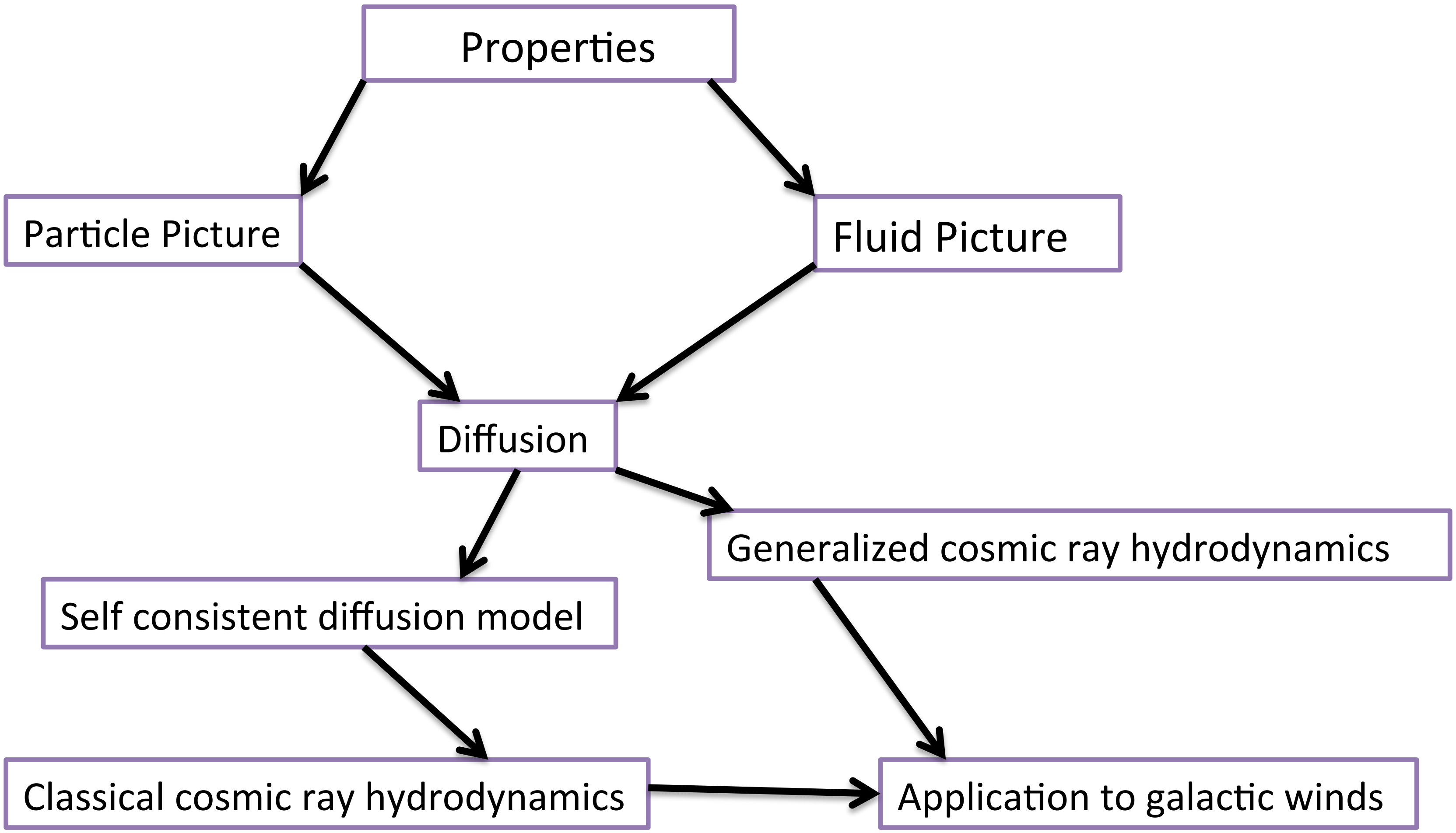
Introduction to Cosmic Rays II: From Kinetic to Fluid Scales

Ellen Zweibel

Plan of Lectures

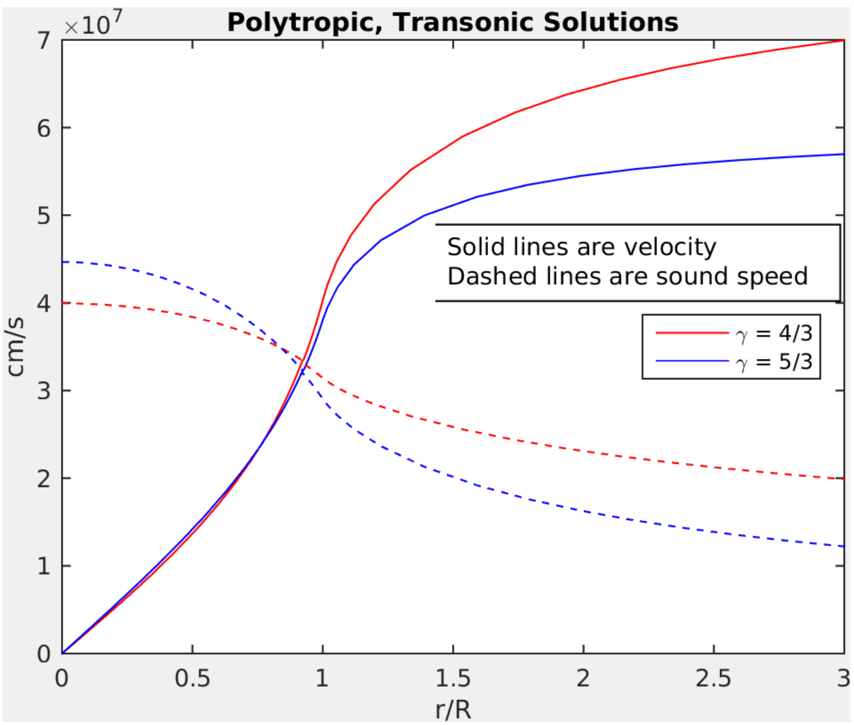


Outcomes of Fluid Theory

- Force on thermal gas due to cosmic rays.
- Transport of cosmic rays (advection & diffusion).
- Energy exchange between cosmic rays & background medium.
- *Fluid behavior will be justified through wave-particle interactions rather than direct particle-particle collisions.*

Originally done by H. Voelk & collaborators to describe nonlinear diffusive shock acceleration.

One Motivation for Including Cosmic Rays in Galactic Winds



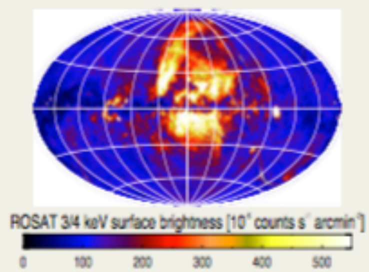
Cory Cotter & Chad Bustard

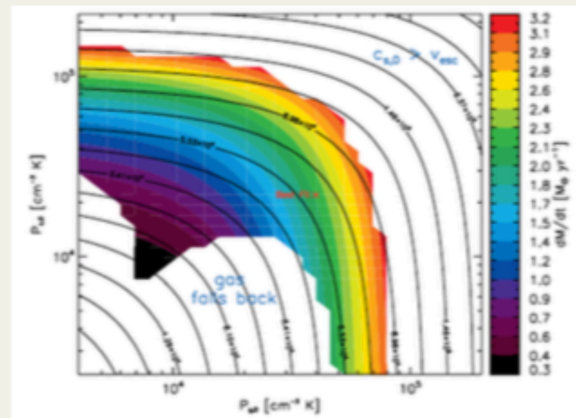
- Steady, spherically symmetric, pressure driven outflow *a la* Chevalier & Clegg 1985 but extended.

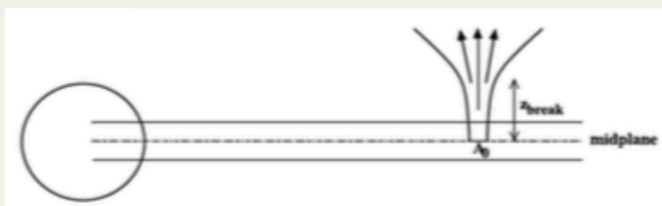
(Bustard, EZ, D'Onghia 2015)

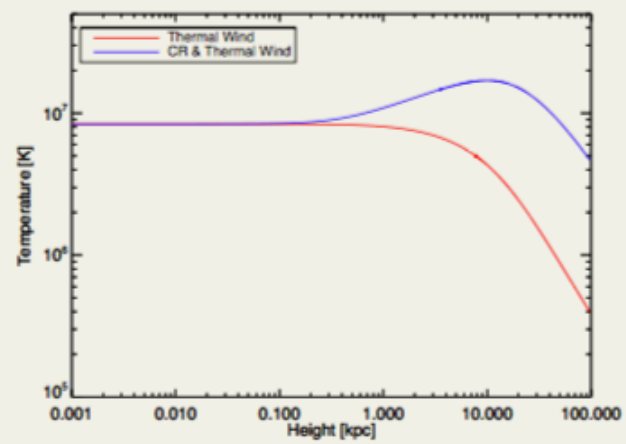
- Relativistic fluid cools more slowly & drives a faster wind.

Milky Way Wind Fits the data Better Than a Static Model







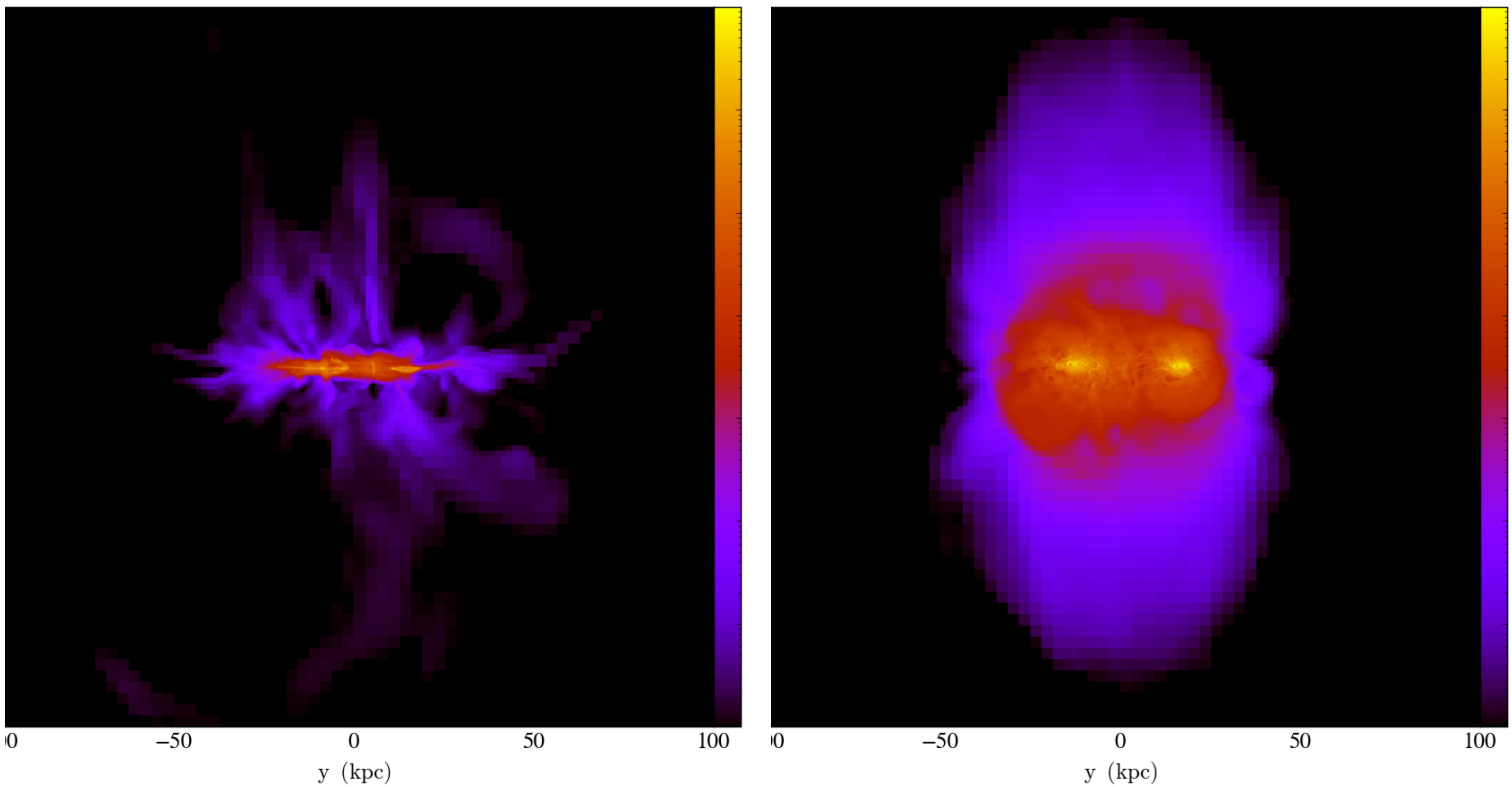


Top left: Soft x-ray sky,
Bottom left: Magnetic flux tube geometry.
Top right: Domains of flow, with mass loss rates
Bottom right: Gas temperature with & without cosmic ray heating.

Everett et al. 2008 ApJ

Transport at v_A relative to fluid, no diffusion (shown to be small).

Recent Simulations of a Star-Forming Disk…Investigation of Feedback



Left: Gas distribution for disk with Initially toroidal magnetic fiel &, no cosmic rays. Right: With cosmic rays. *Ruszkowski, Yang, EZ submitted to ApJ, on astro-ph.*

Cosmic Ray Treatment Matters

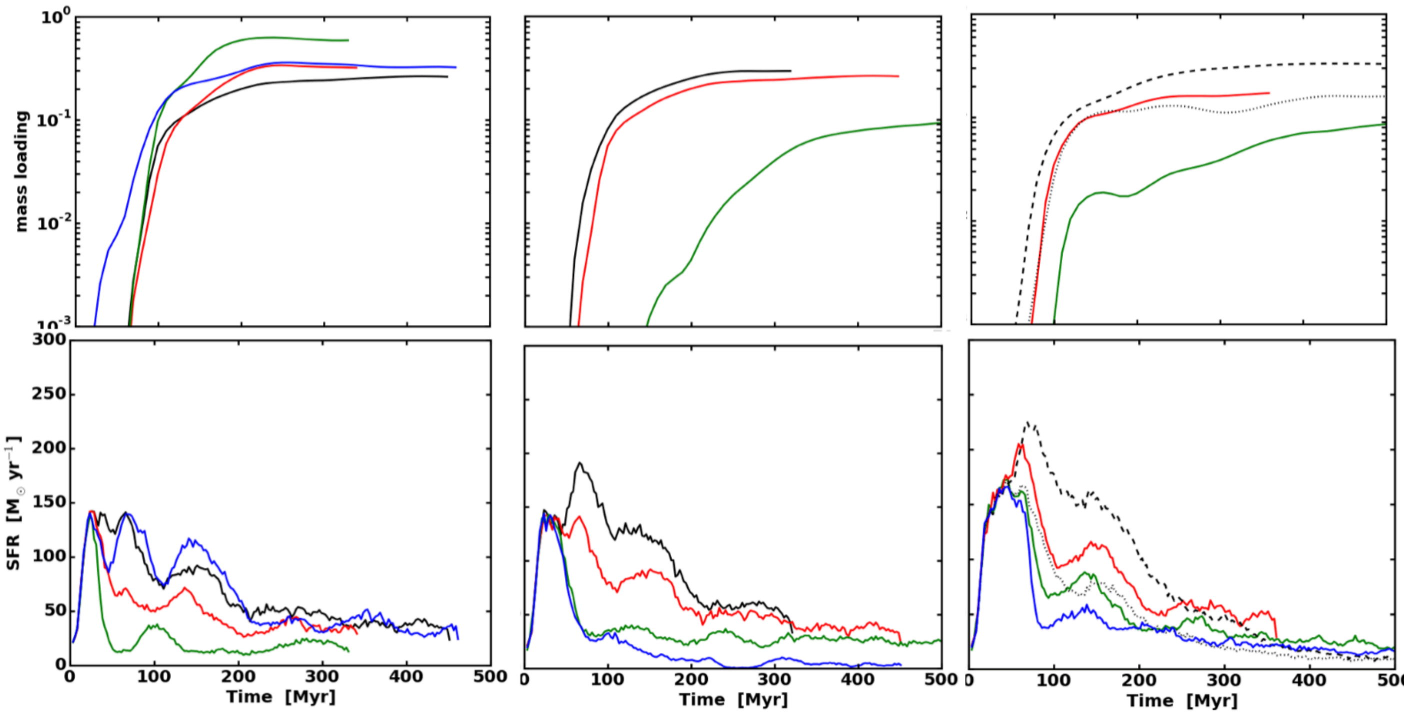


Figure 4. Galactic wind mass loading (top row) and star formation (bottom row). *Left* column (all cases for $f = 4$, SN feedback efficiency of $100 M_\odot$/SN): $f_{cr} = 0.1$ (black); $f_{cr} = 0.15$ (red); $f_{cr} = 0.3$ (green); $f_{cr} = 0.15$, $B_o = 3\mu$G (blue); *Middle* column (all cases for $f_{cr} = 0.1$, SN feedback efficiency of $100 M_\odot$/SN): $f = 8$ (black), $f = 4$ (red), $f = 1$ (green), $f = 0$ (blue); *Right* column (all cases for $f_{cr} = 0.1$, SN feedback efficiency of $185 M_\odot$/SN): $f = 3$ (red), $f = 1$ (green), $f = 0$ (blue), $\kappa_{||} = 10^{28}$cm^{2}s^{-1} (no streaming; dashed), $\kappa_{||} = 3\times 10^{27}$cm^{2}s^{-1} (no streaming; dotted). Note that the mass loading curves in the no-streaming cases ($f = 0$ cases) in the middle and right columns are not shown due to the absence of the wind.

Perpendicular Dynamics are Easy

Cosmic ray force balance:

$$\nabla_\perp P_c = \frac{\boldsymbol{J}_c \times \boldsymbol{B}}{c}$$

Lorentz force on thermal gas: $\boldsymbol{J}_g \times \boldsymbol{B}/c$

$$= \frac{\boldsymbol{J} \times \boldsymbol{B}}{c} - \frac{\boldsymbol{J}_c \times \boldsymbol{B}}{c}$$

Pressure gradient introduced through Lorentz force

$$= \frac{\boldsymbol{J} \times \boldsymbol{B}}{c} - \nabla_\perp P_c.$$

Parallel Dynamics are Subtle

Gist: Scattering transfers momentum and energy.

“Digression” on Waves

Can recover standard Alfven & magnetosonic waves this way, with collisional dissipation replaced by Landau and gyroresonant damping .

Wave with $\boldsymbol{E}_1$, $\boldsymbol{B}_1$, $\propto e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega t}$. From Faraday’s Law

$$\boldsymbol{B}_1 = \frac{c}{\omega}\boldsymbol{k} \times \boldsymbol{E}_1.$$

Solve linearized Vlasov equation for $f_{1\alpha}$ for each species α.

$$\frac{df_{1\alpha}}{dt} + q_\alpha\left(\boldsymbol{E}_1 + \frac{\boldsymbol{v} \times \boldsymbol{B}_1}{c}\right) \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f_{0\alpha} = 0.$$

Current density $\boldsymbol{j}_{1\alpha}$

$$\boldsymbol{j}_{1\alpha}(\boldsymbol{E}_1) \equiv q_\alpha \int \boldsymbol{v} f_{1\alpha}(\boldsymbol{E}_1) d^3v.$$

Dispersion relation from Ampere’s Law

$$(\omega^2 \boldsymbol{I} + c^2\boldsymbol{k} \times \boldsymbol{k}) \cdot \boldsymbol{E}_1 = -4\pi i\omega \sum_\alpha \boldsymbol{j}_{1\alpha}.$$

Cosmic Ray Streaming Instability

- Standard treatment *(standard streaming instability)*
  - Include thermal electrons, thermal ions, & cosmic ray ions.
  - Keep only the gyroresonant cosmic ray contribution.
- More general treatment
  - Keep entire cosmic ray contribution
  - Include additional thermal electrons to balance equilibrium cosmic ray current *(nonresonant instability).*

Gyroresonant Streaming Instability

$$\Gamma_{cr} = \frac{\pi^2 q^2}{2} \frac{v_A^2}{c^2} \sum_{\pm} \int \delta(\omega - kv\mu \pm \omega_c) v(1-\mu^2) \left[\frac{\partial f}{\partial p} + \left(\frac{kv}{\omega} - \mu \right) \frac{1}{p} \frac{\partial f}{\partial \mu} \right] p^2 dp d\mu,$$

resonance condition

damping

excitation by anisotropy

Simple approximation to the growth rate:

$$\Gamma_{cr} \sim C\omega_{ci} \frac{n_{cr}(> p_1)}{n_i} \left(\frac{v_D}{v_A} - 1 \right),$$

$$p_1 \equiv \frac{m\omega_c}{k}$$

Minimum cosmic ray momentum that can resonate with a given k.

Fokker – Planck (F-P) Equation

Back reaction of waves on zero order cosmic ray distribution function f_0

$$\frac{df_0}{dt} = -\left\langle \frac{q}{m}\left(\boldsymbol{E}_1 + \frac{v \times \boldsymbol{B}_1}{c}\right) \cdot \boldsymbol{\nabla}_p f_1 \right\rangle$$
$$= \boldsymbol{\nabla}_p \cdot \boldsymbol{D} \cdot \boldsymbol{\nabla}_p f_0.$$

Small angle Scattering by nearly periodic randomly phased waves

Pitch angle scattering ($D_{\mu\mu}$) dominates:

Scattering frequency $\nu \sim \omega_c(\delta B/B)^2$

$D_{p\mu} = D_{\mu p}$ are order (v_A/c) D_{pp} is order $(v_A/c)^2$

When Pitch Angle Scattering Dominates

$$\frac{\partial f_0}{\partial t} + \mu v \frac{\partial f_0}{\partial z} = \frac{\partial}{\partial \mu} \frac{\nu(1-\mu^2)}{2} \frac{\partial f}{\partial \mu}.$$

This has the same form as the Fokker-Planck equation we derived, but now there's a specific physical basis for it.

Heuristic Derivation of Diffusivity

- In large ν limit, f must be isotropic.

$$\mu v \frac{\partial f_0}{\partial z} = \frac{\partial}{\partial \mu} \frac{\nu(1-\mu^2)}{2} \frac{\partial f_1}{\partial \mu}$$

$$\frac{\partial f_1}{\partial \mu} = -\frac{v}{\nu} \frac{\partial f_0}{\partial z}.$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z} \frac{v^2}{3\nu} \frac{\partial f}{\partial z}.$$

(Ordering parameter based on scattering rate, not wave amplitude).

Energy Equation

Multiply F-P eqn. by particle energy ε &
integrate over momentum space:

$$\frac{\partial U_c}{\partial t} + \nabla \cdot \tilde{W}_c = -\int d\omega dk 2\Gamma_{cr}(\omega, k) I(\omega, k)$$

Energy density Energy flux Energy transfer to waves

Full equation…not equation for pitch angle scattering alone.

Frequent Scattering Approximation
Getting Energy Balance Right

Relate anisotropy to spatial gradient:

$$D_{\mu\mu}\frac{\partial f_0}{\partial \mu} + D_{\mu p}\frac{\partial f_0}{\partial p} = -\frac{v(1-\mu^2)}{2}\frac{\partial f_0}{\partial z}$$

Lets us write Γ_{cr} in terms of density gradient.

Energy equation simplifies to:

$$\frac{\partial U_c}{\partial t} + \nabla \cdot \tilde{\boldsymbol{W}}_c = \boldsymbol{v}_A \cdot \nabla P_c.$$

"frictional heating"

Equation for Waves

From Dewar’s theory:

$$\frac{\partial}{\partial t}\frac{\delta B^2}{4\pi} = -\boldsymbol{\nabla}\cdot\boldsymbol{W}_w + \boldsymbol{u}\cdot\boldsymbol{\nabla}\frac{\delta B^2}{8\pi} - \boldsymbol{v}_A\cdot\boldsymbol{\nabla}P_{cr} - G.$$

$$\boldsymbol{W}_w \equiv \frac{\delta B^2}{4\pi}\left(\boldsymbol{v}_A + \frac{3}{2}\boldsymbol{u}\right)$$

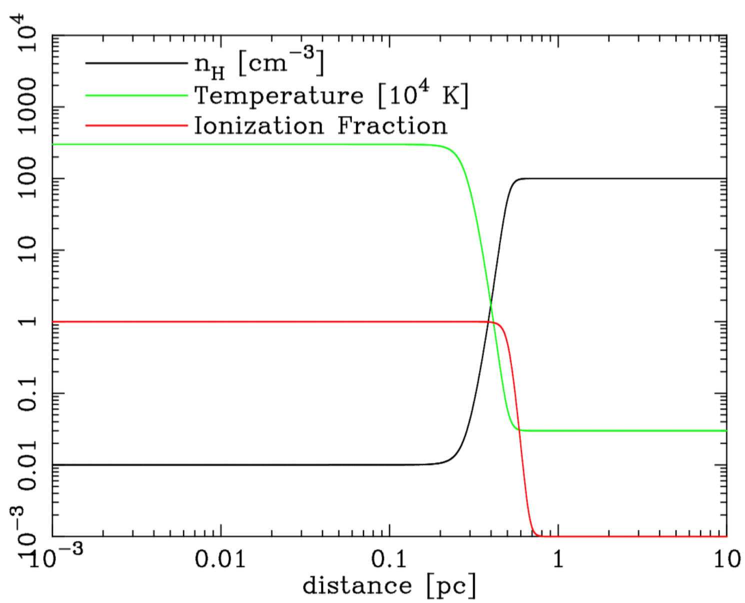
Adiabatic terms

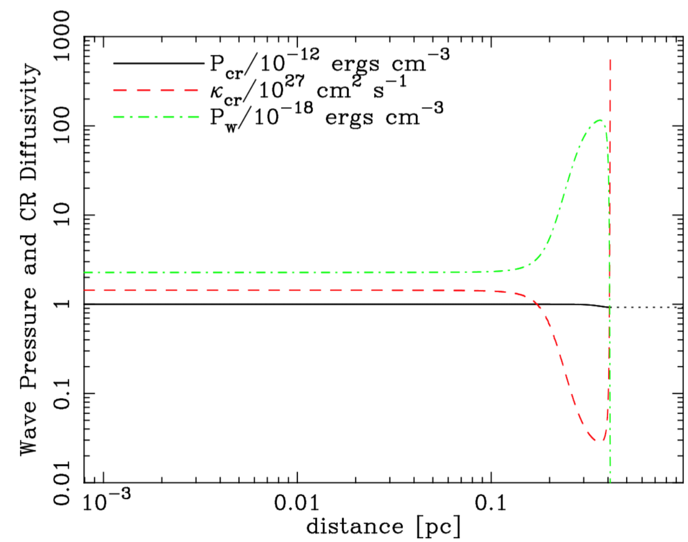
Driving & dissipation

Heating rate

$$\rho\frac{dQ}{dt} = G.$$

Joint Solution for Waves & Cosmic Rays





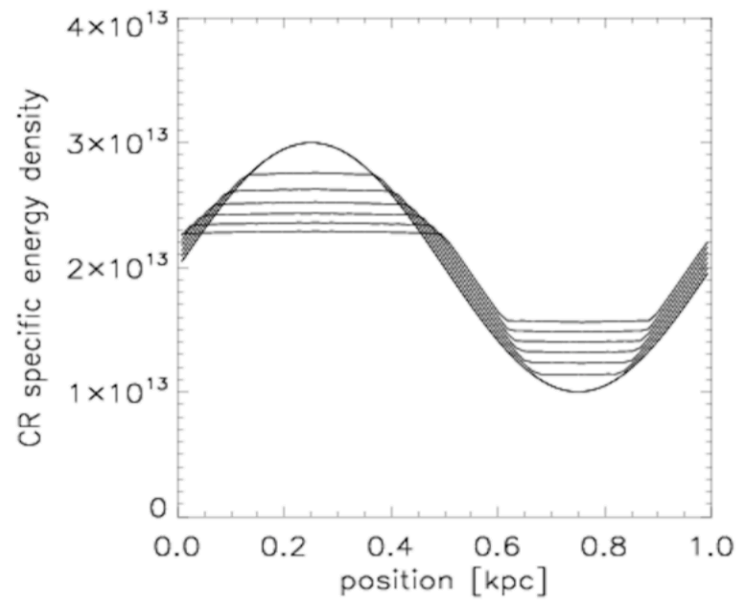
Penetration of cosmic rays into a partially ionized cloud, from Everett & EZ 2011.

Fluid Treatment

- "Classical Cosmic Ray Hydrodynamics (CCRH)
  - Equations developed by Volk & collaborators based on self-confinement model
  - Stream down pressure gradient at $\mathbf{v_A}$ relative to thermal gas *(care needed in implementing this).*
  - Transfer momentum through pressure gradient
  - Heat gas at $-\mathbf{v_A}$ **grad** P_c.
  - Diffusion along B with diffusivity $\kappa \sim v^2/\nu$
- Applied to shocks, galactic winds, ISM heating, intracluster medium.

Implementation of Streaming

- Cosmic rays excite waves which propagate down the cosmic ray density gradient.
- Regularized implementation by Sharma et al:



Model equation:

$$\frac{\partial f}{\partial t} - \frac{\partial}{\partial x}(sgn(\frac{\partial f}{\partial x})f) = 0.$$

Replace $sgn(\partial f/\partial x)$ with $\tanh\left(\epsilon^{-1}\partial f/\partial x\right)$.

The Bottleneck Effect

- Predicted by Skilling in 1971
- When the cosmic rays are perfectly locked to the waves,

$$P_{cr} v_A^{\gamma c}$$

is constant.

- If v_A decreases, P_{cr} should increase, implying the cosmic rays stream up their density gradient.

Numerical Validation

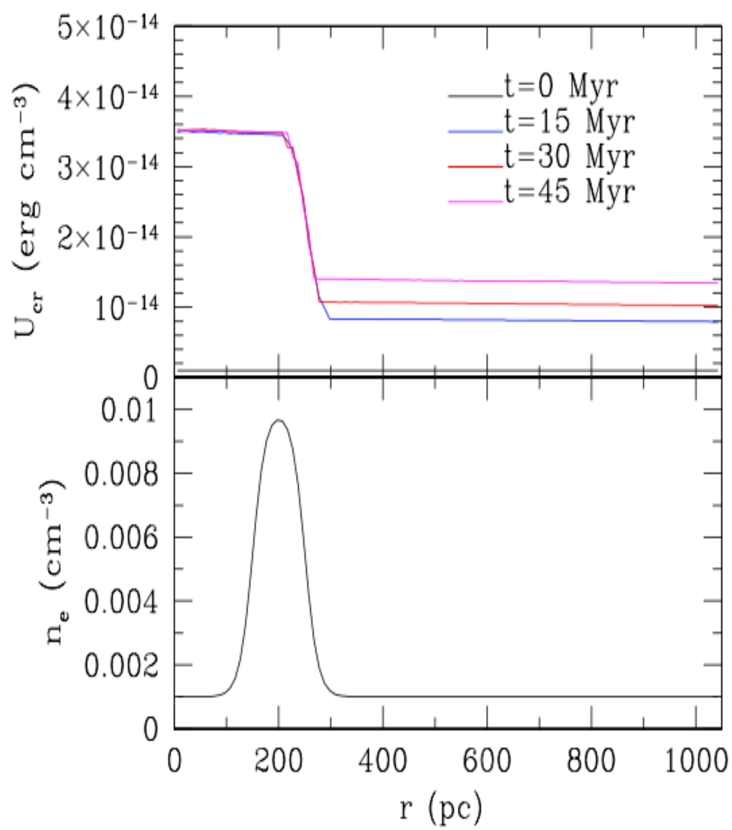


Figure 3. A simple test case. The setup is the same as figure 2, but the gas density is flat at the boundary.

- From Wiener, Oh, & EZ, to be submitted. A cosmic ray source is turned on to the left of a "cloud" with constant B & a density maximum. As predicted, P_{cr} goes flat to the left of the density maximum.

Constrained Diffusion

We saw that for frequent scattering we can relate anisotropy to spatial gradients:

$$D_{\mu\mu}\frac{\partial f_0}{\partial \mu} + D_{\mu p}\frac{\partial f_0}{\partial p} = -\frac{v(1-\mu^2)}{2}\frac{\partial f_0}{\partial z}$$

We can then write the streaming instability growth rate Γ_{cr} in terms of the spatial gradient. But Γ_{cr} should be balanced by Γ_d, the wave damping rate.

The main damping mechanisms are ion-neutral frictional damping, nonlinear Landau damping, & turbulent damping.

Wave Damping

- **Ion-neutral friction**. Tends to wipe out waves in partially ionized gas.
- **Nonlinear Landau Damping**: thermal ions resonantly absorb energy from wave packets. Important in hot gas with β not too small.
- **Turbulent Damping:** Shearing of wave packets by magnetic curvature or turbulence. Not rigorously calculated yet but estimated to be largish.

Upshot: selfconfinement in the Milky Way only works below ~ 100 – 200 GeV.

The f Parameter

- Set streaming velocity to f vA.
- Compute f by balancing wave damping and wave growth.
- Replace diffusion term in transport equation with superalfvenic streaming term.
- *Heuristically, adding a constraint lets us reduce the order of the differential operator, but this is not well tested.*

Convection – Diffusion Equation

$$\frac{\partial f}{\partial t} + \mathbf{u}\cdot\nabla f = \frac{\nabla\cdot\mathbf{u}}{3}p\frac{\partial f}{\partial p} + \nabla\cdot D_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}}\nabla f + \frac{1}{p^2}\frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial f}{\partial p}$$

Replace by something independent of grad *f*, making equation first order instead of second order. It is therefore OK to eliminate diffusion & replace v_A by fv_A, $f > 1$.

Derive a pressure equation by multiplying by pv & integrating over momentum space.

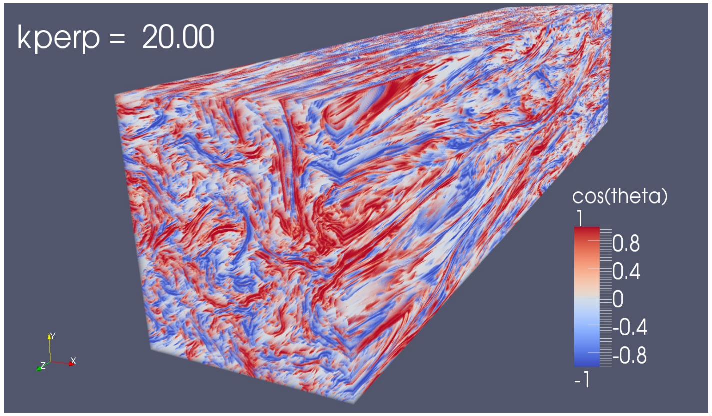
Approximations & Improvisations

- Include grad P_c, advect with fluid, ignore heating, diffusion if any is isotropic.
  - No magnetic field calculation necessary *(implicitly stochastic on gyroradius scale).*
  - No need to ensure streaming is down **grad** P_c.
- Include streaming relative to thermal gas & frictional heating, but replace v_A by thermal sound speed v_S.
  - Same advantages as previous bullet.

These are the main variants in the literature

Generalized Cosmic Ray Hydrodynamics (GCRH)

- Account for non-cosmic ray sources of waves.



MHD turbulence, Boldyrev group

- Generalize F-P equation to include waves traveling in both directions.

Wave Evolution Equations

$$\frac{\partial}{\partial t}\frac{\delta B^{2,\pm}}{4\pi} = -\boldsymbol{\nabla}\cdot\boldsymbol{W}_w^{\pm} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\frac{\delta B^{2,\pm}}{8\pi} \mp \frac{\nu_\pm}{\nu_+ + \nu_-}\boldsymbol{v}_A\cdot\boldsymbol{\nabla}P_{cr} - G^{\pm} + L^{\pm}.$$

$$\boldsymbol{W}_w^{\pm} \equiv \frac{\delta B^{2,\pm}}{4\pi}\left(\pm\boldsymbol{v}_A + \frac{3}{2}\boldsymbol{u}\right);$$

cosmic ray
driving/damping

extrinsic
driver

From Fokker-Planck Equation

- Composite streaming velocity

$$\boldsymbol{w} \equiv \frac{\nu_+ - \nu_-}{\nu_+ + \nu_-}\boldsymbol{v}_A$$

- Pressure gradient force is unchanged

Balance Driving & Damping

Simple model

$$\tau_A \equiv -\frac{P_{cr}}{\boldsymbol{v}_A \cdot \boldsymbol{\nabla} P_{cr}},$$

$$\frac{E_+}{E_+ + E_-}\frac{P_{cr}}{\tau_A} - 2\Gamma E_+ + \frac{\dot{E}}{2} = 0,$$

$$-\frac{E_-}{E_+ + E_-}\frac{P_{cr}}{\tau_A} - 2\Gamma E_- + \frac{\dot{E}}{2} = 0.$$

This is easily solved

Transport Velocity

$$w = \frac{v_A}{x + \sqrt{1 + x^2}}$$

$$x \equiv \frac{\dot{E}\tau_A}{2P_c}$$

w -> 0 when external driving dominates
w -> v_A when cosmic ray driving dominates

Cosmic ray heating is reduced but compensated by turbulent damping

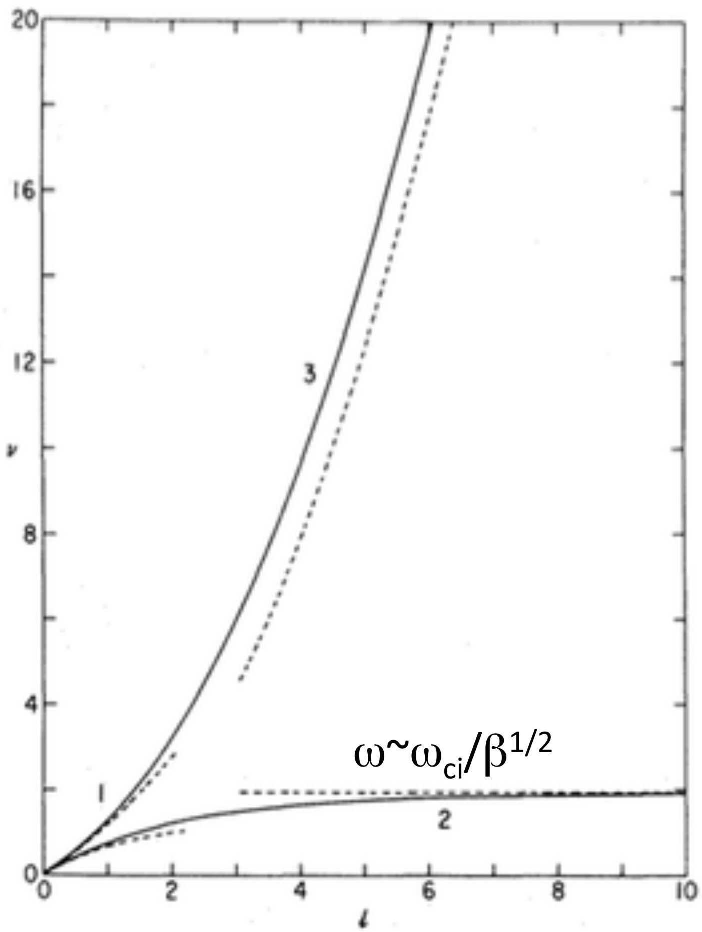
Extrinsic Turbulence Model

- Advect cosmic rays at the fluid speed v.
- Neglect cosmic ray heating.
- Retain pressure gradient force.

All hold in the limit of strongly driven, balanced turbulence.

- But, Alfven turbulence produces anisotropic, field aligned diffusion.
- Isotropic diffusion is produced by a small scale, stochastic field.

Beyond Alfven Waves – High β



- For $\beta = P_G/P_M >> 1$

- Affects waves which scatter cosmic rays with $\mu > \mu_c$

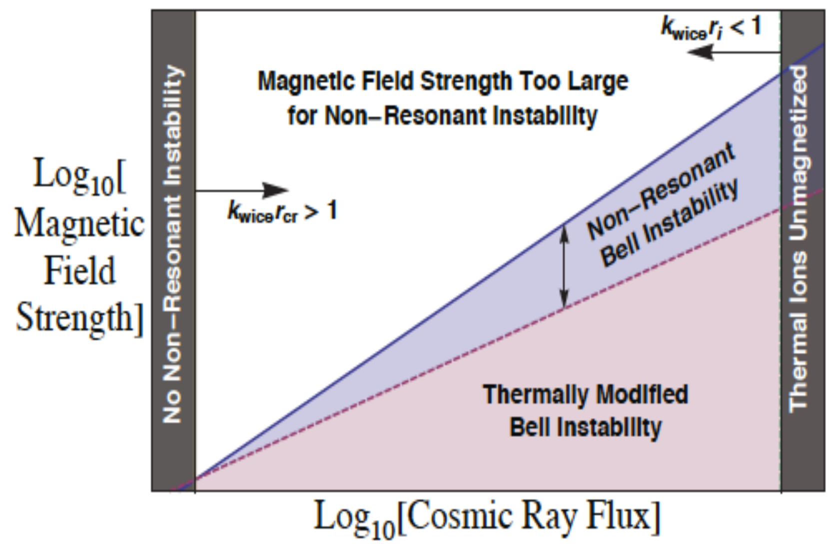
$$\mu_c \sim \frac{v_i}{c}\beta^{1/2}.$$

- Demands very weak fields, e.g B < 10^{-12}G in galaxy clusters.

Enforces sub-Alfvenic streaming

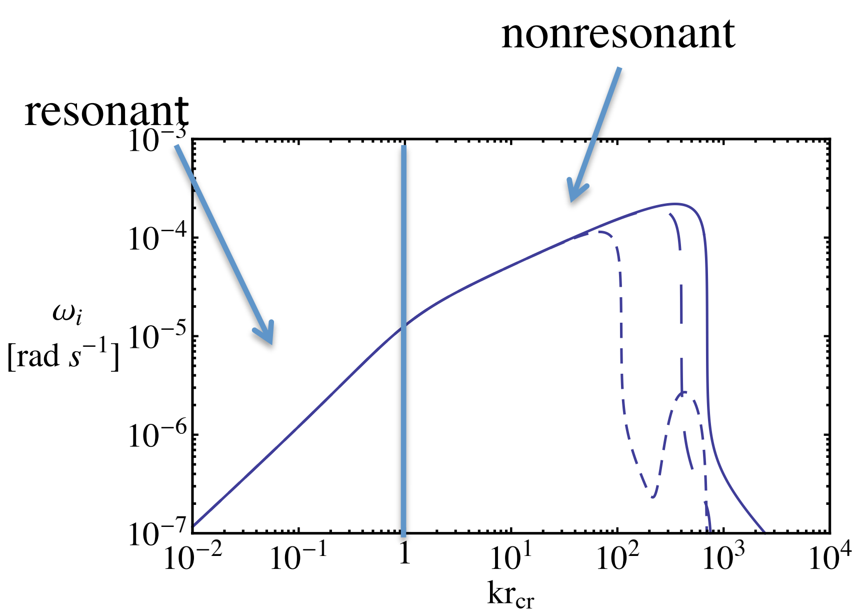
Nonresonant Instabilities

- When $U_{cr}/U_B > c/v_D$ there is a nonresonant instability driven by the electron current that compensates the cosmic ray current *(keep the nonresonant cosmic rays in the dispersion relation)*.
- Conditions are met at shocks, and possibly in young galaxies.



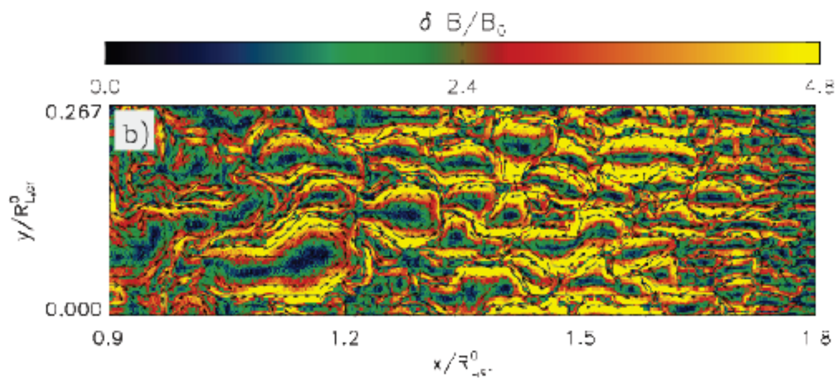
Everett & EZ 2010

Rapid Growth to Nonlinear Amplitude



Linear growth rates for T = 10^3 (solid), 10^4(long dash) & 10^7 (short dash). Zweibel & Everett 2010.

PIC simulation showing magnetic field growth in a shock layer.



Riquelme & Spitkovsky 2010

Beyond Alfven Speed

- Resonant instabilities enforce sub-Alfvenic streaming & require extremely weak fields.
- Nonresonant instabilities require large cosmic ray fluxes and/or weak magnetic fields.
  - Growth rates comparable to frequency require nonstandard treatments
  - Could be very interesting in weak field situations.

Links in the Chain of Feedback

- Good model for the ISM, including its magnetic field & turbulence properties.
- Star formation rate -> supernova rate -> cosmic ray acceleration rate.
- Model for cosmic ray coupling under a variety of ISM conditions.
- Self consistent model of ISM and outflow, if driven, that includes cosmic rays.

Bonuses

- Effect of cosmic rays on circumgalactic, intragroup, intracluster, & intergalactic medium.
  - Heating
  - Magnetization
- Ability to calculate radio & γ-ray spectra, test flow models against observed properties.

Summary

- Cosmic rays appear in diffuse plasmas everywhere, in defiance of thermodynamics.
- They exchange momentum and energy with the background medium, mediated by magnetic fields.
- Advances in observation, computation, & experiment make this a wonderful time to study their acceleration, transport, and feedback.