Solar wind

Dynamic model: spherical V = radial outflow velocity

$$\rho v \frac{dv}{dr} = -\frac{dp}{dr} - \rho \frac{GM}{r^2}$$

$$\nabla \cdot \rho \bar{v} = 0 \rightarrow r^2 \rho v = \text{const}$$

Eugene Parker 1958

$\dot{M} = 4\pi r^2 \rho v$ = mass loss rate (or mass accretion rate)

need energy eqn: let's take T = const. $\equiv$ for simplicity. not right but illustrates all of the key ideas

b.c. let r_0 = base of wind ~ R_*
$T_0 = T(r_0) = T(r)$
$\rho_0 = \rho(r_0)$ etc.

Comments:

1. Model describes ~~isothermal?~~ spherical steady outflow from $*$

   Eqns same if $v \rightarrow -v$

   $\therefore$ model also describes spherical steady accretion of matter by a point mass

   "Bondi" accretion $\rightarrow$ more later
   ~1952

2. real winds more complicated

   - high mass $*$'s radiation pressure & line driven winds
   - heating & accel. by waves (sound & MHD) impt. in sun & other $*$'s

   $\rightarrow$ need additional force in mom. eqn &

   T = const $\rightarrow \rho T \frac{ds}{dt}$ = heating term energy eqn

- too simplistic: real soln would match onto photosphere "r_0" & "ρ_0" a bit arbitrary

$$\frac{dP}{dr} = \frac{kT}{m}\frac{d\rho}{dr} = c_s^2 \frac{d\rho}{dr}$$ $c_s^2 = \frac{kT}{m} =$ isothermal sound speed = const.

$$v\frac{dv}{dr} = -c_s^2 \frac{d\ln\rho}{dr} - \frac{GM}{r^2}$$

$\frac{d\ln\rho}{dr} \to -\frac{2}{r} - \frac{1}{v}\frac{dv}{dr}$ from mass cons.

$$\boxed{\frac{1}{v}\frac{dv}{dr}\left(v^2 - c_s^2\right) = \frac{2c_s^2}{r} - \frac{GM}{r^2}}$$

isothermal wind/accretion eqn.

similar to nozzle eqn

$\exists$ special point where $v^2 = c_s^2$ called sonic point denote r_s

at r_s $\frac{2c_s^2}{r} = \frac{GM}{r^2}$ i.e., $\boxed{r_s = \frac{GM}{2c_s^2}}$

or dv/dr blows up

location of sonic pt.

L'Hopital's rule shows that at $r = r_s$

Wait

$$\left(\frac{dv}{dr}\right)_{r_s} = \pm\frac{2c_s^3}{GM}$$

symmetry of wind/accretion

wind eqn + $T = \text{const} = T_0$ + $\dot{M} = 4\pi r^2 \rho v = \text{const}$ specify our soln.

Consider soln of eqn w/ varying $V_0 \equiv v(r_0)$

r_0 = base of wind

(PP) shows topology of solns

focus on solns w/ $V_0 < c_s$ (physical) winds start nearly at rest

∃ 3 classes of solns.

~~1.~~ # 6 on curve unphysical double valued

~~2.~~ # 3 on curve $v < c_s \ \forall r$

⟶ hydrostatic models should be reasonably accurate

whenever $v \ll c_s$ hydrostatic equil. applies

$\rho \frac{v\,dv}{dr} \sim \frac{\rho v^2}{r}$ $\quad \frac{dp}{dr} \sim \frac{\rho c_s^2}{r}$

Note: these solns pass through $r = r_s$: have $\frac{dv}{dr} = 0$ (like nozzle result)

∴ still have $p(r \to \infty) \gg p_{ISM}$

pressure gradient accelerates flow until $v \to c_s$

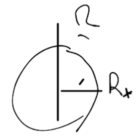
⟶ 3. # 1 on curve flow smooth accelerates through sonic point "transonic soln"

L'Hopital's rule

$\left.\frac{dv}{dr}\right|_{r_s} = \pm \frac{2c_s^3}{\ldots}$

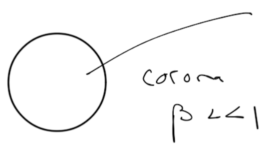
physical wind soln! flow continually

MHD winds

Rotating star.  In hydro $\dot{J} = \dot{M} R_*^2 \Omega$

spindown time = $\frac{J}{\dot{J}} \sim \frac{M}{\dot{M}}$

MHD fundamentally changes this



B-field tied to * & co-rotates
so long as $B^2/8\pi \gg \rho v^2, P_{gas}$
gas also co-rotates

$\Rightarrow$ ∡ mom of gas ↑ as it moves out

$v_\phi = R\Omega$ ↑ & $l = R^2 \Omega$ ↑

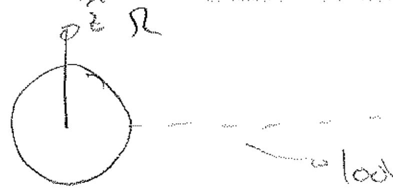
at expense of * rot KE & ∡ mom

continues until outflow not B dominated

say $\rho v^2 \sim B^2/8\pi$ (Alfven radius)

$\Rightarrow$ outflow & B stresses remove far more ∡ mom. & rot. KE than in hydro

Key to * spindown, jets, ∡ mom. ~~transport~~ in disks by large-scale B-fields full 3D theory hard: instead use "equatorial" wind i.e. axisymmetry



look at outflow here

assume $V_\theta = 0$, $B_\theta = 0$

+ equatorial symmetry $\Rightarrow V_\theta = B_\theta = 0$

$$\bar{V} = V_r(r)\,\hat{r} + V_\phi(r)\,\hat{\phi}$$

rotation of $*$ $\to V_\phi$

$$\bar{B} = B_r(r)\,\hat{r} + B_\phi(r)\,\hat{\phi}$$

Basic eqns: mass, mom, induction, EOS

mass: $\rho V_r r^2 = \text{const}$ $\qquad \nabla \cdot \bar{B} = 0 \Rightarrow r^2 B_r = \text{const}$

(see next page)

split monopole can't be true everywhere or else $\exists$ monopole

induction $\quad \dfrac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{V} \times \bar{B})$

$$\nabla \times (\bar{V} \times \bar{B}) = 0$$

$\bar{V} \times \bar{B}$ has no r, ϕ comp. only θ

writing out $\Rightarrow$ $\dfrac{d}{dr}\Big(r\,(V_r B_\phi - V_\phi B_r)\Big) = 0$

set $B_\phi = 0$ or $V_r = 0$

$V_\phi = R\Omega$

at "surface"

use $B_r R_*^2 = B_r r^2$

algebra $\Rightarrow$ $\boxed{\dfrac{B_\phi}{B_r} = \dfrac{V_\phi - r\Omega}{V_r}}$ $\quad$ field freezing

Int: solid body rotation $V_\phi = r\Omega \Rightarrow B_\phi = 0$

field is radial & gas + field corotate together at Ω

$E \times B$ drift

int: $$\boxed{\vec{V} = \kappa \bar{B} + r\Omega \hat{\phi}}$$ $\kappa = \text{const}$

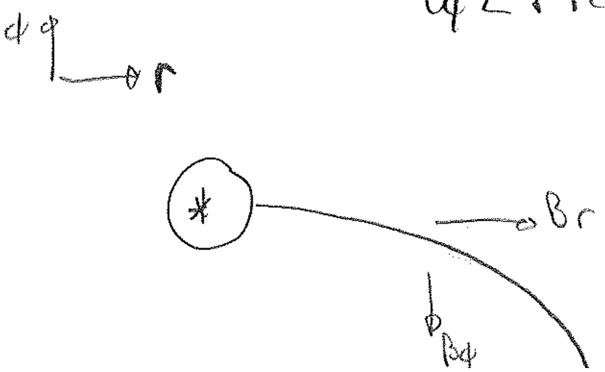
$$V_r = \kappa B_r$$
$$V_\phi = \kappa B_\phi + r\Omega$$

$$V_\phi = V_r \frac{B_\phi}{B_r} + r\Omega \Rightarrow \boxed{\frac{B_\phi}{B_r} = \frac{V_\phi - r\Omega}{V_r}}$$

in inertial frame, B is unchanging but gas rotating around w/ $r\Omega\hat{\phi}$ → due to $\bar{E}$ field → $\bar{E} \times \bar{B}$ drift. In addition to this particle can move along field lines freely ($\kappa \bar{B}$ part)

Notice solid body rotation $B_\phi = 0$ $V_\phi = r\Omega$
corotating magnetosphere

$V_\phi < r\Omega$ $B_\phi < 0$



gas "lags behind" corotation → drags field w/ it → get mom eqn: $(\bar{v}\cdot\bar{\nabla})\bar{v} = -\bar{\nabla}p - \frac{GM_*}{r^2}\hat{r} + \frac{1}{4\pi}(\bar{\nabla}\times\bar{B})\times\bar{B}$

$$B\text{-force} = -\frac{1}{4\pi}\left(\frac{B_\phi}{r}\frac{d}{dr}(rB_\phi)\right)\hat{r} + \frac{1}{4\pi}\left(\frac{B_r}{r}\frac{d}{dr}(rB_\phi)\right)\hat{\phi}$$

$\bar{B}\cdot\bar{\nabla}\bar{B}$
$= B_r\frac{d}{dr}B_\phi + \frac{B_\phi B_r}{r}$

radial accel.

pressure + tension

$\Rightarrow$ torque on outflow

(mag. tension)

ϕ comp. of momentum eqn. $\rightarrow$

$$\rho v_r \frac{d}{dr}(rv_\phi) = \frac{B_r}{4\pi}\frac{d}{dr}(rB_\phi)$$

lagrangian change in ∡ mom

$\rightarrow$ B torque

mult. by r^2 + integrate $\rho r^2 v_r$ + $r^2 B_r$ = const.

$$\Rightarrow \boxed{L = rv_\phi - \frac{rB_rB_\phi}{4\pi\rho v_r} = \text{const.}}$$

L = ∡ mom. per unit mass carried by outflow

$L = L_{gas} + L_{mag}$

— $B_\phi < 0$ so L_{mag} !

substitute in $\frac{B_\phi}{B_r} = \frac{u_\phi - r\Omega}{u_r}$ into

L = const eqn $\Rightarrow$ gets rid of B_ϕ in favor of B_r

$$\boxed{u_\phi = r\Omega \left(\frac{\frac{u_r^2 L}{r^2 \Omega} - V_{Ar}^2}{u_r^2 - V_{Ar}^2} \right) \qquad V_{Ar}^2 = \frac{B_r^2}{4\pi\rho}}$$

Note: $V_{Ar} \propto \frac{B_r}{\sqrt{\rho}} \sim \frac{1}{r}$

as $r \to \infty$

$B_r \propto r^{-2}$

$\rho \propto r^{-2}$

as flow radially accelerated, passes through critical pt. where $\boxed{u_r = V_{Ar}}$ Alfven pt. radius $r = r_A$

smooth transition requires Num $= 0$

$$\frac{u_r^2 L}{r_A^2 \Omega} = V_{Ar}^2 \Rightarrow \boxed{L = r_A^2 \Omega}$$

~~$\measuredangle$ momentum extracted from $\star$ as if gas corotates out to Alfven pt.~~

~~in reality doesn't quite corotate out to r_A but B stresses carry $\measuredangle$ momentum $\Rightarrow$~~

~~$L = r_A^2 \Omega$~~

w/ $L = r_A^2 \Omega$ in V_ϕ eqn.

$$V_\phi = r\Omega \left(\frac{V_r^2 \frac{r_A^2}{r^2} - V_{Ar}^2}{V_r^2 - V_{Ar}^2} \right)$$

2 limits

(2.) $r \to \infty$ $\quad V_{Ar} \to 0$

$V_r \to$ const

$\Rightarrow V_\phi \propto 1/r$ so, $\ell = r V_\phi =$ const

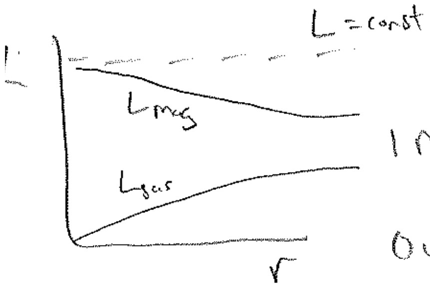
ang. mom. of gas conserved

B weak as $r \to \infty$ no torque

(1.) $r \to R_*$ $\quad V_r \to 0$

$V_\phi = r\Omega$ $\quad \Omega =$ const. Corotation

Impl.: $L = r_A^2 \Omega$ ang. mom. removed as if gas corotates out to Alfven pt.



In reality gas doesn't quite corotate out to r_A but B stresses carry ang. mom. $\Rightarrow L = r_A^2 \Omega$

$\sim 2/3 - 1$

$\Rightarrow$ B stresses dominate

$V_\phi = V_r (r \to \infty$

Implication: $\dot{J} = \dot{M} L = \dot{M} r_A^2 \Omega$ $\quad \frac{r_A^2}{R_*^2}$ larger than hydro model

$\therefore \quad \tau_J =$ spindown time $\approx \frac{J}{\dot{J}} = \frac{M R_*^2 \Omega}{\dot{M} r_A^2 \Omega}$

$$\boxed{\tau_J \approx \frac{M}{\dot{M}} \frac{R_*^2}{r_A^2}}$$

even if only a small amount of mass lost large lever arm out to $r_A >> R_*$

$\Rightarrow$ significant spindown

e.g. current solar models have $r_A = 30 R_*$

$\dot{M} \sim 2 \times 10^{-14} M_\odot \, yr^{-1}$

$\Rightarrow \tau_J \approx 5 \times 10^{10}$ years

But $\dot{M}$ larger in past so that spindown significant & ~ accounts for current slow rotation of sun

We've discussed in detail ℓ mom + rotation of wind: what about energetics?

in hydro $\exists$ conserved quantity

enthalpy $= \frac{\gamma}{\gamma-1}\frac{kT}{m}$

$$Be = \frac{1}{2}(v_r^2 + v_\phi^2) + h - \frac{GM_*}{r}$$

energy per unit mass flowing in r direction

In MHD must worry about B transport of energy

Poynting flux $\quad \bar{S} = \frac{c}{4\pi}(\bar{E}\times\bar{B})$ $\quad$ energy per unit area per unit time

$$\bar{E} = -\frac{\bar{v}\times\bar{B}}{c} \Rightarrow \bar{S} = \bar{B}\times(\bar{v}\times\bar{B})/4\pi$$

$$\bar{S} = \frac{\bar{v}B^2}{4\pi} - \frac{\bar{B}(\bar{v}\cdot\bar{B})}{4\pi}$$

Subs. in for $\bar{v}\cdot\bar{B}$

$\frac{v_\phi}{v_r}, \frac{B_\phi}{B_r}\ldots$

$$\Rightarrow \vec{S} = \underbrace{-\frac{r\Omega B_\phi B_r}{4\pi}\hat{r}}_{\text{key}} + \frac{r\Omega B_r^2}{4\pi}\hat{\phi}$$

$\therefore$ In MHD conserved radial energy flux is

$$F_r = (Be)r^2\rho v_r + r^2 S_r = \text{const.}$$

$$\text{"} \quad Be + \frac{S_r}{\rho v_r} = \text{const} \qquad \text{since } r^2\rho v_r = \text{const}$$

cons. energy per unit mass

$$\mathcal{E} \equiv \underbrace{\frac{1}{2}(v_r^2 + v_\phi^2) + h - \frac{GM_*}{r}}_{\mathcal{E}_{gas}} - \underbrace{\frac{r\Omega B_\phi B_r}{4\pi \rho v_r}}_{\mathcal{E}_{mag}} = \text{const}$$

$$\mathcal{E} = \mathcal{E}_{gas} + \mathcal{E}_{mag}$$

$$L = L_{gas} + L_{mag} = r v_\phi - \frac{r B_r B_\phi}{4\pi \rho v_r}$$

$$\mathcal{E}_{mag} = \Omega L_{mag}$$ — interp. energy is rot. KE of *

$\dot{E} = \frac{1}{2} I \dot{\Omega}^2 = I\Omega\dot{\Omega}$

$\dot{J} = I\dot{\Omega}$ $\therefore \dot{E} = \Omega \dot{J}$

energetics of wind:

hydro: $r \to 0$ $\mathcal{E} \simeq h - \frac{GM_*}{r}$

$r \to \infty$ $\mathcal{E} \simeq \frac{1}{2} v_r^2$

energy comes from thermal energy of hot gas close to * (+

MHD $\mathcal{E}_{mag} = \Omega L_{mag} \approx R_A^2 \Omega^2$ (rough bec. L_{mag} not const

2 kinds of MHD winds

$h(R_A) \approx a^2|_{R_A} >> R_A^2 \Omega^2$ → energetics same as hydro $\vec{B}$ impl. for spindown & ang mo but wind thermally

2. $R_*^2 \Omega^2 \gg h$ gas pressure irrelevant

magneto-centrifugal winds wind accelerated by $\vec{B}$-field

energy ultimately from rotational KE of $*$ → transfer of Poynting flux to bulk KE

solar wind firmly in class 1. B-field cont. ~ 9% of final KE (not nec. true early in life, ...)

rapidly rotating $*$'s, pulsars, jets from accretion disks all class 2. energy from rotation mediated by B-field

We now und. basic dynamics — energy &

& mom — of MHD wind.

can calculate v_ϕ, B_ϕ given v_r

Lastly need eqn. for v_r

Derive as in hydro — radial mom. eqn can be manipulated $\Rightarrow$ $\frac{dv_r}{dr}$ = algebraic no derivatives

(sonic pt. eqn. in hydro)

For MHD wind, result is $a^2 = \gamma \frac{kT}{m}$

$$\frac{r}{v_r}\frac{dv_r}{dr} = \frac{(v_r^2 - v_{Ar}^2)\left(2a^2 + v_\phi^2 - \frac{GM_*}{r}\right) + 2 v_r v_\phi v_{Ar} v_{A\phi}}{(v_r^2 - v_{Ar}^2)(v_r^2 - a^2) - v_r^2 v_{A\phi}^2}$$

Den $= 0$ when

$$v_r^4 - v_r^2 (\underbrace{v_A^2}_{v_{A\phi}^2 + v_{Ar}^2} + a^2) + v_{Ar}^2 a^2 = 0$$

2 solns:

$$v_r^2 = \frac{1}{2}(v_A^2 + a^2)\left(1 \pm \left(1 - \frac{4 v_{Ar}^2 a^2}{(v_A^2 + a^2)^2}\right)^{1/2}\right)$$

v_r^2 = radial phase speed of fast & slow magnetosonic waves

$\rightarrow$ 2 more critical pts. generalization of

Radiation Pressure Driven Winds

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP_{gas}}{dr} + \frac{1}{c}\kappa F - \frac{GM}{r^2}$$

$$\chi = \text{flux mean opacity}$$

$$= \frac{1}{F}\int \kappa_\nu F_\nu \, d\nu$$

$$F = L/4\pi r^2 \qquad \text{drop } P_{gas}$$

$$v\frac{dv}{dr} = \frac{\kappa}{c}\frac{L}{4\pi r^2} - \frac{GM}{r^2}$$

$$v\frac{dv}{dr} = \frac{\kappa}{c\,4\pi r^2}\left(L - L_{EDD}\right)$$

$4\pi r^2 \rho \times$

$$\dot{M}\frac{dv}{dr} = \frac{\kappa\rho}{c}\left(L - L_{EDD}\right)$$

τ = total optical depth through wind

$$\dot{M}V_\infty = \tau\frac{(L - L_{EDD})}{c} \simeq \tau L/c$$

Note: momentum flux <u>not</u> limited to L/c

multiple scatterings

max τ is when all of the photon energy is used to accelerate the wind

$$\frac{1}{2}\dot{M}V_\infty^2 \simeq \frac{\tau}{2}\frac{V_\infty}{c}L \lesssim L$$

$$\tau_{max} \sim c / V_{esc}$$

"photon tired"

WR stars $\dot{M}V_\infty \sim 10\text{s}\ L/c$ observed
& AGB " in multiple scattering regime

Line Driven Winds

In hydrostatic situation, net opacity due to lines set by # of lines & thermal broadening

expanding wind this changes

l = photon mfp = $1/\kappa\rho$

velocity difference across $l \simeq \left|\frac{dv}{dr}\right| l$

when $\left|\frac{dv}{dr}\right| l > v_{th}$ bulk Doppler shifts

enhance opacity by increasing line broadening

$$t \equiv \frac{v_{th}}{\kappa_e \left|\frac{dv}{dr}\right|} = \frac{\kappa_e \rho\, v_{th}}{|dv/dr|}$$

dimensionless measure of importance of bulk optical depths

$t \ll 1$ large Doppler shifts

$$F_{rad} \equiv \frac{\kappa_e F}{c} \overbrace{M(t)}^{\text{Force multiplier}}$$

often see $M(t) = k\, t^{-\alpha}$

$k \sim 0.1 - 1 \quad \alpha \sim 1/2$

depending on * T_{eff}

LTE non-LTE

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP_{gas}}{dr} - \frac{GM}{r^2} + \frac{\kappa F}{c} M(t)$$

$$t = \frac{\kappa_e \rho v_{th}}{\frac{dv}{dr}}$$

critical pt. structure diff't but

get wind soln w/

analogous derivations