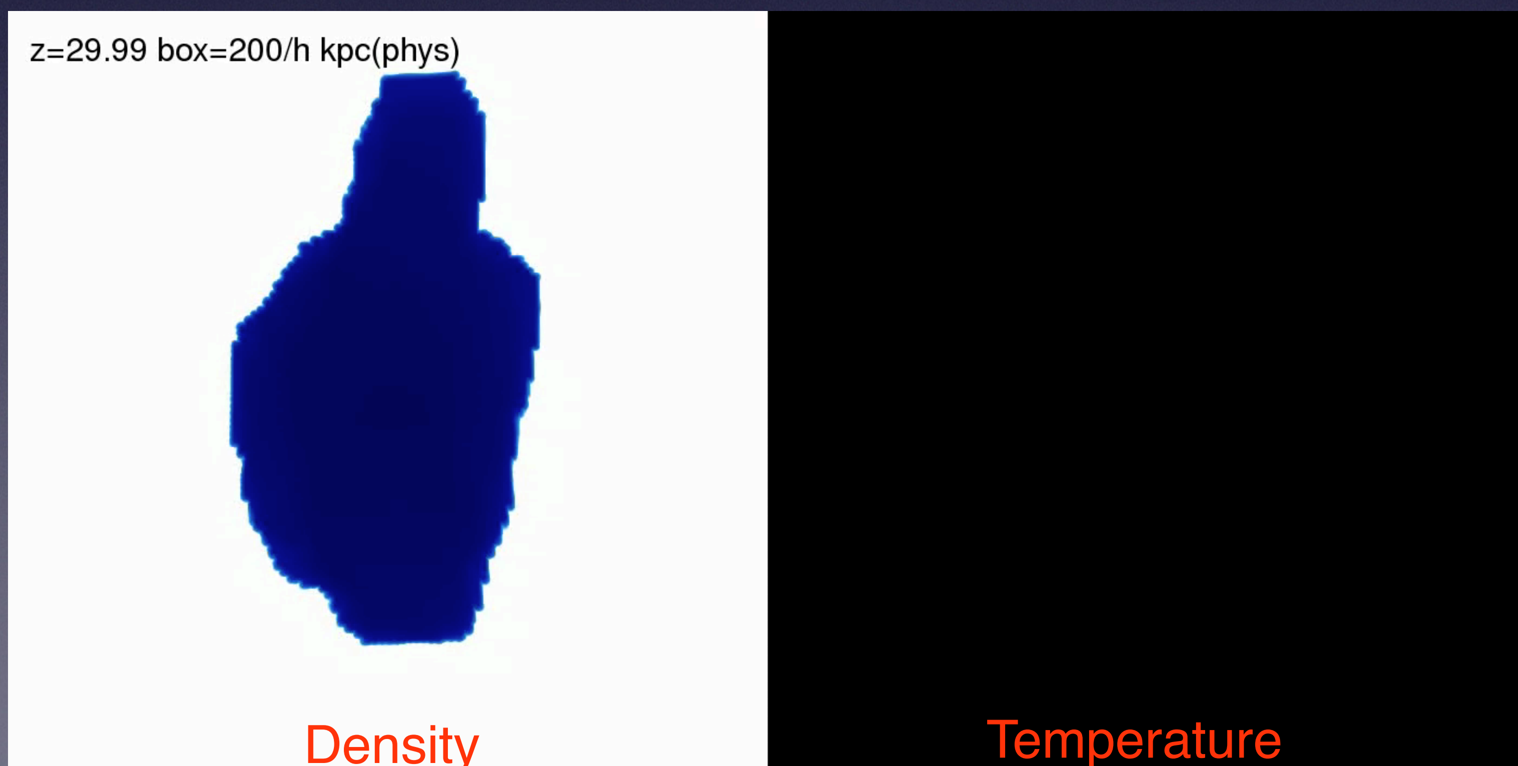
- Range of Astrophysical Plasmas and Relevant Techniques
- Stellar Winds (Lecture I)
  - Thermal, Radiation, and Magneto-Rotational Driven Winds
  - Connections to Other Areas of Astrophysical Fluids/Plasmas
- Instabilities In Ideal Fluids and Dilute Plasmas (Lecture II)
  - Ideal Fluid theory of Convection and MRI
  - How do Anisotropic Conduction & Viscosity Modify Convection and MRI
  - Astrophysical Context: Clusters and Accretion Disks

Instabilities In Ideal Fluids and Dilute Plasmas

- Who Cares About Linear Theory? Let's Simulate!
- Buoyancy Instabilities
  - Hydrodynamic Convection
  - Convection Induced by Anisotropic Thermal Conduction
    - Important for the intracluster plasma in galaxy clusters
- Instabilities Driven by Differential Rotation
  - The Magnetorotational Instability (MRI)
  - Non-ideal Effects on the MRI
    - collisional fluids (e.g., protostellar disks)
    - low collisionality plasmas (e.g., hot accretion flows onto BHs)

Role of Linear Instabilities

- Extremely Instructive for Identifying Key Physics in Problems of Interest
  - can't simulate everything; need to know what physics to include
- Produce Turbulent Transport of Mass, Momentum, Energy, B-Fields, …
  - accretion disks, stars, intracluster medium, …
  - physics of linear theory often imprinted on nonlinear state (buoyancy, B-tension …)
- Fundamentally Rearrange the Structure and Dynamics of the System



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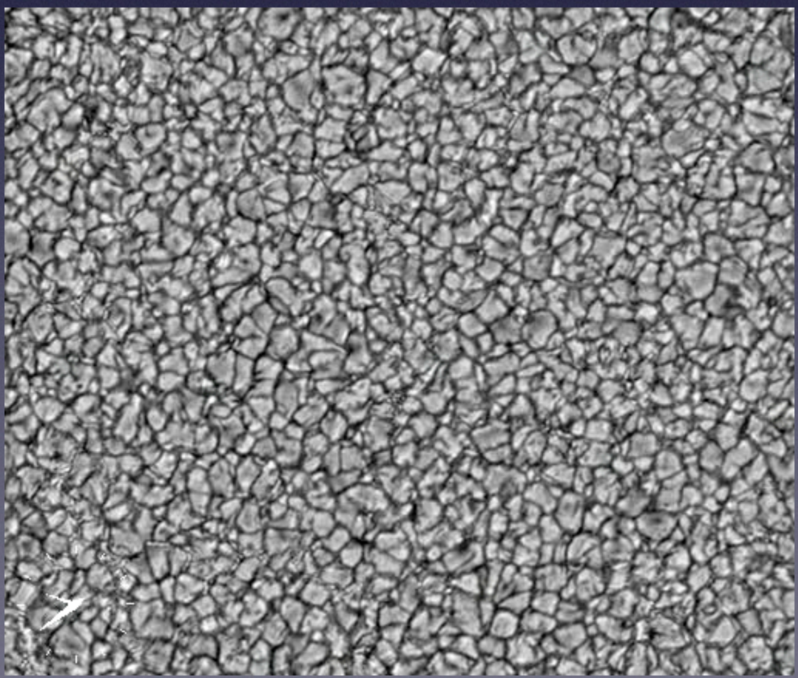
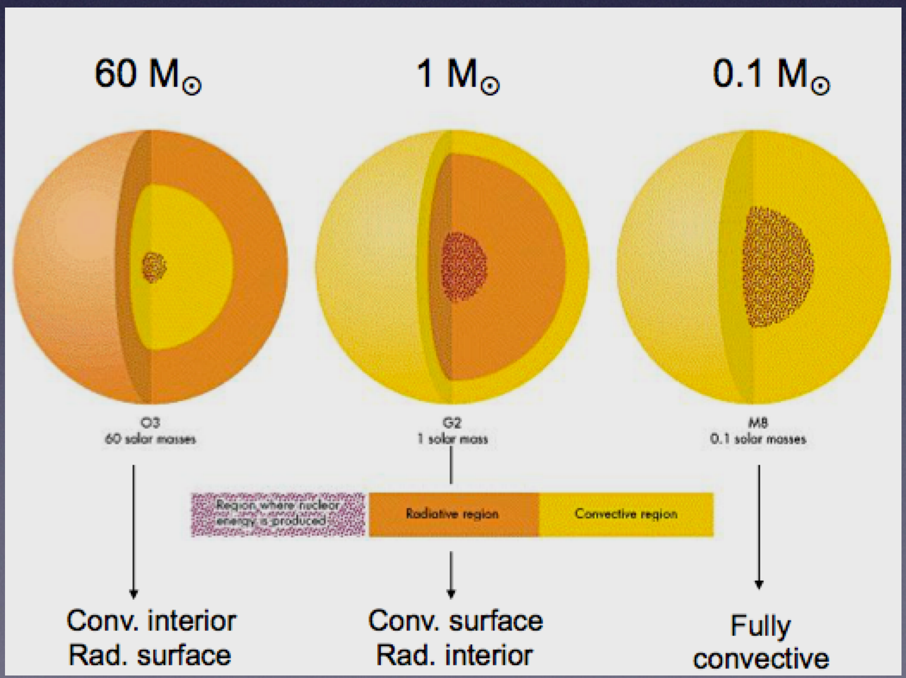
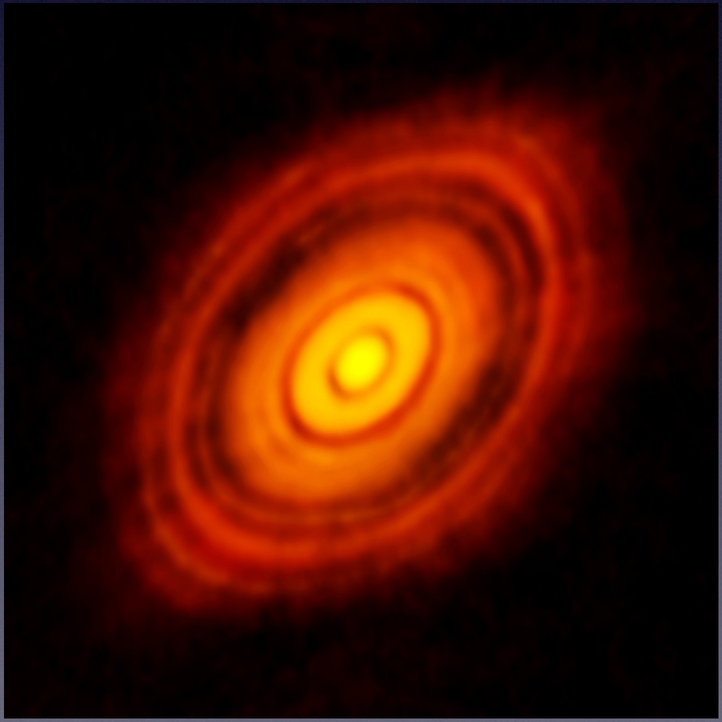


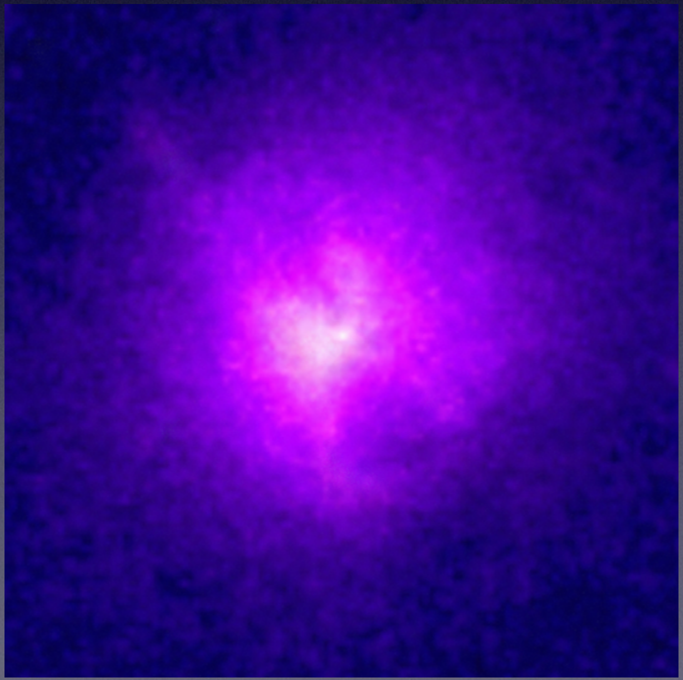


Diversity of Astrophysical Plasmas

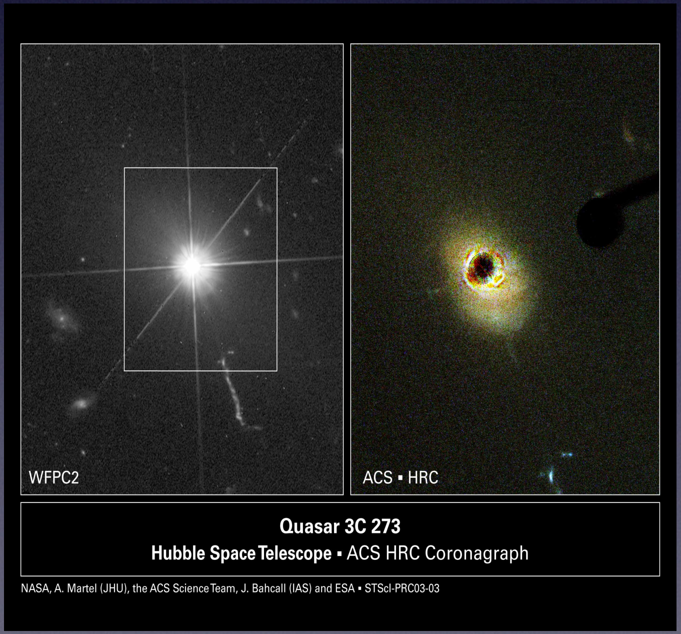
- Ideal Single Fluid (M)HD a Useful Starting Point for Astrophysical Plasmas
    - encapsulates mass, momentum, energy conservation; often does better than expected
- But Non-Ideal and Multi-Fluid Effects are Critical in many Systems



Star Formation, Planet Formation: Gas Cool, Dense, Largely Neutral (Multi-Fluid MHD + Dust)



Intracluster Plasma in Galaxy Clusters is Hot & Dilute (anisotropic conduction, viscosity, …)



Luminous Accreting Black Holes Radiation Pressure Dominated (2 fluid: radiation MHD)

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Hydrodynamic Convection

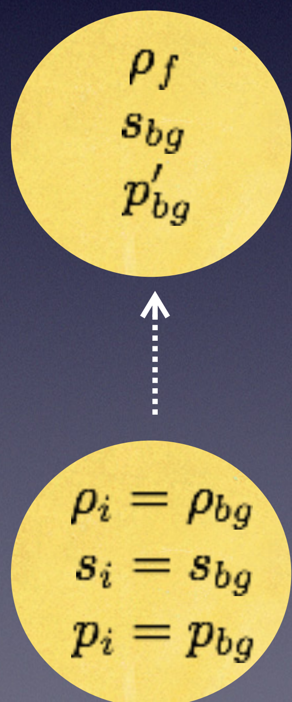
- Schwarzschild criterion for convection: **ds/dz < 0**
- Motions slow & adiabatic: **pressure equil, s ~ const**

solar interior: t_{sound} ~ hr << $t_{buoyancy}$ ~ month << $t_{diffusion}$ ~ 10^4 yr

low entropy (s)

gravity

high s



background fluid

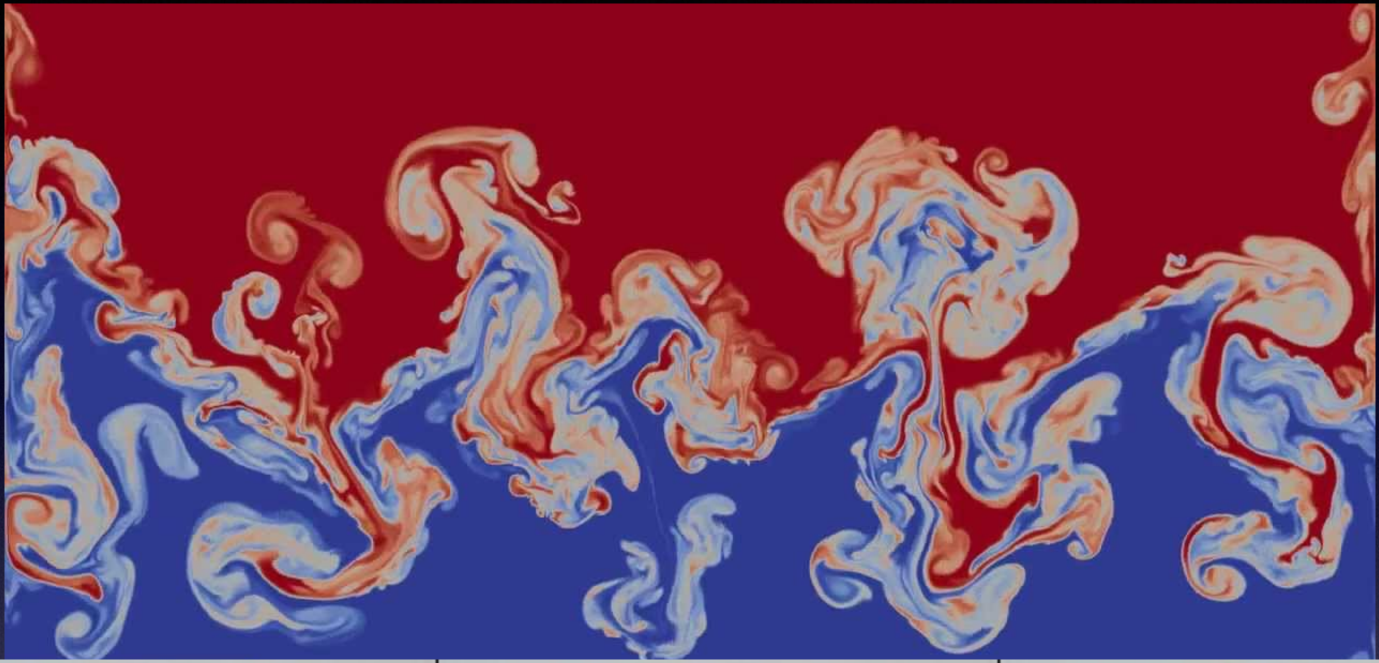
$$s'_{bg} \quad \rho'_{bg} \quad p'_{bg}$$

$$s(p, \rho) \propto \ln[p/\rho^\gamma]$$

$$\mathrm{if\ ds/dz} < 0 \rightarrow \rho_\mathrm{f} < \rho'_\mathrm{bg}$$

convectively unstable

What about Differences in Composition?



What about Differences in Composition?

- Schwarzschild criterion for convection: **ds/dz < 0**

gravity

$$s(p,\rho) \propto \ln[p/\rho^{\gamma}]$$

$$p = \sum_j n_j kT \equiv \frac{\rho kT}{\mu m_p}$$

μ = mean molecular weight
μ = 1/2 (ionized H)
μ = 4/3 (ionized He)
μ = 0.62 (solar metallicity)

$$\frac{ds}{dz} = \frac{d\ln p}{dz} - \gamma\frac{d\ln \rho}{dz} = \frac{d\ln T}{dz} - (\gamma - 1)\frac{d\ln \rho}{dz} - \frac{d\ln \mu}{dz}$$

dμ/dz > 0 (heavy on top of light) is destabilizing

(continuous version of Rayleigh-Taylor instability)

Impact of Isotropic (Photon) Diffusion on Convection in Stars

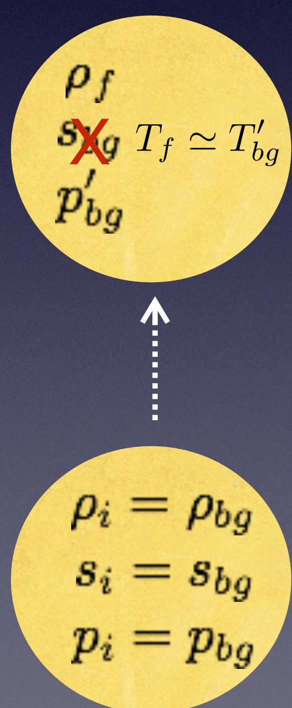
$t_{diff} \sim H^2/\ell c \sim \tau H/c$ $\qquad$ $t_{conv} \gtrsim H/c_s$

$t_{diff} \lesssim t_{conv}$ if $\tau \lesssim c/c_s$ $\Rightarrow$ *surface layers non-adiabatic*

low entropy (s)

gravity

high s



background fluid

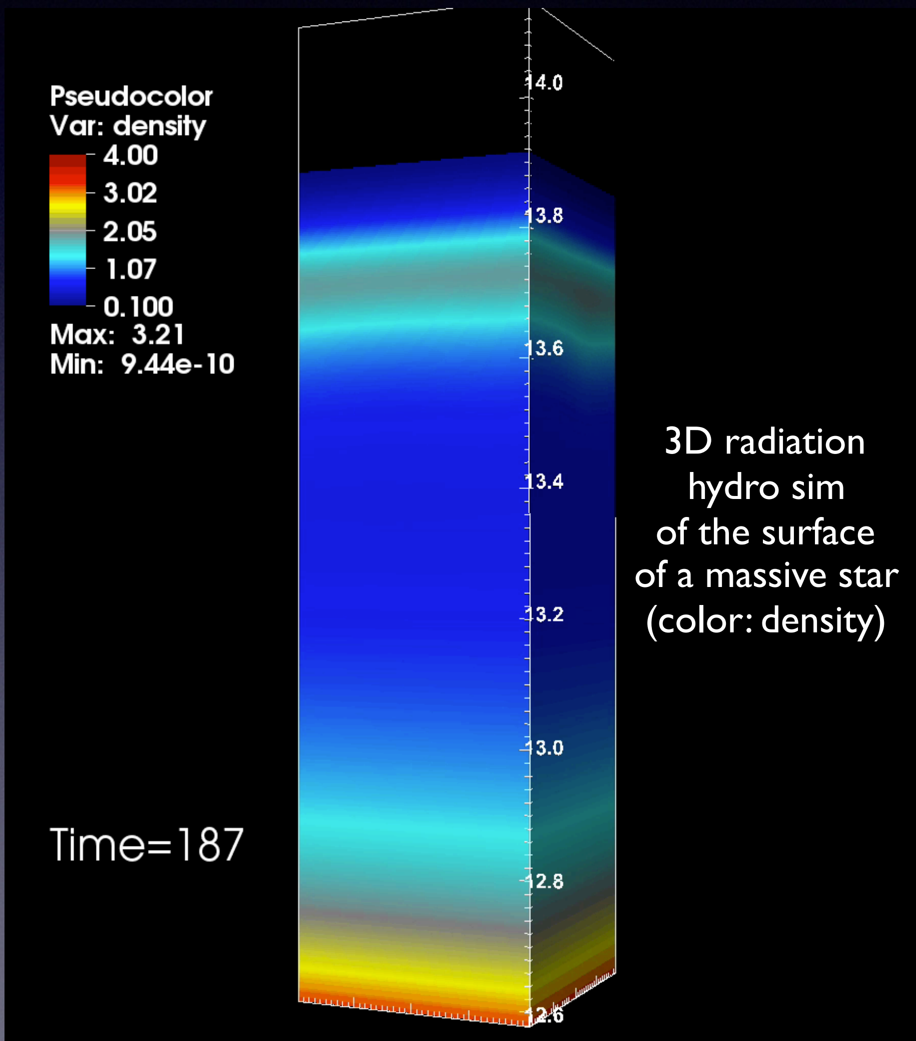
$$s'_{bg} \ \rho'_{bg} \ p'_{bg} \ T'_{bg}$$

$$\rightarrow \rho_f \simeq \rho'_{bg}$$

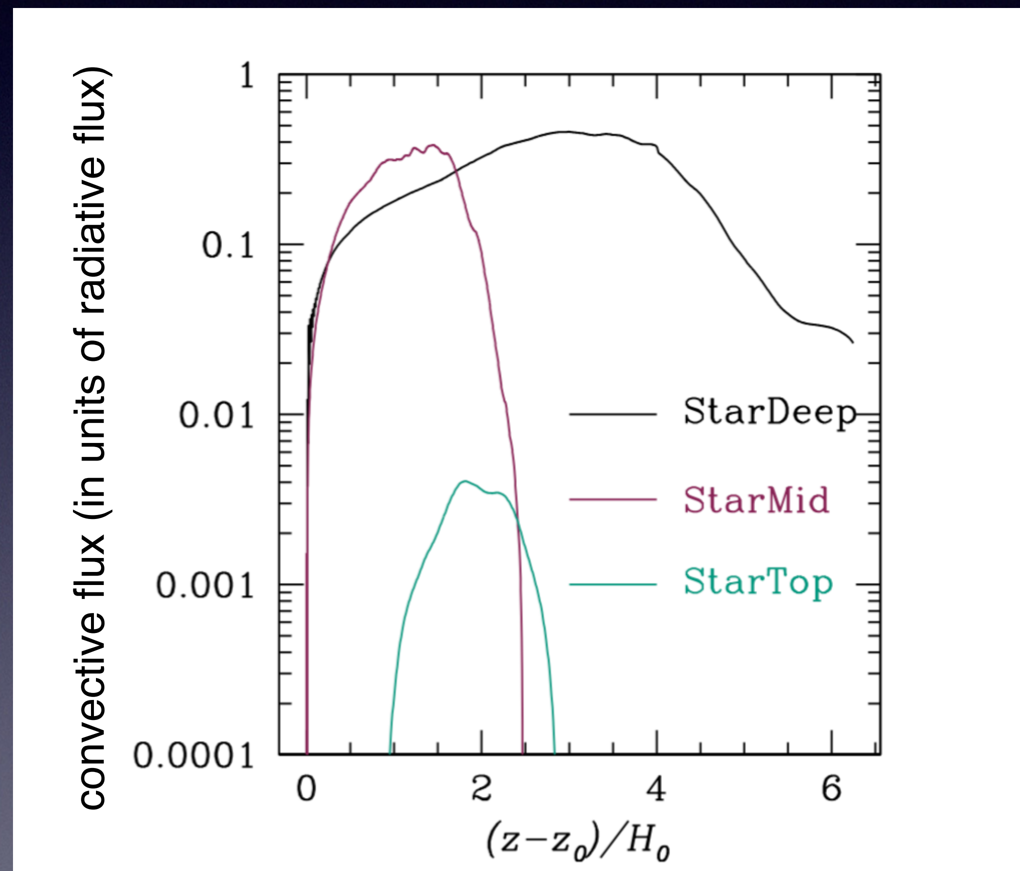
buoyancy weakened by rapid isotropic diffusion

Radiation Hydro Sims of Convection in the Atmospheres of Massive Stars

Jiang+ 2015



3D radiation hydro sim of the surface of a massive star (color: density)



Microscopic Energy Transport

- Photons dominate in non-degenerate dense plasmas w/ $l_{photon} << $ system size
  - e.g., stars

$$\sigma_{Coulomb} \simeq \frac{\pi e^4 \ln \Lambda}{(kT)^2} \simeq 10^{-18} \left(\frac{T}{10^7\,\mathrm{K}}\right)^{-2} \mathrm{cm}^2 \gg \sigma_{es} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\,\mathrm{cm}^2$$

- Thermal conduction dominates in
  - degenerate plasmas: white dwarfs and neutron stars

$$l_e > \rho_e \text{ for } B \gtrsim 10^9\,\mathrm{G}\ (\mathrm{non-relativistic\ degenerate\ plasma})$$

    - conduction typically ~ isotropic for WDs, but ~ anisotropic for NS surfaces
  - dilute, hot non-degenerate plasmas
    - e.g., solar corona & wind, clusters of galaxies, hot accretion flows onto black holes
    - $l_e >>> \rho_e \Rightarrow$ conduction highly anisotropic $\kappa_\perp \simeq \kappa_{||}\left(\frac{\rho_e}{l_e}\right)^2 \ll \kappa_{||}$

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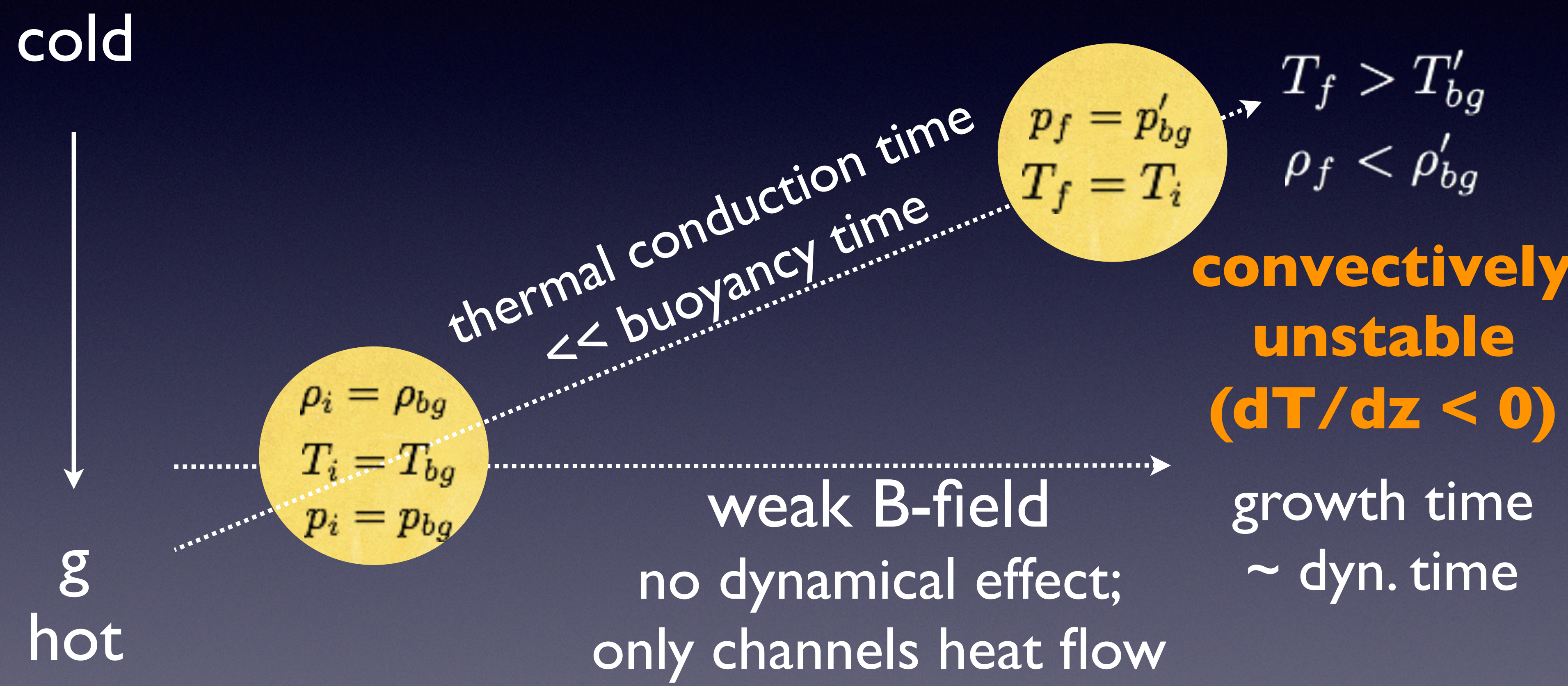
The Magnetothermal Instability (MTI)

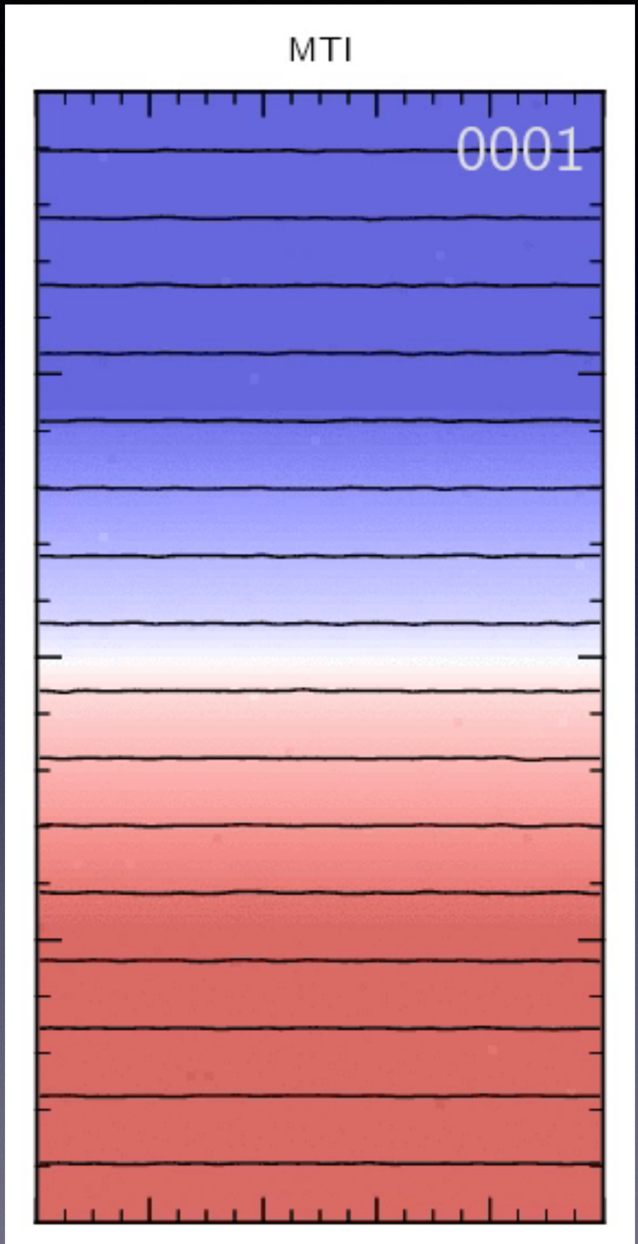


The Magnetothermal Instability (MTI)

cold

g

hot



McCourt+ 2011

B-field lines & Temp

instability saturates
by generating sustained
convection & amplifying
the magnetic field
(analogous to hydro convection)

The Heat Flux-Driven Buoyancy Instability (HBI)

hot

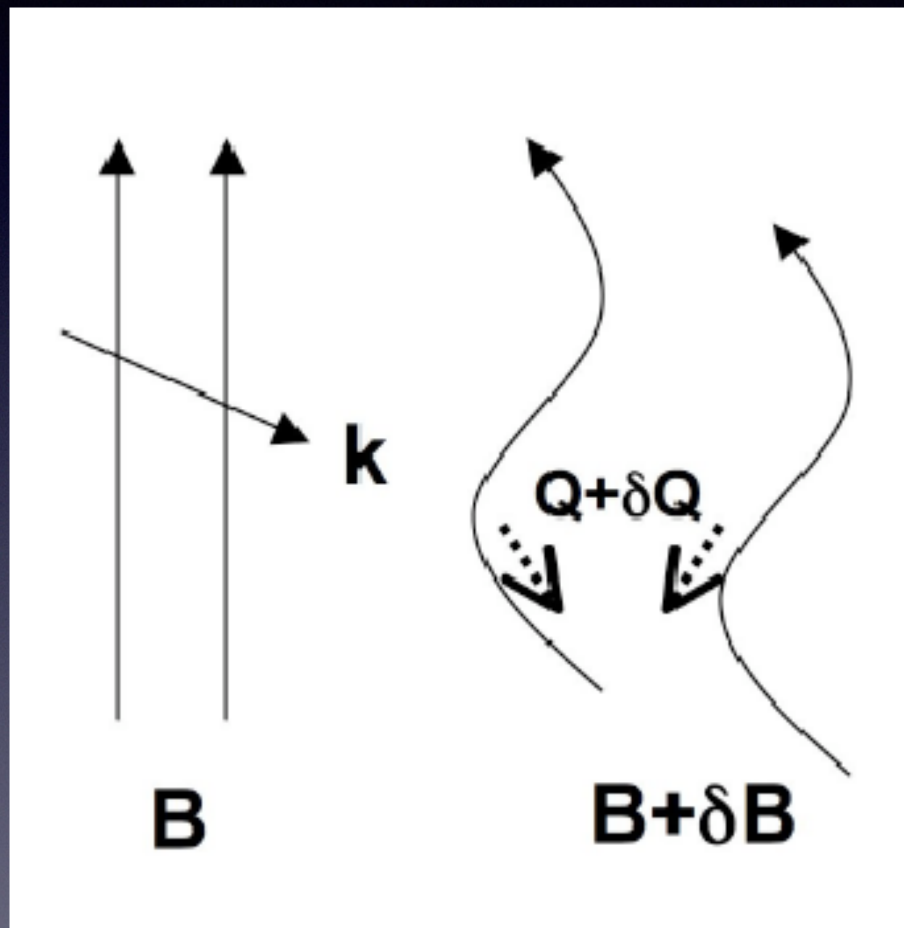
cold

g, Q_z

heat flux

weak

B



pert to field tap into heat flux

$\Rightarrow$

conductive heating & cooling

for **$dT/dz > 0$** upwardly displaced fluid heats up & rises, bends field more, ...

convectively unstable

The Heat Flux-Driven Buoyancy Instability (HBI)

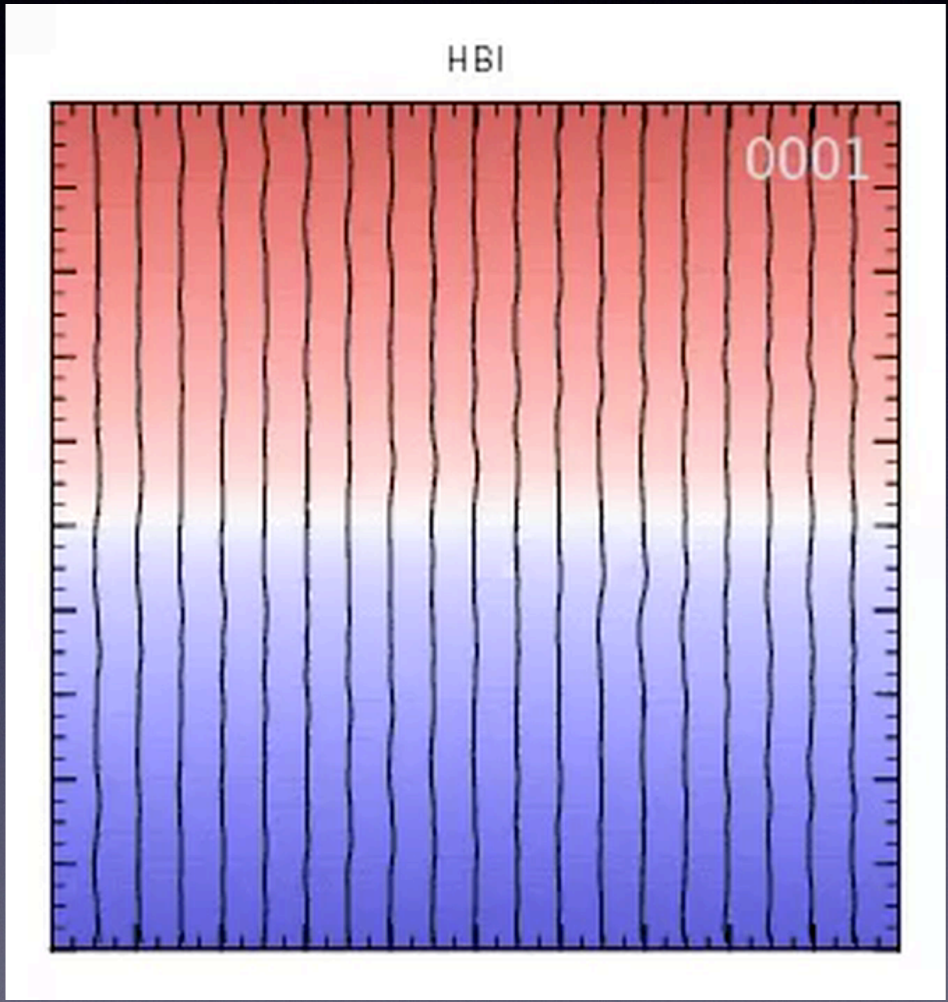
magnetic field lines

hot

cold

g

initial heat flux



saturates by rearranging the magnetic field &

suppressing heat flux through plasma

Role of Anisotropic Viscosity

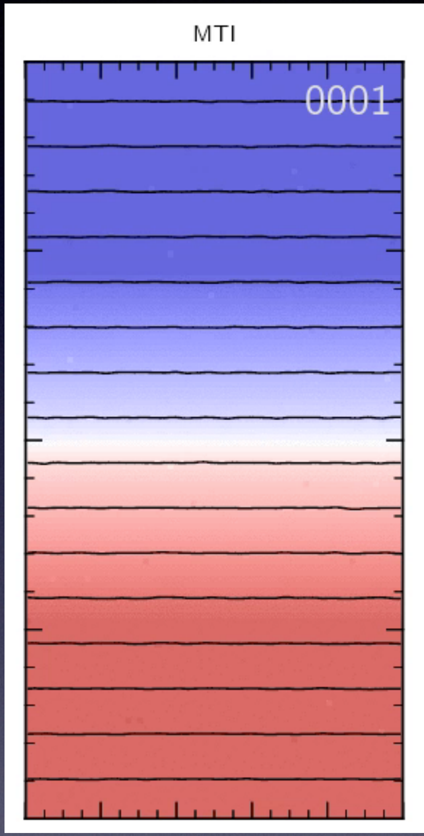
- Anisotropic Conduction and Viscosity Come Together
  - conduction somewhat faster: $\tau_{cond} \sim (m_e/m_p)^{1/2}\,\tau_{visc}$ (electrons vs. protons)

$$\Pi = -\Delta P \left[\hat{b}\hat{b} - \frac{I}{3}\right] \quad \Delta P = \rho \nu_{\parallel} \frac{d}{dt} \ln\left(\frac{B^3}{\rho^2}\right)$$

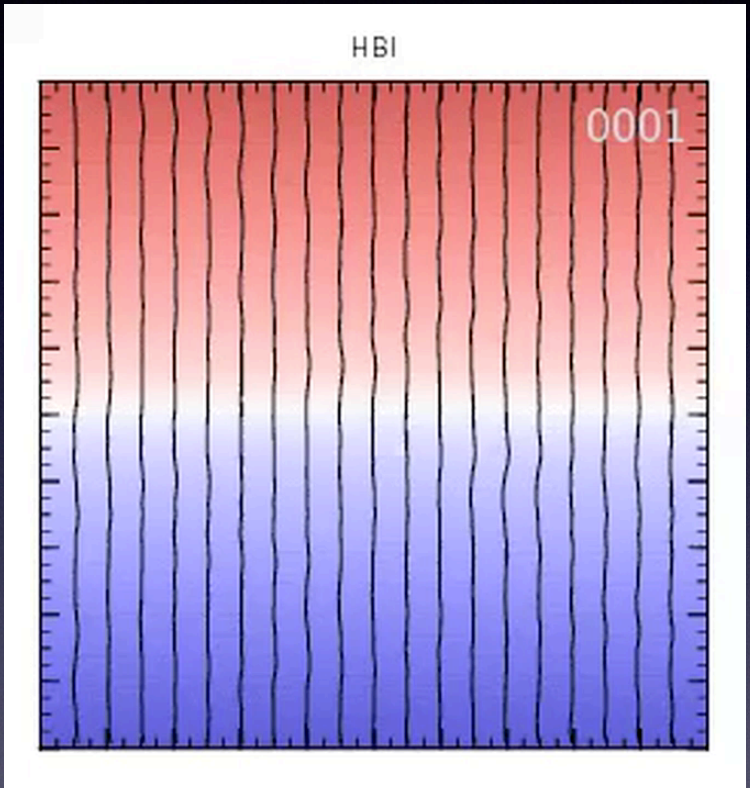
  - ⇒ in magnetized plasma, viscosity resists changes in magnetic field strength
- MTI: $\delta B = 0$ HBI: $\delta B \neq 0$ (simplest setups)
  - ⇒ viscosity can suppress growth rates of HBI

Buoyancy Instabilities in Low-Collisionality Plasmas

MTI ($dT/dz < 0$)



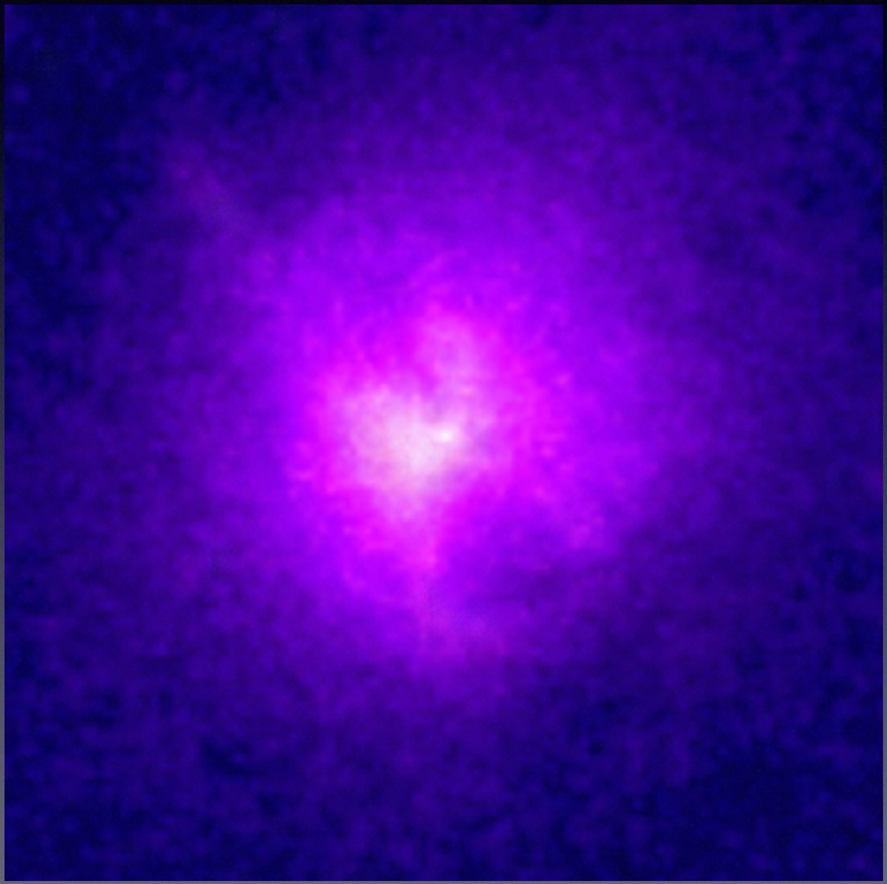
HBI ($dT/dz > 0$)



a weakly magnetized plasma w/ anisotropic heat transport is always buoyantly unstable, independent of dT/dz

Instabilities suppressed by 1. strong B ($\beta < 1$; e.g., solar corona) or 2. isotropic heat transport >> anisotropic heat transport (e.g., solar interior)

Hot Plasma in Galaxy Clusters



$L_x \sim 10^{43\text{-}46}$ erg s^{-1}

$n \sim 10^{-4}$-1 cm^{-3}

$T \sim$ 1-15 keV

$M_{gas} \sim 10^{13\text{-}14}\, M_\odot$

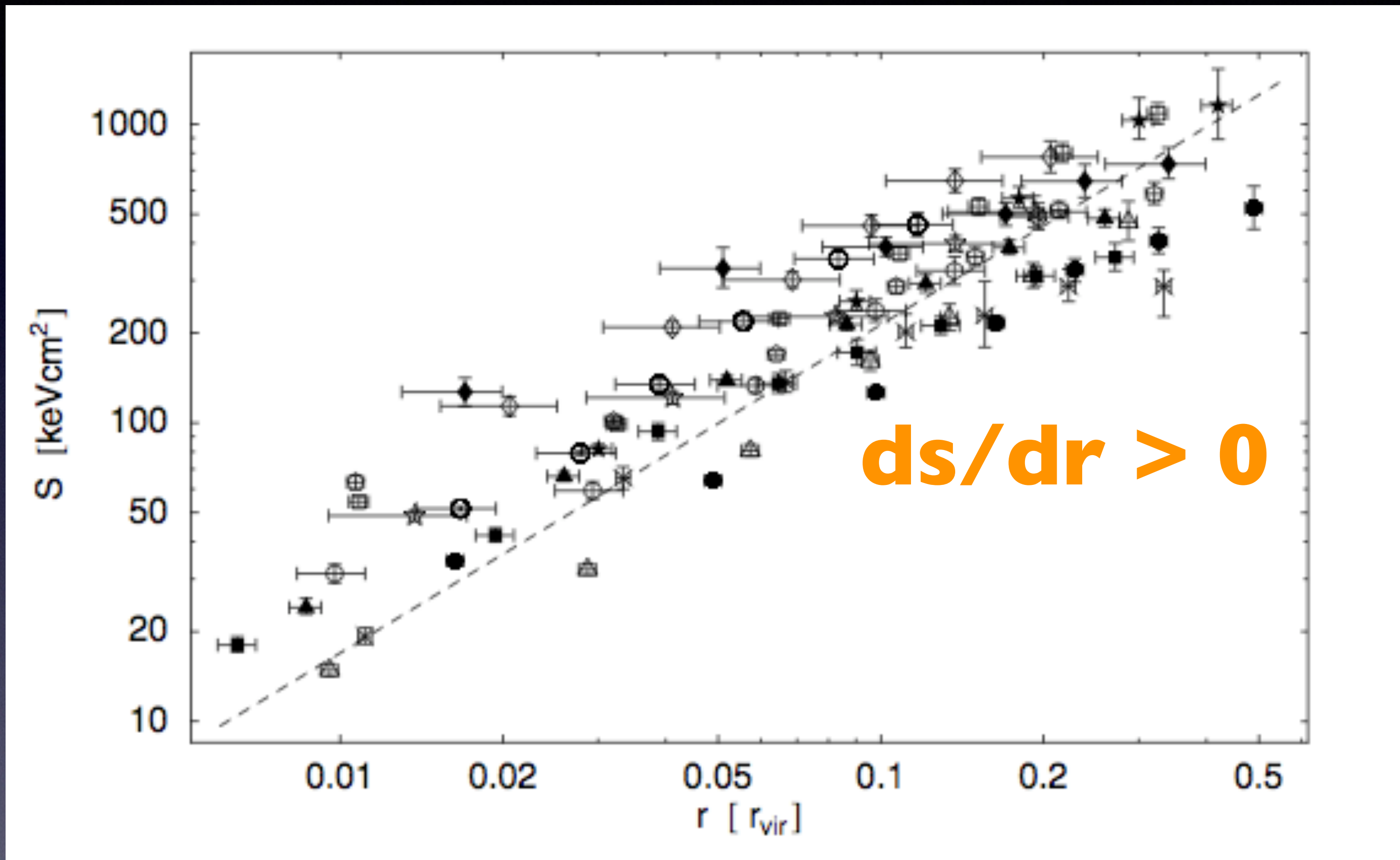
large electron mean free path:

$$\ell_e \simeq 2 \left(\frac{T}{3\,\mathrm{keV}}\right)^2 \left(\frac{n}{0.01\,\mathrm{cm}^{-3}}\right)^{-1} \mathrm{kpc}$$

→ thermal conduction and viscosity are important

Cluster Entropy Profiles

Entropy



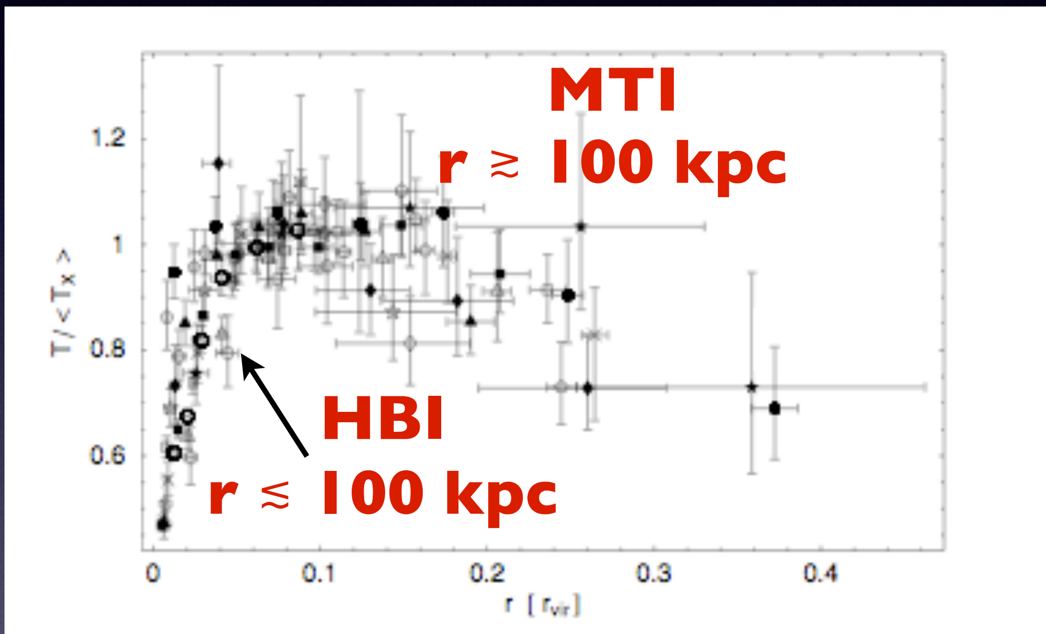
Piffaretti et al. 2005

Radius (R_{vir})

Schwarzschild criterion → clusters are buoyantly **stable**

The MTI & HBI in Clusters

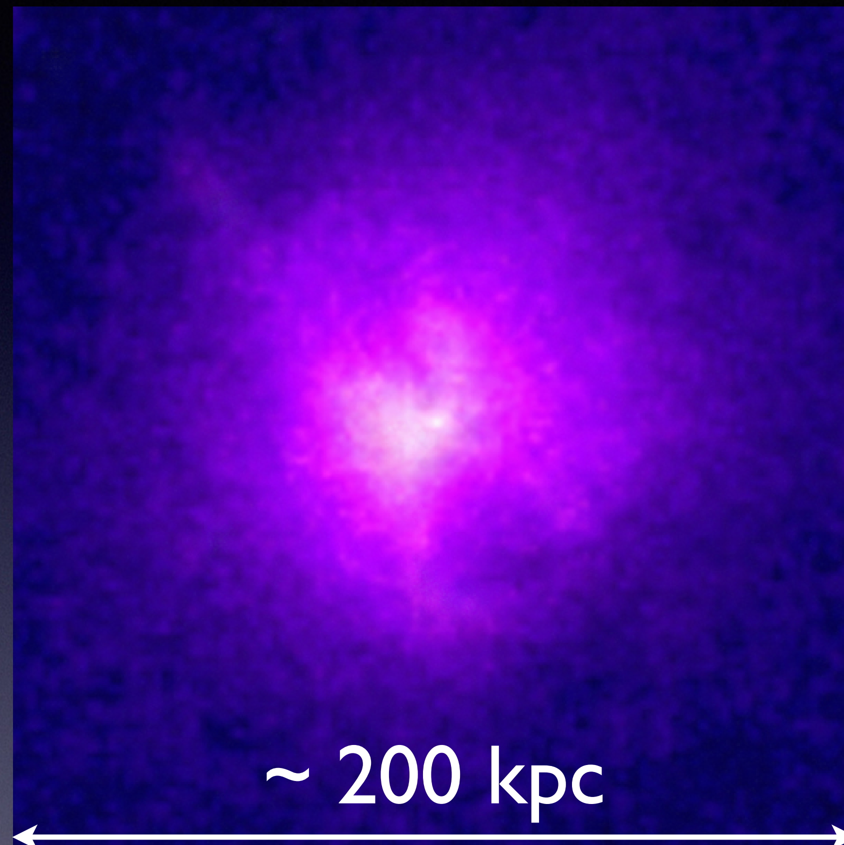
cool core cluster temperature profile



T/<T>

Radius (R_{vir})

Piffaretti et al. 2005



The entire cluster is convectively unstable, driven by anisotropic thermal conduction

Important implications for the thermal evolution of clusters, cluster B-fields, cooling flows, …

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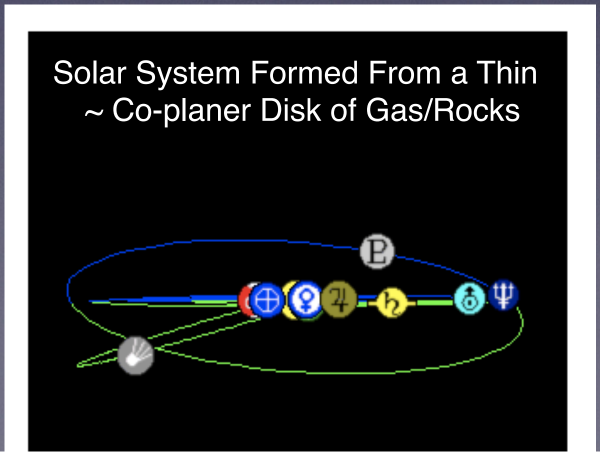
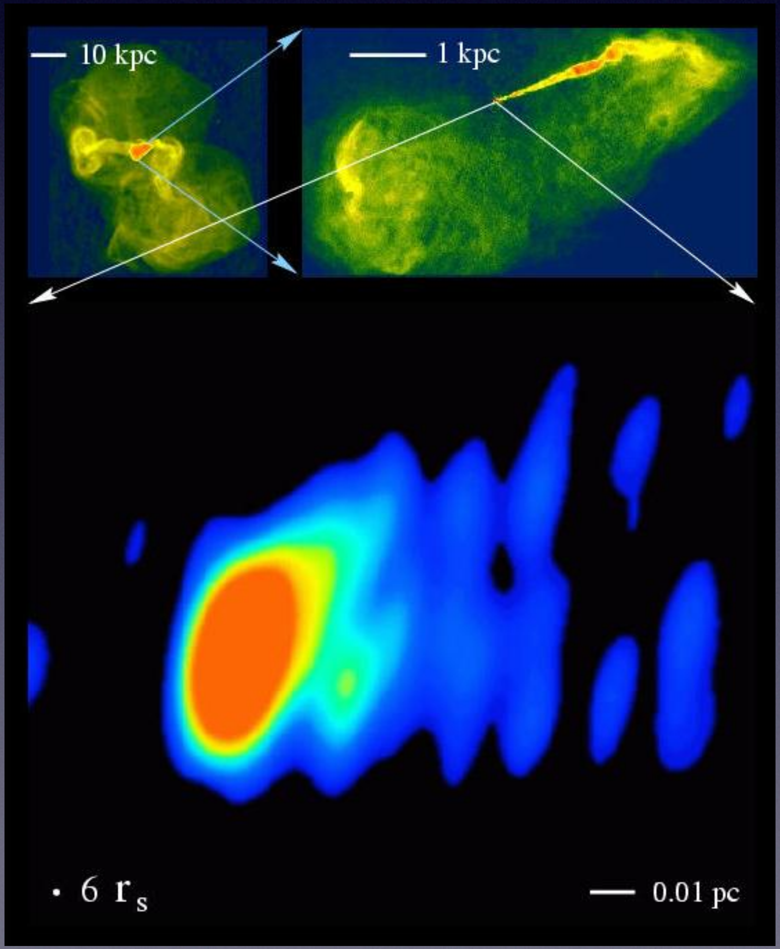
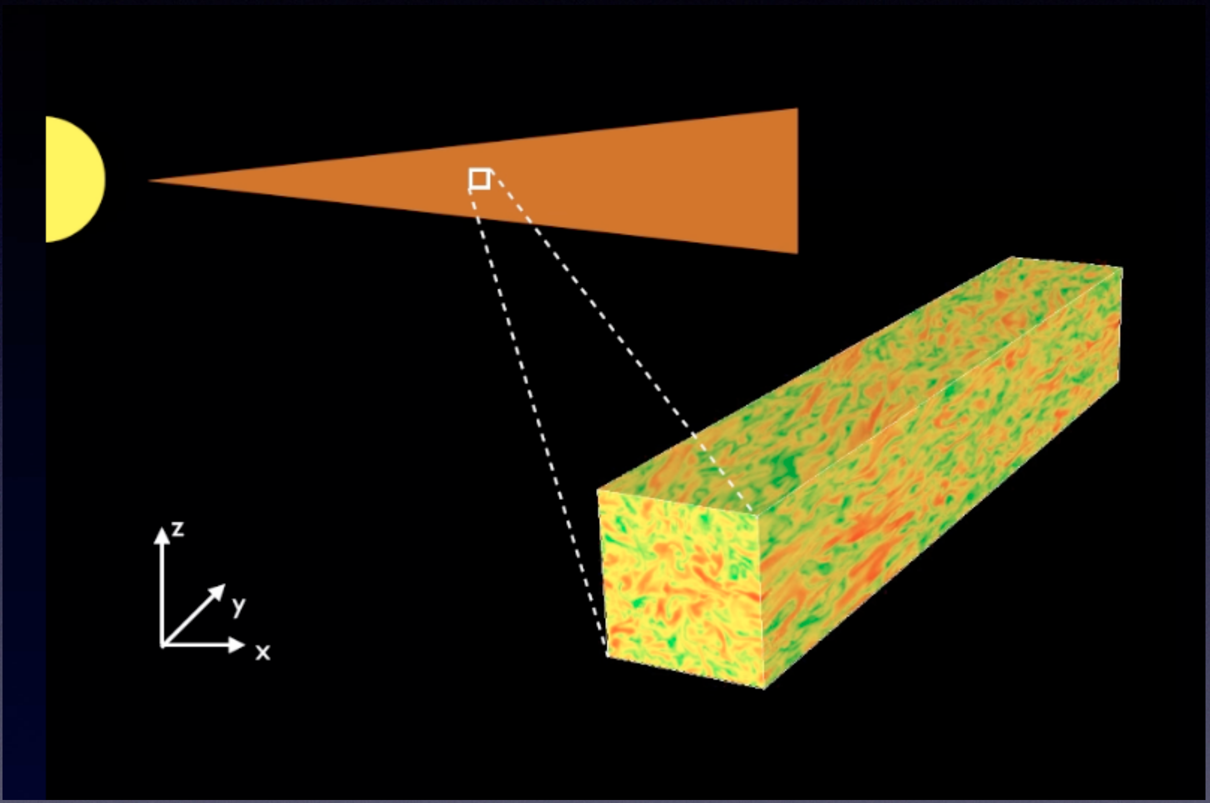
Accretion Disks

- Central to Planet, Star, & Galaxy Formation, Compact Objects
- Turbulence Generated by Linear Instabilities Transports Angular Momentum, Allowing Accretion to Proceed





Local Instabilities Driven by Differential Rotation



Assumed Equilibrium $$\Omega^2 \simeq \frac{1}{R}\frac{d\phi}{dR} \simeq \frac{GM}{R^3}$$

- In Hydrodynamics ∃ a Linear Axisymmetric Instability if

$$\kappa^2 \equiv \frac{1}{R^3}\frac{d}{dR}R^4\Omega^2 < 0$$

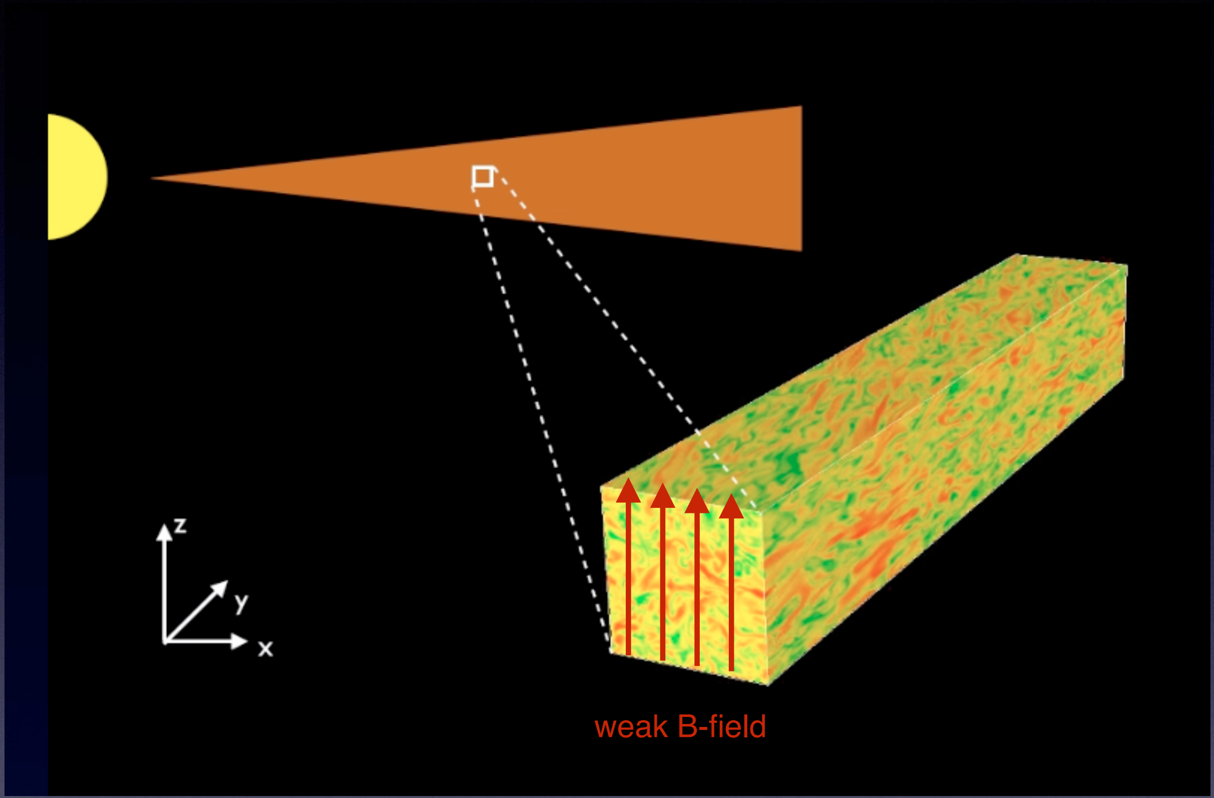
  - κ = epicyclic frequency (= Ω for pt mass)

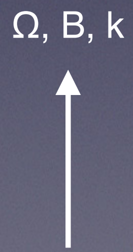
$a_{gravity} = GM/R^2$ $a_{centrifugal} = \Omega^2 R = \ell^2/R^3$ $(\ell = R^2\Omega)$

$R \rightarrow R + \delta R$ $\Rightarrow$ $a_{net} = -\kappa^2 \delta R$ Unstable if $\kappa^2 < 0$

$(\ell = \text{const})$

MRI in Ideal MHD





axisymmetric nearly incompressible instability with weak B_z ($\beta >> 1$)

MRI in Ideal MHD

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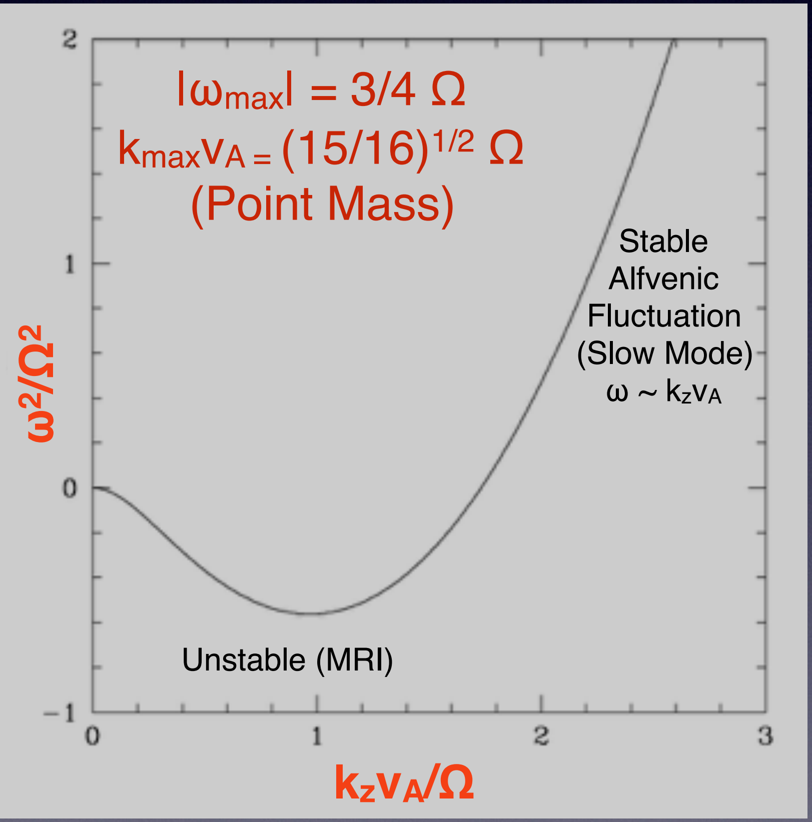
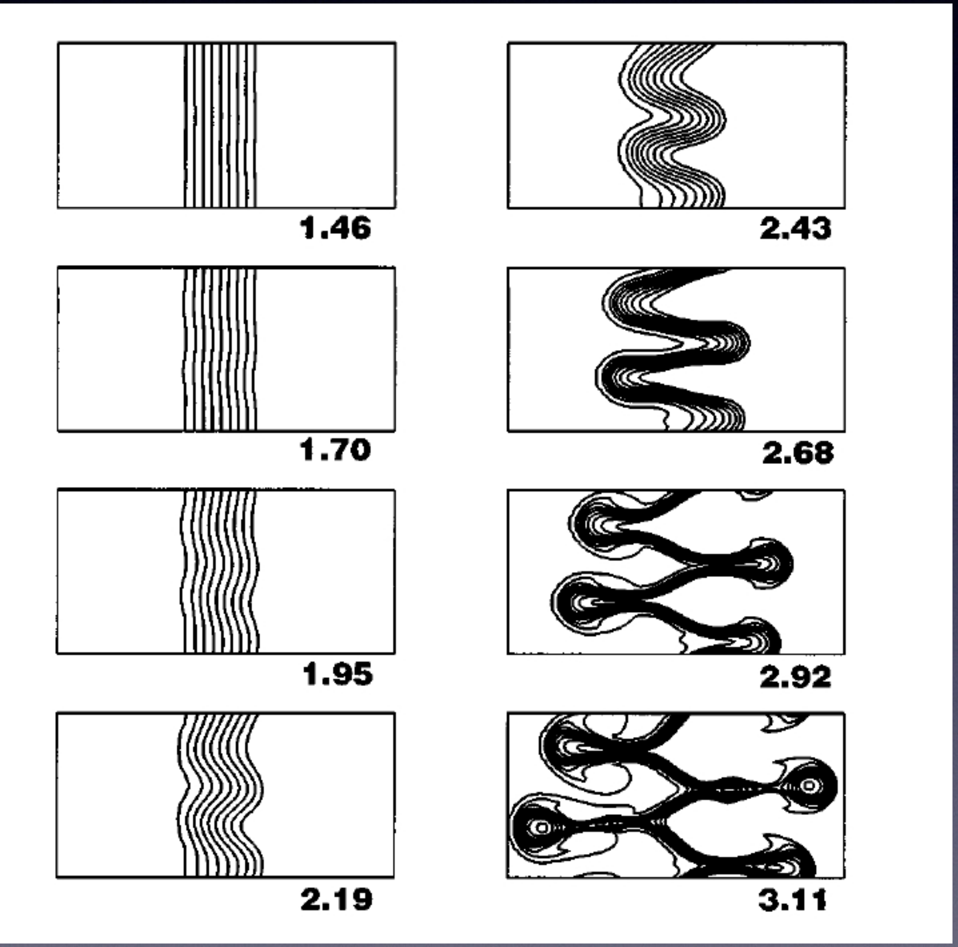


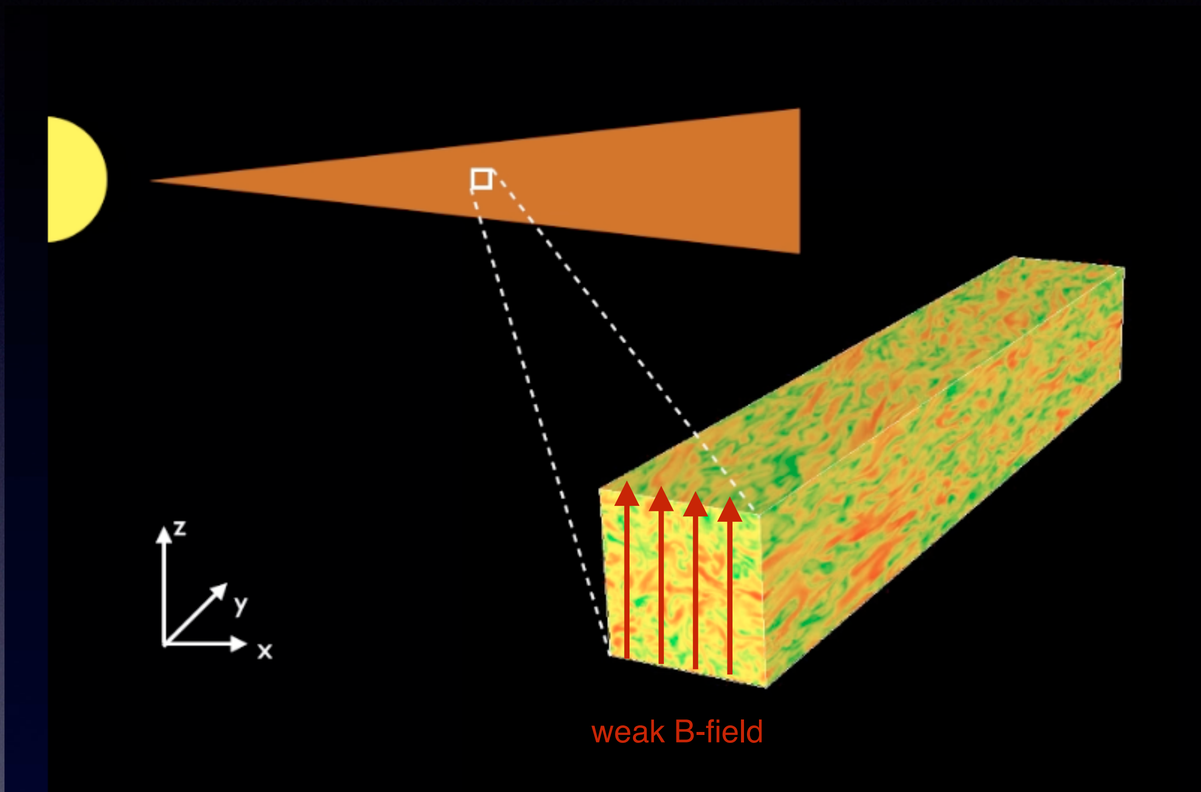
Unstable when Alfven freq ~ Rotation Freq



$$F_\phi = \mathbf{B} \cdot \nabla \mathbf{B}_\phi = i B_0 k_z \delta B_\phi$$

tension redistributes angular momentum

Local Instabilities Driven by Differential Rotation



Assumed Equilibrium $\Omega^2 \simeq \frac{1}{R}\frac{d\phi}{dR} \simeq \frac{GM}{R^3}$

In MHD ∃ a Linear Axisymmetric Instability if $d\Omega^2/dR < 0$

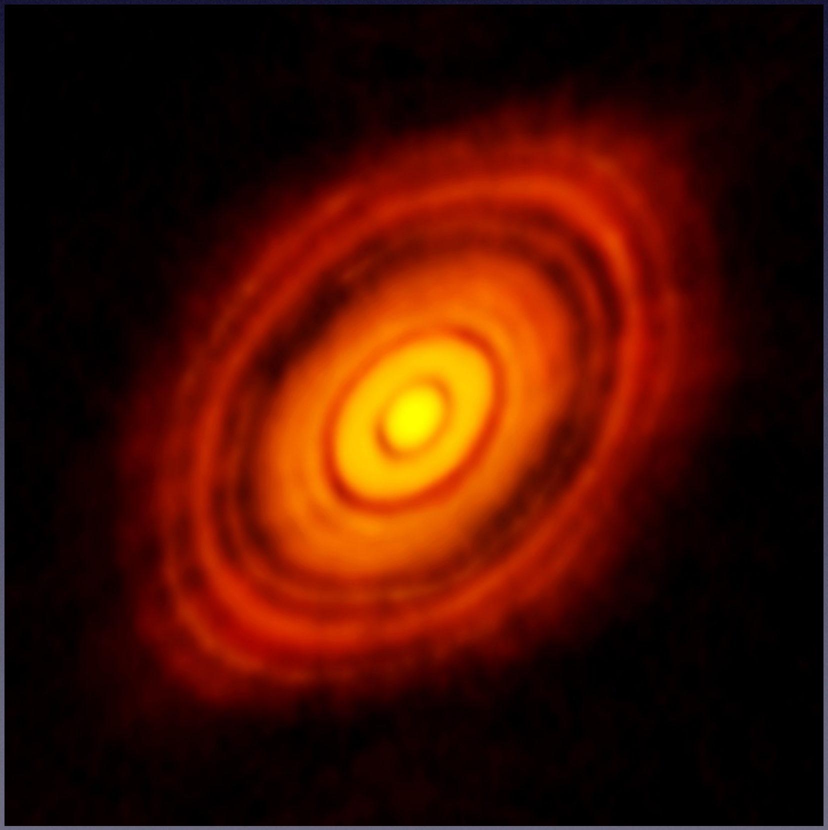
$a_{gravity} = GM/R^2$ $a_{centrifugal} = \Omega^2 R$

$R \rightarrow R + \delta R$ (Ω = const) $\Rightarrow$ $a_{net} = -d\Omega^2/d\ln R \, \delta R$ Unstable if $d\Omega^2/dR < 0$

Non-Ideal Effects on the MRI: Collisional Plasmas

key non-ideal effects: resistivity, viscosity, ambipolar diffusion, Hall effect

Non-ideal effects Critical in Protostellar Disks
(low temperatures and ionization fractions)



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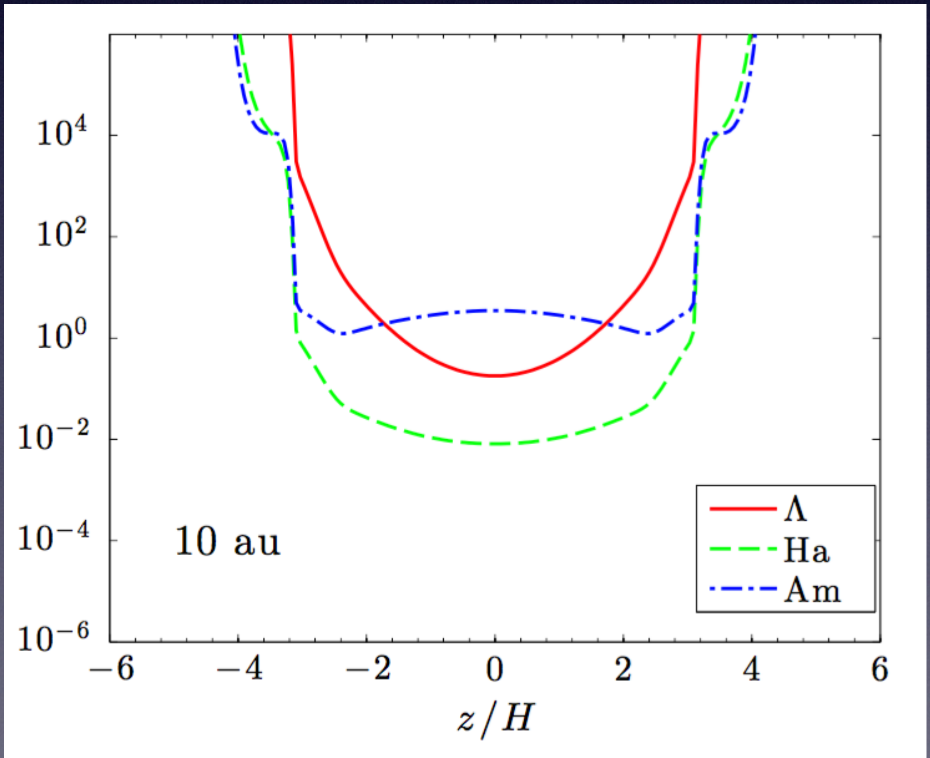


$$Am \equiv \frac{\nu_{ni}}{\Omega} = \frac{\text{neutral ion collision freq.}}{\text{rotation freq}}$$

MRI suppressed when Am ≲ 1

let η = isotropic viscous, resistive diffusion coef:

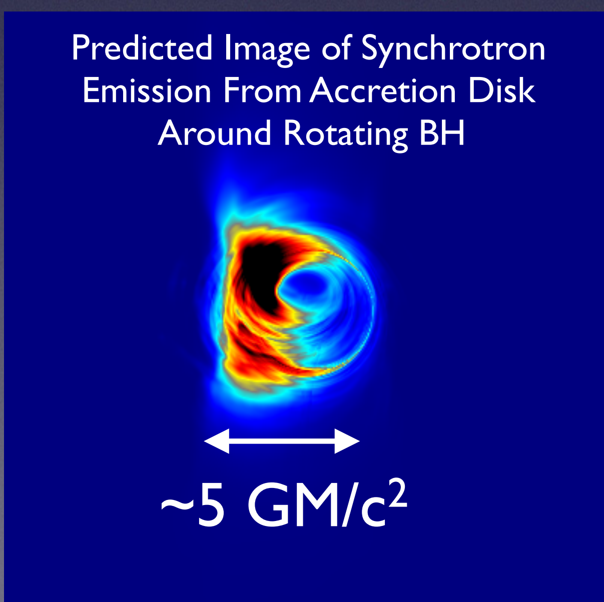
[η]= $cm^2\ s^{-1}$ viscosity, resistivity suppress MRI when

$k^2\eta \sim \Omega \quad kv_A \sim \Omega \quad \Rightarrow \quad \eta \gtrsim v_A^2/\Omega$, i.e., $\Lambda = v_A^2/\eta\Omega \lesssim 1$

MRI modified by the Hall effect when
$Ha = v_A^2/\eta_H\Omega \lesssim 1$, though rapid growth remains

Non-Ideal Effects on the MRI: Low Collisionality Plasmas

- Radiatively Inefficient Accretion Flows
  - At low densities (accretion rates), cooling is inefficient
  - kT ~ GMm_p/R (virial): $T_p \sim 10^{11\text{-}12}$ K $\gtrsim T_e \sim 10^{10\text{-}11}$ K near BH
  - collisionless plasma: e-p equil. time > inflow time for $\dot{M} \lesssim 10^{-2}\,\dot{M}_{Edd}$



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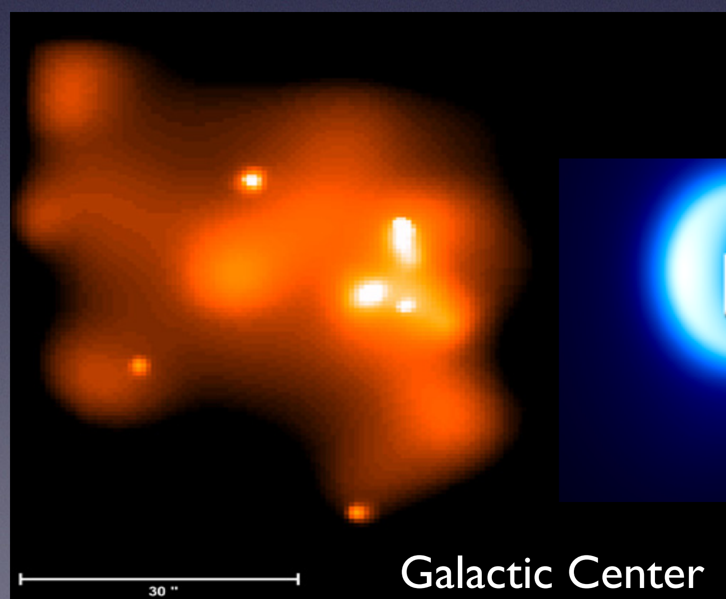
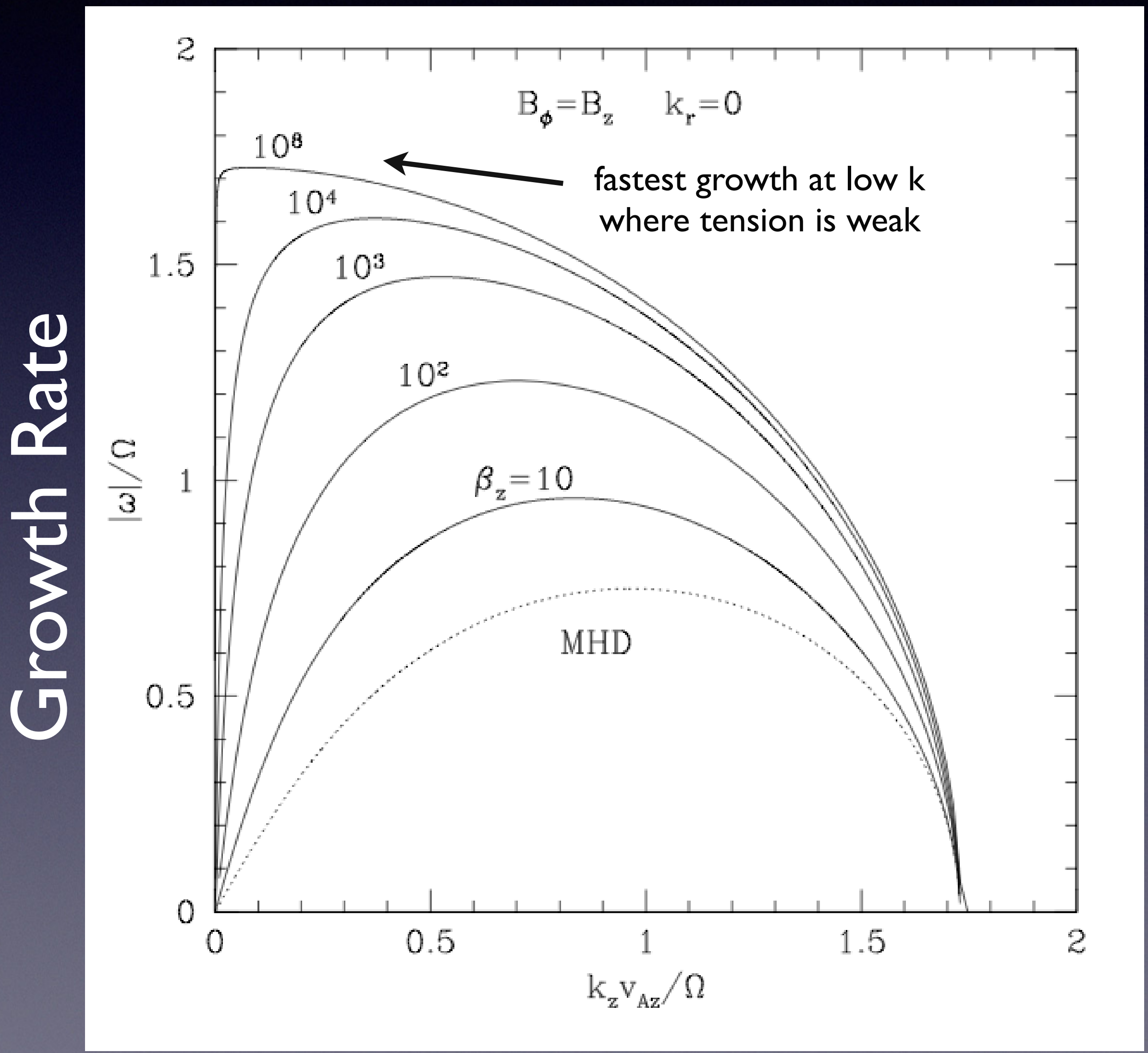


Event Horizon Telescope

Non-Ideal Effects on the MRI: Low Collisionality Plasmas



key non-ideal effects: anisotropic conduction and viscosity

$F_\phi \sim k_z \Delta p (b_z b_\phi)$

viscous torque transports angular momentum in addition to magnetic stress (dominates when $B^2 << p$)

anisotropic viscosity is **destabilizing**, unlike isotropic viscosity

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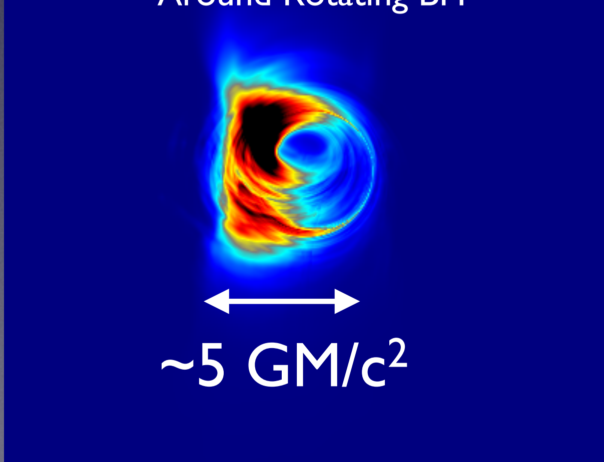
Predicted Image of Synchrotron Emission From Accretion Disk Around Rotating BH



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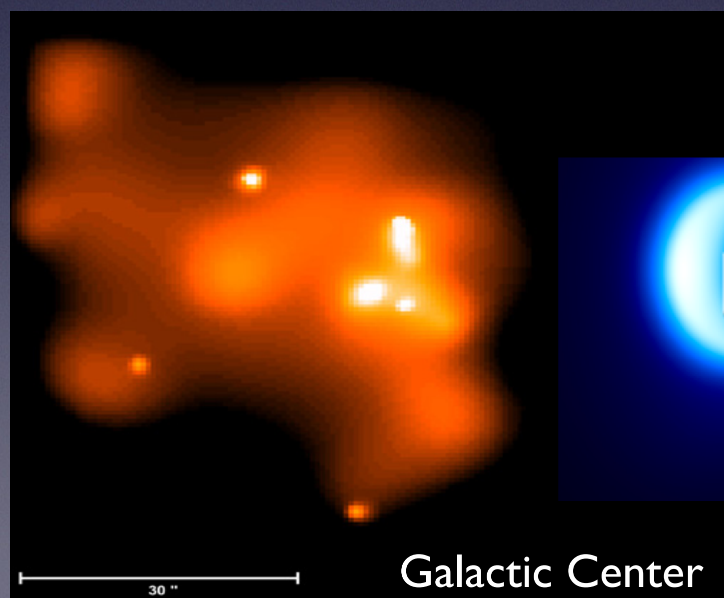

Galactic Center

EventHorizonTelescope

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