The Strong CP Problem and Its Implications

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Lecture 1: The Strong CP Problem
Lecture 2: Solutions of the Strong CP Problem: An Assessment
Lecture 3: Axion Cosmology and Axion Searches: Old and New Ideas
The lectures will be self contained. But some suggested reading:
Suggested Reading: Supersymmetry and String Theory: Beyond the Standard Model by M. Dine, sections 5.4, 5.5, 19.2
There are many recent reviews. Some examples:
J. Kim:
https://arxiv.org/abs/1703.03114
D. Marsh:
https://arxiv.org/abs/1510.07633
Kawasaki et al:
https://arxiv.org/abs/1301.1123
P. Sikivie:
https://arxiv.org/abs/astro-ph/0610440
Helpful to do some review of anomalies. Choose your favorite textbook (Peskin and Schroder, Schwarz, Dine)

Lecture 1: The Strong CP Problem

1. Our understanding of QCD: Lagrangian parameters, symmetries
2. Anomalies; CP violating parameters, U(1) problem and their connections
3. Instantons as an indicator of θ dependence and consequences of the anomaly
4. Consequences of θ-dependence: the Neutron electric Dipole moment.
5. Possible Solutions

Our present understanding of the Strong Interactions

QCD: very simple lagrangian, governed by symmetries and particle content:

$$\mathcal{L} = -\frac{1}{4g^2}(F^a_{\mu\nu})^2 + \sum_f \bar{q}_f(i\not{D} - m_f)q_f \quad (1)$$

Here $F_{\mu\nu}$ is the field strength of QCD,q_f the various quark flavors.

Extremely successful.

- At very high energies, detailed, precise predictions, well verified at Tevatron and LHC (dramatically in the context of Higgs discovery.
- At low energies, precision studies with lattice gauge theory.

QCD Parameters from Lattice Gauge Theory

The parameters of the theory: α_s at some scale; quark masses. Current results from lattice simulations (summarized by the FLAG working group)

$$m_u = 2.16\ (9)(7)\mathrm{MeV}\quad m_d = 4.68\ (14)(7)\mathrm{MeV}\, m_s = 93.5(2.5)\mathrm{MeV} \tag{2}$$

(Numbers are in $\overline{MS}$ scheme at 2 GeV.)

$\alpha_s(M_Z) = 0.1192 \pm 0.0011$ from lattice gauge theory; compatible with other measurements.

Would seem we know all we could want to know

Symmetries of QCD

The u, d, and to a lesser extent the s quarks are light. Working with left-handed fields,

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad \bar{q} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix} \tag{3}$$

For zero quark mass, the theory has symmetry $U(3) \times U(3)$:

$$q \to Uq; \quad \bar{q} \to V\bar{q} \tag{4}$$

The symmetry is spontaneously broken to a vector subgroup, $SU(3) \times U(1)_B$:

$$\langle \bar{q}_{\bar{f}} q_f \rangle = a\Lambda^3 \delta_{\bar{f}f} \tag{5}$$

Goldstone bosons: π, K, η (eight particles). These particles behave as expected for GB's ("current algebra").

Where's the ninth? The η' is much heavier than the others. E.g.

$$m'_\eta = 958 \text{ MeV}; \; m_\eta = 548 \text{ MeV} \tag{6}$$

One feature of the axial $U(1)$ current which might account for this: it is not strictly conserved. $q \to e^{i\alpha q} \quad \bar{q} \to e^{i\alpha q}$ In four component language,

$$j_5^\mu = \bar{q}\gamma^\mu\gamma^5 q; \quad \partial_\mu j_5^\mu = \frac{N_f}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{7}$$

Here $\tilde{F}$ (“F-dual") is

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}; \quad F\tilde{F} = 2\vec{E}\cdot\vec{B}. \tag{8}$$

The right hand side is a total derivative, $\frac{1}{16\pi^2}\partial_\mu K^\mu$

$$K_\mu = \epsilon_{\mu\nu\rho\sigma}\left(A^a_\nu F^a_{\rho\sigma} - \frac{2}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma\right) \tag{9}$$

So one can define a new conserved current,

$$\tilde{j}^\mu = j^\mu_5 - \frac{N_f}{16\pi^2} K^\mu \tag{10}$$

But this current is not gauge invariant, so its status requires investigation. We'll say much more about it shortly, but the charge associated with this current is not conserved. Configurations with $A^\mu \sim \frac{1}{r}$ are already important in a semi-classical treatment. So there is no mystery in the absence of a ninth Goldstone boson.

Strong CP Problem

But the assertion that $F\tilde{F}$ solves the $U(1)$ problem raises a puzzle: one can now add to the QCD lagrangian

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2} F\tilde{F}. \tag{11}$$

This term violates *parity*, and thus CP.

Again, while

$$F\tilde{F} = \partial_\mu K^\mu; \tag{12}$$

K^μ is not gauge invariant. If there are important configurations, say, in the path integral, for which the A^3 term falls off as $1/r^3$, then we can't drop the surface term. Examples of such configurations will be discussed shortly (instantons).

The possibility of a non-zero θ is related to another possible source of *CP* violation. In writing the QCD Lagrangian, we did not commit to two or four component fermions. In the language of two component fermions, we might have taken complex masses. With four component fermions, we might have included $\bar{q}\gamma_5 q$ terms in the quark lagrangian.

One might have said: what's the big deal? If we had written (two component form)

$$\bar{q}_f m_{\bar{f}f} q_f + \mathrm{c.c.} \tag{13}$$

we could, by separate $U(3)$ transformations of the q and $\bar{q}$ fields, have rendered m (the quark mass matrix) diagonal and real.

But we have just asserted that this redefinition has an anomaly. Indeed, if we think about the effect of such a transformation on the lagrangian,

$$\delta\mathcal{L} = \alpha\partial_\mu j_5^\mu = \alpha\frac{1}{16\pi^2}F\tilde{F}. \quad (14)$$

So we can trade a phase in the quark mass matrix for θ, or vice versa. The invariant quantity is often called $\bar{\theta}$,

$$\bar{\theta} = \theta - \arg\ \det m_q. \quad (15)$$

Anomaly in the current $j^\mu = \bar{q}\gamma^\mu\gamma^5 q$: A calculation

We are interested in a mass term,$me^{i\alpha}\bar{q}q$ There are many ways to think about the anomaly and I cannot do justice to the subject in ten minutes. But I can at least indicate how this effect comes about. In four component language, we have a coupling, for small α:

$$m\,\alpha\bar{q}\gamma^5 q \tag{16}$$

Let's replace m by a field (this might be one of the pseudoscalar mesons, or, as we will see later, the axion). This allows some momentum flow through the diagram.

It is convenient to work with the graph in four component notation since the fermion is massive. The basic expression has the form, after introducing Feynman parameters:

$$N_f m(q)\alpha \int\int dx_1 dx_2 f(x_1, x_2) \frac{d^4k}{(2\pi)^4[k^2+m^2]^3} \mathrm{Tr}\gamma_5 \tag{17}$$

$$(\not p_1 \not A(p_1) ptwoslash \not A(p_2))$$

$$= \frac{1}{16\pi^2}\alpha\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu A(p_1)^\rho A(p_2)^\sigma.$$

This is the anticipated result.

Exercise: Verify the expression above

Some things to note:

1. If m is a constant, the result vanishes ($p_1 = -p_2$). This is related to the total derivative, and the rather trivial (plane wave) nature of the external fields.
2. If we did the computation using the background field formalism, we would obtain, also, $\partial_\mu j^\mu = \frac{N_f}{16\pi^2} F\tilde{F}$. This result would hold for non-trivial backgrounds (magnetic monopoles in a $U(1)$ theory, instantons in the non-abelian theory) for which $\int d^4x F\tilde{F}$ is non-zero.
3. It is interesting that $\alpha F\tilde{F}$ term appears in the effective action. Surprising since would seem one could just rotate away.

Instantons

QCD is a strongly coupled theory, and providing a reliable answer to this question requires strong coupling methods (lattice gauge theory). But a semiclassical analysis, while not reliable, indicates that one cannot neglect $\int d^4x \partial_\mu K^\mu$. As a result, the axial current is not conserved and θ is physical.

In the Euclidean functional integral

$$Z = \int [dA][dq][d\bar{q}]e^{-S} \tag{18}$$

it is natural to look for stationary points of the effective action, i.e. finite action, classical solutions of the theory in imaginary time. These *instanton* solutions can be found rather easily. The following tricks simplify the construction, and turn out to yield the general solution. First, note that the Yang–Mills action satisfies an inequality, the Bogomolnyi bound:

$$\int (F \pm \tilde{F})^2 = \int (F^2 + \tilde{F}^2 \pm 2F\tilde{F}) = \int (2F^2 \pm 2F\tilde{F}) \geq 0. \tag{19}$$

So the action is bounded by $|\int F\tilde{F}|$, with the bound being saturated when

$$F = \pm\tilde{F} \tag{20}$$

i.e. if the gauge field is (anti-) self-dual.

This is a first order differential equation. Comparatively easy to solve.

The Instanton Solution

't Hooft presented the instanton in a fashion which is useful for actual computations. Defining the symbol η:

$$\eta_{aij} = \epsilon_{aij}; \ \eta_{a4i} = -\eta_{ai4} = -\delta_{ai}; \quad \bar{\eta}_{a\mu\nu} = (-1)^{\delta_{a\mu}+\delta_{a\nu}} \eta_{a\mu\nu} \tag{21}$$

the instanton takes the simple form:

$$A^a_\mu = \frac{2\eta_{a\mu\nu} x^\nu}{x^2 + \rho^2} \tag{22}$$

while the field strength is given by:

$$F^a_{\mu\nu} = \frac{4\eta_{a\mu\nu}\rho^2}{(x^2 + \rho^2)^2} \tag{23}$$

That this configuration solves the equations of motion follows from:

$$\eta_{a\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\eta_{a\alpha\beta}. \tag{24}$$

so $F = \tilde{F}$.
The η symbols are connected to the embedding of $SU(2)$ of the gauge group in an SU(2) subgroup of $O(4) = SU(2) \times SU(2)$. This can be understood by noting:

$$\eta_{a\mu\nu} = \frac{1}{2}\mathrm{Tr}(\sigma^a\sigma_{\mu\nu}) \quad \bar{\eta} = \mathrm{Tr}(\sigma^a\bar{\sigma}_{\mu\nu}). \tag{25}$$

Since $F = \tilde{F}$, the equations of motion are satisfied. Note the $1/r$ falloff of A^μ, as opposed to the $1/r^4$ falloff of $F_{\mu\nu}$.

Topological Charge

Asymptotically, A^μ is a pure gauge. The gauge transformation maps the three sphere onto the gauge group. $\frac{1}{16\pi^2}\int d^4x F\tilde{F}$ measures the number of times that the sphere is mapped into the gauge group.

$$\frac{1}{16\pi^2}\int d^4x F\tilde{F} = \frac{1}{16\pi^2}\int d^4x \partial_\mu K^\mu = 1 \qquad (26)$$

At large x,

$$A_\mu = \frac{\eta_{a\mu\nu} x^\nu \tau^a}{x^2} \tag{27}$$

$$= i g^{-1} \partial_\mu g$$

[**Exercise:** show what $g = \frac{x_4 + i\vec{x}\cdot\vec{\tau}}{r}$].

Consequences of the Instanton

Having found a classical solution, we want to integrate about small fluctuations about it. Including the θ term, these have the form

$$\langle \mathcal{O} \rangle = e^{-\frac{8\pi^2}{g^2}} e^{i\theta} \int [d\delta A][dq][d\bar{q}] \exp\left(-\frac{\delta^2 S}{\delta A^2} \delta A^2 - S_{q,\bar{q}} \right) \mathcal{O}. \quad (28)$$

Now S contains an explicit factor of $1/g^2$. As a result, the fluctuations are formally suppressed by g^2 relative to the leading contribution. The one-loop functional integral yields a product of determinants for the fermions, and of inverse square root determinants for the bosons.

Both the bosonic and fermionic quadratic fluctuation operators have zero eigenvalues. For the bosons, these potentially give infinite contributions to the functional integral, and they must be treated separately. The difficulty is that among the variations of the fields are symmetry transformations: changes in the location of the instanton (translations), rotations of the instanton, and scale transformations.

More explicitly

$$A_\mu^a(x) = \Omega \frac{2\eta_{a\mu\nu}(x - x_0)^\nu}{x^2 + \rho^2} \tag{29}$$

where Ω denotes a global gauge transformation (or rotation).

Consider translations. For every solution, there is an infinite set of solutions obtained by shifting the origin (varying x_0). Instead of integrating over a coefficient, c_0, we integrate over the *collective coordinate* x_0 (one must also include a suitable Jacobian factor). The effect of this is to restore translational invariance in Green's functions. Similarly, the instanton breaks the rotational invariance of the theory. Correspondingly, we can find a three-parameter set of solutions and zero modes. Integrating over these rotational collective coordinates restores rotational invariance. (The instanton also breaks a global gauge symmetry, but a combination of rotations and gauge transformations is preserved.)

Finally, the classical theory is scale invariant; this is the origin of the parameter ρ in the solution. Again, one must treat ρ as a collective coordinate, and integrate over ρ. There is a power of ρ arising from the Jacobian, which can be determined on dimensional grounds. If d_O is the dimension of the operator $\mathcal{O}$, then, on dimensional grounds, one expects for the ρ dependence:

$$\int d\rho \rho^{-d_0-1}. \tag{30}$$

However, there is additional ρ-dependence because the quantum theory violates the scale symmetry. This can be understood by replacing $g^2 \to g^2(\rho)$ in the functional integral, and using

$$e^{-\frac{8\pi^2}{g^2(\rho)}} \approx (\rho M)^{b_0} \tag{31}$$

for small ρ. For three-flavor QCD, for example, $b_0 = 9$, the leading operator has dimension 9, and the ρ integral diverges logarithmically for large ρ. This is just the statement that the integral is dominated by the infrared, where the QCD coupling becomes strong.

Fermion functional integrals introduce a new feature. In four-component language, it is necessary to treat q and $\bar{q}$ as independent fields. (In two-component language, this corresponds to treating q and q^* as independent fields.) So at one-loop order, we need to study:

$$\not{D} q_n = \lambda_n q_n \quad \not{D} \bar{q}_n = \lambda_n \bar{q}_n \tag{32}$$

$$q(x) = \sum a_n q_n(x), \quad (33)$$

$$S = \sum \lambda_n a_n^* a_n. \quad (34)$$

Then

$$\int [dq][d\bar{q}]e^{-S} = \prod_{n=0}^{\infty} da_n da_n^* e^{-\sum_{n\neq 0} \lambda_n a_n^* a_n}. \quad (35)$$

Zero eigenvalues of the Dirac operator are special. Because the zero modes do not contribute to the action, many Green functions vanish. For example, $\langle 1 \rangle = 0$. In order to obtain a non-vanishing result, we need enough insertions of q to "soak up" all of the zero modes.

The explicit form of the zero modes is not complicated. For $SU(2)$, for simplicity:

$$\not{D} q = 0 \quad \not{D} \bar{q} = 0 \tag{36}$$

and

$$q_0 = \frac{\rho}{(\rho^2 + (x - x_0)^2)^{3/2}} \zeta, \tag{37}$$

where ζ is a constant spinor.

q here might be $u, \bar{u}, d, \bar{d}$, etc.

We can put all of this together to evaluate a Green function which violates the classical $U(1)$ symmetry of the massless theory, $\langle \bar{u}(x)u(x)\bar{d}(x)d(x)\bar{s}(x)s(x)\rangle$. There is one zero mode for each of $u, d, s, \bar{u}, \bar{d}, \bar{s}$. The fields in this Green's function can soak up all of these zero modes. The effect of the integration over x_0 is to give a result independent of x, since the zero modes are functions only of $x - x_0$. The integration over the rotational zero modes gives a non-zero result only if the Lorentz indices are contracted in a rotationally invariant manner (the same applies to the gauge indices). The integration over the instanton scale size – the conformal collective coordinate – is more problematic, exhibiting precisely the infrared divergence we discussed earlier.

So we have provided some evidence that the $U(1)$ problem is solved in QCD, but no reliable calculation. What about θ-dependence? Let us ask first about θ-dependence of the vacuum energy. In order to get a non-zero result, we need to allow that the quarks are massive. Treating the mass as a perturbation, noting the path integral contains a term $e^{i\theta} \frac{1}{16\pi^2} \int d^4x F\tilde{F} = e^{i\theta}$ we obtain a result of the form:

$$E(\theta) = C\Lambda_{\rm QCD}^9 m_u m_d m_s \cos(\theta) \int d\rho \rho^7. \tag{38}$$

So we have evidence for θ-dependence, but again cannot do a reliable calculation. That we cannot do a calculation should not be a surprise. There is no small parameter in QCD to use as an expansion parameter. Fortunately, we can use other facts which we know about the strong interactions to get a better handle on both the $U(1)$ problem and the question of θ-dependence.

Neutron Electric Dipole Moment

A particularly sensitive test of CP conservation in the strong interactions is provided by the neutron electric dipole moment. This corresponds to an operator $\bar{n}\gamma_5\sigma_{\mu\nu}nF^{\mu\nu}$ (here F is the gauge field of electrodynamics). We might first guess that d_n is of order $e\theta \, \mathrm{Fm} \approx \theta 10^{-13} cm$. Then we might expect suppression by powers of quark mass. Our instanton analysis would suggest three factors of quark mass (m_q/m_n, say). We'll see in a moment that there is only one such factor; $m_u/m_n \sim 0.002$, so from the limit on $d_n < 10^{-26}\ \mathrm{e\ cm}$, we would have $\theta < 10^{-10}$. We'll give a sharper estimate now.

Consider, first, the coupling of pions to nucleons (for simplicity we'll consider a limit of approximate $SU(2) \times SU(2)$ symmetry, i.e. we'll just treat the u and d quarks as light. Also for simplicity I'll take the u and d quark masses identical. I'll quote general formulas in the end, and leave the derivations to you (or to an examination of the literature).

$$\mathcal{L}_{\pi NN} = \vec{\pi} \cdot \bar{N}(\vec{\tau} i \gamma_5 g_{\pi NN} + \bar{g}_{\pi NN})N. \tag{39}$$

The second term is CP violating. Its effects are directly measurable, in principle, but we will take this to a far more sensitive test in a moment.

As an aside, we can ask what this lagrangian means. We are used to the notion of an effective action for Goldstone bosons. This makes sense; the Goldstone bosons are the light fields, and we can obtain their lagrangian by integrating out heavy fields, such as nucleons and vector mesons. But in QCD, because baryon number is conserved, we can consider a sector with a fixed, non-zero baryon number. Baryon number one is the simplest. For low momenta, we can treat the nucleons as non-relativistic and ignore nucleon-anti-nucleon pairs, and ask about pion scattering amplitudes. This subject was developed extensively some time ago by Weinberg.

For example, in the baryon number one sector, we can study low momentum pion-nucleon processes by thinking of matrix elements such as $\langle \pi\pi \ldots | \bar{q}q | \pi\pi \ldots \rangle$ in terms of a background pion field, and obtain the amplitude from the non-linear chiral lagrangian.

The CP violating term in the underlying lagrangian is obtained from the quark mass term, $m\bar{q}q + \mathrm{c.c.}$, and performing the transformation $q \rightarrow e^{i\frac{\vec{\pi}\cdot\vec{\tau}}{2f_\pi}} q$:

$$\delta\mathcal{L} = \frac{m_q\theta}{2}\bar{q}\frac{\vec{\pi}}{f_\pi}\cdot\vec{\tau}q \tag{40}$$

(compare the chiral lagrangian, $\bar{q}mq \rightarrow \mathrm{Tr}mU$, $U = e^{i\frac{\vec{\pi}}{2f_\pi}}$, now $\bar{N}UN$. We need the matrix element between an initial and final nucleon state:

$$\frac{\theta m_q}{2f_\pi}\langle N_f|\bar{q}\tau^a q|N_i\rangle \tag{41}$$

The matrix element can be obtained from standard $SU(3)$ global symmetry (Gell-Mann) arguments.

Now this coupling induces a neutron electric dipole moment. The diagram is infrared divergent as $m_\pi \to 0$, and this term is readily extracted.

$$d_n = g_{\pi NN} \bar{g}_{\pi NN} \frac{\log(M_N/m_\pi)}{4\pi^2 M_N} \tag{42}$$

Working through the details:

$$d_n = g_{\pi NN} \frac{\theta m_u m_d}{f_\pi (m_u + m_d)} \langle N_f | \bar{q} \tau^a q | N_f \rangle \ln(m_p/m_\pi) \frac{1}{4\pi^2 m_p} \tag{43}$$

$$= 5.2 \times 10^{-16} \theta cm$$

(this is calculated in an approximation which becomes more and more reliable as the masses of the light quarks become smaller).

From the experimental limit, $d_n < 3 \times 10^{-26}$ e cm, one has $\theta < 10^{-10}$.

The Strong CP Problem

This is a puzzle. Why such a small dimensionless number?

$\theta \to 0$: strong interactions preserve CP. If not for the fact that the rest of the SM violates CP, would be *natural*.

Among naturalness problems, the strong CP problem is special in that it is of almost no consequence. We don't have to invoke anthropic selection to realize that if the cosmological constant was a few orders of magnitude larger than observed, the universe would be dramatically different. The same is true for the value of the weak scale and of the light quark and lepton masses. But if θ were, say, 10^{-3}, nuclear physics would hardly be different than we observe, since effects of θ are shielded by small quark masses.

Possible Resolutions

1. $m_u = 0$ If true, $u \to e^{-i\frac{\theta}{2}\gamma_5} u$ eliminates θ from the lagrangian. An *effective* m_u might be generated from non-perturbative effects in the theory (Georgi, McArthur; Kaplan, Manohar) Could result as an accident of discrete flavor symmetries (Banks, Nir, Seiberg), or a result of “anomalous" discrete symmetries as in string theory (M.D.)
2. CP exact microscopically, $\theta = 0$; spontaneous breaking gives the CKM phase but leads, under suitable conditions, to small effective θ (Nelson, Barr). In critical string theories, CP is an exact (gauge) symmetry, spontaneously broken at generic points in typical moduli spaces. A plausible framework.
3. A new, light particle called the axion dynamically cancels off θ.

Lecture 2: Solutions of the Strong CP Problem: An Assessment

1. $m_u = 0$: how plausible as an idea? Confrontation with lattice gauge theory
2. Small radiative corrections to θ in Standard Model. Possible realization through spontaneous CP violation (Barr-Nelson mechanism). Models. Virtues and problems.
3. Axions. Basic ideas. Virtues, problems, constraints.
4. Basics of Axion Physics
5. Astrophysical Constraints on Axions

Problems with each of these solutions:

1. $m_u = 0$. Lattice computations seem to rule out (the required non-perturbative effects do not seem to be large enough).
2. Spontaneous CP: special properties required to avoid large θ once CP is spontaneously broken. What would single out such theories?
3. Axions: promise and limitations.

If $m_u = 0$, one can rotate away θ. More precisely, one requires, since $d_n \propto \theta m_u$ in this limit,

$$\frac{m_u}{\Lambda_{QCD}} < 10^{-10} \tag{44}$$

at the scale Λ_{SM}. There are two issues with this proposal:

1. Why might m_u be so small?
2. We can measure m_u (with the help of the lattice). Is this consistent with lattice results?

Accounting for small m_u

Banks, Nir, Seiberg put forward models which, in accounting for quark flavor, gave rise to small or zero m_u.

A simple possibility is suggested by string theory, which often exhibits anomalous discrete symmetries; more precisely, the chiral content of the theory is anomalous, with the anomaly being cancelled by the non-linear transformation of an axion-like field. In the supersymmetric case, this means that one has a modulus field, coupling to the $\bar{u}$ quark as ($\Phi = \phi + ia$)

$$e^{-\Phi} Q H_U \bar{u}. \tag{45}$$

One requires that the exponential be very small, but this is plausible. One can speculate as to whether or not a suitable discrete symmetry structure is typical of underlying theories.

How might $m_u = 0$ be consistent with known facts of hadron physics

Instantons suggestive (Georgi-McArthur). With three light quarks, generate an effective u quark mass (two point function) proportional to $m_d m_s$. Simple dimensional analysis suggests the effect goes as

$$\frac{m_d m_s}{\Lambda} \tag{46}$$

with Λ a suitable QCD scale. This could easily be of order the few MeV expected from current algebra. Kaplan and Manohar expressed this as an ambiguity in current algebra, i.e. they isolated a term and second order in quark masses which could mimic the effects of a u quark mass.

Summary of lattice results for light quark masses

Current results from lattice simulations (summarized by the FLAG working group) are inconsistent with $m_u = 0$.

$$m_u = 2.16\ (9)(7)\mathrm{MeV} \quad m_d = 4.68\ (14)(7)\mathrm{MeV} \tag{47}$$

$$m_s = 93.5(2.5)\mathrm{MeV}$$

Numbers are in $\overline{MS}$ scheme at 2 GeV.

So m_u is many standard deviations from zero. Probably end of story, but some proposals for dedicated tests (Kitano), calibrations (Dine, Draper, Festuccia).

Loop Corrections at Low Energies in the Standard Model

Loop corrections to θ in the Standard Model are highly suppressed. Focussing on divergent corrections, one requires Higgs loops. These involve the Hermitian matrices

$$A = y_d^\dagger y_d; \quad B = y_u^\dagger y_u \tag{48}$$

Contributions to θ are proportional to traces of the form

$$Tr(ABA^2B\ldots) \tag{49}$$

one additional matrix factor for each loop.

It is easy to check that the first complex combination involves six matrices, e.g.

$$\mathrm{Tr}(ABA^2B^2) \tag{50}$$

but this and its complex conjugate both appear with the same coefficient. It is necessary to add a $U(1)$ gauge loop (which distinguishes u and d) to have the possibility of a complex traces. [Ellis, Gaillard]

So if θ is small at some scale Λ_{SM}, further corrections are extremely tiny (finite corrections are also very small).

One has the feeling that this might not be such a big problem. The question is: why might $\theta(\Lambda_{SM}$ be small?

Spontaneous CP Violation: The Nelson-Barr mechanism

Invokes spontaneous CP violation to argue “bare θ" is zero. Constructs a mass matrix such that spontaneous CP breaking gives a large CKM angle (as observed, $\delta = 1.2$) with $\arg \det m_q = 0$.

Bare θ is tree level θ (presumes some perturbative approximation). Must insure that $\theta(\Lambda_{SM})$ is small.

Unlike axion, $m_u = 0$ solutions, no obvious low energy consequences.

Attempts to achieve a setup where θ at the scale Λ_{SM} is extremely small.

Such a structure is perhaps made plausible by string theory, where CP is a (gauge) symmetry, necessarily spontaneously broken. At string scale, $\theta = 0$ a well-defined notion. Some features of the required mass matrices appear, e.g., in Calabi-Yau compactifications of the heterotic string.

Simple realization of the NB structure

Complex scalars η_i with complex (CP-violating) vev's. Additional vectorlike quark with charge $1/3$.

$$\mathcal{L} = \mu \bar{q} q + \lambda_{if} \eta_i \bar{d}_f q + y_{fg} Q_f \bar{d}_g \phi \tag{51}$$

where ϕ is Higgs; y, λ, μ real.

$$M = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix} \tag{52}$$

$B_f = \lambda_{if} \eta_i$ is complex. M has real determinant.

The structure is reminiscent of an E_6 gauge theory, which has the requisite vector-like quarks and singlets.

Requirements for a successful NB Solution

1. Symmetries: It is important that η_i not couple to $\bar{q}q$, for example. So, e.g., η's complex, subject to a Z_N symmetry.
2. Coincidences of scale: if only one field η, CKM angle vanishes (can make d quark mass matrix real by an overall phase redefinition). Need at least two, and their vev's (times suitable couplings) have to be quite close:

$$\delta_{CKM} \propto \frac{B_{small}}{B_{large}} \quad (53)$$

3. Similarly, μ (which might represent vev of another field) can not be much larger than η_i, and if much smaller the Yukawa's and B's have to have special features.

Constraints on the Overall Scale

Before considering radiative effects, possible higher dimension operators in $\mathcal{L}$ constrain the scales η_i, μ. E.g.

$$\frac{\eta_i^* \eta_j}{M_p} \bar{q} q \tag{54}$$

requires $\frac{|\eta|}{M_p} < 10^{-10}$.

Barr-Nelson With/Without Supersymmetry

Without supersymmetry, highly tuned. Two light scalars and μ (or three light scalars), with masses 10 orders of magnitude below M_p. Far worse than θ.

Even ignoring that, require close coincidence of scales.

Supersymmetry helps. Allows light scalars. Coincidences still required (and more chiral multiplets to achieve desired symmetry breakings – typically at least seven). Some of the high dimension operators better controlled (e.g. if μ, η_i much larger than susy breaking scale, don't have analogs of the $\eta_i^* \eta_j \bar{q} q$ operator).

Loop Corrections in Nelson-Barr: Non-Supersymmetric case

In the non-supersymmetric case, in the simplest model, potential corrections arise at one loop order. Consider, in particular, couplings of the form

$$\lambda_{ij}\eta_i\eta_j|H|^2$$

give rise to one loop contributions.

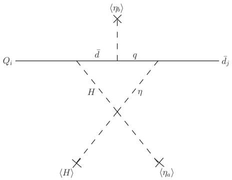


Figure 1: Example threshold correction to Arg det m_d.

If the new couplings are of order one these are six or seven orders of magnitude too large.

In the past these have sometimes been dismissed on the grounds that these couplings contribute to the Higgs mass, but this is just part of the usual fine tuning problem.

Supersymmetry breaking and Nelson-Barr

Many possible phases once allow soft breaking **Note: these effects don't decouple for large susy-breaking scale**. E.g. is susy breaking described by Goldstino superfield, X, superpotential couplings

$$\frac{\mathcal{O}_d}{M_p^{d-2}} X \tag{55}$$

where $\langle \mathcal{O} \rangle$ is complex can lead to large phases in soft breakings. Similarly phases in W. Phases in gaugino masses feed directly into θ.

Loop Corrections in Supersymmetric Nelson-Barr

If tree level phases in soft terms suppressed, loops still pose a problem (Kagan, Leigh, M.D.). Loop corrections to gaugino mass from loops with q, $\bar{q}$, fields. Require, e.g., A terms small or proportional to Yukawas. Gauge mediation (with real F) most plausible solution (A terms small). (Luty, Schmaltz in a slightly different context)

The Peccei-Quinn Symmetry

In a somewhat streamlined language, the Peccei-Quinn proposal was to replace θ by a dynamical field: $\theta \to \frac{a(x)}{f_a}$

It is assumed that $a \to a + \omega f_a$ is a good symmetry of the theory, *violated only by effects of QCD*. Without QCD, θ can take any value.

In QCD *by itself*, the energy is necessarily stationary when

$$\theta_{eff} = \langle \frac{a}{f_a} \rangle = 0. \tag{56}$$

This is simply because *CP* is a good symmetry of QCD if $\theta = 0$, so the vacuum energy (potential) must be an odd function of θ.

One can do better, calculating, again using what we know about chiral symmetry in QCD, the axion potential:

$$V(a) = m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{a^2}{2 f_a^2} \tag{57}$$

This gives, for the axion mass:

$$m_a = 0.6 \text{ meV} \left(\frac{10^{10} \text{ GeV}}{f_a} \right). \tag{58}$$

[**Exercise**: Derive the expression for $V(a)$. To do this, integrate out the “heavy" fields (the pions) by solving the π_0 equation of motion, and subsituting back in the action. Simplify by working to second order in a, π^0.]

Peccei and Quinn actually constructed a model for this phenomenon, which was a modest extension of the Standard Model with an extra Higgs doublet. They didn't phrase the problem in quite the way I did above, and didn't appreciate that their model had a light, pseudoscalar particle, *a*. This was recognized by Weinberg and Wilczek, who calculated its mass and the properties of its interactions. It quickly become clear that the original axion idea was not experimentally viable.

The Invisible Axion

But in the more general picture described above, the problems with the axion are easily resolved. The strength of the axion's interactions are proportional to $1/f_a$. This is because of the Peccei-Quinn symmetry. The symmetry requires that axion interactions appear only with derivatives of the axion field; on dimensional grounds, these come with powers of $\frac{\partial_\mu}{f_a}$ (momenta – q^μ/f_a). QCD terms which break the symmetry also come with powers of $1/f_a$. So if f_a is large enough, the axion will be hard to detect (it becomes “harmless" or “invisible").

The scale, f_a, might be associated with some high scale of physics (M_{gut}? M_p? – more later).

Sample couplings

1. Axion to two photons (notation of PDG):

$$\mathcal{L}_{\gamma\gamma} = \frac{1}{4} G_{a\gamma\gamma} \, a \, F\tilde{F} \tag{59}$$

where now F is the *electromagnetic* field strength.

$$G_{a\gamma\gamma} = \frac{\alpha}{2\pi}\left(\frac{E}{N} - \frac{4}{3}\frac{4+z}{1+z}\right)\frac{1+z}{\sqrt{z}}\frac{m_a}{m_\pi f_\pi} \quad z = \frac{m_u}{m_d} \tag{60}$$

$E, \; N$ are the electromagnetic and QCD anomalies of the PQ current.

2. Axion to quarks, leptons:

$$\mathcal{L}_{aff} = \sum_f \frac{C_f}{2f_a} \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f \, \partial_\mu a. \tag{61}$$

The detailed coefficients depend on the model.

Two Benchmark models

DFSZ

Add to the Standard Model an additional Higgs doublet (e.g. as in supersymmetry), i.e. two doublets, H_u, H_d, plus a singlet, ϕ. Impose the Peccei-Quinn symmetry:

$$\phi \to e^{i\alpha}\phi;\; H_u \to e^{-i\frac{\alpha}{2}} H_u;\; H_d \to e^{-i\frac{\alpha}{2}} H_d \tag{62}$$

Require potential such that H_U, H_D, ϕ have expectation values, where the ϕ vev is very large.

$$\langle \phi \rangle \approx \frac{f_a}{\sqrt{2}} \gg \text{TeV}. \tag{63}$$

This breaks the PQ symmetry spontaneously. (Pseudo-)Goldstone boson:

$$\text{Im}\;\phi = \frac{a}{\sqrt{2}}.$$

a couples to $G\tilde{G}$, $F\tilde{F}$. Also couples to leptons, quarks.

$$\frac{E}{N} = 8/3; \quad C_e = \frac{\cos^2\beta}{3} \quad \tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}. \tag{64}$$

As expected, as f_a becomes large, the axion's interactions with other particles become weaker. Once $f_a >> \mathrm{Tev}$, unobservable in accelerator experiments.

KSVZ Model

Here one has a field, ϕ, and a new quark, q and $\bar{q}$, which will be very heavy. q and $\bar{q}$ are assumed to carry color but to be $SU(2) \times U(1)$ singlets. In two component language, the Peccei-Quinn symmetry is assumed to be

$$\phi \to e^{i\alpha}\phi \quad q \to e^{-i\frac{\alpha}{2}} q\, '\bar{q} \to e^{-i\frac{\alpha}{2}} \bar{q}. \quad \mathcal{L}_{\phi\bar{q}q} = \lambda \phi \bar{q} q \tag{65}$$

ϕ is assumed to have an expectation value:

$$\langle \phi \rangle = \frac{f_a}{\sqrt{2}}. \tag{66}$$

The imaginary part of ϕ is the axion:

$$\phi = \frac{1}{\sqrt{2}} \left(f_a + ia \right). \tag{67}$$

But these are just two of a wealth of possible models, characterized by the coefficients E, N, C_i above. These two, however, are often used as benchmarks to characterize the capabilities of different experimental detection schemes, as well as to illustrate the range of possible astrophysical phenomena.

Physical Processes Associated with Axions

1. Production in accelerators
2. Decay
3. Production in stars
4. Production in strong magnetic fields (ADMX and other experiments

Astrophysical Constraints

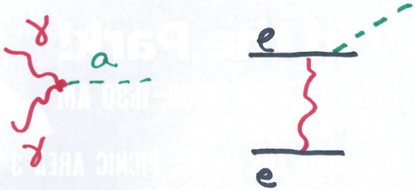
Axion interactions are “semi weak", in the sense that cross sections go as $1/f_a^2$, as opposed to weak interactions which behave as $1/v^4$. So even for large f_a, reaction rates can be comparable to those for neutrinos. This raises a worry about stars, where various processes can produce axions. If interaction rates are large compared to those for neutrinos, excessive amounts of energy will be carried off by axions. More detailed studies in particular astrophysical environments place lower limits on f_a.

Sources of Astrophysical Constraints

Partial list:

1. The sun
2. Red Giants, Globular Clusters
3. SN 1987a
4. White dwarfs

Primakoff process, axion bremstrahlung.



Axion Luminosity

In sun:

$$L_a = G_{a\gamma\gamma}^2 \times 1.85 \times 10^{17} L_\odot \tag{68}$$

so

$$G_{a\gamma\gamma} < 7 \times 10^{-10}. \tag{69}$$

Stronger constraint from globular clusters, $7 \to 1$.

Lecture 3: Axion Cosmology and Axion Searches: Old and New Ideas

1. Cosmology of Axions: conventional
2. Theory of axion detection
3. The Problem of Axion Quality
4. Axions in String Theory
5. Cosmology of Axions: non-conventional
6. Concluding thoughts on likelihood of axions, where to look

Axions In the Early Universe

Paradoxically, because the axion is so weakly interacting, it can play a significant role in the early universe. $H \gg m_a$ for $t < m_a^{-1}$. At this time, the axion potential (due to QCD) is of no importance. So the initial axion expectation value (initial θ, or "misalignment angle", θ_0, may be a randome variable.

Is there a PQ phase transition, and if so when?

First question: Is there a PQ Phase transition and does it occur before/after inflation?

1. Presumably PQ symmetry is an accident. This accident might not hold at very early times (high curvatures, etc.). In this case, no PQ transition at all.
2. Simplest: PQ transition before inflation. Current observable universe started out with a single value of θ.
3. More complex: PQ Transition after inflation. Initial θ a random variable. Topological objects important: domain walls, cosmic strings.

Axion Evolution

Simplify by assuming a homogeneous axion field, with an

In an FRW universe:

$$\ddot{a} + 3\,H\dot{a} + m_a^2\,a = 0 \tag{70}$$

a is overdamped for $H > m_a$; oscillates for $H < m_a$.

m_a is small; e.g. for

$$f_a = 10^{16}\mathrm{GeV}, m_a = 10^{-9}\ \mathrm{eV}; \quad f_a = 10^{11}\mathrm{GeV} \quad m_a = 6 \times 10^{-4}\ \mathrm{eV}. \quad (71)$$

$H = 10^{-9},\ 10^{-4}$ eV when the temperature of the universe is about $1,\ 10^2$ GeV. This is late compared to, e.g., the likely times of inflation.

Solving the axion equation of motion

$$\ddot{\theta} + 3\, H\dot{\theta} + m_a^2\, \theta = 0 \tag{72}$$

Seek a solution of form, for large t:

$$\theta(t) = \theta_0 f(t) \cos(m_a t) \tag{73}$$

with $f(t)$ slowly varying. For radiation/matter dominated eras:

$$H = \frac{1}{2t}; f = \left(\frac{t_0}{t}\right)^{3/4} \quad H = \frac{2}{3t}; f = \left(\frac{t_0}{t}\right) \tag{74}$$

Each of these solutions falls off as $1/R(t)^3$. In other words, the system behaves like pressure-less dust, a collection of zero momentum axions.

When the axion starts to oscillate, it constitutes a fraction of the energy density of order $\theta_0^2 \frac{f_a^2}{M_p^2}$; with $f_a = 10^{11}$ this is about 10^{-14}.

During the radiation dominated era, the energy density in matter (axions) falls off as T^3, as opposed to the radiation, which falls off as T^4. The usual matter-radiation equality occurs for $T \approx$ eV. At this time, the energy density in axions, indeed, is of order the energy density in radiation.

Larger f_a: matter domination too early. Smaller: axions only a fraction of the dark matter.

Temperature Dependence of the Axion Mass

The previous discussion treated the axion mass as a constant. But the axion mass falls off rapidly with temperature.

Including Temperature Dependence

We might guess that the axion mass would fall off rapidly with temperature. As we'll see in a moment, the instanton computation at high temperature does not suffer from infrared divergences, and as a result is characterized by $g^2(T)$. At high temperatures, we have approximate zero modes for the light quarks, so we might expect that the free energy, as a function of θ, behaves as:

$$F(\theta, T) = C m_u m_d m_s \; T e^{-\frac{8\pi^2}{g^2(T)}} \propto \frac{\Lambda^9}{T^8} m_u m_d m_s \qquad (75)$$

A detailed computation (to be described) gives, for the full expression:

$$F(\theta, T) = -\chi(T_0) \left(\frac{T_0}{T}\right)^8 \quad T_0 = 1.5 \text{ GeV} \;\; \chi(T_0) = 1.6 \times 10^{-12} \text{ GeV}^4.$$

A good approximation is obtained by treating the axion as frozen until a temperature, T_{osc}:

$$m_a(T_{osc}) = 3H(T_{osc}). \tag{77}$$

At this point, the axion begins to oscillate with a time (temperature) dependent mass. Calling

$$\rho(t) = \frac{1}{2}\dot{a}^2 + \frac{1}{2}m_a^2(T)a^2 \tag{78}$$

one can show:

$$\rho(T) = \rho(T_{osc})\left(\frac{R^3(T_{osc})}{R^3(T)}\right)\frac{m_a(T)}{m_a(T_{osc})}. \tag{79}$$

So indeed for $f_a \approx 10^{11}$ GeV and $\theta_0 \approx 1$, the axions come to dominate the energy density of the universe at the approximate time of matter-radiation equality. More careful calculation takes into account the temperature dependence of the axion mass, and yields:

$$\Omega_a h^2 = 0.11\ \theta_0^2 \left(\frac{f_a}{5 \times 10^{11}\ \mathrm{GeV}} \right)^{1.184} . \tag{80}$$

So from the combination of astrophysical and cosmological considerations, the axion decay constant/mass lies in a rather narrow range. At the high end of this range, the axion constitutes the dark matter.

Uncertainties in Axion Energy Density Computation

Recently, some concern that large QCD uncertainties could modify the estimate significantly, e.g. for a fixed dark matter density, very different prediction of f_a. Arise from lattice gauge computations.

We will shortly turn to understanding this computation, and to attempting to estimate the errors. To summarize our analysis and results:

1. At very high temperatures, the instanton computation of the free energy is reliable, yielding the formula above. This has been questioned recently; we will explain the origin of the skepticism and its resolution.
2. From current algebra, one knows the form of the zero temperature potential.
3. The dominant temperatures for the computation lie in the strong coupling region. To assess the associated errors, we will consider a range of possible interpolations. We will see that there is very little sensitivity of Ω_{axion} to these uncertainties. Alternatively, the value of m_a which accounts for a given dark matter density (and for a given "misalignment angle", θ_0) has only a few percent uncertainty.
4. Results from lattice gauge theory from lattice gauge theory for the topological susceptibility differ from each other and the instanton computation by orders of magnitude. We will conclude that reproducing the instanton results is an important test.

The Standard Dilute Gas Computation

At high temperatures, the θ dependence of the free energy is controlled by instantons. Classically, even at finite temperature, there are instantons of all scale sizes. But at one loop, there are two sources of scale invariance violation: the usual ultraviolet divergences familiar in the zero temperature theory, and the finite temperature itself. Both correct the effective action, rendering finite the scale size integral at small and large ρ.

GPY provided a heuristic explanation of the infrared cutoff. They noted the existence of a screening length, or mass, associated with the field A_4 in the finite temperature theory.

$$m_d^2 = \frac{1}{3}(g^2 T^2)(N + \frac{N_f}{2}). \tag{81}$$

They then computed

$$\int d^4x \frac{1}{2g^2} m_d^2 A_4^2 = \frac{\pi^2}{2g^2} m_d^2 \rho^2 \tag{82}$$

Note the g^{-2} in front of m_d^2, reflecting the $1/g^2$ in front of the whole action, and the fact that the actual screening length is of order $\frac{1}{gT}$.

If this were the complete result for the correction to the effective action, the ρ integration for the free energy would take the form, in the case of three flavors,

$$F(T) \propto m_u m_d m_s \int \frac{d\rho}{\rho^2} (\Lambda\rho)^9 e^{-3\pi^2 \rho^2 T^2}. \quad (83)$$

This integral is finite, and dominated by $\rho \sim (\pi T)^{-1}$.

However, as the dominant scale is of order T^{-1}, the effective action cannot be expanded in powers of ρ; in a derivative expansion of the background field effective action, terms of the form

$$\frac{g^2}{T^{n-2}} A_4(\vec{x}) \partial_{i_1} \ldots \partial_{i_n} A_4(\vec{x}) \tag{84}$$

are all of the same order, $g^2 T^2$, in the instanton background.

GPY indeed computed the full one-loop determinant. At small ρ, in particular, the above expression for the action is modified:

$$\delta S = \frac{1}{3}\pi^2\rho^2 T^2(2N + N_f) - \frac{1}{18}\pi^2\rho^2 T^2(N - N_f). \tag{85}$$

For $N_f = 0$, for example, this correction is not parametrically smaller than the Debye screening term, though it is numerically smaller.

At one loop, the complete expression for the free energy in the presence of a single instanton is given by

$$F(\theta, T) = \int \frac{d\rho}{\rho^5} \left(\frac{4\pi^2}{g^2}\right)^{2N} e^{-\frac{8\pi^2}{g^2(\rho)}} C_N e^{-1/3\lambda^2(2N+N_F)-12A(\lambda)[1+\frac{1}{6}(N-N_f)]+i\theta} \tag{86}$$

$$\times \prod_{i=1}^{N_f} (\xi \rho m_i)$$

where

$$A(\lambda) = -\frac{1}{12}\ln(1+\lambda^2/3) + \alpha(1+\gamma\lambda^{-2/3})^{-8} \tag{87}$$

$$\lambda = \pi\rho T \quad C_N = 0.097163; \quad \xi = 1.3388 \quad \alpha = .01290 \quad \gamma = 0.1586 \tag{88}$$

and $N_F = 3$ in temperature regimes where three quarks are excited.

At a temperature of $T = 1.5$ GeV and using a renormalization scale $\mu = T$, we obtain

$$F_0(1.5) = 3.7 \times 10^{-14}\ \mathrm{GeV}^{-4} \tag{89}$$

where the subscript indicates $\theta = 0$.

The expansion is exponentially sensitive, for example, to the value of g^2, to small corrections to the action. Our goal is to assess the reliability of this estimate.

Sources of uncertainty:

1. higher loop effects (partly a question of the scale of α_s
2. effects of heavier quarks
3. Infrared divergences

To be conservative about errors, we have considered the range $\mu = T$ to $\mu = \pi T$ as the scale of the coupling, since the typical ρ is of order $(\pi T)^{-1}$. We have also considered the possibility that there are three or four active quarks.

In the literature, UV-sensitive two-loop corrections to the free energy are incorporated using renormalization group considerations. But lacking a two-loop computation of the θ-dependent part of the free energy, the only principled approach is to use the one-loop expression with μ of order T (or πT).

Higher order contributions *can* serve as an uncertainty estimator.

3F, 1L, T	3.6
3F, 2L, T	10
3F, 1L, $\sqrt{\pi}$T	4.9
3F, 2L, $\sqrt{\pi}$T	7.2
4F, 1L, $\sqrt{\pi}$T	3.2
4F, 2L, $\sqrt{\pi}$T	5.2
3F, 1L, πT	6.0
3F, 2L, πT	5.5
4F, 1L, πT	4.0
4F, 2L, πT	3.8

Table : The instanton-induced free energy in units of 10^{-14} GeV^{-4} at $\theta = 0$ and $T = 1.5$. Rows correspond to a variety of computations: (3F,4F) = three or four light flavors; (1L,2L) = one-loop complete or partial two-loop; (T,$\sqrt{\pi}$T,πT) = renormalization scale.

Including also uncertainties in the value of α_s at these low scales, yields a range:

$$F_0(1.5) \in (2.2, 10) \times 10^{-14}\ \mathrm{GeV}^{-4}\ . \qquad (90)$$

The lattice result for $F_0(1.5)$ obtained by Borsanyi et al, $F_0(1.5) \approx 4 \times 10^{-13}\ \mathrm{GeV}^{-4}$, lies somewhat outside this range. While we do not have an explanation for this discrepancy, part of the purpose of this work is to illustrate the *insensitivity* of the axion relic density to the details of $F(T)$. We will include the value of $F_0(1.5)$ by Borsanyi et al in our calculations of Ω below.

It has been suggested that the uncertainty on the dilute gas computation might be much larger than estimated in the previous section, due to the presence of infrared divergences in finite temperature QCD.

In particular, there are two (overlapping) sources of concern that finite temperature corrections might be large. These are associated with the Debye screening length and with the appearance of uncontrolled infrared divergences at high order in the perturbative expansion. The latter are connected with the effective three dimensional theory which controls long wavelength modes at high temperature and signal a breakdown of perturbation theory.

The Debye screening length has been shown to receive large corrections beyond leading order (Rebhan, Arnold and Yaffe, others). A simple, heuristic understanding of these corrections can be obtained by considering Π_{44} as a function of (spatial) momentum, $\vec{q}$, for small $\vec{q}$. Already at one loop, for example, there is a contribution to the first derivative with respect to q^2 which diverges linearly as $m_d \to 0$, at $q = 0$. The full correction is

$$m_d^2 = (m_d^2)_0 + \frac{2Ng^2}{4\pi} T (m_d)_0 \ln(m_d/g^2 T). \qquad (91)$$

This correction is of order g^3, signaling a breakdown of the perturbation expansion. Numerically, at 1.5 GeV, the correction is quite large, corresponding to approximately $m_d^2 \to 2m_d^2$. Were m_d the cutoff on the instanton computation, there would be a correction of order 2^4, a substantial effect.

However, we have seen that it is not momenta of order gT that are relevant for the instanton computation, but momenta of order T. At these scales, the individual diagrams contributing to Π_{44} are well behaved, yielding corrections proportional to $g^2 T^2$. The expansion parameter would appear to be $g^2(T)/(4\pi)^{3/2}$ (the loop counting parameter of the three dimensional theory).

There is also the question of actual infrared divergent contributions to the instanton action, associated with very low momentum $\vec{A}$ fields. Such infrared divergences arise in ordinary perturbation theory for the free energy (i.e. in the zero instanton sector) first at four loop order. It is believed that they are cut off at a scale of order $g^2 T$, i.e. the presumed mass gap of the *three* dimensional gauge theory.

For an instanton of scale size of order T^{-1}, the infrared divergence should be similar. At zero temperature, the propagators are known (Creamer et al, U..W.), and at distances large compared to ρ, they are similar to free field propagators. At finite temperatures, when all coordinates except x_4 are large compared to T^{-1} and ρ, we expect something similar. So we expect an infrared divergent correction at the same order as at zero temperature. At, say 3 GeV, this is of order a part in 10^3. (This is still under investigations).

Based on these remarks, we know, with some confidence the behavior at temperatures a few GeV and above, at the order of magnitude level.

The relevant regime of temperature for cosmology is likely about $0.7 - 0.9$ GeV If we simply assume a smooth interpolation between the two regimes, as we now argue, the axion energy density is not very sensitive to the form of the interpolation, for a broad range of possible interpolating functions.

Modeling $\chi(T)$ At Intermediate Temperatures

At very low temperatures, the θ dependence of the vacuum energy is known reliably from current algebra,

$$F(\theta, 0) = -3.6 \times 10^{-5}\ \mathrm{GeV}^4 \cos(\theta). \tag{92}$$

We will adopt a class of models for $F(\theta, T)$:

$$F(\theta, T) = \begin{cases} -\chi(0)\cos\theta, & 0 < T < T_2 \\ -\chi(T_0)\left(\frac{T_0}{T}\right)^n \cos\theta, & T_2 < T < T_0 \\ -\chi(T_0)\left(\frac{T_0}{T}\right)^8 \cos\theta, & T > T_0 \end{cases} \tag{93}$$

We take $T_0 = 1.5$ GeV and vary $\chi(T_0)$ within the uncertainty on the instanton computation. T_2, the anchor point for ChPT, is related to T_0 and the slope of the power law in the model by

$$T_2^n = T_0^n \times \frac{\chi(T_0)}{\chi(0)}. \qquad (94)$$

Axion Relic Density from Misalignment

We can now assess the sensitivity of the axion relic density to uncertainties in $\chi(T)$, including both the uncertainties in the instanton computation and the behavior at intermediate temperatures.

To obtain the axion density, having adopted a form for the potential, in the case that the Peccei-Quinn transition occurs before inflation, one solves the equation of motion for the axion in a homogeneous cosmology. The axion obeys the equation

$$\ddot{a} + 3H\dot{a} + V'(a) = 0. \tag{95}$$

Using this formula, we can compute $\Omega_a h^2$ as we vary the parameters n and $\alpha_s(T_0)$ ($\chi(T_0)$). The result is:

$$h^2\Omega_{axion}(n,\chi,m_a) = \frac{0.957(8.29\times 10^{-4} m_a^2 \chi_0)^{-\frac{1}{4+n}}\theta_0^2}{m_a} \qquad (96)$$

where m_a is expressed in $\mu\mathrm{eV}$, θ_0 is the initial misalignment angle, and $\chi_0 = \frac{\chi(1.5\ \mathrm{GeV})}{1.62\times 10^{-12}(\mathrm{GeV})^4}$. For fixed $\Omega_{axion} = \Omega_{\mathrm{dark\ matter}}$,

$$m_a = \frac{(8.92\times 10^{-4}\chi_0)^{-\frac{1}{6+n}}}{(1.05\Omega_{\mathrm{dark\ matter}}h^2)^{\frac{4+n}{6+n}}}\theta_0^{\frac{8+2n}{6+n}} \qquad (97)$$

This again yields only modest variation over a broad range of n and χ_0.

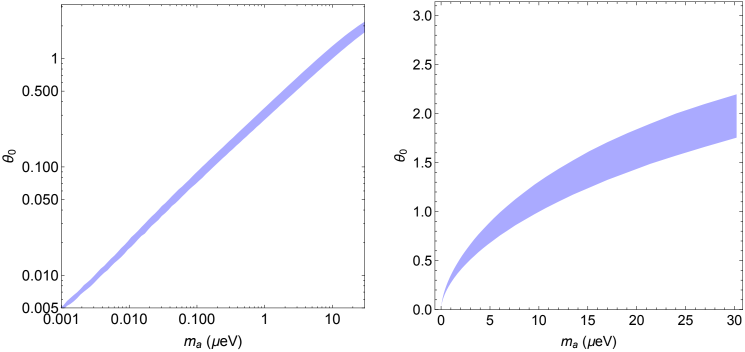


Figure : Axion relic density from misalignment in the pre-inflationary scenario. The curve shows the misalignment angle needed to obtain $\Omega = 0.258$. The band reflects the uncertainty in the instanton computation of the free energy, and the anchor point for ChPT has been fixed to $T_2 = 140$ MeV.

$$\Omega h^2/\theta^2;\ \chi_0$$

n	$\chi_0 = 1$	$\chi_0 = 0.14$	$\chi_0 = 0.021$
2	0.14	0.20	0.27
4	0.13	0.17	0.21
8	0.11	0.14	0.16

Table : Axion density as a function of model parameters

Peccei-Quinn transition before or after inflation

The axion dark energy density is quite sensitive to the question: does the Peccei Quinn transition occur before or after inflation. We should start by noting that there is not necessarily a Peccei-Quinn transition at all (A. Anisimov, M.D.). The Peccei-Quinn symmetry, if it exists, is almost certainly an accident. This accident may not occur at the high temperatures or small curvature which characterize the early universe. In this case, the initial value of θ is a fixed number, or possibly one of a set of discrete numbers. This number might well be small, or alternatively might be of order π. Either has significant implications for the final dark matter density.

Alternatively, as usually assumed, there may be an approximate symmetry both for low and high temperature (or curvature). The question of whether the symmetry is broken during or after inflation then depends on questions such as the coupling of the inflaton to the field responsible for PQ symmetry breaking. For example, there might be an effective mass term for this field, of either sign. There seems no particular reason to believe that one or the other outcome is favored.

These different possibilities have been extensively studied in the literature. In the case of symmetry breaking before inflation, one can simply take our results above. If the symmetry breaking occurs after inflation, one has to average over random initial misalignment angles, yielding an increase in the density by a factor of order 4. In this case, there are also additional sources of axion dark matter, such as cosmic strings.

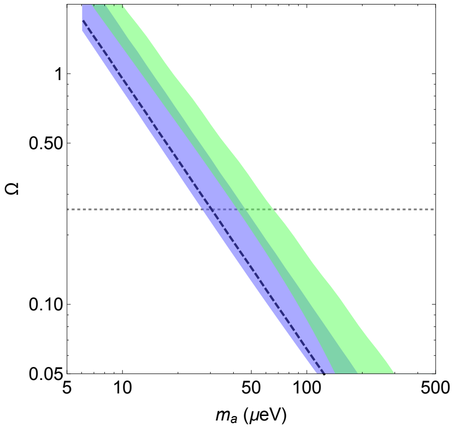


Figure : Axion relic density from misalignment in the post-inflationary scenario. Purple, blue, green, orange) curves correspond to $T_2 = (100, 200, 300, 600)$ MeV, respectively. The width of each band reflects the uncertainty in the instanton computation of the free energy. The dashed line corresponds to the value of $F_0(1.5)$ obtained in the lattice calculation of Borsanyi et al.

So we see that the density is rather weakly sensitive to

1. The overall size of the dilute gas result
2. The form of the interpolation between low and high temperature

This is clear from the formula for Ω, which depends, even for $n = 2$, on $\chi_0^{1/6}$, and is roughly linear in m_a. So substantial uncertainties in χ_0 translate into modest uncertainties in m_a. Numerical solution of the equations, including non-linear effects, again over the parameter range, does not alter these results by a factor larger than about 1.5.

The lattice results of Borsanyi et al, for example, differ from our estimates of χ in the case $n = 8$ by factors of order 10. Allowing for such a variation makes a 10% change in the value of the axion mass required to account for the observed dark matter density.

In the post-inflationary scenario, the result is less than 50 μ eV. Additional sources of axion production (cosmic strings) can force a larger axion mass. These masses are, indeed, at the edge of capability of cavity experiments such as ADMX, and are the focus of much future planning.

Conclusion: Axion Energy Density Estimates are Robust

Within the conventional picture of axion cosmology, computations of the axion energy density appear robust. While improved computations of the topological susceptibility would lead to improvements in the density computations, these would be modest. In particular, the computations which define the parameters of the ADMX experiment seem on a solid footing.

Axion *Quality*

Finally, we turn to a theoretical question: Why are there axions at all? More precisely, why should there be a Peccei-Quinn symmetry, and how good a symmetry does this have to be?

General belief (supported by studies of string theory): *a theory of quantum gravity does not possess (exact) global symmetries.*

Then hopeless? No: symmetry might be an accidental consequence of other symmetries.
Example: discrete symmetries.

$$\phi \to \phi e^{\frac{2\pi i}{N}}. \tag{98}$$

So leading symmetry breaking terms in potential might take the form:

$$\mathcal{L}_{symm-breaking} = \frac{\phi^N}{M_p^{N-4}} \tag{99}$$

If N is large, these terms would seem very small. But they have to be *extremely* small to insure the smallness of θ. One needs, e.g., the linear term in the a potential

$$V = \frac{1}{2} m_a^2 \, a^2 + \Gamma a + \ldots \tag{100}$$

to be such that

$$\frac{\Gamma}{m_a^2} < 10^{-10} f_a \tag{101}$$

This translates into a requirement that $N > 12$, if $f_a = 10^{11}$; even larger for larger f_a

Why should this be?

Axions in String Theory

We have seen that axions, from the perspective of effective field theory, are surprising. It has long been known that axions are common in string theory, indeed axion-like objects seem ubiquitous. What insight do they give and what expectations do they lead to?

Examples:

1. Heterotic string contains an axion (always) which couples universally to all of the gauge groups.
2. In string theories, antisymmetric tensor fields in higher dimensions become pseudoscalars in four dimensions with axion type couplings.
3. All of these fields exhibit approximate, continuous shift symmetries, $a \to a + \omega f_a$. They typically exhibit *exact* discrete shift symmetries, $a \to a + 2\pi f_a$. The breaking of the continuous symmetries is suppressed, at weak coupling, by $e^{-2\pi/\alpha}$.

Might expect large axion decay constants ($f_a \sim 10^{16}$ GeV?).
What about the cosmological limits?

Unconventional cosmologies

It has long been recognized that the underlying cosmological assumptions of the standard calculation may not hold, and that there is good theoretical motivation to consider lighter axions with larger decay constants. If there is an underlying supersymmetry, for example, the *saxion*, the scalar partner of the axion, leads to severe problems with nucleosynthesis, unless there is significant entropy production at a relatively late stage of evolution. In such a picture, the universe was likely never much hotter than 10 MeV.

So string theory might be a setting for the axion. If there is an exponentially small parameter (e.g. accounting for hierarchies) this parameter might explain why the Peccei-Quinn symmetry is of the necessary quality.

Suggests high f_a, modified cosmology.

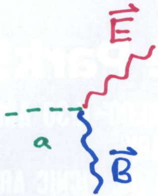
Searching for the Axion

The conventional cosmology suggests a strategy for axion detection. For $f_a > 10^8$ GeV, the axion is extremely weakly interacting. In scattering experiments, it is produced rarely and detection is essentially impossible.

However, if we assume that the axion constitutes the dark matter, we are living in a sea of axions, and we might hope to detect them. The main interaction at our disposal is the interaction with the electromagnetic field characterized by $G_{a\gamma\gamma}$: In particular, in a strong magnetic field, an axion can convert into a photon. If the magnetic field is in a cavity, this means we can hope to produce a cavity excitation.

Axion Detection Process

$$G_{a\gamma\gamma}\, F\tilde{F} = G_{a\gamma\gamma}\vec{E}\cdot\vec{B}.$$



There are many challenges. The axion is quite narrow and we don't know it's mass with anything like precision. So one needs to be able to sweep through many small frequency steps. One needs a cavity of very high quality. The most impressive effort of this type is the ADMX experiment. All of this you will hear about in Professor Graham's lectures.

Basic idea is that cavity is superconducting with high Q (10^5). B field is large ($8T$). Cavity also large volume

$$g_{a\gamma\gamma} = \frac{\alpha g_\gamma}{\pi f_a}; \quad f_\gamma = -0.97 \text{ KSVZ}; \quad g_\gamma = 0.36 \text{ DFSZ}. \tag{102}$$

From what we have seen, $\Omega_a = \left(\frac{6\mu\text{eV}}{m_a}\right)^{\frac{7}{6}}$.
So tells us the ball park mass to search for (20 μeV for $\Omega_a = 0.23$. $m_a < 16$ meV from stellar constraints.

But now argue that the cosmological constraint is rather soft. Searches for lighter axions interesting and potentially important. One proposal: Casper (next week from Peter Graham).

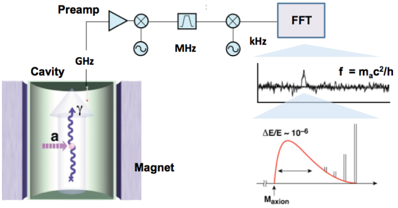


Figure 2

Schematic of the microwave cavity search for dark matter axions. Axions resonantly convert to a quasi-monochromatic microwave signal in a high-Q cavity in a strong magnetic field; the signal is extracted from the cavity by an antenna, amplified, mixed down to the audio range, and the power spectrum calculated by a FFT. Possible fine structure on top of the thermalized axion spectrum would reveal important information about the formation of our galaxy.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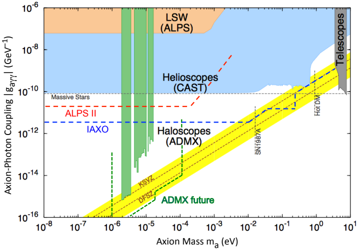
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DFSZ
ADMX future
Axion-Photon Coupling $|g_{a\gamma\gamma}|$ (GeV^{-1})
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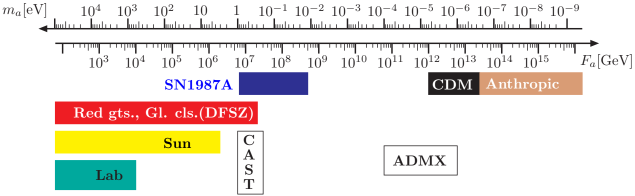

FIG. 15: A cartoon for the F_a bounds.

How Robust are the Cosmological Limits on the Axion

The axion cosmology we have described assumes that the universe was in thermal equilibrium at very early times, times much smaller than the scale set by the axion mass.

There are reasons to question this assumption. For example, suppose that nature is approximately supersymmetric. Then the axion has a *scalar* superpartner, the saxion. This particle is long lived. If it decays through the two photon interaction (or its superpartners), its lifetime is of order

$$\Gamma = \left(\frac{\alpha}{4\pi}\right)^2 \frac{m_{saxion}^3}{f_a^2} = \left(10^5 \text{ s}\right)^{-1} \frac{m_{saxion}^3}{\text{TeV}^3} \left(\frac{10^{16}}{f_a}\right)^2 \qquad (103)$$

where we have taken the grand unified scale as our benchmark axion decay constant. Even at 10^{11} GeV, this is after the axions start to oscillate.

The saxion, when it decays, “reheats" the universe. This temperature should be higher than the temperature at which nucleosynthesis occurs (say 10 MeV).

When the axion starts to oscillate, the universe is dominated by the saxion. It’s energy density is of order $m_a^2 f_a^2$, while the total energy is of order $m_a^2 M_p^2$, so the axion energy fraction is approximately

$$f_a^2/M_p^2 \tag{104}$$

After the saxion decays to radiation, the fractional energy density grows with $1/T$. So between 10 MeV and 1 eV, it grows by 10^7. This gives

$$f_a < 10^{15} \tag{105}$$

a much weaker limit than before. Could be weaker still.

Cosmology of String Theoretic Axions

We have seen that string theory has axions suitable for solving the strong CP problem. But what about their cosmology?

1. f_a large; in what is imagined a typical string phenomenology, $f_a \geq 10^{15}$ GeV.
2. If approximate supersymmetry, axions accompanied by saxions, other “moduli". Can dilute axions as above. May require surprisingly low f_a.

Detecting string scale axions

Clearly challenging. Recently pursued by P.Graham, S. Rajendran, and others.

Strategies involve noting that if axions are the dark matter, $\theta \propto \cos(m_a t)$, and using (searching for) time varying dipole moments. Some prototype experiments under discussion. More in Peter Graham's lectures here.

Outlook

Axions are a well-motivated dark matter candidate.

The most straightforward approach suggests they should be detectable in cavity experiments.

Theoretical arguments, however, suggest that in a more complete cosmology, axions might be lighter.

In either case, their detection and study would be an extraordinary development.