Selected Topics in Plasma Astrophysics

- Range of Astrophysical Plasmas and Relevant Techniques
- Stellar Winds (Lecture I)
  - Thermal, Radiation, and Magneto-Rotational Driven Winds
  - Connections to Other Areas of Astrophysical Fluids/Plasmas
- Instabilities In Ideal Fluids and Dilute Plasmas (Lecture II)
  - Ideal Fluid theory of Convection and MRI
  - How do Anisotropic Conduction & Viscosity Modify Convection and MRI
  - Astrophysical Context: Clusters and Accretion Disks
Instabilities In Ideal Fluids and Dilute Plasmas

• **Who Cares About Linear Theory? Let’s Simulate!**

• **Buoyancy Instabilities**
  - Hydrodynamic Convection
  - Convection Induced by Anisotropic Thermal Conduction
    - Important for the intracluster plasma in galaxy clusters

• **Instabilities Driven by Differential Rotation**
  - The Magnetorotational Instability (MRI)
  - Non-ideal Effects on the MRI
    - collisional fluids (e.g., protostellar disks)
    - low collisionality plasmas (e.g., hot accretion flows onto BHs)
Role of Linear Instabilities

- Extremely Instructive for Identifying Key Physics in Problems of Interest
  - can’t simulate everything; need to know what physics to include
- Produce Turbulent Transport of Mass, Momentum, Energy, B-Fields, …
  - accretion disks, stars, intracluster medium, …
  - physics of linear theory often imprinted on nonlinear state (buoyancy, B-tension …)
- Fundamentally Rearrange the Structure and Dynamics of the System
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Diversity of Astrophysical Plasmas

- Ideal Single Fluid (M)HD a Useful Starting Point for Astrophysical Plasmas
  - encapsulates mass, momentum, energy conservation; often does better than expected
- But Non-Ideal and Multi-Fluid Effects are Critical in many Systems

Star Formation, Planet Formation: Gas Cool, Dense, Largely Neutral (Multi-Fluid MHD + Dust)

Intracluster Plasma in Galaxy Clusters is Hot & Dilute (anisotropic conduction, viscosity, …)

Luminous Accreting Black Holes Radiation Pressure Dominated (2 fluid: radiation MHD)
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Hydrodynamic Convection

- Schwarzchild criterion for convection: $\frac{ds}{dz} < 0$
- Motions slow & adiabatic: pressure equil, $s \sim$ const
  
  Solar interior: $t_{\text{sound}} \sim \text{hr} \ll t_{\text{buoyancy}} \sim \text{month} \ll t_{\text{diffusion}} \sim 10^4 \text{ yr}$

Low entropy ($s$)

Gravity

High $s$

Background fluid

$s_{bg}', \rho_{bg}', p_{bg}'$

$s(p, \rho) \propto \ln[p/\rho^\gamma]$

If $\frac{ds}{dz} < 0 \rightarrow \rho_f < \rho_{bg}'$

Convectively unstable
What about Differences in Composition?
What about Differences in Composition?

- Schwarzschild criterion for convection: \( \frac{ds}{dz} < 0 \)

\[
p = \sum_j n_j kT \equiv \frac{\rho kT}{\mu m_p}
\]

\[
\frac{ds}{dz} = \frac{d \ln p}{dz} - \gamma \frac{d \ln \rho}{dz} = \frac{d \ln T}{dz} - (\gamma - 1) \frac{d \ln \rho}{dz} - \frac{d \ln \mu}{dz}
\]

\( d\mu/dz > 0 \) (heavy on top of light) is destabilizing (continuous version of Rayleigh-Taylor instability)

\( \mu = \text{mean molecular weight} \)
\( \mu = 1/2 \) (ionized H)
\( \mu = 4/3 \) (ionized He)
\( \mu = 0.62 \) (solar metallicity)
Impact of Isotropic (Photon) Diffusion on Convection in Stars

\[ t_{\text{diff}} \sim H^2/\ell c \sim \tau H/c \quad t_{\text{conv}} \gtrsim H/c_s \]

\[ t_{\text{diff}} \lesssim t_{\text{conv}} \text{ if } \tau \gtrsim c/c_s \Rightarrow \text{surface layers non-adiabatic} \]

low entropy (s)

gravity

high s

background fluid

\[ s'_{bg} \quad \rho'_{bg} \quad p'_{bg} \quad T'_{bg} \]

\[ \rightarrow \rho_f \approx \rho'_{bg} \]

buoyancy weakened by rapid isotropic diffusion
Radiation Hydro Sims of Convection in the Atmospheres of Massive Stars

Jiang+ 2015

3D radiation hydro sim of the surface of a massive star (color: density)

Time=187

3D radiation hydro sim of the surface of a massive star (color: density)

Convective flux (in units of radiative flux)

- StarDeep
- StarMid
- StarTop

$(z-z_0)/H_0$
Microscopic Energy Transport

- Photons dominate in non-degenerate dense plasmas w/ \( I_{\text{photon}} \ll \text{system size} \)
  - e.g., stars

  \[ \sigma_{\text{Coulomb}} \approx \frac{\pi e^4 \ln \Lambda}{(kT)^2} \approx 10^{-18} \left( \frac{T}{10^7 \text{ K}} \right)^{-2} \text{ cm}^2 \gg \sigma_{es} = \frac{8\pi}{3} \left( \frac{e^2}{m_e c^2} \right)^2 = 6.65 \times 10^{-25} \text{ cm}^2 \]

- Thermal conduction dominates in
  - degenerate plasmas: white dwarfs and neutron stars
    \[ l_e > \rho_e \text{ for } B \gtrsim 10^9 \text{ G (non – relativistic degenerate plasma)} \]
    - conduction typically \( \sim \) isotropic for WDs, but \( \sim \) anisotropic for NS surfaces
  - dilute, hot non-degenerate plasmas
    - e.g., solar corona & wind, clusters of galaxies, hot accretion flows onto black holes
    - \( l_e \gg \rho_e \Rightarrow \text{conduction highly anisotropic} \quad \kappa_\perp \approx \kappa_\parallel \left( \frac{\rho_e}{l_e} \right)^2 \ll \kappa_\parallel \)
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The Magnetothermal Instability (MTI)

- Weak B-field: no dynamical effect; only channels heat flow
- Thermal conduction time << buoyancy time
- Hot cold
- Convectively unstable ($dT/dz < 0$)
- Growth time $\sim$ dyn. time
- $T_f > T_{bg}$
- $\rho_f < \rho_{bg}'$
- $p_f = p_{bg}'$
- $T_f = T_i$
- $\rho_i = \rho_{bg}$
- $T_i = T_{bg}$
- $p_i = p_{bg}$
The Magnetothermal Instability (MTI)

instability saturates by generating sustained convection & amplifying the magnetic field (analogous to hydro convection)

B-field lines & Temp
The Heat Flux-Driven Buoyancy Instability (HBI)

pert to field tap into heat flux
⇒ conductive heating & cooling

for $dT/dz > 0$
upwardly displaced fluid heats up & rises, bends field more, ...

convectively unstable
The Heat Flux-Driven Buoyancy Instability (HBI)

magnetic field lines

cold
  ↓
hot
  ↓

initial heat flux

g

saturates by rearranging the magnetic field & suppressing heat flux through plasma
Role of Anisotropic Viscosity

- Anisotropic Conduction and Viscosity Come Together
  - conduction somewhat faster: $\tau_{\text{cond}} \sim (m_e/m_p)^{1/2} \tau_{\text{visc}}$ (electrons vs. protons)

\[
\Pi = -\Delta P \left[ \hat{b}\hat{b} - \frac{I}{3} \right] \quad \Delta P = \rho \nu_\parallel \frac{d}{dt} \ln \left( \frac{B^3}{\rho^2} \right)
\]

- in magnetized plasma, viscosity resists changes in magnetic field strength

- MTI: $\delta B = 0$  HBI: $\delta B \neq 0$ (simplest setups)
  - $\Rightarrow$ viscosity can suppress growth rates of HBI
Buoyancy Instabilities in Low-Collisionality Plasmas

MTI \( (dT/dz < 0) \)

HBI \( (dT/dz > 0) \)

A weakly magnetized plasma with anisotropic heat transport is always buoyantly unstable, independent of \( dT/dz \).

Instabilities suppressed by 1. strong B \( (\beta < 1\); e.g., solar corona) or 2. isotropic heat transport >> anisotropic heat transport (e.g., solar interior).
Hot Plasma in Galaxy Clusters

\[ L_x \sim 10^{43-46} \text{ erg s}^{-1} \]
\[ n \sim 10^{-4-1} \text{ cm}^{-3} \]
\[ T \sim 1-15 \text{ keV} \]
\[ M_{\text{gas}} \sim 10^{13-14} \text{ M}_\odot \]

large electron mean free path:

\[ \ell_e \sim 2 \left( \frac{T}{3 \text{ keV}} \right)^2 \left( \frac{n}{0.01 \text{ cm}^{-3}} \right)^{-1} \text{ kpc} \]

→ thermal conduction and viscosity are important
Cluster Entropy Profiles

Schwarzschild criterion $\Rightarrow$ clusters are buoyantly stable

Entropy $S$ [keV cm$^{-2}$]

Radius ($R_{vir}$)

$ds/dr > 0$
The entire cluster is convectively unstable, driven by anisotropic thermal conduction.

Important implications for the thermal evolution of clusters, cluster B-fields, cooling flows, …
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Accretion Disks

- Central to Planet, Star, & Galaxy Formation, Compact Objects
- Turbulence Generated by Linear Instabilities Transports Angular Momentum, Allowing Accretion to Proceed

Solar System Formed From a Thin ~ Co-planer Disk of Gas/Rocks
Local Instabilities Driven by Differential Rotation

- In Hydrodynamics, there exists a Linear Axisymmetric Instability if

\[ \kappa^2 \equiv \frac{1}{R^3} \frac{d}{dR} R^4 \Omega^2 < 0 \]

- \( \kappa \) = epicyclic frequency (\( \Omega \) for point mass)

**Assumed Equilibrium**

\[ \Omega^2 \approx \frac{1}{R} \frac{d\phi}{dR} \approx \frac{GM}{R^3} \]

\[ \frac{\partial}{\partial R} \int_{R}^{R'} \frac{\rho}{\rho} dR = \frac{2}{\rho} \int_{R}^{R'} \frac{\rho}{\rho} dR = \frac{2}{\rho} \left( \frac{\rho}{\rho} R^2 - \frac{\rho}{\rho} R' \right) \]

\[ \int_{R}^{R'} \frac{\rho}{\rho} dR = \frac{2}{\rho} \left( \frac{\rho}{\rho} R^2 - \frac{\rho}{\rho} R' \right) \]

**Gravity and Centrifugal Accelerations**

\[ a_{\text{gravity}} = \frac{GM}{R^2} \quad a_{\text{centrifugal}} = \Omega^2 R = \ell^2/R^3 \quad (\ell = R^2 \Omega) \]

\[ R \to R + \delta R \quad \Rightarrow \quad a_{\text{net}} = -\kappa^2 \delta R \quad \text{Unstable if} \quad \kappa^2 < 0 \]
axisymmetric nearly incompressible instability with weak $B_z$ ($\beta \gg 1$)
MRI in Ideal MHD

axisymmetric nearly incompressible instability with weak $B_z (\beta >> 1)$

$$|\omega_{\text{max}}| = 3/4 \Omega$$
$$k_{\text{max}}v_A = (15/16)^{1/2} \Omega$$
(Point Mass)

Stable Alfvenic Fluctuation (Slow Mode)
$$\omega \sim k_z v_A$$

Unstable (MRI)

$k_z v_A/\Omega$
$$\omega^2/\Omega^2$$

Unstable when Alfven freq $\sim$ Rotation Freq

$F_\phi = B \cdot \nabla B_\phi = iB_0 k_z \delta B_\phi$

tension redistributes angular momentum
Local Instabilities Driven by Differential Rotation

In MHD, there exists a Linear Axisymmetric Instability if $\frac{d\Omega^2}{dR} < 0$

Assumed Equilibrium: $\Omega^2 \approx \frac{1}{R} \frac{d\phi}{dR} \approx \frac{GM}{R^3}$

In MHD, there exists a Linear Axisymmetric Instability if $d\Omega^2/dR < 0$

Weak B-field

$\begin{align*}
R &\rightarrow R + \delta R \\
(\Omega = \text{const})
\end{align*}$

$\begin{align*}
a_{\text{gravity}} &= \frac{GM}{R^2} \\
a_{\text{centrifugal}} &= \Omega^2 R \\
a_{\text{net}} &= -\frac{d\Omega^2}{d\ln R} \delta R \\
&\text{Unstable if } d\Omega^2/dR < 0
\end{align*}$
Non-Ideal Effects on the MRI: Collisional Plasmas

key non-ideal effects: resistivity, viscosity, ambipolar diffusion, Hall effect

Non-ideal effects Critical in Protostellar Disks
(low temperatures and ionization fractions)
Non-Ideal Effects on the MRI: Collisional Plasmas

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Non-ideal effects Critical in Protostellar Disks (low temperatures and ionization fractions)

\[
Am = \frac{\nu_{ni}}{\Omega} = \text{neutral ion collision freq.} \over \text{rotation freq}
\]

MRI suppressed when \( Am \ll 1 \)

let \( \eta = \text{isotropic viscous, resistive diffusion coef.} \)

\([\eta]= cm^2 s^{-1} \) viscosity, resistivity suppress MRI when

\( k^2 \eta \sim \Omega \quad \kappa v_A \sim \Omega \quad \Rightarrow \quad \eta \gg v_A^2/\Omega, \text{ i.e., } \Lambda = v_A^2/\eta \Omega \ll 1 \)

MRI modified by the Hall effect when

\( Ha = v_A^2/\eta_H \Omega \ll 1 \), though rapid growth remains
Non-Ideal Effects on the MRI: Low Collisionality Plasmas

- Radiatively Inefficient Accretion Flows
- At low densities (accretion rates), cooling is inefficient
  \[ kT \sim GMm_p/R \text{ (virial)}: \quad T_p \sim 10^{11-12} \text{ K} \gtrsim T_e \sim 10^{10-11} \text{ K near BH} \]
- collisionless plasma: e-p equil. time > inflow time for \( \dot{M} \lesssim 10^{-2} \dot{M}_{\text{Edd}} \)

Predicted Image of Synchrotron Emission From Accretion Disk Around Rotating BH

\[ \sim 5 \text{ GM/c}^2 \]
Non-Ideal Effects on the MRI: Low Collisionality Plasmas

key non-ideal effects:
- anisotropic conduction and viscosity

\[ F_\phi \sim k_z \Delta p(b_z b_\phi) \]

viscous torque transports angular momentum in addition to magnetic stress (dominates when \( B^2 \ll p \))

anisotropic viscosity is **destabilizing**, unlike isotropic viscosity

fastest growth at low \( k \) where tension is weak
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