L2

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INTRODUCTION TO NEARLY-AdS_2

J. MALDACENA

REFERENCES

- ALMHEIRI - POLCHINSKI - 1402.6334
- JENSEN - 1605.06098
- ENGELSÖY - MERTENS VERLINDE 1606.03438
- MALDACENA - STANFORD - YANG 1606.01857

REFERENCE FOR QUANTIZATION OF SCHWARZIAN THEORY:

1-d LIOUVILLE: - BAGRETS, ALTLAND, KAMENEV. 1607.00694 - 1702.08902

LOCALIZATION: - STANFORD, WITTEN. 1703.04612

2-d LIOUVILLE - MERTENS, TURIACI, VERLINDE. 1705.08408

PARTICLE IN MAGNETIC FIELD: KITAEV - SUH - TO APPEAR.

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(1)

- DECOUPLING LIMITS.

BH

- LOOK AT BLACK HOLE FROM OUTSIDE
- WE WOULD LIKE TO ISOLATE THE B.H. DYNAMICS.
- BH IS COUPLED TO EXTERNAL FIELDS.

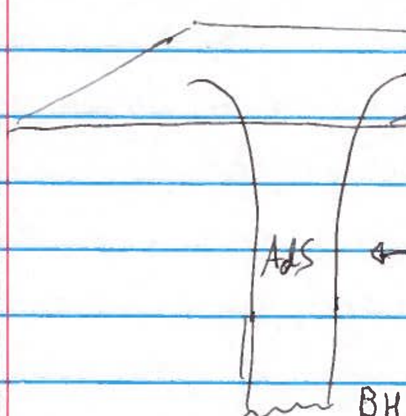
. OPERATORS ACTING ON THE BH HILBERT SPACE.

. LEAD TO CORRECTIONS TO B.H. DYNAMICS.

THAT ARE RELATIVELY LARGE

(i.e. LARGE MODIFICATION TO THE PRECISE ENERGY LEVELS).

- DECOUPLING LIMITS :



← LONG AdS THROAT

↓

QUANTUM THEORY FOR FULL AdS.

(ASLO NON-AdS... CASES).

AdS_{D+1}/CFT

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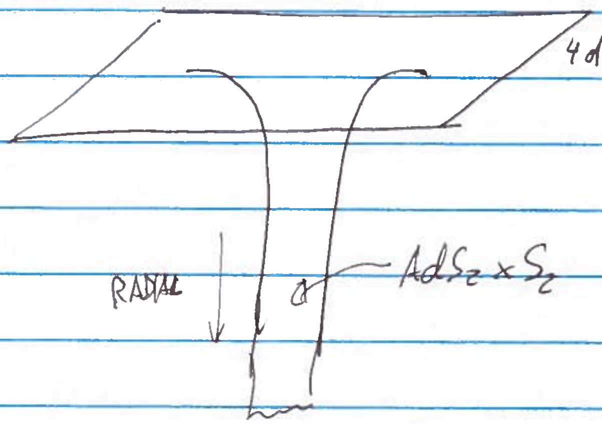
WHAT ABOUT AdS_2/CFT_1 ?



CHARGED BLACK HOLES

$M \gtrsim Q.$

$M \sim Q.$

ROTATING / KERR BH $\rightarrow$ SIMILAR.

- CRITICAL SYSTEM

$$S = \underset{\substack{\uparrow \\ \text{EXTREMAL} \\ \text{VALUE}}}{S_0} + \underbrace{\text{CONST } T}_{\substack{\text{NEAR} \\ \text{EXTREMAL.}}} \underset{\uparrow \, l_p = 1}{\approx} Q^2 + Q^3 T + \ldots \approx \frac{r_e^2}{G_N} + \frac{r_e^3}{G_N} T + \ldots$$

$\downarrow$

IS FINITE!

WHAT COULD BE $\rho(E)$ FOR A SCALE INV. SYSTEM?

$$\rho(E) = \begin{cases} \delta(E) & \rightarrow \text{NO DYNAMICS (ALL ZERO ENERGY STATES)} \\ \dfrac{1}{E} & \rightarrow \text{DIVERGENT.} \end{cases}$$

$\rightarrow$ PURELY EXTREMAL LIMIT $\rightarrow$ ONLY GOOD TO STUDY EXACTLY DEGENERATE STATES $\sim$ INDEX, BPS...

$\rightarrow$ DYNAMICS $\rightsquigarrow$ REQUIRES INCLUDING BREAKING OF SCALING SYMMETRY

$\rightsquigarrow$ NEARLY-AdS_2. OR $NAdS_2$

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– ASYMPTOTIC SYMMETRIES OF PURELY AdS_2.

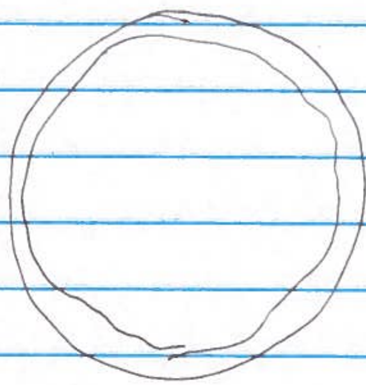
$$\frac{dz^2+dt^2}{z^2}. \qquad t \to f(t) \qquad z \to z f'(t)$$

⇝ LEADING TERMS IN THE METRIC REMAIN INVARIANT.

→ SYMMETRIES OF $AAdS_2$. BOUNDARY CONDITIONS.

IF f IS $SL(2)$ TRANSF → AdS_2 IS INVARIANT.

• SIMILAR POINT OF VIEW



AdS_2.

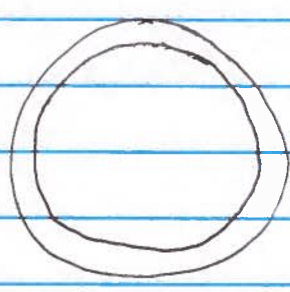
TAKE SOME BOUNDARY WHERE WE WILL GLUE IT TO SOME OTHER SPACE – OR SOME UV CUTOFF.

• SET BC. FIXING LONGITUDINAL METRIC.

$$g_{uu} = \frac{1}{\varepsilon^2} \qquad ds^2 = \frac{du^2}{\varepsilon^2}$$

u = BDY TIME " PROPER TIME (RESCALED)

⇒ ANY CURVE WITH SAME PROPER TIME WILL DO



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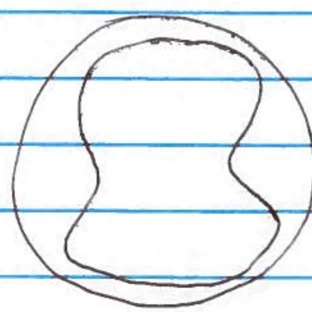

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SAME GEOMETRIES LOCALY.
BUT DIFFERENT - CUT OUTS.

CLASSICAL ACTION.

$$\phi_0\left(\int_{M_2} \sqrt{g}\, R\, d^2x - 2\int_{\partial M_2} K\right) \rightsquigarrow \text{TOPOLOGICAL INVARIANT}$$

$\rightsquigarrow$ GIVES THE EXTREMAL ENTROPY.

- IF WE HAD BULK QUANTUM FIELDS.
  WE GET $\neq$ ANSWERS IF CUTOFF @ FINITE SIZE.

BUT AS $\varepsilon \to 0$ $\to$ SAME ANSWERS.

- RELATED BY THE ACTION OF THE ASYMPTOTIC SYMMETRY GROUP.

- WE CAN PARAMETRIZE A CURVE AS.

$$t(u) \qquad\qquad ds^2 = \frac{dz^2 + dt^2}{z^2}$$
$$z(u)$$

$$ds = \frac{du}{\varepsilon} \to \text{SET} \quad z(u) \equiv \varepsilon\, t'(u)$$

- $t(u)$ PARAMETRIZES THE CURVE.

- WE CAN ADD AN EXTRA TERM TO THE ACTION

$$\underbrace{\int K}_{} \qquad\qquad ds K = \frac{1}{\varepsilon}\, du \left(1 + \varepsilon^2 \{t, u\}\right)$$

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$$\frac{\text{CONST}}{\varepsilon}\int K \approx \frac{\text{CONST}}{\varepsilon^2}\int du + \text{CONST}\int du\,\{t,u\}$$

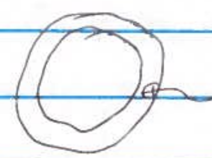
WE EXPECT THAT THIS IS THE LEADING CORRECTION.

→ LOCAL ACTION ⇝ LOCAL DYNAMICS FOR THE MOTION OF THE BOUNDARY. LOCAL IN u. (NOT NECESSARILY LOCAL AT RAdS SCALES)!

⇝ FIXES BOUNDARY SHAPE

⇝ BREAKS GENERAL REPARAMETRIZATION SYMMETRY.

$t \to \frac{at+b}{ct+d}$ ⇝ GAUGE SYM. (NOT-LOCAL) UNPHYSICAL = REDUNDANCY OF THE DESCRIPTION.



MOVE RIGIDLY IN AdS_2.

$u \to \frac{au+b}{cu+d}$ **NOT** A SYMMETRY.

⇝ **NO** PHYSICAL CONFORMAL SYMMETRY

- BUT MANY CONSEQUENCES OF THIS PATTERN OF APROXIMATE SYMMETRIES.

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GRAVITY IN 2-d.

$$S = \int \tilde{F}(\tilde{\phi})\tilde{R} + \tilde{U}(\tilde{\phi}) + H(\tilde{\phi})(\nabla\tilde{\phi})^2. \qquad \text{GENERAL}$$

$\downarrow$ FIELD REDEFINITIONS.

$\phi = F(\tilde{\phi}) \qquad \tilde{g}_{\mu\nu} = G(\phi)\, g_{\mu\nu}$

$$S = \int \phi R + U(\phi)$$

$\rightarrow$ FIX BC. $\phi = \phi_b$ $\rightarrow$ CAN CHOOSE $U(\phi)$ TO FIT ANY THERMODYNAMICS.

SIMPLEST CASE: ~~(STUDIED IN 90's)~~ $\rightarrow$ NEAR HORIZON REGION OF ANY BLACK HOLE...

$$\cancel{S =} \int \phi R + \text{CONST} + \phi_0 \int \sqrt{g} R$$

NEXT SIMPLEST: NEARLY AdS_2:

$$S = \int d^2x\, \phi (R + 2) + \phi_0 \int \sqrt{g} R$$

$$\phi_b 2 \int K + \phi_0 2 \int K + S_{matt}[g, \chi]$$

NO ϕ.

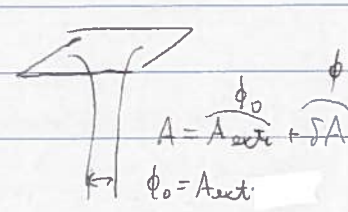
EOM:

$R = -2 \rightarrow$ METRIC FIXED TO AdS_2

$\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \nabla^2 \phi + g_{\mu\nu} \phi = 0$

$\Big\}$ NO PROPAGATING MODES.

$\phi \propto$ VOLUME OF REST OF DIMENSIONS.



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- THIS IS THE EXPECTED ACTION FOR **ANY** NEAR EXTREMAL BLACK HOLE SOLUTION

(ALMHEIRI - POLCHINSKI)

- THE FACT THAT MATTER FIELDS DO NOT COUPLE TO ϕ, AT LEADING ORDER, IS RELATED TO THEIR KK ORIGIN.

$$\text{AREA} = \phi_0 + \phi \qquad \text{AND} \qquad \phi_0 \gg \phi.$$

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. WE FIRST INTEGRATE OUT ϕ

$R = -2$

AND

$$I = \underbrace{2\phi_b \int K}_{\text{WHAT WE DISCUSED BEFORE.}}$$

$\rightarrow$ SCH. ACTION.

$\rightsquigarrow$ PARTICLE IN ELECTRIC FIELD IN AdS_2.

$$2\int_{\partial M} K = \chi - \underbrace{\int_M \sqrt{g} R}$$

AREA: $\int_M \sqrt{g} = \int_M F = \int_{\partial M} A$

$$\underbrace{\int ds}_{\text{FROM}}$$ LOCAL COUNTER TERM.

. ALSO:

GENERAL SOLUTION FOR ϕ:

1. $\varepsilon^{\alpha\beta}\partial_\beta \phi$ IS A KILLING VECTOR.

2. $ds^2 = d\rho^2 + sh^2\rho \, d\tau^2$ WHERE: $\rho = 0 \rightarrow \partial_\alpha \phi = 0$

3. $\phi = \phi_h \, ch\, \rho$ $\rightarrow$ ϕ INCREASES TOWARDS BOUNDARY.

$\phi_{ren} = \frac{N}{J}$ $\quad \frac{\phi_{ren}}{\varepsilon} = \phi_b = \phi_h \frac{e^{\rho_c}}{2}$ $\quad d\tau \frac{e^{\rho_c}}{2} = \frac{1}{\varepsilon} du$ $\quad \frac{\varepsilon e^{\rho_c}}{2} = \frac{\beta}{2\pi}$

$$\phi_h = \frac{\phi_{ren}}{\varepsilon e^{\rho_c}/2} = \frac{\phi_{ren} 2\pi}{\beta} \propto N/\beta J$$

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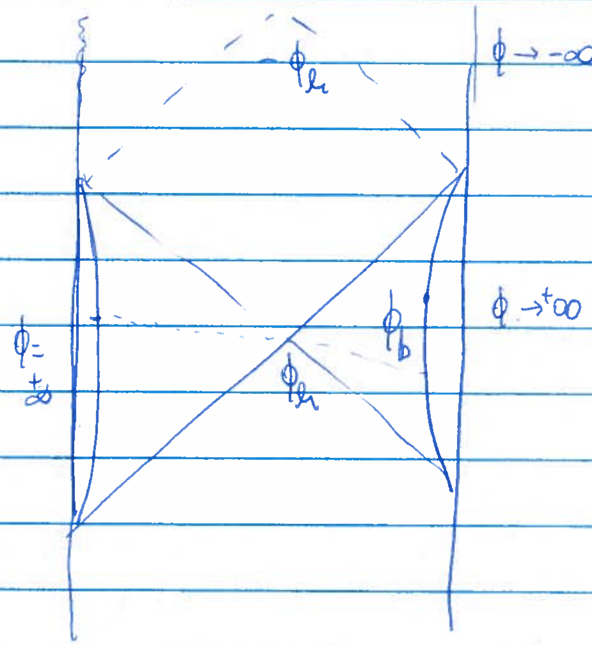
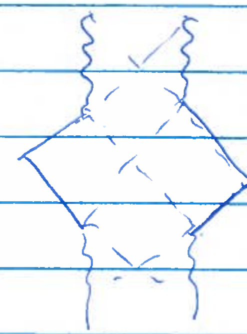
LORENTZIAN





- PHYSICAL PARAMETERS. :

ϕ_h , ~~LOCATION OF $u_L = 0$ VS $u_R = 0$~~] CONJUGATE VARIABLES.

$\mathrm{MASS} \approx \phi_h^2$

GENERAL STORY → ANY EXTENDED (ETERNAL) BH.

- EACH BDY ⇝ VIEW AS PARTICLES WITH
  - SL(2) CHARGES.

$$Q^a \qquad a = -1, 0, 1$$

- $$E = \frac{Q^2}{\phi_r} = \frac{Q^a Q^a \eta_{ab}}{\phi_r} = -\{f, u\} \times \phi_r$$

- $Q^a_L + Q^a_R = 0$ CONSTRAINT (GAUGE).

→ HILBERT SPACE DOES NOT FACTORIZE.
(OF BDY DOF
OR JT GAUU)

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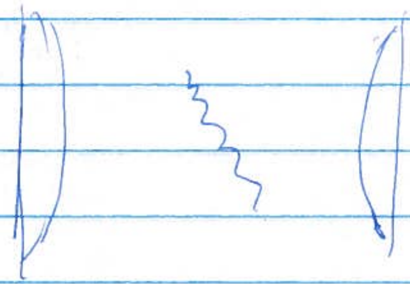
ADD MATTER.



- MATTER MOVES ON A RIGID METRIC.
- MATTER CARRIES SL(2) CHARGES q^a

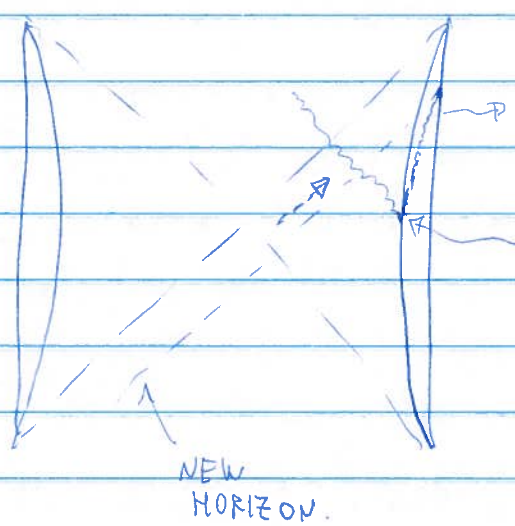
$$Q_L^a + q^a + Q_R^a = 0$$

$$H_R = Q_R^2$$
$$H_L = Q_L^2$$

BUT CONSTRAINT LINKS BULK DYNAMICS TO H_R & H_L

IS A TOY MODEL FOR WHAT WE USUALLY HAVE.

- IS A VERY USEFUL WAY TO UNDERSTAND DYNAMICS:



$\rightsquigarrow$ EXPONENTIAL DEVIATION FROM PREVIOUS TRAJECTORY - BUT NOT SEEN FOR LOCAL OBSERVABLES.

SL(2) CHARGE CONSERVATION.
$\approx$ MOMENTUM CONSERVATION
$\rightsquigarrow$ BDY KICKED OUTWARDS.

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SUMMARY

- THE GRAVITATIONAL DYNAMICS OF $NAdS_2$ REDUCES TO A BDY GRAVITON.
  → IS A SINGLE QM DEGREE OF FREEDOM.

- IN ADDITION WE HAVE BULK MATTER.

- S_{extr} IS JUST A PARAMETER IN A TOPOLOGICAL TERM IN THE ACTION.

- THE ACTION FOR BDY GRAVITON IS ALSO SCHWARZIAN.

- SCHWARZIAN ACTION BREAKS PHYSICAL $SL(2)$ SYMMETRY
  ⇝ SPONT. BREAKING OF $Diff(S_1) \to SL(2)$
  + EXPLICIT BREAKING.

- SAME PATTERN AS SYK.

SYK: Sch. + $G\Sigma$

IDENTICAL

Sch = GRAVITON + MATTER INSIDE

JT GRAVITY $NAdS_2$

NOT IDENTICAL, BUT SAME SYMMETRIES.

$$S_{extr} = \# N = \frac{r_+^2}{G_N} \sim Q^2 , \quad \frac{N}{J} = \frac{r_e^2}{G_N} r_e = \phi_{REN}$$

- MANY PROBLEMS HAVE A COMMON SOLUTION.

- WE EXPECT THAT ANY NCFT$_1$ HAS THIS PATTERN

WE ARE GETTING SOME INSIGHTS INTO TIME TOWARDS INTERIOR

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SYK @ Low energies — NAdS$_2$ GRA

$G_c = \frac{1}{|t_{12}|^{2\Delta}}$ $\longleftrightarrow$ AdS$_2$ METRIC

REPARAMETRIZATION SYMMETRY $\longleftrightarrow$ ASYMPTOTIC SYMMETRIES OF AdS$_2$

$\cup$ — $\cup$

SL(2) $\longleftrightarrow$ SL(2)

- SPONT BROKEN TO ~~Diff(S')~~ SL(2)

- EXPLICITLY BROKEN BY

UV — BOUNDARY COND (DILATON)

- $\{f, \tau\}$

PHYSICAL TIME — BDY PROPER TIME (RESCALED)

BROKEN {NO SCALE INVARIANCE IN $\mathcal{M}$

$\frac{N}{J}$ $\sim$ $\frac{r_e^2}{G_N} \cdot r_e$ FOR R-N BH.

S_0 $\sim$ $\frac{r_e^2}{G_N}$.

$G_\perp$ = OTHER EXCITATIONS $\not\Leftrightarrow$ PARTICLES MOVING IN AdS$_2$

(SEEMS NOT LOCAL) — (LOCAL IN GRAVITY).

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PHYSICAL CONTENT OF 'SCHW. THEORY.

$$f = \tanh\left(\tfrac{\varphi(u)}{2}\right)$$

$$\{f, u\} = \{\varphi, u\} - \frac{1}{2}\varphi'^2 =$$

$\varphi = u$ $\quad(\beta = 2\pi)$

$\varphi = u + \varepsilon(u).$

$$\{f, u\} = -\frac{1}{2}\left[(\varepsilon'')^2 + (\varepsilon')^2\right]$$

EOM: $\varepsilon'''' - \varepsilon'' = 0$

$$\varepsilon = \alpha_0 + \alpha_1 u + \beta e^{u} + \beta e^{-u}.$$

COME FROM ACTING w/ $SL(2)_R$ ON THE ORIG. SOLUTION.

CHANGES THE ENERGY

PHYSICAL CHANGE. → CHANGE IN ADM MASS.

NAIVELY WE EXPECTED ONLY ONE GRAVITY MODE → TOTAL ENERGY

⇝ CONSERVED QUANTITY.

BUT ⇝ EXP. GROWING TERMS CAN LEAD TO PHYSICAL EFFECTS SOMETIMES ⇝ NEED SPECIAL OBSERVABLES.

e.g. OTOC OR CORRELATORS IN THE TFD STATE

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QUANTIZATION

WHEN IS IT IMPORTANT.
e.g. $\Delta S = S - S_{ext} = \frac{N}{\beta J} = \mathcal{E}T$ $\beta \sim 1 \rightarrow$ Quantum effects are important

- IT IS POSSIBLE TO EXACTLY SOLVE THE Q.M. OF THE SCHWARZIAN THEORY

$$\int \{f,u\} \longrightarrow \int \underbrace{\cancel{\left(\frac{f''}{f'}\right)'}}_{\text{TOT. DER.}} - \frac{1}{2}\left(\frac{f''}{f'}\right)^2$$

$$\mathfrak{z} = \log f'$$

$$\int \mathfrak{z}'^2 + \lambda\left(e^{\mathfrak{z}} - f'\right)$$

. EOM FOR $f \Rightarrow \lambda = $ CONST.
$\Rightarrow$ $\mathfrak{z} = $ LIOUVILLE $\rightarrow$ SOLVABLE.

. THERE ARE **MANY** OTHER WAYS **TO** SOLVE IT.

- PATH INTEGRALS USING LOCALIZATION.
- FROM EXACT 2d LIOUVILLE.
- EQUIVALENCE WITH PARTICLE IN ELECTRIC FIELD, ETC.

. RESULTS

$$Z(\beta) \propto e^{S_0} \frac{1}{(\beta J)^{3/2}} e^{+\frac{N}{\beta J}}$$ $\rightarrow$ EXACT. IN SCH. THEORY

BUT WRONG! FOR $\beta \sim e^{S_0}$

WHAT CORRECTIONS FIX THIS?

$\rightsquigarrow$ SEE – SSS = SAAD, – , SHENKER, STANFORD

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LORENTZIAN CORRELATORS.

- CONF SYMMETRY

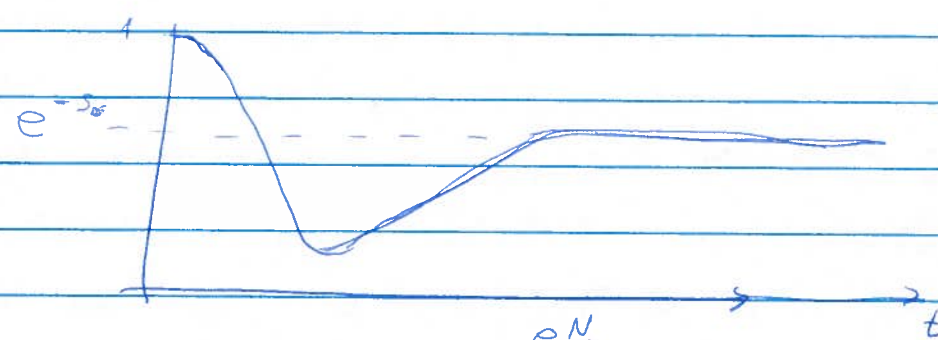
QUASI-NORMAL MODE

$$\propto \frac{1}{\left(\beta_c \sinh\left(\frac{t\pi}{\beta}\right)\right)^{2\Delta}} \sim e^{-\frac{t\pi 2\Delta}{\beta}} \quad @ \quad \begin{array}{l} N \gg J\beta \gg 1 \\ N \gg Jt \gg 1 \end{array}$$

$$\propto e^{-\frac{2\pi^2 N}{\beta J}} \frac{1}{t^3} \qquad e^N \gg Jt \gg N$$

SLOWER DECAY

→ INTERESTING FURTHER BEHAVIOR AT LARGER TIMES: → EXPONENTIALLY LARGER TIMES.



REFERENCES FOR LONG TIMES IN SYK:

- COTLER, GUR-ARI, HANADA, POLCHINSKI, SAAD, SHENKER, STANFORD, STREICHER, TEZUKA - 1611.04650
- SAAD, SHENKER, STANFORD, 1806.06840