(A)

FOR MHV AMPLITUDES

AT GENUS ZERO, WE

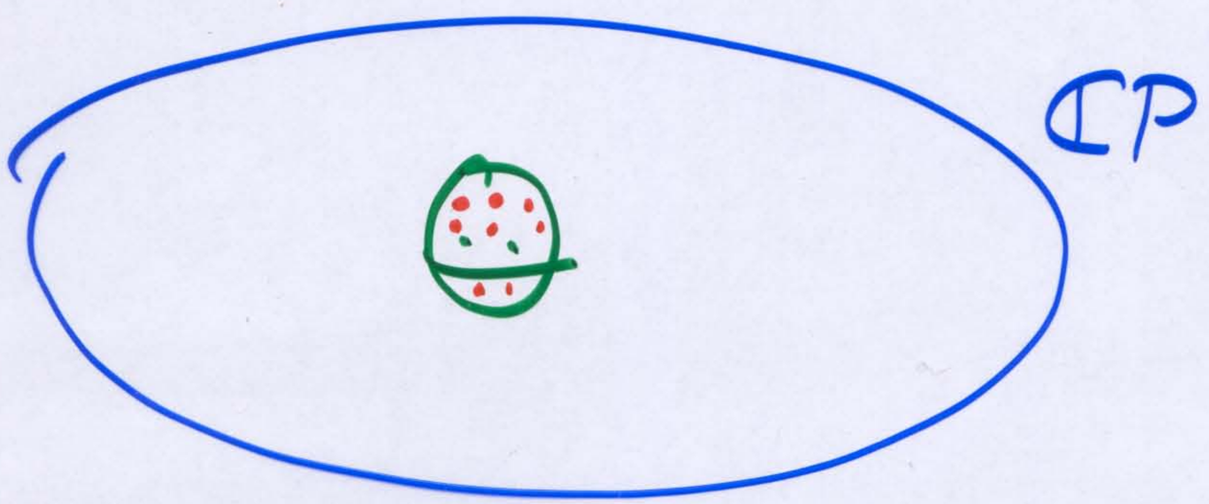
USED S^2's OF DEGREE ONE

$$\mu_{\dot{a}} + x_{a\dot{a}}\lambda^{a} = 0$$

LAST TIME WE SHOWED THAT TREE LEVEL MHV AMPLITUDES

$++ \ldots ++--$

ARE DESCRIBED BY $\mathbb{C}P^1$'s IN $\mathbb{C}P^3$



AND WE CONJECTURED THAT, MORE GENERALLY, TREE AMPLITUDES WITH ANY NUMBER OF HELICITIES

$\underbrace{+++\ldots+}_{p}\underbrace{---\ldots\ldots-}_{q}$

ARE DESCRIBED BY GENUS ZERO CURVES OF DEGREE $q-1$

MY MAIN GOAL TODAY IS TO TELL YOU WHAT KIND OF STRING THEORY GIVES THIS RESULT.

THE S^2's OF DEGREE ONE ARE THE ONES WE FOUND LAST TIME

$$\mu_{\dot{a}} + x_{a\dot{a}} \lambda^a = 0$$

TO DESCRIBE THE NEXT

AMPLITUDES

$$+ + \cdots + \underbrace{- - -}_{q=3}$$

WE NEED S^2's OF DEGREE 2

THESE CAN BE CONSTRUCTED

AS I TOLD YOU ~~LAST TIME~~

linear $\quad f(z_1, \dots, z_4) = 0$

quadratic $\quad g(z_1, \dots, z_4) = 0$

THE DEGREE IS

$$d = 1 \cdot 2 = 2$$

IN THIS CASE, AN ELEMENTARY WAY TO PROVE THAT THE GENUS IS ZERO IS TO TAKE

$$f = z_4$$

$$g = z_1^2 + z_2^2 + z_3^2$$

THE MANIFEST $SO(3)$ SYMMETRY SHOWS THIS MUST BE A 2-SPHERE, AND WE CAN SEE THIS MORE DIRECTLY AS FOLLOWS:

WE SIMPLY SOLVE THE EQUATIONS $f = g = 0$, i.e.

$$0 = z_4 = z_1^2 + z_2^2 + z_3^2$$

BY

$$u = \frac{z_3}{z_1 + i z_2}, \quad \frac{z_1 - i z_2}{z_1 + i z_2} = -u^2$$

SO OUR CURVE IS THE COMPLEX u-PLANE PLUS A POINT AT INFINITY

(THE LATTER IS $(z_1, z_2, z_3) = (1, i, 0)$)

THESE, THEN, ARE THE "NEXT" CURVES AFTER THE ONES THAT GIVE MHV AMPLITUDES.

IF THERE'S TIME AT THE END, I'LL EXPLAIN A SYSTEMATIC WAY TO CONSTRUCT GENUS ZERO CURVES IN $\mathbb{CP}^3$ OF ANY DEGREE.

FIRST, THOUGH, I WANT TO EXPLAIN HOW TO CONSTRUCT THE STRING THEORY WHOSE WORLDSHEET INSTANTON EXPANSION REPRODUCES PERTURBATIVE GAUGE THEORY IN $D=4$.

(I'LL EXPLAIN ONE OF TWO KNOWN APPROACHES, THE SECOND BEING DUE TO BERKOVITS.)

NOW THE STRING THEORY....

WE NEED TO DISCUSS SOMETHING A LITTLE MORE TECHNICAL THAN WHAT WE'VE DISCUSSED SO FAR...

TOPOLOGICAL FIELD THEORY

START WITH $N=2$ SUPERSYMMETRY IN $D=2$

$i, j = 1, 2$

$$\{Q_{\alpha i}, Q_{\beta j}\} = \delta_{ij}\, \gamma^{\mu}_{\alpha\beta}\, P_{\mu}$$

IN $D=2$, THIS ALGEBRA CAN BE "DIAGONALIZED" USING LIGHT CONE VARIABLES

$$\{Q_{+i}, Q_{+j}\} = \delta_{ij} P_+$$

$$\{Q_{-i}, Q_{-j}\} = \delta_{ij} P_-$$

$$\{Q_{+i}, Q_{-j}\} = 0$$

WHERE $P_\pm = P_0 \pm P_1$

WE LET

$$Q = Q_{+1} + i Q_{+2} + Q_{-1} \pm i Q_{-2}$$

WITH EITHER CHOICE OF SIGN

SO $Q^2 = 0$

WE NOW GIVE THE THEORY A NEW INTERPRETATION, SAYING THAT A PHYSICAL STATE Ψ MUST OBEY $Q\Psi = 0$

// $\Psi = Q\chi$
$Q^2 = 0$

AND Ψ IS EQUIVALENT TO Ψ' IF $\Psi - \Psi' = Q\chi$ FOR SOME χ.

i.e. WE TREAT Q AS A BRST OPERATOR

AS I'VE DESCRIBED IT, THIS CONSTRUCTION VIOLATES LORENTZ INVARIANCE SINCE

$$Q = Q_{+1} + i Q_{+2} + Q_{-1} \pm i Q_{-2}$$

IS NOT LORENTZ INVARIANT.

IF THE THEORY HAS LEFT AND RIGHT R-SYMMETRIES R_+ AND R_- WE CAN FIX THIS IF WE REPLACE THE LORENTZ GENERATOR M BY

$$M - \frac{1}{2} R_+ \mp \frac{1}{2} R_-$$

AT A MORE FUNDAMENTAL LEVEL, THIS CHANGE IN THE LORENTZ GENERATOR ARISES IF WE REPLACE THE STRESS TENSOR $T_{\mu\nu}$ BY

$$\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\left(\partial_\mu J_\nu^+ + \partial_\nu J_\mu^+\right)$$
$$\mp \frac{1}{2}\left(\partial_\mu J_\nu^- + \partial_\nu J_\mu^-\right)$$

WHERE $J_\nu^\pm$ ARE THE RIGHT, LEFT R-CURRENTS.

THE THEORY WITH STRESS TENSOR $\tilde{T}_{\mu\nu}$ AND BRST OPERATOR Q IS CALLED A TOPOLOGICAL FIELD THEORY...

THE BASIS FOR THE NAME IS THAT ONE CAN USE THE SUSY ALGEBRA TO SHOW

$\tilde{T}_{\mu\nu} = \{Q, \Lambda_{\mu\nu}\}$ FOR SOME $\Lambda_{\mu\nu}$ ... SO IN A SENSE THE WORLDSHEET METRIC IS IRRELEVANT.

CORRELATION FUNCTIONS

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\ldots\mathcal{O}_n(x_n)\rangle_\Sigma$$

OF PHYSICAL OPERATORS $\mathcal{O}_i$:

(OBEYING $[Q,\mathcal{O}_i\}=0$) ON A FIXED

RIEMANN SURFACE Σ ARE

INDEPENDENT OF METRIC ON Σ AS

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\int_\Sigma \delta(\sqrt{g}\,g^{\mu\nu})\,\tilde{T}_{\mu\nu}\rangle$$

$$=\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\int_\Sigma \delta(\sqrt{g}\,g^{\mu\nu})\{Q,\Lambda_{\mu\nu}\}\rangle$$

$$=0$$

MORE IMPORTANTLY FOR US, ONE CAN ALSO CONSTRUCT A "TOPOLOGICAL STRING THEORY" IN WHICH ONE OBTAINS CORRELATION FUNCTIONS BY INTEGRATING OVER THE MODULI OF Σ, USING $\Lambda_{\mu\nu}$ ~~AS~~ WHERE THE "ANTIGHOST," $b_{\mu\nu}$, USUALLY APPEARS IN DEFINING THE STRING MEASURE.

PHYS. STR. THEORY

$T_{\mu\nu} = \{Q, b_{\mu\nu}\}$ $\qquad$ $\tilde{T}_{\mu\nu} = \{Q, \Lambda_{\mu\nu}\}$

FOR AN $N=2$ SUSY MODEL IN $D=2$ WITH ANOMALY-FREE LEFT AND RIGHT R-SYMMETRIES WE GET TWO TOPOLOGICAL STRING THEORIES, DEPENDING ON THE CHOICE OF SIGN.

I WANT TO CONSIDER THE CASE THAT THE $N=2$ MODEL IS A SIGMA MODEL WITH TARGET SPACE A COMPLEX MANIFOLD X....

IN THIS CASE, THE TWO R-SYMMETRIES EXIST CLASSICALLY, SO CLASSICALLY WE CAN CONSTRUCT THE TWO TOPOLOGICAL STRING THEORIES - CALLED THE A-MODEL AND THE B-MODEL.

QUANTUM MECHANICALLY, HOWEVER, THERE IS AN ANOMALY, AND THE B-MODEL ONLY EXISTS IF X IS A CALABI-YAU MANIFOLD.

IN THIS B-MODEL, WE WANT TO CONSIDER SPACE-FILLING D-BRANES... N OF THEM, WRAPPED ON X.

RESULTING THEORY

$\Leftrightarrow$ TOPOLOGICAL VERSION OF LOW ENERGY D-BRANE THEORY ~~AS~~

~~PRESENTED BY KIRITSIS AND OTHER LECTURERS.~~

U(N) GAUGE FIELD $A_{\bar{i}}$

(U(N) IF WE HAVE N D-BRANES)

{<u>WITH</u> <u>NO</u> <u>A_i</u>}

WITH ACTION

$$I = \int d\bar{y}^{\bar{i}} d\bar{y}^{\bar{j}} d\bar{y}^{\bar{k}} \, \mathrm{Tr}\left(A_{\bar{i}} \partial_{\bar{j}} A_{\bar{k}} + \frac{2}{3} A_{\bar{i}} A_{\bar{j}} A_{\bar{k}}\right) \wedge \Omega$$

Ω = CALABI-YAU VOLUME FORM.

THERE IS ACTUALLY AN UNRESOLVED POINT ABOUT THIS THEORY ... BY POWER COUNTING IT IS UNRENORMALIZABLE

BUT IT HAS A COMPLEXIFIED GAUGE INVARIANCE THAT DOESN'T ALLOW ANY LOCAL COUNTERTERMS...

BASICALLY SINCE ONLY $A_{\bar{i}}$ AND NOT A_i APPEARS IN THE ACTION

WE'D LIKE TO CONSIDER A MODEL LIKE THIS WITH TARGET SPACE $\mathbb{CP}^3$ = TWISTOR SPACE, BUT WE CANNOT, SINCE $\mathbb{CP}^3$ ISN'T A CALABI-YAU MANIFOLD.

SO WE NEED TO INTRODUCE <u>SPACE-TIME</u> <u>SUPERSYMMETRY</u>

WE CONSIDER, INSTEAD OF $\mathbb{CP}^3$, WHICH HAS HOMOGENEOUS COORDINATES Z^I, $I = 1, \dots, 4$

A SUPERMANIFOLD

$\mathbb{CP}^{3|N}$ WITH BOSONIC

AND FERMIONIC COORDINATES

$$Z^I,\ \psi^A$$

$$I = 1 \ldots 4,\quad A = 1 \cdots N.$$

$$(Z^I, \psi^A) \simeq (tZ^I, t\psi^A)$$

$$t \in \mathbb{C}^*$$

IS $\mathbb{CP}^{3|N}$ A CALABI-YAU

SUPERMANIFOLD?

THE ANSWER IS "YES" IF AND ONLY IF $N=4$

WE SET

$$\Omega_0 = dz^1 dz^2 \dots dz^4 \, d\psi^1 d\psi^2 \dots d\psi^N$$

AND SEE IT IS INVARIANT TO

$$(z^I, \psi^A) \to (t z^I, t \psi^A)$$

IF AND ONLY IF $N=4$. $dz \to t\,dz$, $d\psi \to t^{-1} d\psi$

WHEN Ω_0 IS $\mathbb{C}^*$-INVARIANT, WE CAN DIVIDE BY $\mathbb{C}^*$ AND GET A CALABI-YAU MEASURE Ω ON $\mathbb{CP}^{3|N}$

$\mathbb{CP}^3$ HAS SU(2,2) SYMMETRY

AND $\mathbb{CP}^{3|N}$ HAS SU(2,2|N)

(OR PSU(2,2|4)) SYMMETRY... i.e.

SUPERCONFORMAL SYMMETRY WITH

N SUPERSYMMETRIES.

NOW, SUPER YANG-MILLS THEORY

IN 4 DIMENSIONS EXISTS

WITH VARIOUS NUMBERS OF

SUPERSYMMETRIES....

BUT $N=4$ IS SPECIAL

FOR MANY REASONS:

- MAXIMUM SUPERSYMMETRY
- β-FUNCTION VANISHES
- THERE IS A TOPOLOGICAL B-MODEL OF $\mathbb{CP}^{3|N}$

IN THIS TOPOLOGICAL B-MODEL, WITH SPACE-FILLING BRANES, THE BASIC FIELD IS NOW

$$a_{\bar{i}}(x,\bar{x},\psi) = A_{\bar{i}}(x,\bar{x}) + \psi^A \chi_{\bar{i}A}(x,\bar{x}) + \psi^A\psi^B \phi_{\bar{i}AB}(x,\bar{x}) + \dots$$

THE ACTION IS THE SAME,
EXCEPT IT DEPENDS ON ψ

$$\int d\bar{x}^{\bar{i}}\, d\bar{x}^{\bar{j}}\, d\bar{x}^{\bar{k}}$$
$$\mathrm{Tr}\left(a_{\bar{i}}\, \partial_{\bar{j}}\, a_{\bar{k}} + \tfrac{2}{3}\, a_{\bar{i}}\, a_{\bar{j}}\, a_{\bar{k}}\right)$$
$$\Omega$$

WHERE, ROUGHLY,

$$\Omega = d^3x\, d^4\psi$$

100

THIS ACTION HAS THE AMAZING PROPERTY THAT ITS SPECTRUM IS THE SAME AS THAT OF $\mathcal{N}=4$ SUPER YANG-MILLS IN MINKOWSKI SPACE

$$a_{\bar{\iota}} = \underset{1}{A_{\bar{\iota}}} + \underset{1/2}{\psi^A \chi_{\bar{\iota} A}} + \underset{0}{\psi^A \psi^B \phi_{\bar{\iota} AB}}$$

I LABELED THE HELICITIES!

$$+ \underset{-1/2}{\psi^A \psi^B \psi^C \varepsilon_{ABCD} \tilde{\chi}^D_{\bar{\iota}}}$$

$$+ \underset{-1}{\psi^A \psi^B \psi^C \psi^D \varepsilon_{ABCD} G_{\bar{\iota}}}$$

IT ALSO DESCRIBES SOME OF THE INTERACTIONS OF $\mathcal{N}=4$ SUPER YANG-MILLS - BUT NOT ALL!

IT CANNOT DESCRIBE ALL THE INTERACTIONS, BECAUSE AN EXTRA $U(1)$ R-SYMMETRY GETS IN THE WAY

$$(Z^I, \underbrace{\psi^A}_{})$$

$U(4)_R = SU(4)_R \times U(1)_R$ ACTS ON $\mathbb{CP}^{3|4}$

BUT $N=4$ SUPER YANG MILLS HAS ONLY $SU(4)_R$ SYMMETRY

THE EXTRA $U(1)_R$, WHICH I CALL S, IS ANOMALOUS IN THE B-MODEL, SINCE IT DOESN'T LEAVE FIXED THE MEASURE

$$\Omega \sim d^3x \, d\psi^1 \ldots d\psi^4$$

BUT AS WE'VE SET THINGS UP SO FAR, THE ANOMALY IS TOO TRIVIAL TO AGREE WITH N=4 SUPER YANG-MILLS THEORY.

WITH A CERTAIN NORMALIZATION OF S, THE $\mathcal{N}=4$ SUPER YM ACTION IS A SUM OF TERMS WITH $S=-4$ AND $S=-8$.

THE ACTION

$$\int d^3\bar{x}\,\mathrm{Tr}\,(a\,\bar\partial\bar{a}+\ldots)\,\Omega$$

HAS $S=-4$, AND WE NEED AN INSTANTON CORRECTION TO GET THE TERMS OF $S=-8$.

THE INSTANTONS IN QUESTION ARE D1-BRANES, WRAPPED ON HOLOMORPHIC CURVES IN $\mathbb{CP}^{3|4}$.

THAT IS HOW HOLOMORPHIC CURVES ENTER THE STORY

OUR DIFFERENT AMPLITUDES HAVE DIFFERENT S-CHARGE

$$a_i = A_i + \ldots + \psi^4 G_i$$

A_i: HELICITY 1, S-CHARGE ZERO

$\psi^4 G_i$: HELICITY -1, S CHARGE -4

SO AMPLITUDES WITH MORE AND MORE NEGATIVE HELICITY GLUONS VIOLATE S BY MORE AND MORE ...

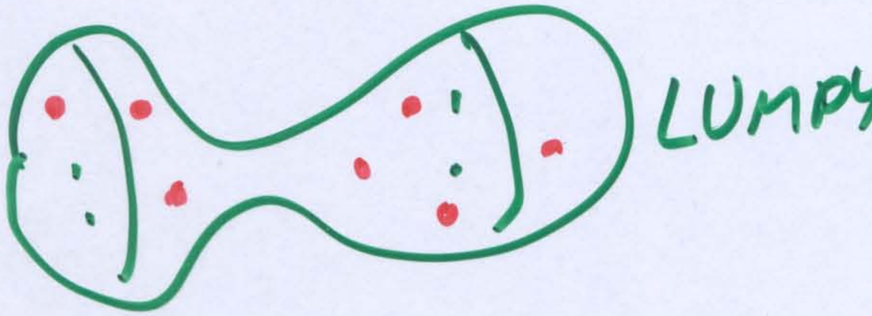
BY $-4q$ WHERE $q = \#$ OF NEGATIVE HELICITY GLUONS,

THIS LEADS TO YESTERDAY'S FORMULA FOR THE INSTANTON NUMBER d CONTRIBUTING TO A GIVEN AMPLITUDE:

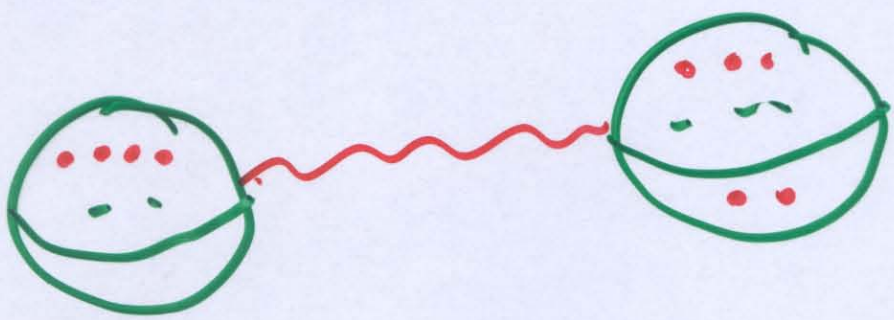
$$d = q - 1$$

(TREE LEVEL)

HOWEVER, SUPPOSE (FOR EXAMPLE) $g=3$. WE CAN MAKE TWO TYPES OF CONFIGURATION OF $d=2$:



OR



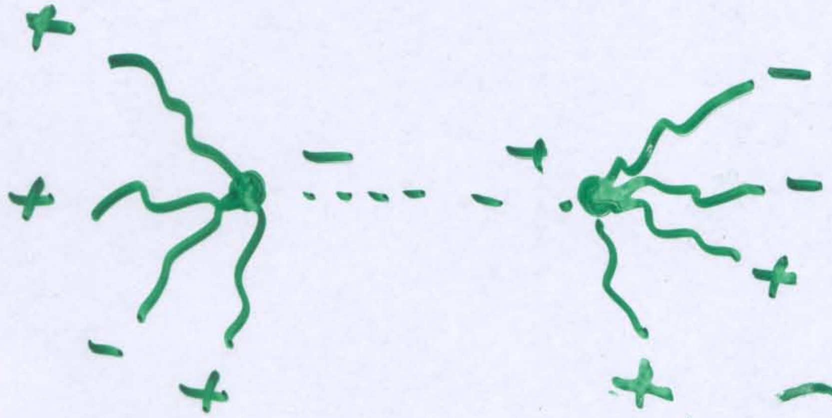
I ORIGINALLY EXPECTED TO ADD THE TWO, BUT IT TURNS OUT THAT EITHER ONE BY ITSELF GIVES THE COMPLETE ANSWER

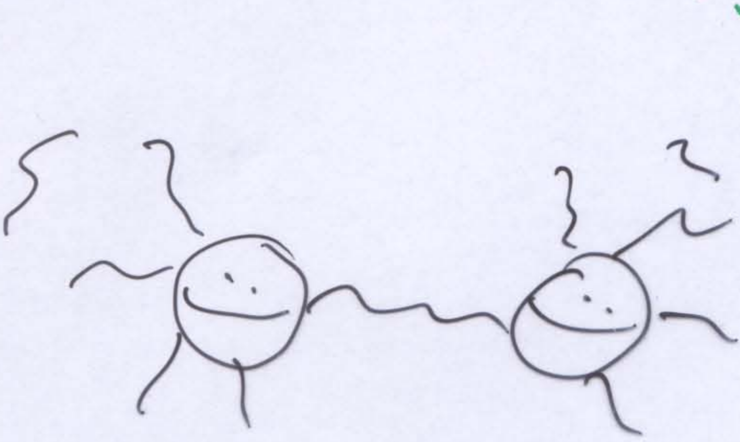
connected	Roiban Spradlin Volovich
disconnected	Cachazo, Svrcek, Witten
relation between them	Gukov, Motl, Neitzke

THE DISCONNECTED INSTANTONS

LEAD TO VERY EXPLICIT FORMULAS:

"MHV TREE AMPLITUDES"

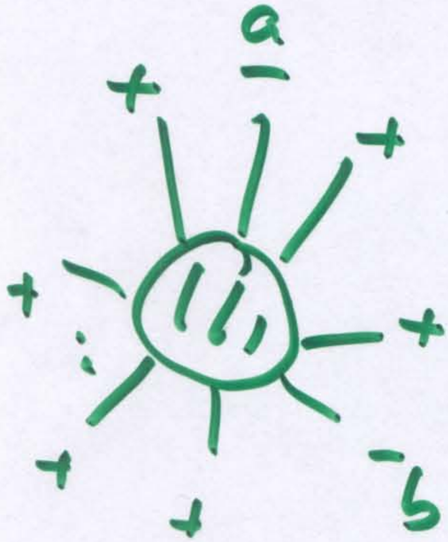




THE D1-D5 STRINGS ARE FERMIONS $\alpha^I, \beta_{\dot{J}}$ IN $N + \bar{N}$ OF $U(N)$ AND THE COUPLING OF A TWISTOR FIELD $a_{\bar{z}}{}^I{}_J$ TO D1-BRANE IS

$$\int_\Sigma d\bar{z}^{\bar{I}}\, a_{\bar{z}}{}^I{}_J \underbrace{\alpha^J \beta_I(z)}_{J_I^J = \text{CURRENT}} dz$$

IF WE GO BACK TO THE MHV TREE AMPLITUDE, THE FACTORS ARE AS FOLLOWS:



1)

$$= -i g^{n-2} (2\pi)^4 \delta^4\left(\sum p_i\right)$$

2)

$$\cdot \langle \lambda_a, \lambda_b \rangle^4$$

3)

$$\cdot \frac{1}{\prod_{i=1}^{n} \langle \lambda_i, \lambda_{i+1} \rangle}$$

WHERE

1) $(2\pi)^4 \delta^4(\sum p_i)$ COMES FROM BOSONIC COLLECTIVE COORDINATES OF INSTANTON, AS WE SAW YESTERDAY

2) $\langle \lambda_a, \lambda_b \rangle^4$ COMES SIMILARLY FROM FERMIONIC COLLECTIVE COORDINATES

3) $\prod \frac{1}{\langle \lambda_i, \lambda_{i+1} \rangle}$ IS THE CURRENT CORRELATION FUNCTION ON THE D1-BRANE (NAIR)