Supersymmetry I

Hitoshi Murayama (Berkeley)

PiTP 05, IAS

Contents as given by Michael

- Review of global SUSY formalism
- SUSY models of particle physics
- experimental signatures of SUSY.
- Spectroscopy measurements in supersymmetry.
- Models of supersymmetry breaking and their experimental implications.
- Can we test string theory at colliders?

Plan

Mon: Non-technical Overview

what SUSY is supposed to give us

Tue: From formalism to the MSSM

Global SUSY formalism, Feynman rules, soft SUSY breaking, MSSM

Wed: SUSY breaking

how to break SUSY, mediation mechanisms

Thu: SUSY at colliders

basic reactions, signatures, and how do we know it is SUSY?

Fri: SUSY as a telescope

supersymmetry breaking, GUT, string

Standard Model

A Long History

- Since Fermi and Yukawa to the "Standard Model," it took almost 40 years to build
- Since deep inelastic scattering and J/ψ to precision measurements, it took almost 30 years to test
- See Michael Peskin's lectures for its beautiful experimental tests
- Yet not completely established

Renormalizable Quantum Field Theory

- $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory

	Q	d	u	L	e	B	W	g	H	G
$SU(3)_C$	3	3	3	1	1	1	1	8	1	1
$SU(2)_L$	2	1	1	2	1	1	3	1	2	1
$U(1)_Y$	+1/6	−1/3	+2/3	−1/2	+1	0	0	0	−1/2	0
spin	−1/2	+1/2	+1/2	−1/2	+1/2	1	1	1	0	2
flavor	3	3	3	3	3	1	1	1	1	1
seen?	Y	Y	Y	Y	Y	Y	Y	Y	N	N

Anomaly Cancellation

$U(1)^3$ $$3\times 2\left(\frac{1}{6}\right)^3+3\times\left(-\frac{2}{3}\right)^3+3\times\left(\frac{1}{3}\right)^3+2\left(-\frac{1}{2}\right)^3+(+1)^3=0$$

$U(1)(\text{gravity})^2$ $$3\times 2\left(\frac{1}{6}\right)+3\times\left(-\frac{2}{3}\right)+3\times\left(\frac{1}{3}\right)+2\left(-\frac{1}{2}\right)+(+1)=0$$

$U(1)(SU(2))^2$ $$3\times 2\left(\frac{1}{6}\right)+2\left(-\frac{1}{2}\right)=0$$

$U(1)(SU(3))^2$ $$3\times 2\left(\frac{1}{6}\right)+3\times\left(-\frac{2}{3}\right)+3\times\left(\frac{1}{3}\right)=0$$

$(SU(3))^3$ $$\#3-\#3^*=2-1-1=0$$

$(SU(2))^3$, $(SU(3))^2SU(2)$, $SU(3)(SU(2))^2$ $$0$$

$SU(2)$ $$\#2=3+1=4=\text{even}$$

Non-trivial connection between q & l

General

- The most general renormalizable Lagrangian with the given particle content

$$\mathcal{L} = -\frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4g^2} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4g_s^2} G^a_{\mu\nu} G^{\mu\nu a}$$
$$+ \bar{Q}_i i \not{D} Q_i + \bar{u}_i i \not{D} u_i + \bar{d}_i i \not{D} d_i + \bar{L}_i i \not{D} L_i + \bar{e}_i i \not{D} e_i$$
$$+ |D_\mu H|^2 + Y_u^{ij} \bar{Q}_i u_j \tilde{H} + Y_d^{ij} \bar{Q}_i d_j H + Y_l^{ij} \bar{L}_i e_j H$$
$$- \lambda (H^\dagger H)^2 + \lambda v^2 H^\dagger H + \frac{\theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$$

Parameters

- 3 gauge coupling constants + θ_{QCD}
- 2 parameters in the Higgs potential (G_F, m_H)

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4g^2}W^a_{\mu\nu}W^{\mu\nu a} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{\mu\nu a} \\
& + \bar{Q}_i i \not{D} Q_i + \bar{u}_i i \not{D} u_i + \bar{d}_i i \not{D} d_i + \bar{L}_i i \not{D} L_i + \bar{e}_i i \not{D} e_i \\
& + |D_\mu H|^2 + Y_u^{ij}\bar{Q}_i u_j \tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i e_j H \\
& - \lambda (H^\dagger H)^2 + \lambda v^2 H^\dagger H + \frac{\theta}{64\pi^2}\varepsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}
\end{aligned}
$$

$g' \sim 0.36$, $g \sim 0.65$, $g_s \sim 1.2$

$G_F \sim (300\ \text{GeV})^{-2}$, m_H unknown, $\theta_{QCD} < 10^{-10}$

Parameters

- 3x3 complex Y_u^{ij}, Y_d^{ij}, Y_l^{ij}: 54 real params
- reparameterization $U(3)_{Q,d,u,L,e}{}^5/U(1)_{B,e,\mu,\tau}{}^4=41$

$$\begin{aligned}\mathcal{L}= & -\frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu}-\frac{1}{4g^2}W^a_{\mu\nu}W^{\mu\nu a}-\frac{1}{4g_s^2}G^a_{\mu\nu}G^{\mu\nu a}\\ & +\bar{Q}_i i\not{D}Q_i+\bar{u}_i i\not{D}u_i+\bar{d}_i i\not{D}d_i+\bar{L}_i i\not{D}L_i+\bar{e}_i i\not{D}e_i\\ & +|D_\mu H|^2+Y_u^{ij}\bar{Q}_i u_j\tilde{H}+Y_d^{ij}\bar{Q}_i d_j H+Y_l^{ij}\bar{L}_i e_j H\\ & -\lambda(H^\dagger H)^2+\lambda v^2 H^\dagger H+\frac{\theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}\end{aligned}$$

$54-41=13=3_u+3_d+3_l+(3+1)_{CKM}$

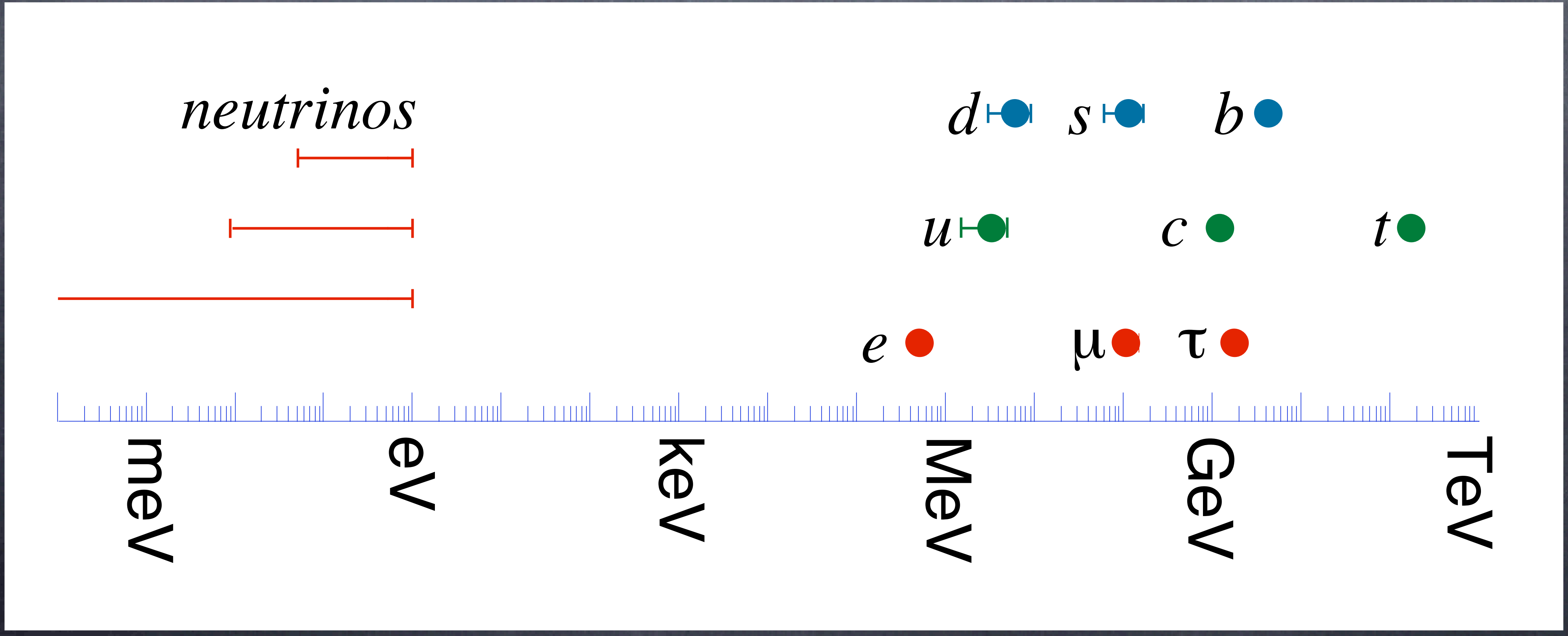
Masses and Mixings

- Choose masses and mixings as observed

$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho + i\eta) \\ -\lambda & 1 & A\lambda^2 \\ -\lambda^3(1+\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

A, ρ, η≈O(1)



Standard Model is extreeemely successful

- Take Particle Data Group "Reviews of Particle Physics" with 400+ pages
- With only a few exceptions, all numbers in the book are consistent with the Standard Model with suitably chosen 19 parameters
- Some of them tested at 10^{-9} –10^{-12} level
- Many at 10^{-3} level

Standard Model is extreeemely successful

- baryon and lepton number conserved (apart from anomaly $\propto e^{-8\pi^2/g^2}$) $\tau(t \to e^+\bar{\nu}_\mu\bar{\nu}_\tau) \sim 10^{150}\text{years}$
- flavor approximately conserved (apart from small mixing in V_{CKM})
- especially flavor-changing neutral current small (e.g. $s \to d$ vanishes at tree-level, suppressed by m_c^2/m_W^2 at one-loop)

So,
what's the problem?

empirically incomplete

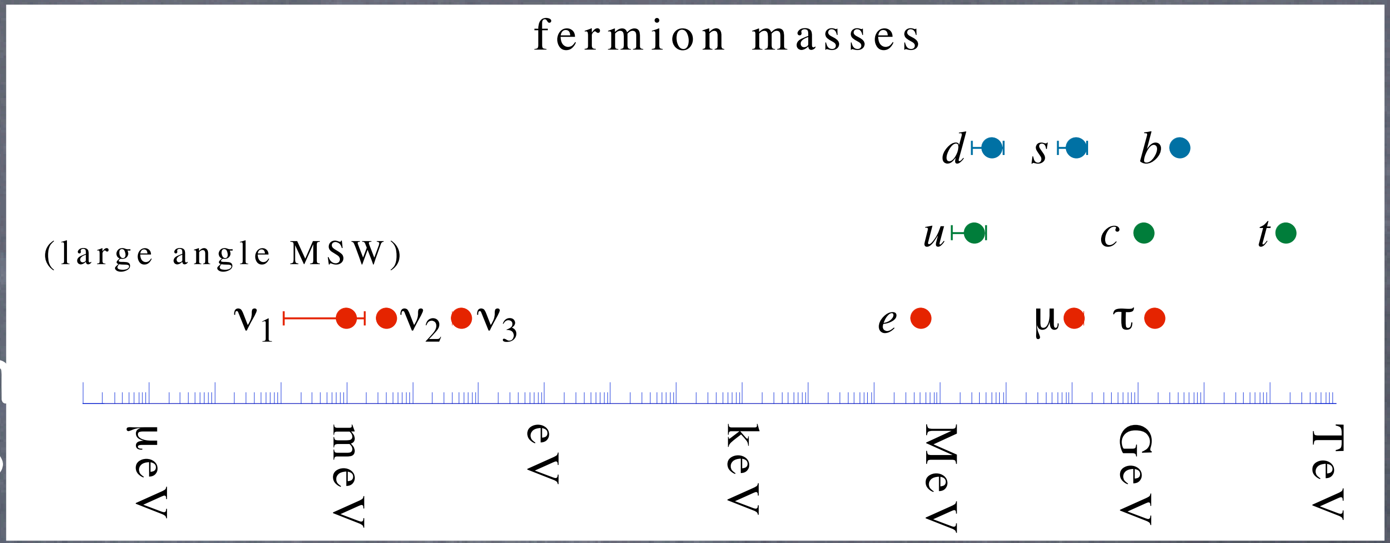
- neutrino mass
- dark matter
- dark energy
- nearly scale-invariant apparently acausal density fluctuation
- baryon asymmetry

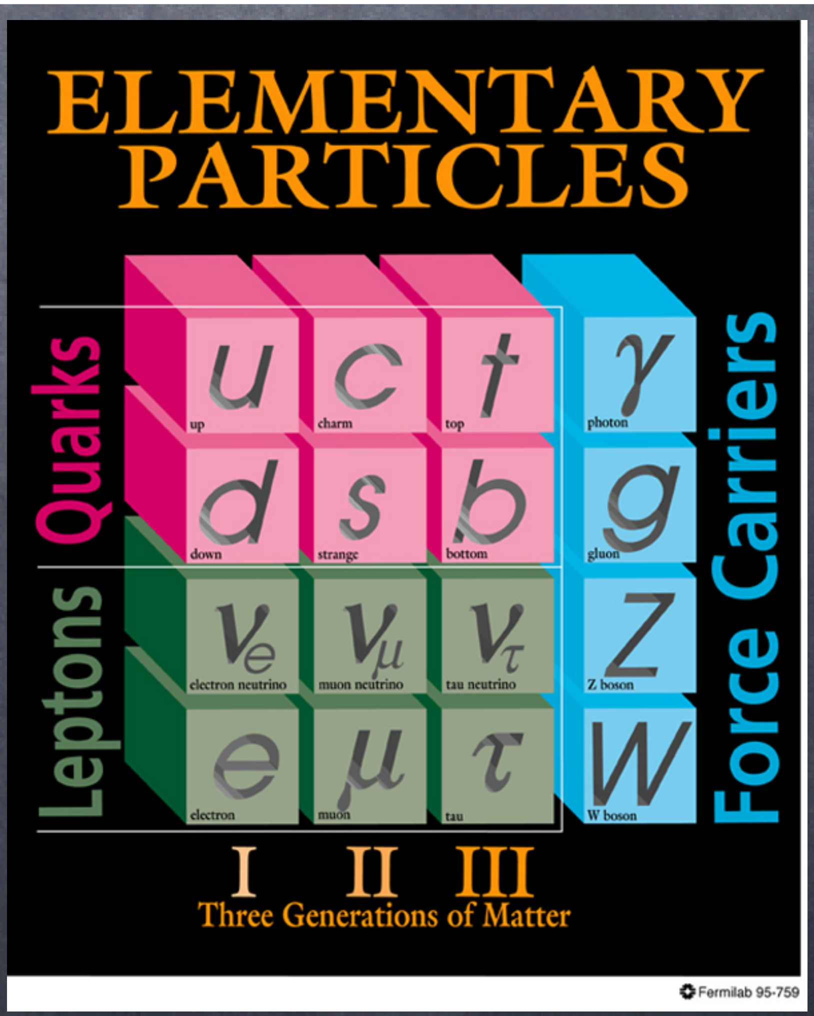
aesthetically unacceptable

- structure is quite complicated
- many naturalness problems
- no quantum gravity
- questions in four categories

Big Questions
-Horizontal-

- Why are there three generations?
- What physics determines the pattern of masses and mixings?
- Why do neutrinos have mass yet so light?
- What is the origin of CP violation?
- Why $\theta_{QCD} \ll 10^{-10}$?
- What is the origin of matter anti-matter asymmetry in Universe?



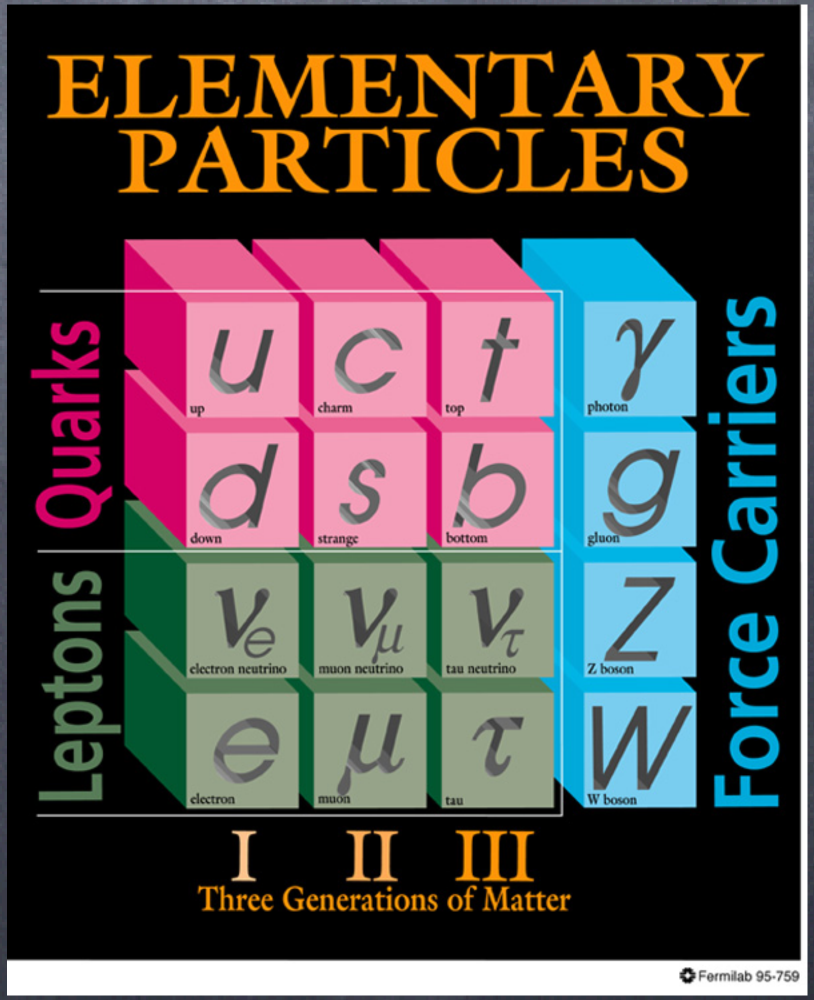


Big Questions
-Vertical-

- Why are there three unrelated gauge forces?
- Why is strong interaction strong?
- Charge quantization
- anomaly cancellation
- quantum numbers
- Is there a unified description of all forces?
- Why is $m_W \ll M_{Pl}$?

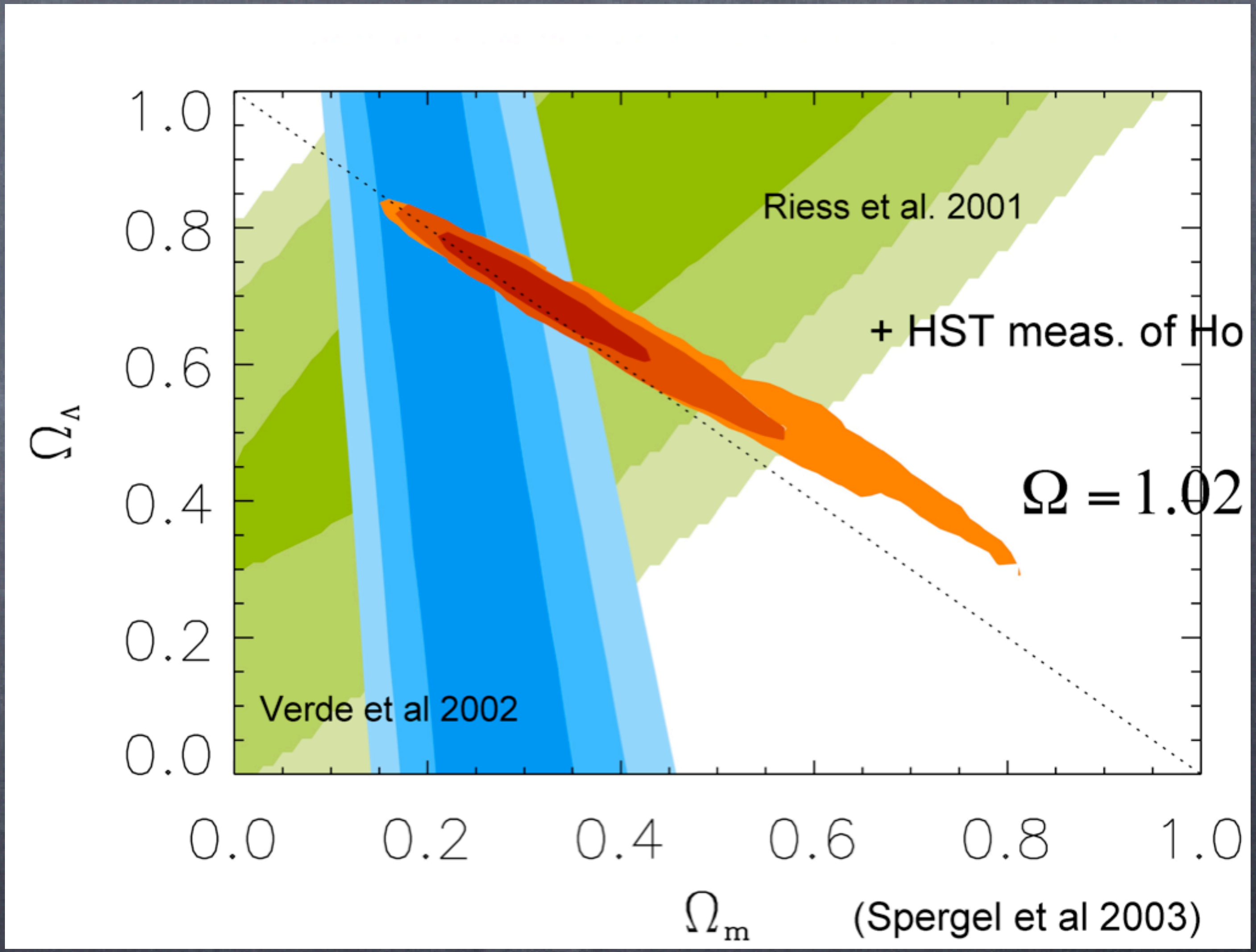
(Hierarchy Problem)

$$Q(\mathbf{3},\mathbf{2},+\frac{1}{6}),\quad u(\mathbf{3},\mathbf{1},+\frac{2}{3}),\quad d(\mathbf{3},\mathbf{1},-\frac{1}{3}),$$
$$L(\mathbf{1},\mathbf{2},-\frac{1}{2}),\quad e(\mathbf{1},\mathbf{1},-1)$$

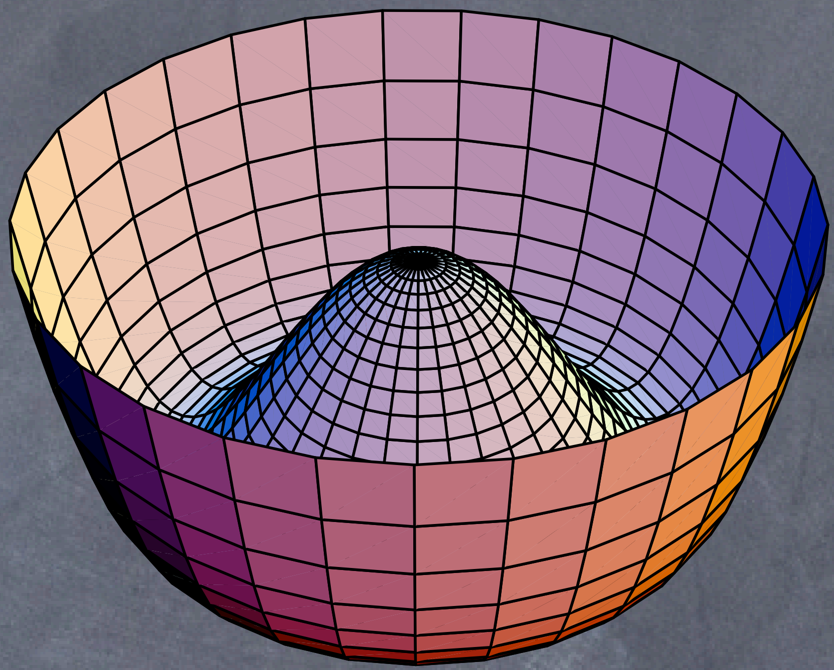


Big Questions
-From the Heaven-

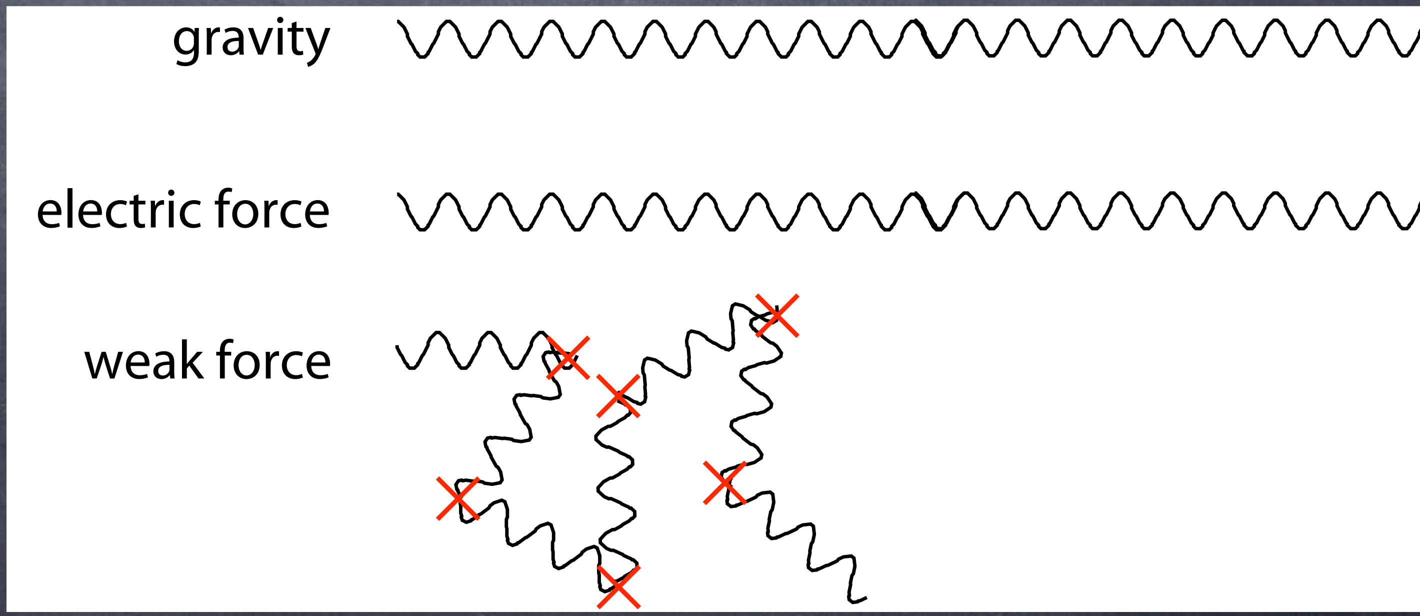
- What is Dark Matter?
- What is Dark Energy?
- Why now? (Cosmic coincidence problem)
- What was Big Bang?
- Why is Universe so big? (flatness problem, horizon problem)
- How were galaxies and stars created?



Big Questions
-From the Hell-



- What is the Higgs boson?
- Why does it have negative mass-squared?
- Why is there only one scalar particle in the Standard Model?
- Is it elementary or composite?
- Is it really condensed in our Universe?



Standard Model is fragile

The minute you allow for additional fields and/or gauge groups, much of the success is destroyed

- suppressed flavor-changing neutral currents
- no proton decay
- no neutrino mass either (good&bad)
- consistency with precise electroweak data
- no excessive CP violation (e/n EDM)
- no charge/color breaking

Standard Model is fragile

- The minute you allow for parameters to vary, it exhibits very different physics
- take $m_d<m_u$, all protons decay to neutrons and there are no atoms
- take $m_e>4m_p-m_\alpha$, Sun doesn't burn
- if $m_H^2>0$, EWSB still occurs by QCD, but the world is too radioactive to live
- If $m_c\sim m_t$, no J/ψ before the end of cold war

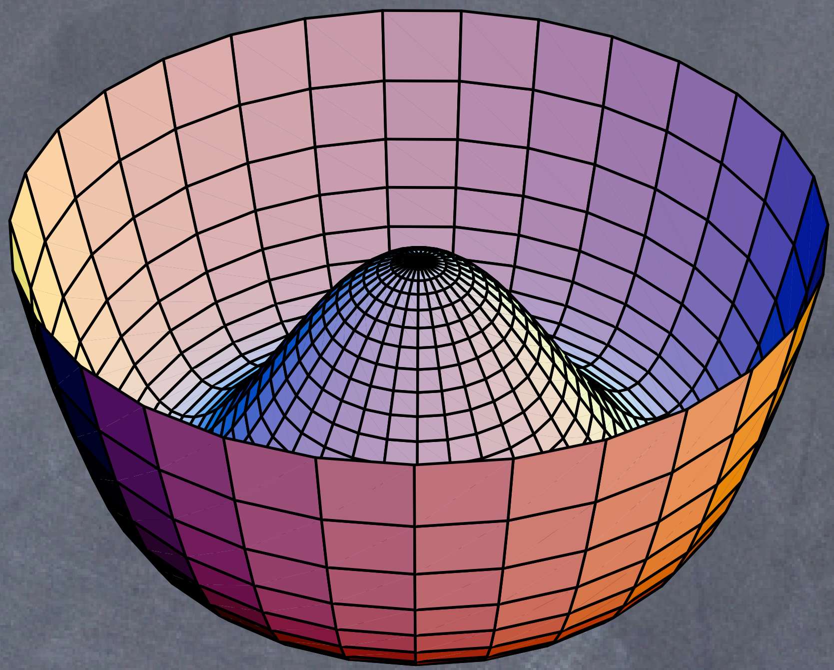
  and no high-energy physics funding by now

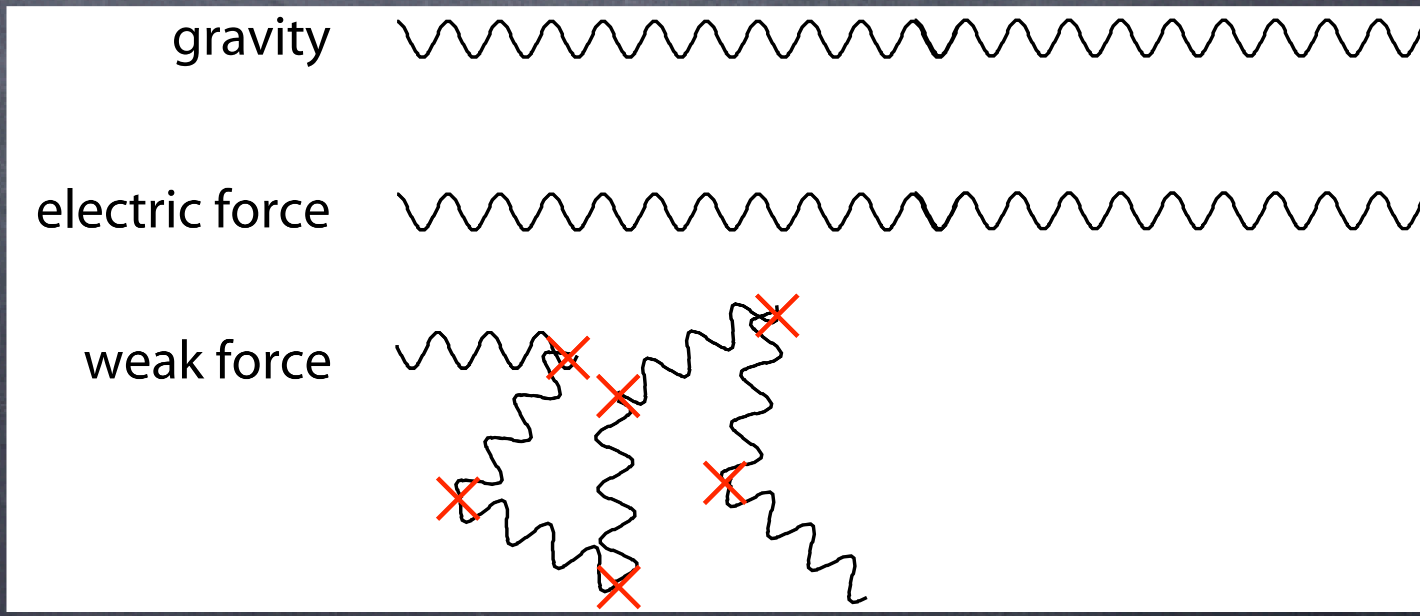
Dark Field =

cosmic superconductor

Big Questions
-From the Hell-

- What is the Higgs boson?
- Why does it have negative mass-squared?
- Why is there only one scalar particle in the Standard Model?
- Is it elementary or composite?
- Is it really condensed in our Universe?



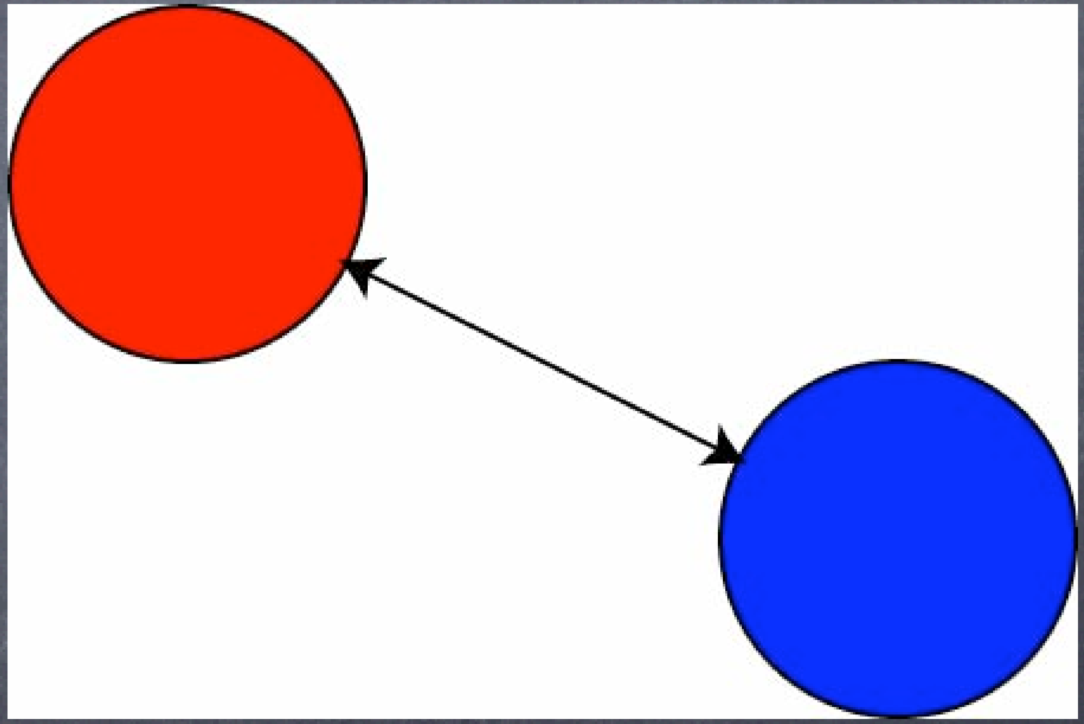


Mystery of the weak force

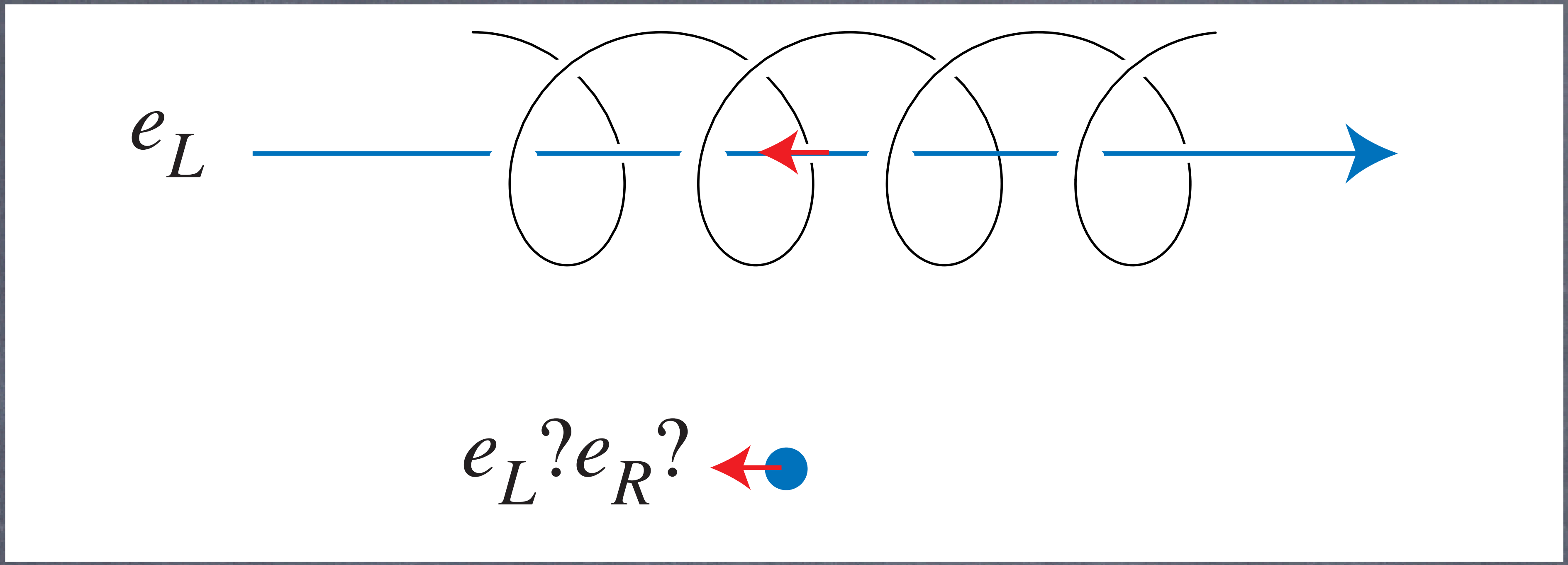
- Gravity pulls two massive bodies (long-ranged)
- Electric force repels two like charges (long-ranged)
- Weak force pulls protons and electrons (short-ranged) acts only over 0.000000001 nanometer

[need it for the Sun to burn!]

- We know the energy scale: 0.3 TeV



Mystery deepens

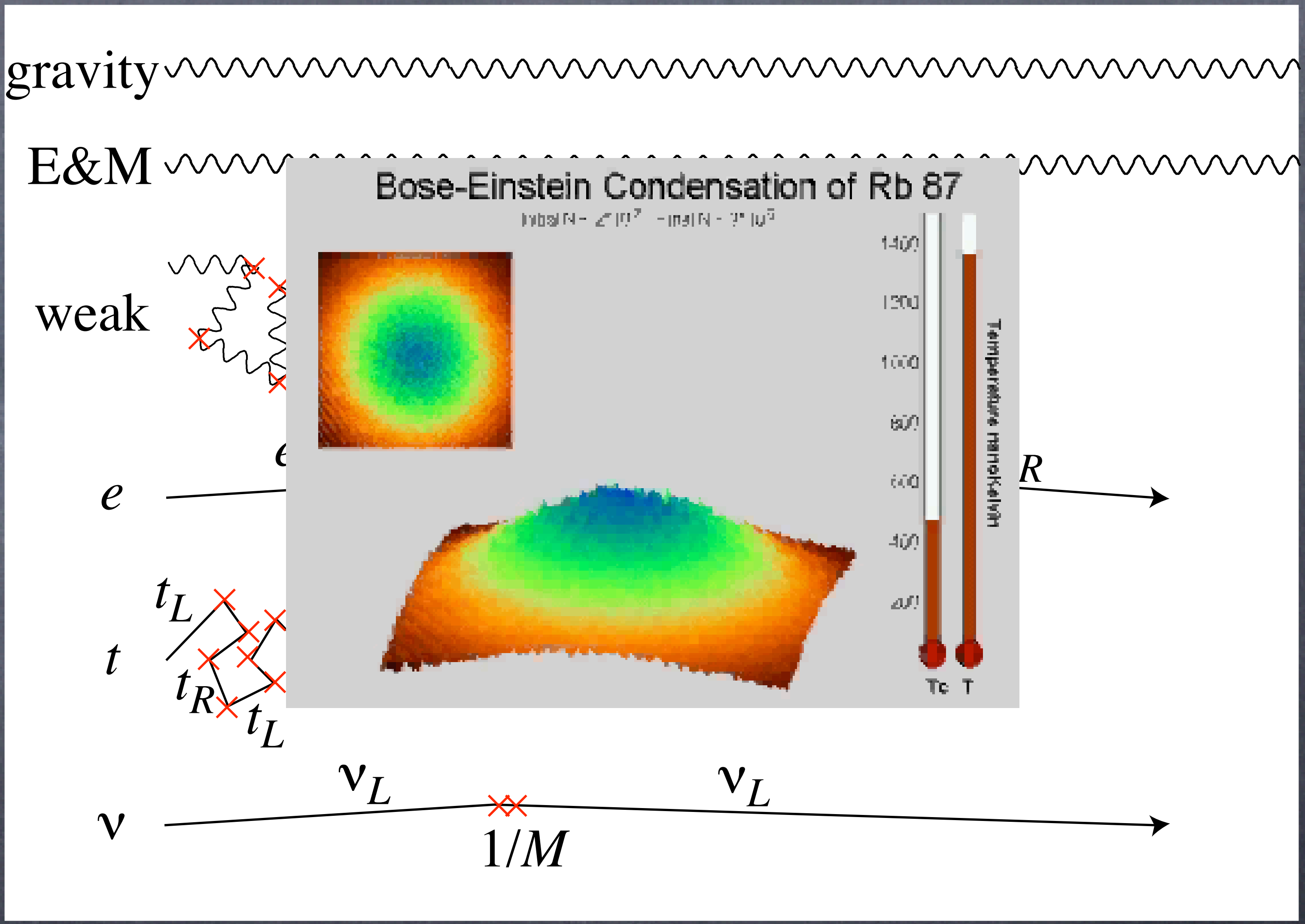


- Nuclear beta decay is due to a yet another force, the weak force
- Strangely, only left-handed particles participate in the weak force
- That sounds OK as long as they are moving
- but when they stop???

We are swimming in Dark Field

- There is quantum liquid filling our Universe
- It doesn't disturb gravity or electric force
- It does disturb weak force and make it short-ranged
- It slows down all elementary particles from speed of light
- What is it??

Extremely bizarre theory!



Cosmic Superconductor

- In a superconductor, magnetic field gets repelled (Meißner effect), and penetrates only over the "penetration length"

  ⇒ Magnetic field is short-ranged!

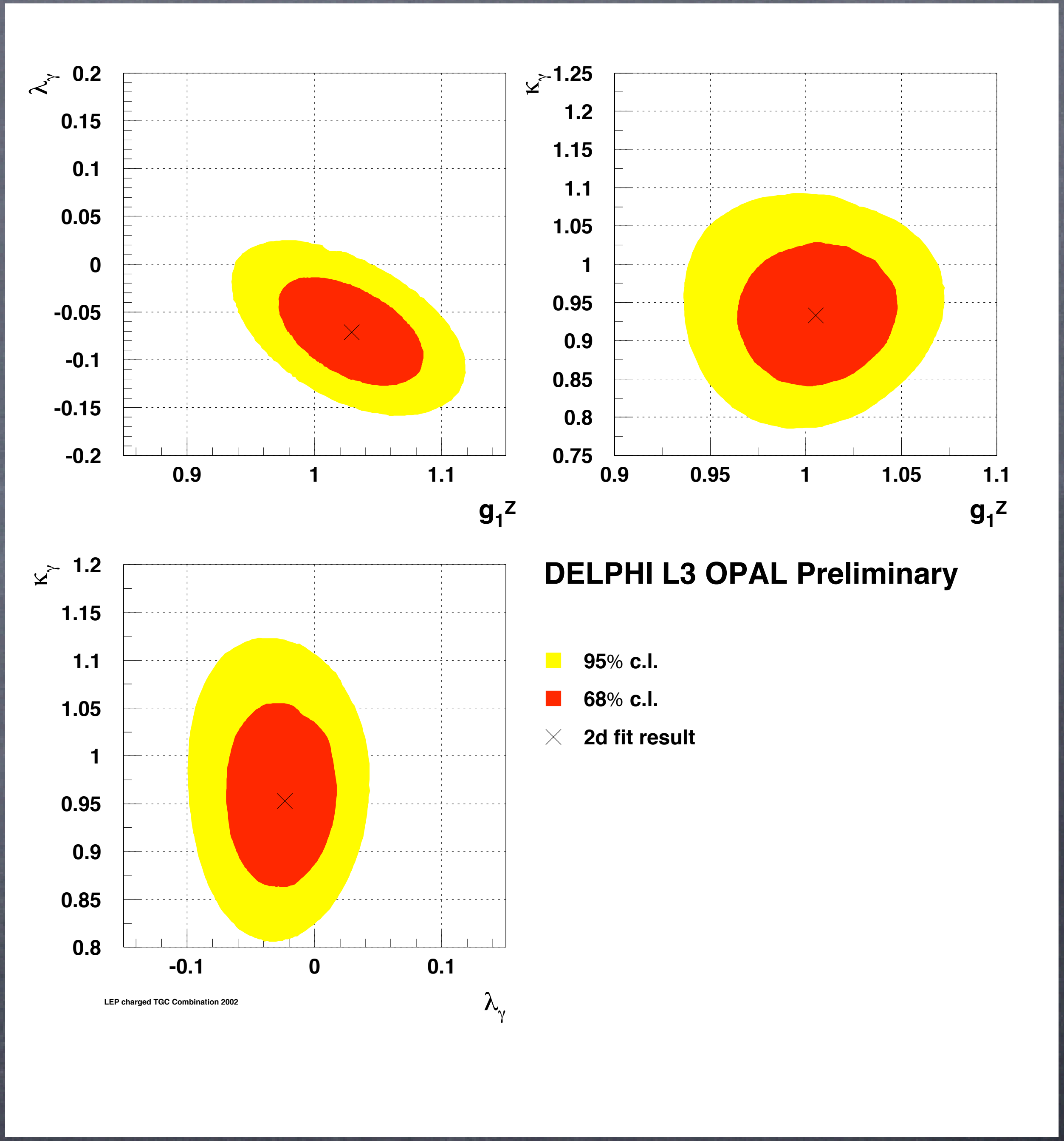
Imagine a physicist living in a superconductor

- She finally figured:
  - magnetic field must be long-ranged
  - there must be a mysterious charge-two "Dark Field" in her "Universe"
  - But doesn't know what the Dark Field is, nor why it is there
  - Doesn't have enough energy (gap) to break up Cooper pairs

That's the stage where we are!

Textbook

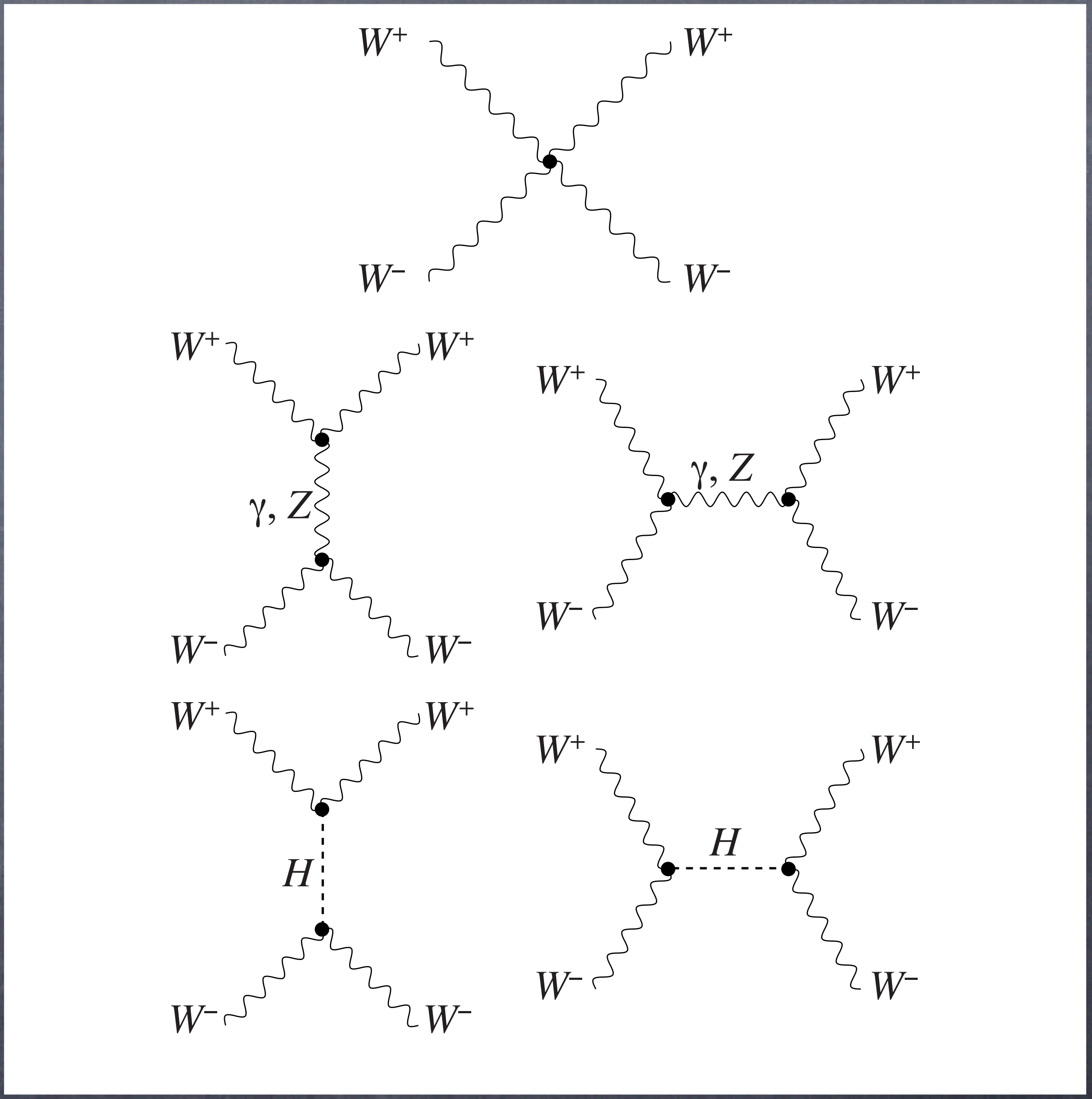
- W and Z are massive vector bosons
- Only known consistent (renormalizable) quantum field theory of massive vectors is gauge theory with Higgs mechanism
- Therefore, W and Z bosons must be gauge bosons, broken by a Higgs



Now added evidence of vector boson self-couplings

unitarity

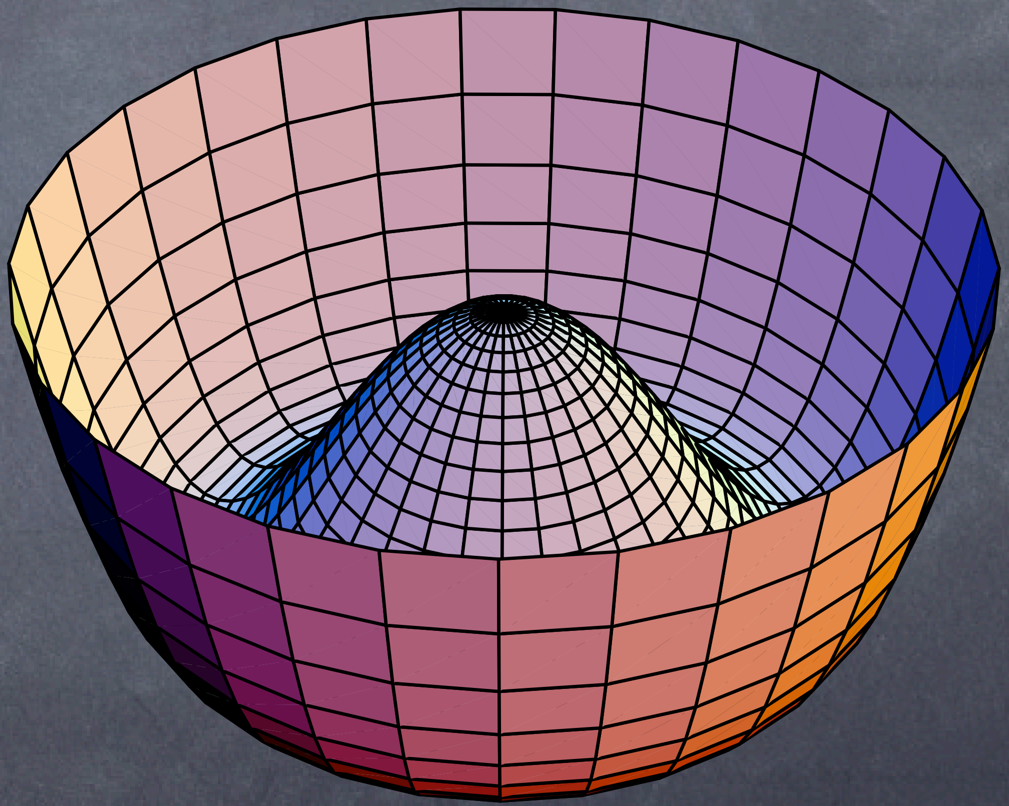
- W-boson scattering grows with energy $A \sim G_F E^2$ and violates unitarity at 1.8TeV
- If you allow only one extra particle beyond what we know to restore unitarity, the only possibility is to add a spin zero particle whose couplings are precisely those of the SM Higgs



C. H. Llewellyn Smith; D. A. Dicus and V. S. Mathur;
J. M. Cornwall, D. N. Levin and G. Tiktopoulos

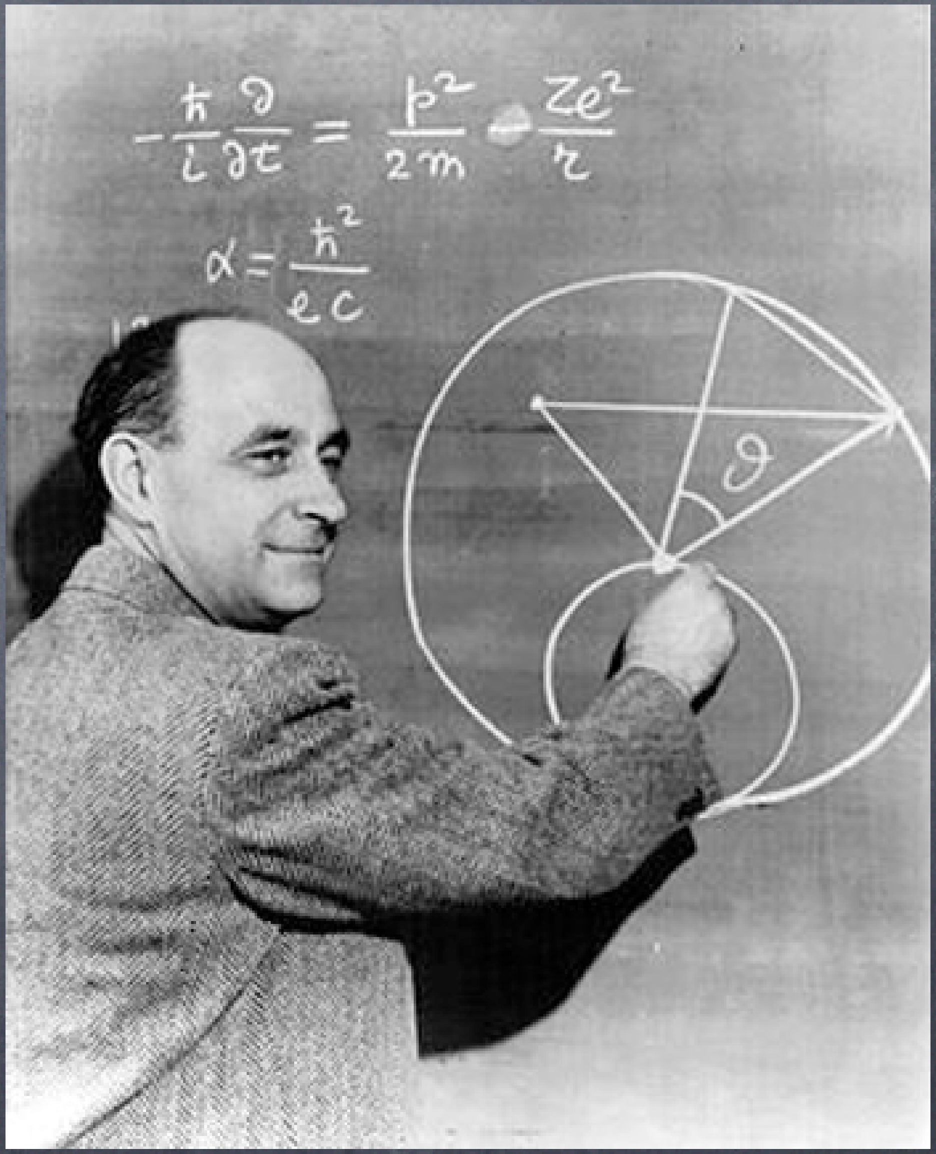
ugly

- $V=\lambda|H|^4-\mu^2|H|^2$
- Why negative mass-squred?
- Why only one scalar in the SM?
- Hierarchy problem because of its quadratic divergence
- does not appear fundamental, i.e. Ginzburg-Landau vs BCS



Fermi's dream era

- Fermi formulated the first theory of the weak force (1933)
- The required energy scale to study the problem known since then: ~TeV
- We are finally getting there!

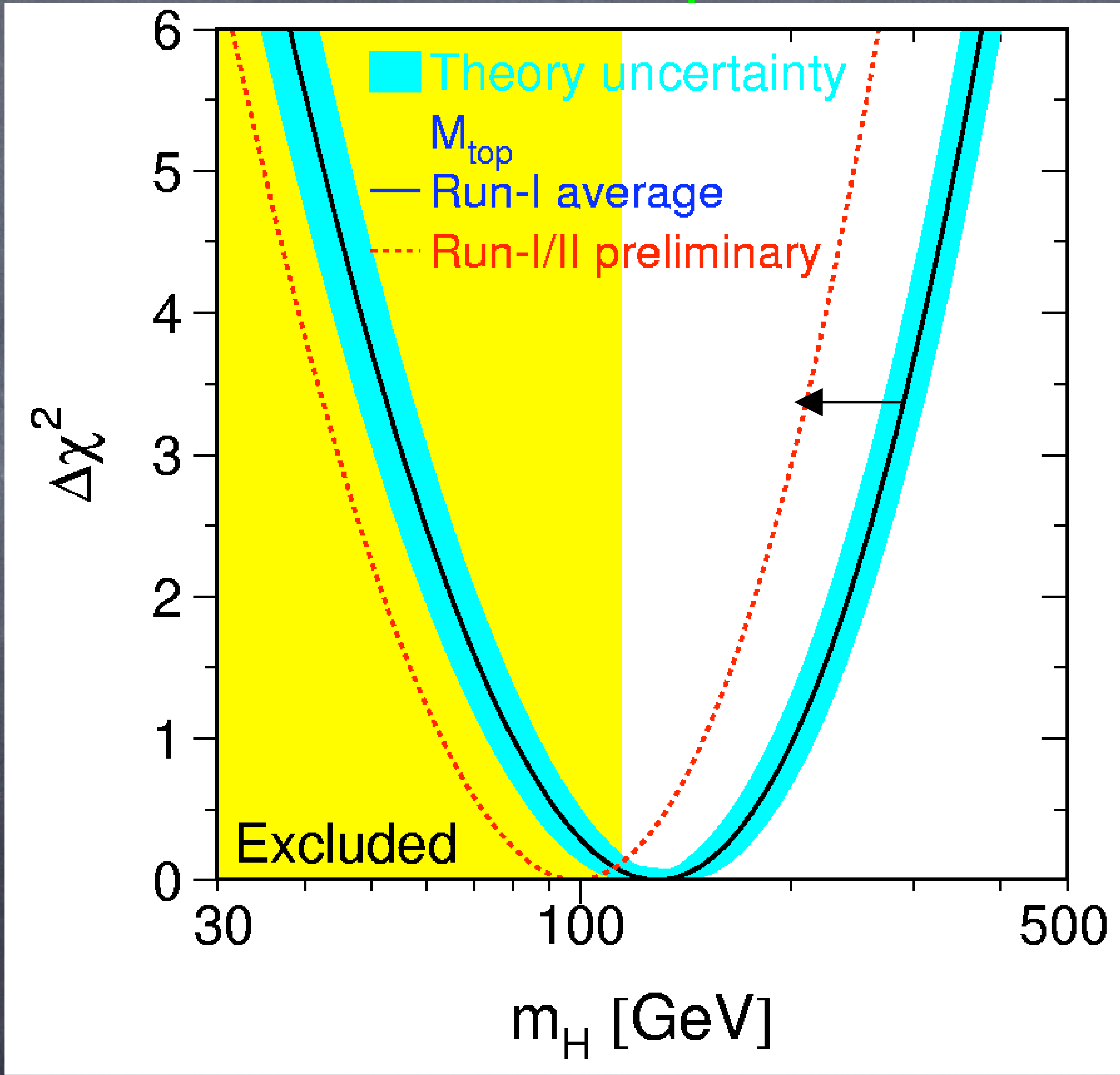


Gap Excitation

- We know the energy scale of the problem:
  $G_F \approx (300\ \mathrm{GeV})^{-2}$
- the gap excitation is called "Higgs boson"
- Current data combined with the Standard Model theory predict

$m_H < 208\mathrm{GeV}$ (95%CL)

the top mass is a crucial input data



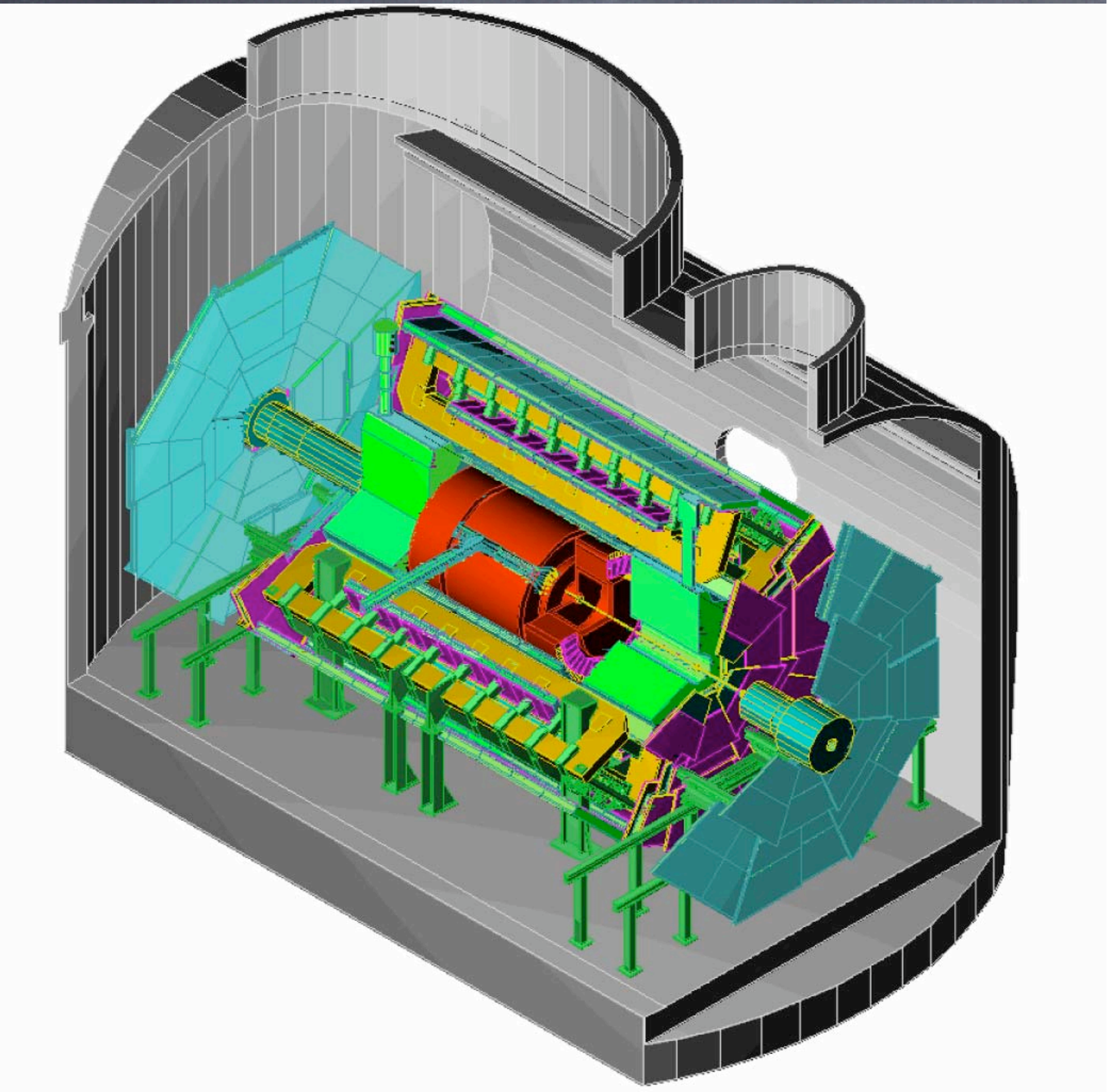
Kick out Dark Field from the vacuum

- We know the energy scale of the problem:

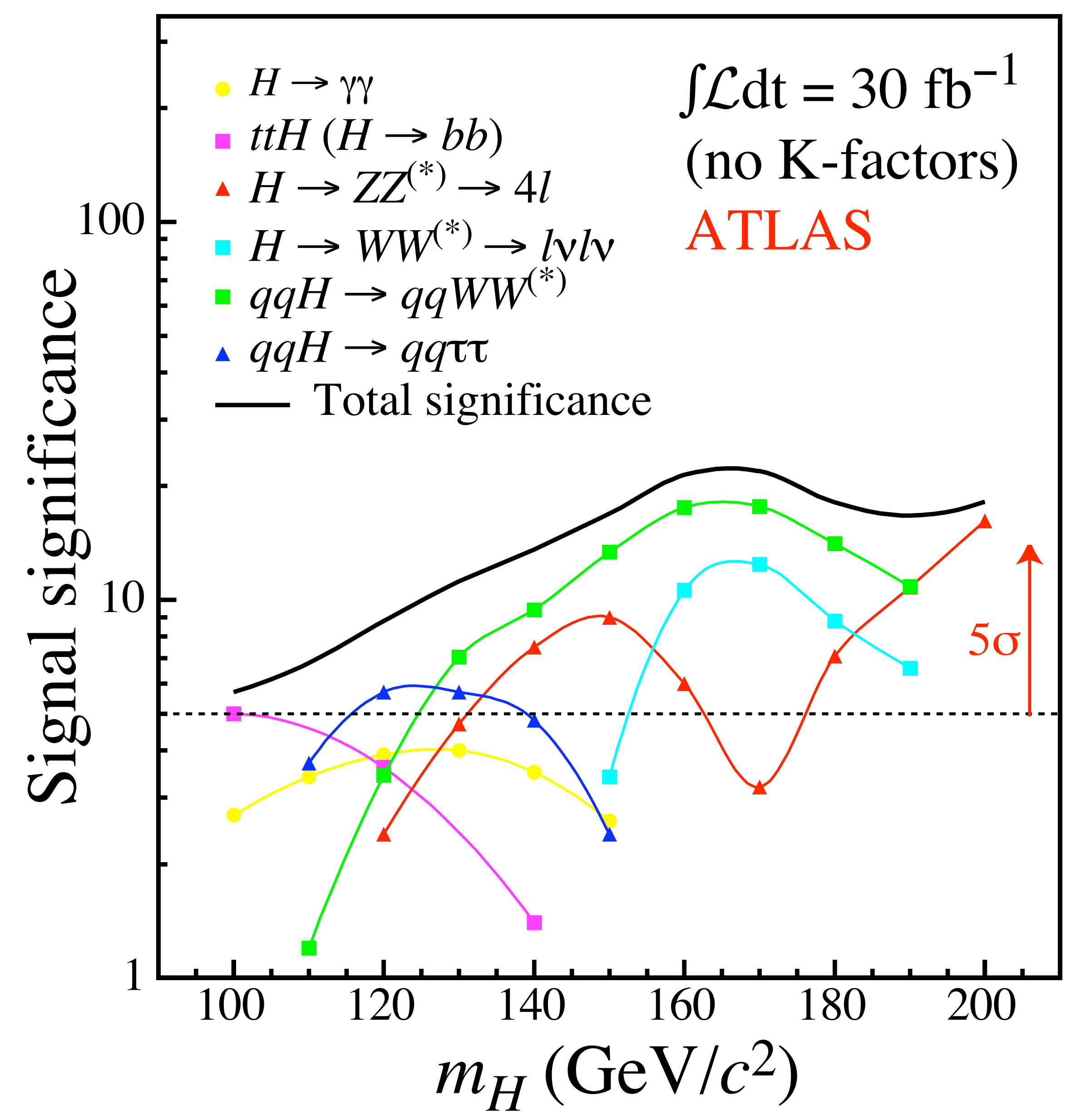
  0.3 TeV

- pump energy into empty space to kick out whatever makes Dark Field: "Higgs boson"
- LHC will find it!!!!!

Higgs at ATLAS



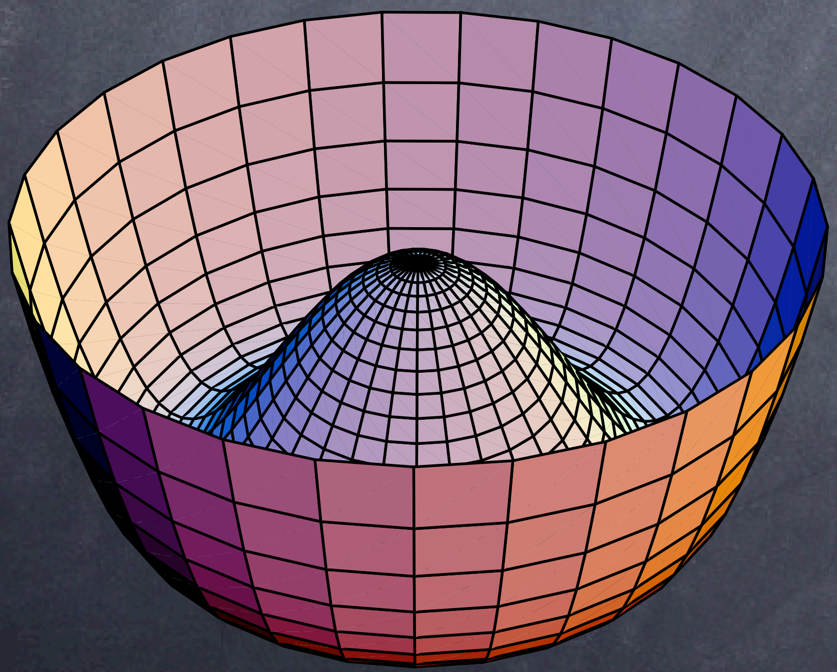
Robust discovery

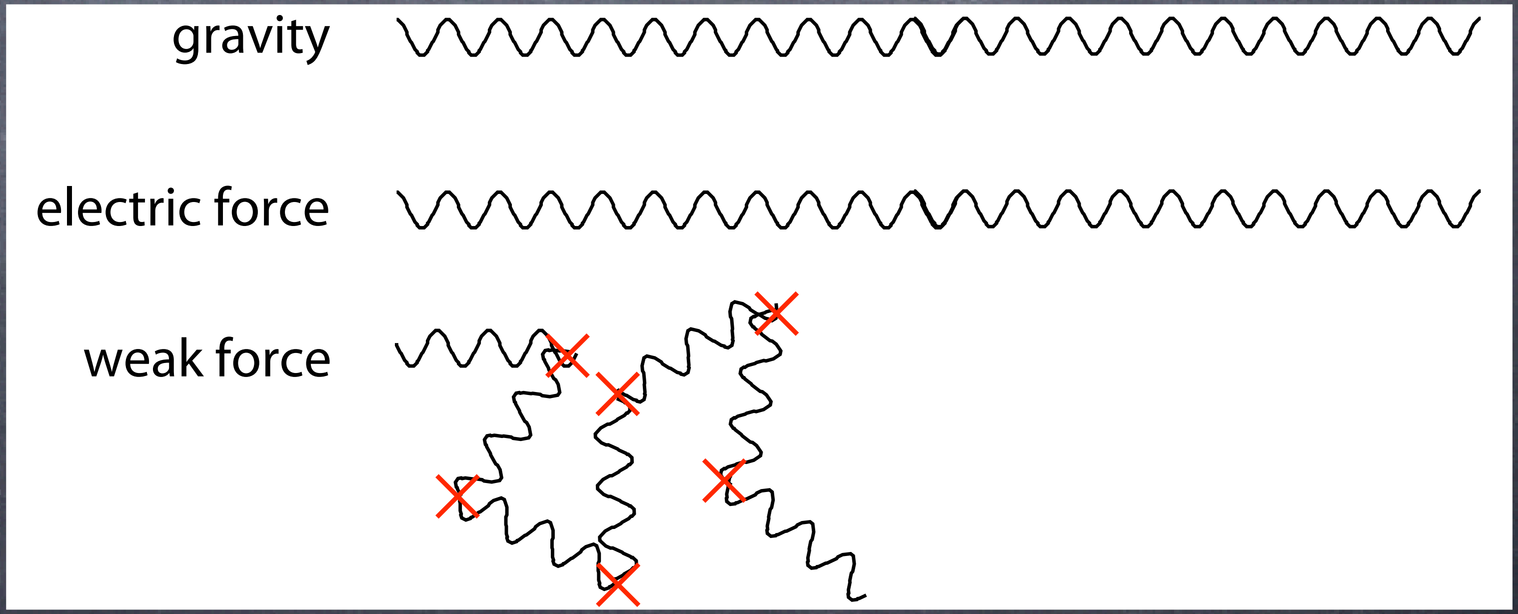


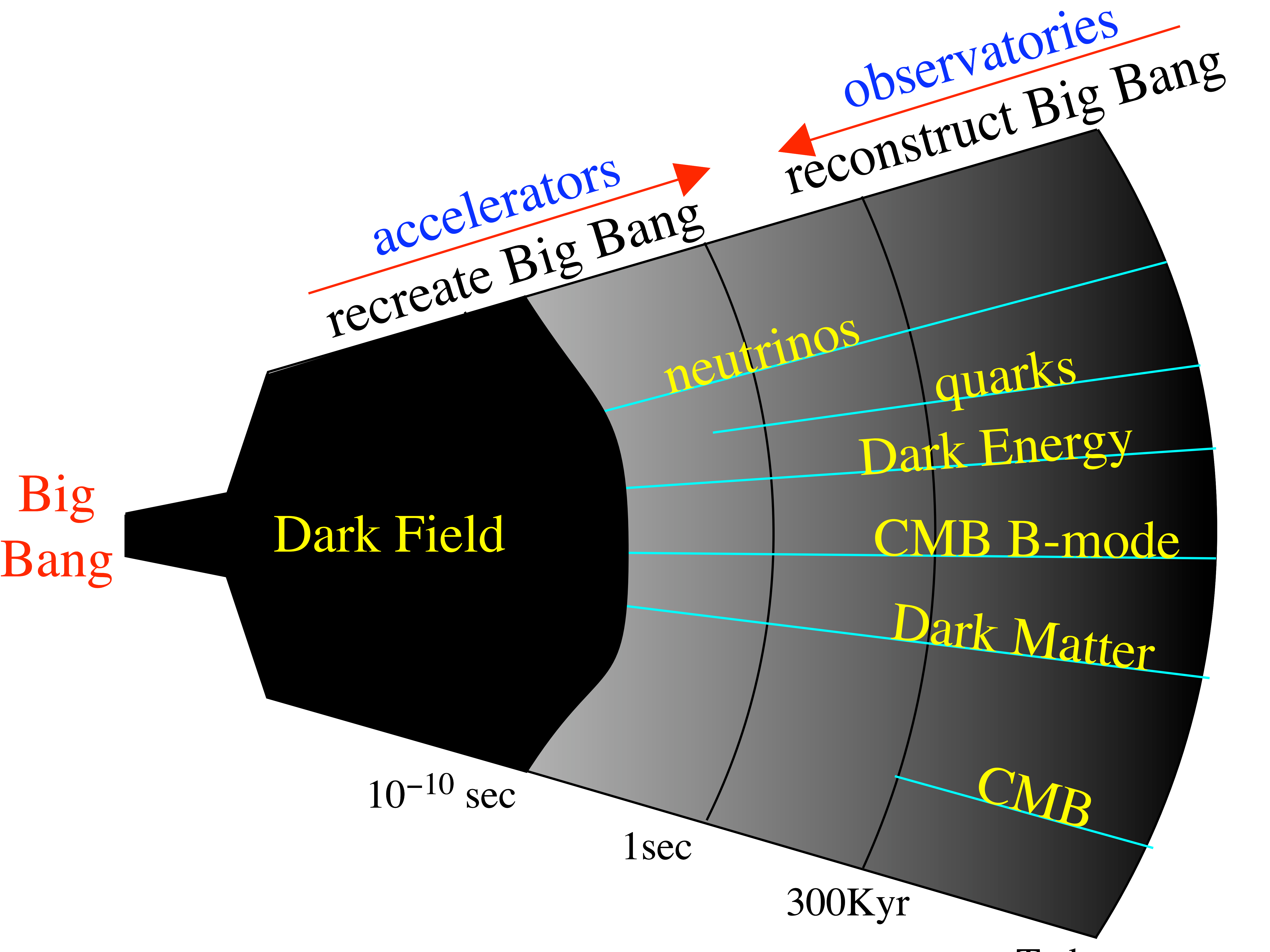
Post-Higgs Problem

- We see "what" is stuck in our universe
- But we still don't know "why"
- Two problems:
  - Why anything is condensed at all
  - Why is the scale of Dark Field 0.3TeV much much smaller than the scale of gravity ~10^{15} TeV
- Explanation most likely to be at ≤TeV scale because this is the relevant energy scale

Hierarchy Problem

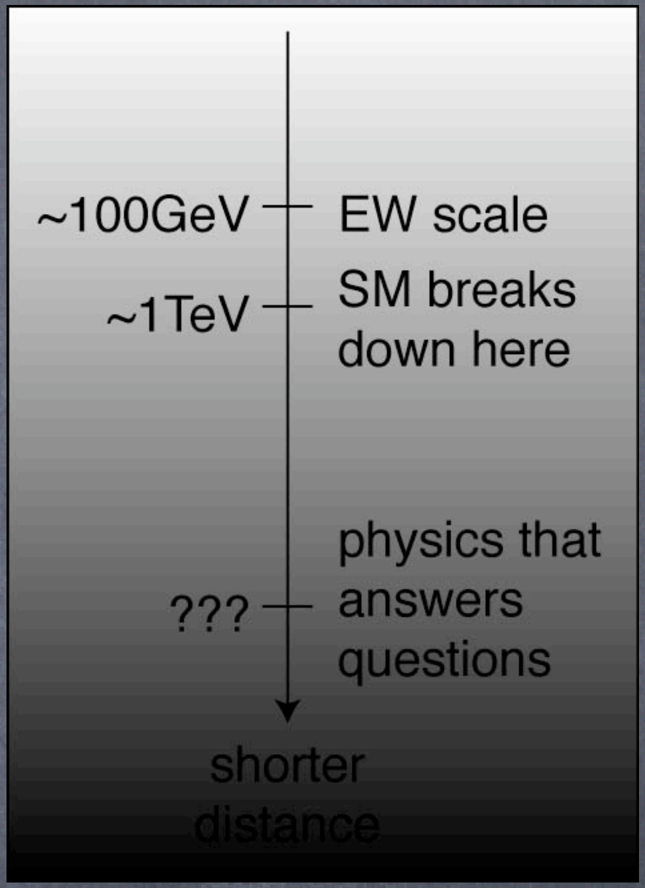





observatories
reconstruct Big Bang
accelerators
recreate Big Bang
Big
Bang
Dark Field
neutrinos
quarks
Dark Energy
CMB B-mode
Dark Matter
CMB
10^{-10} sec
1sec
300Kyr

The Main Obstacle

- We look for physics beyond the Standard Model that answers big questions about early universe
- By definition, that is physics at shorter distances
- Then the Standard Model must survive down to whatever shorter distance scale
- Hierarchy problem is the main obstacle to do so



Once upon a time, there was a hierarchy problem...

- At the end of 19th century: a "crisis" about electron
  - Like charges repel: hard to keep electric charge in a small pack
  - Electron is point-like
  - At least smaller than 10^{-17}cm
- Need a lot of energy to keep it small!

$$\Delta m_e c^2 \sim \frac{e^2}{r_e} \sim \mathrm{GeV}\frac{10^{-17}\mathrm{cm}}{r_e}$$

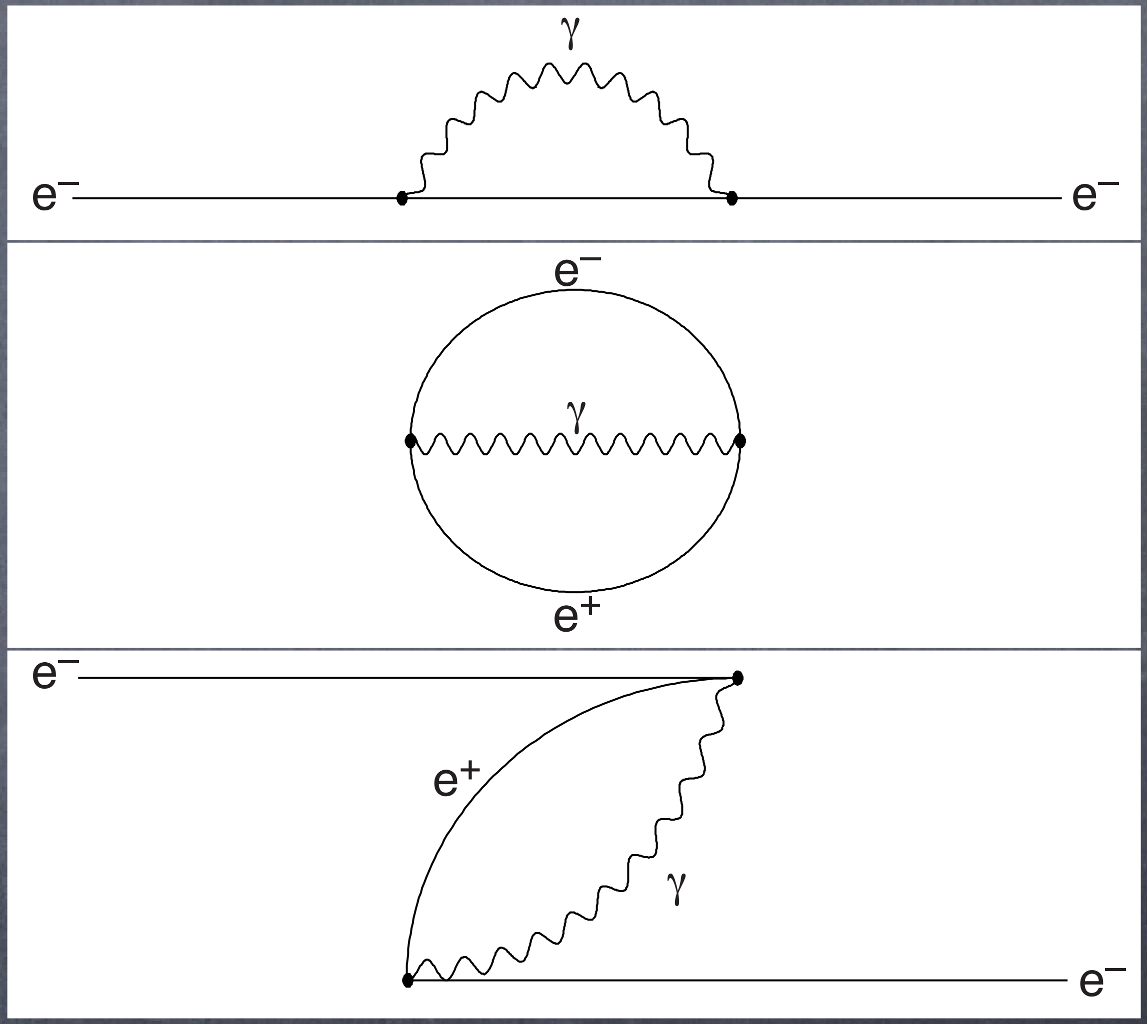
- Correction $\Delta m_e c^2 > m_e c^2$ for $r_e < 10^{-13}$cm
- Breakdown of theory of electromagnetism

$\Rightarrow$ Can't discuss physics below 10^{-13}cm

Anti-Matter Comes to Rescue by Doubling of #Particles

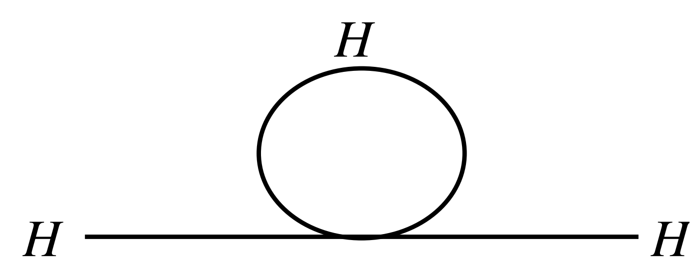
- Electron creates a force to repel itself
- Vacuum bubble of matter anti-matter creation/annihilation
- Electron annihilates the positron in the bubble

⇒ only 10% of mass even for Planck-size r_e~10^{-33}cm



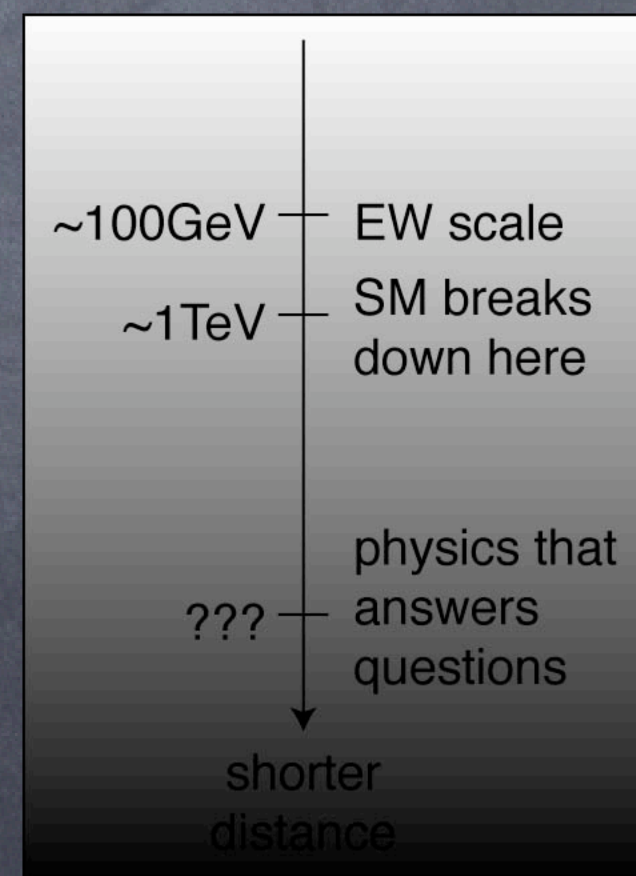
$$\Delta m_e \sim m_e \frac{\alpha}{4\pi} \log(m_e r_e)$$

Higgs repels itself, too



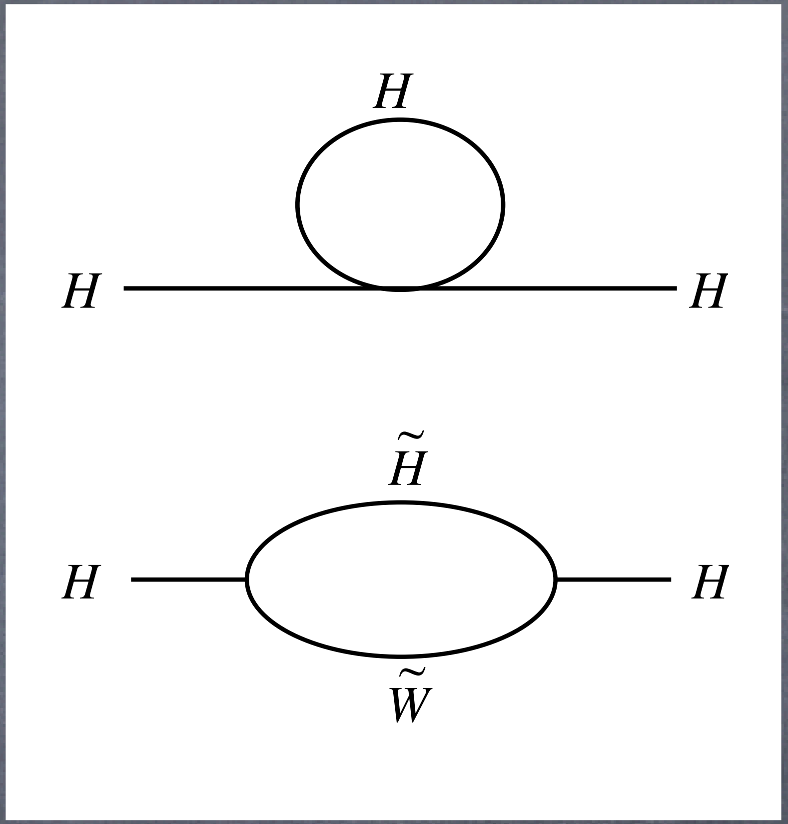
$$\Delta m_H^2 c^4 \sim \left(\frac{\hbar c}{r_H}\right)^2$$

- Just like electron repelling itself because of its charge, Higgs boson also repels itself
- Requires a lot of energy to contain itself in its point-like size!
- Breakdown of theory of weak force
- Can't get started!



History repeats itself?

- Double #particles again ⇒ superpartners
- "Vacuum bubbles" of superpartners cancel the energy required to contain Higgs boson in itself
- Standard Model made consistent with whatever physics at shorter distances

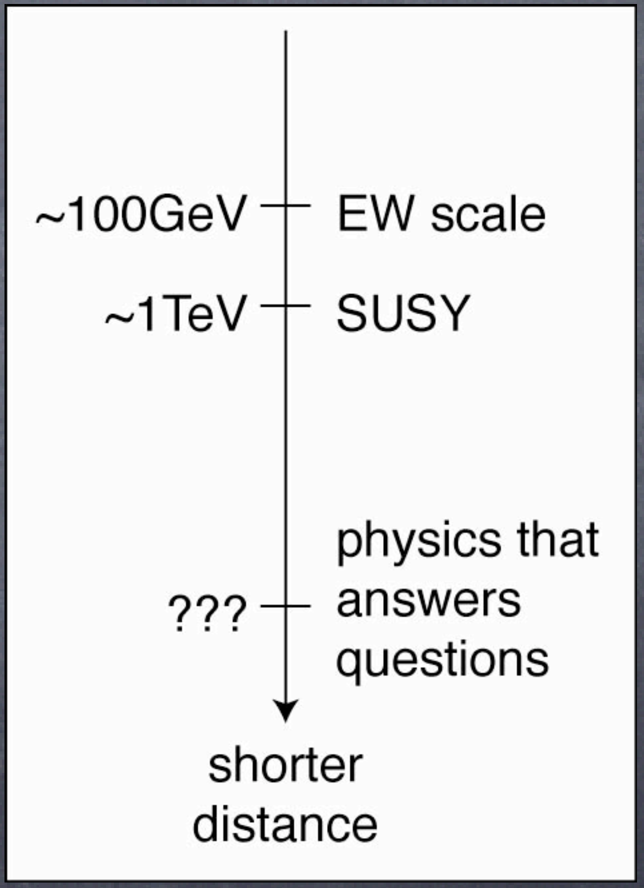


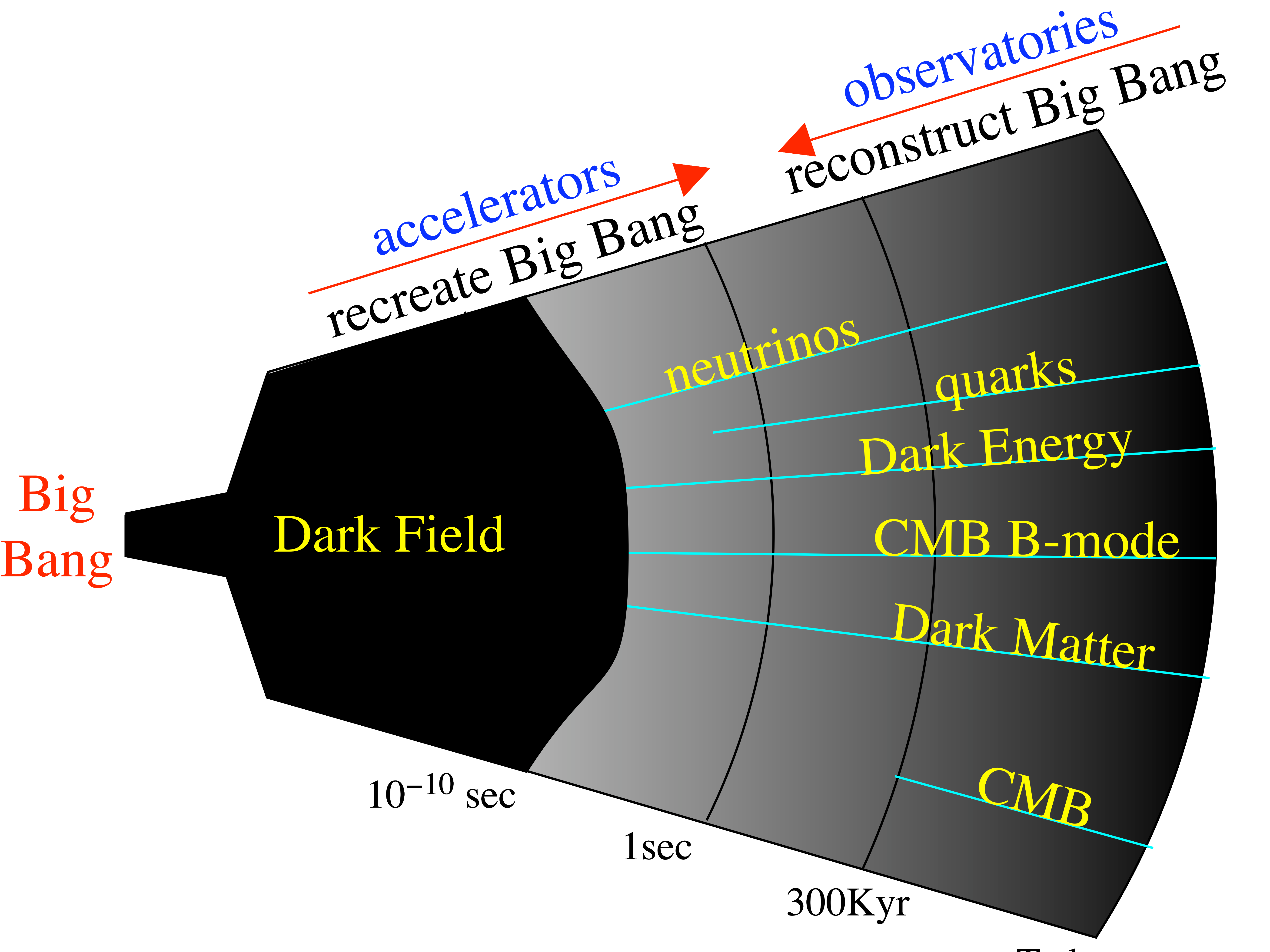
$$\Delta m_H^2 \sim \frac{\alpha}{4\pi} m_{SUSY}^2 \log(m_H r_H)$$

Opening the door

- Once the hierarchy problem solved, we can get started to discuss physics at shorter distances and earlier universe.
- It opens the door to the next level:

  Hope to answer big questions
- The solution to the hierarchy problem itself, e.g., SUSY, provides additional probe to physics at short distances




accelerators
recreate Big Bang
observatories
reconstruct Big Bang
Big
Bang
Dark Field
neutrinos
quarks
Dark Energy
CMB B-mode
Dark Matter
CMB
10^{-10} sec
1sec
300Kyr

Supersymmetry

Supersymmetry

- SUSY
  - Higgs only one of many scalars that happen to acquire negative mass-squared
  - SUSY stabilizes the hierarchy
- easily consistent with the EW precision observables because it is “decoupling” physics
- fully consistent, renormalizable, calculable theory
- can be connected to GUT, string, etc

A Broken Symmetry

- Supersymmetry predicts boson and fermion to have the same mass
- Clearly not true in nature
- It has to be broken, partners heavier than the SM particles
- Once broken, it is natural for partners to be heavier as their masses allowed by $SU(2)\times U(1)$, while quark, lepton, W, Z masses forbidden

Soft supersymmetry breaking

- Purpose of supersymmetry is to protect hierarchy
- Arbitrary terms in Lagrangian that break supersymmetry reintroduce power divergences
- “Soft supersymmetry breaking” classified:

$m\lambda\lambda$, $m^2{}_{ij}\phi^{i*}\phi^j$, $A_{ijk}\phi^j\phi^j\phi^k$, $B_{ij}\phi^j\phi^j$, $C_i\phi^j$

gaugino mass, squark/slepton mass-squared, etc

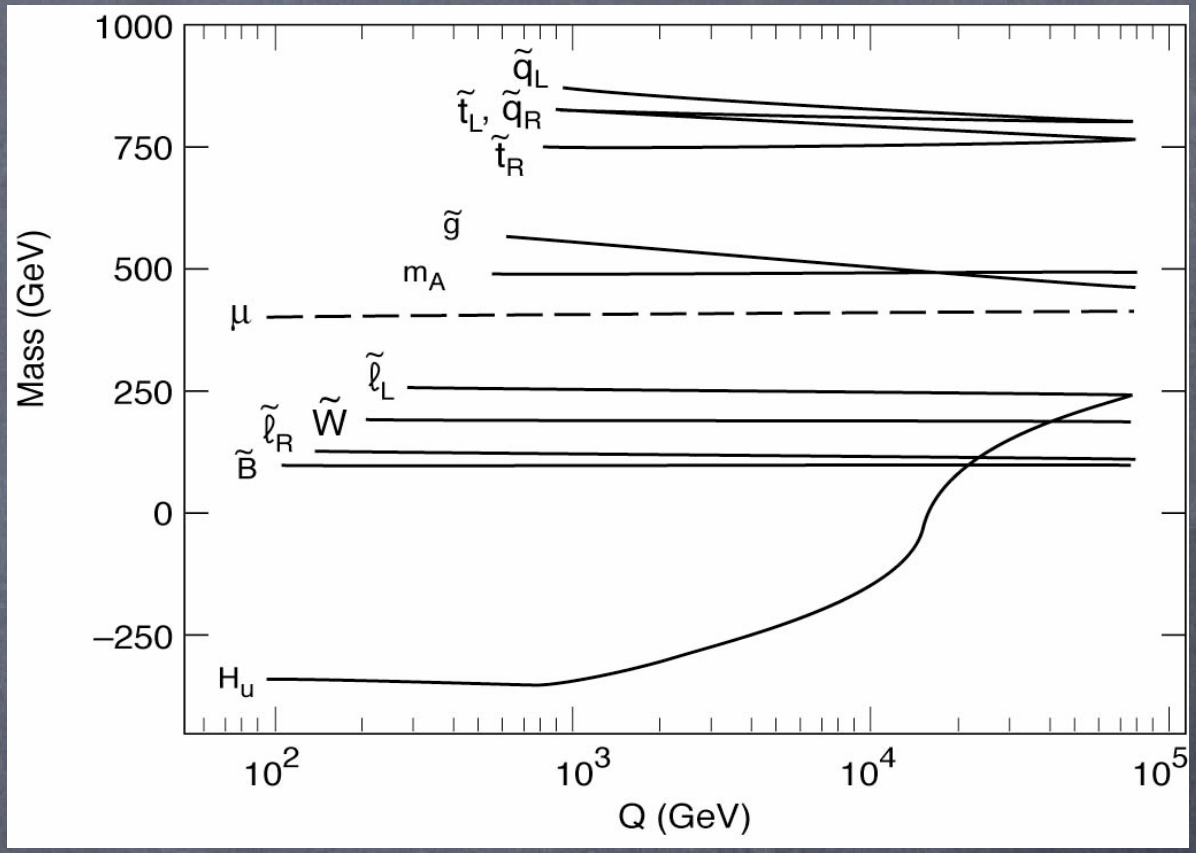
- Dark horse terms (not always allowed):

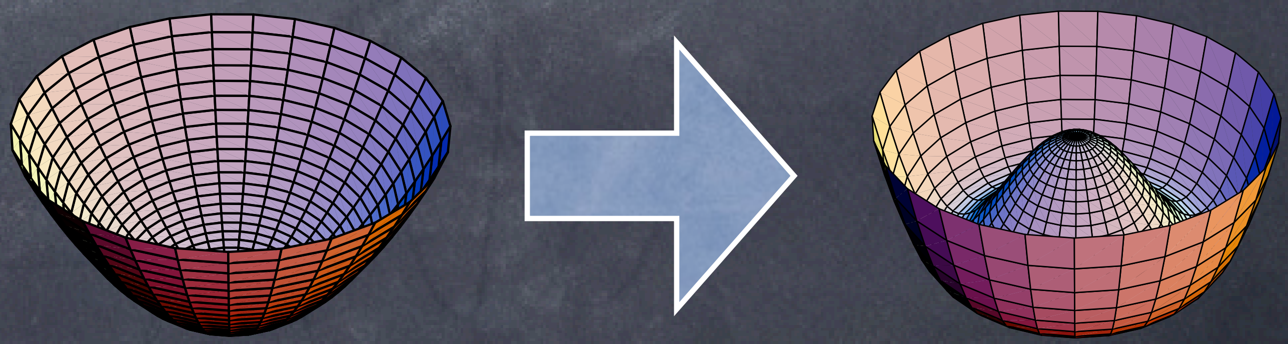
$\phi^{j*}\phi^j\phi^k$, $\lambda\psi^j$, $\psi^i\psi^j$

Radiative Symmetry Breaking

(Inoue et al; Alvarez-Gaumé et al; Ibañez-Ross)

- In the MSSM, electroweak symmetry does not get broken
- Only after supersymmetry is broken, Higgs can obtain a VEV $v \sim m_{\mathrm{SUSY}}$
- EWSB is as a consequence of supersymmetry breaking
- EW symmetry and hierarchy "protected" by supersymmetry





Origin of Hierarchy
(Witten)

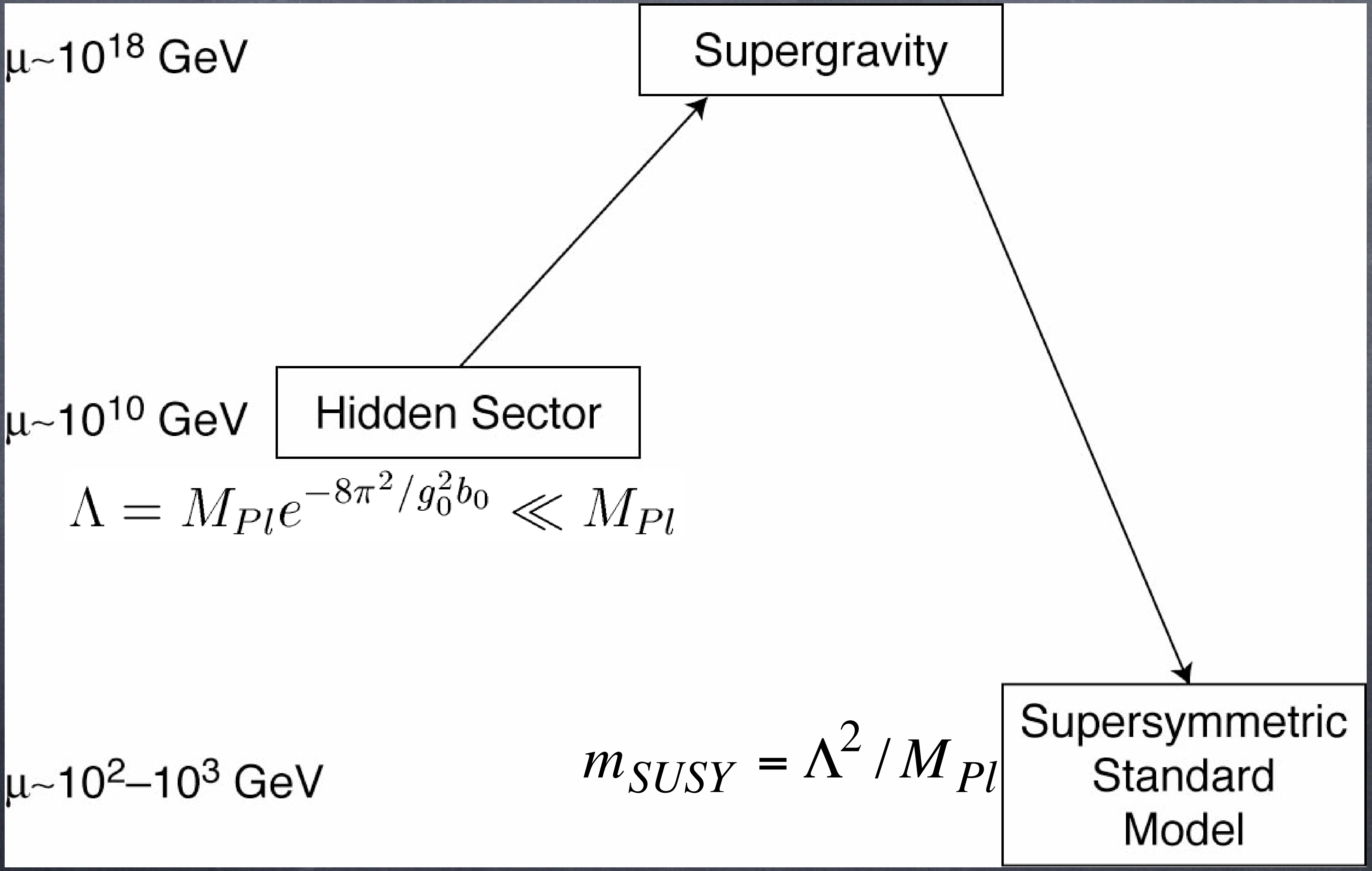
- $v \ll M_{\rm Pl}$ because $v \sim m_{\rm SUSY} \ll M_{\rm Pl}$
- Why $m_{\rm SUSY} \ll M_{\rm Pl}$?
- Idea: running coupling constant
- SUSY broken by strong gauge dynamics with
- "Dynamical supersymmetry breaking"

$$\Lambda = M_{Pl} e^{-8\pi^2/g_0^2 b_0} \ll M_{Pl}$$

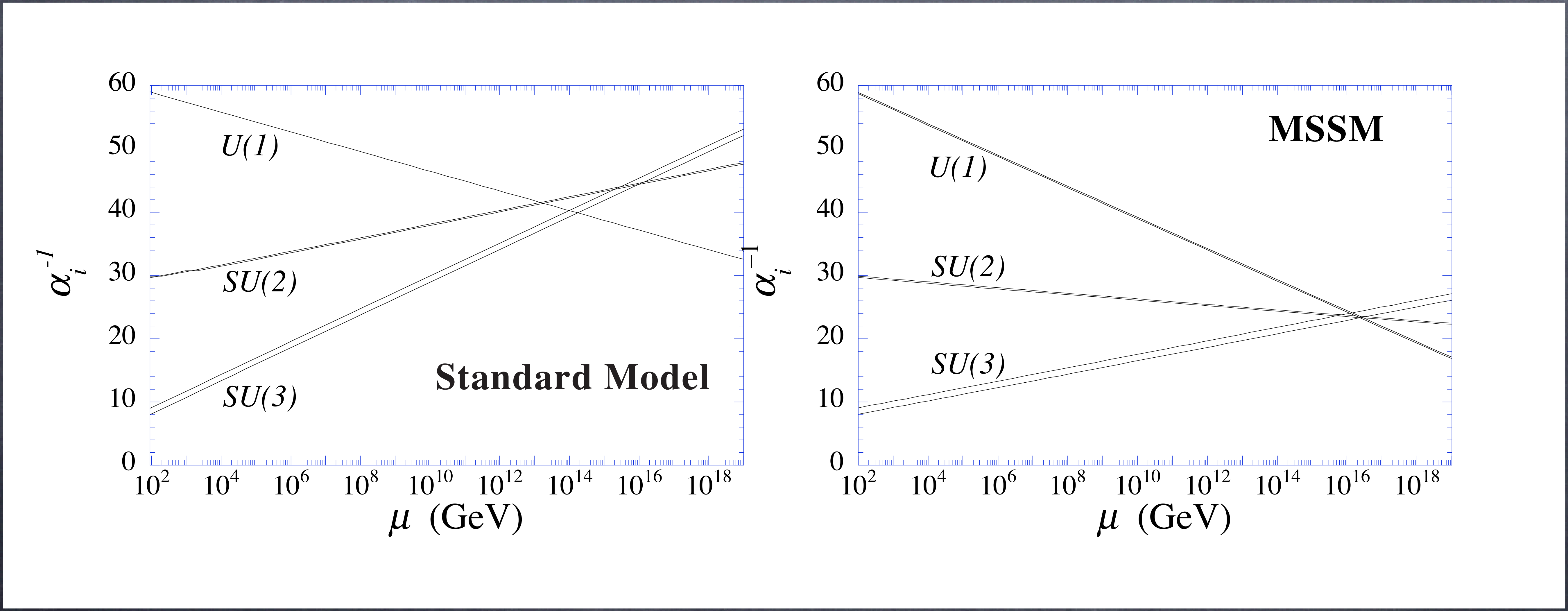
- e.g., SO(10) with one 16

(Affleck, Dine, Seiberg; HM)

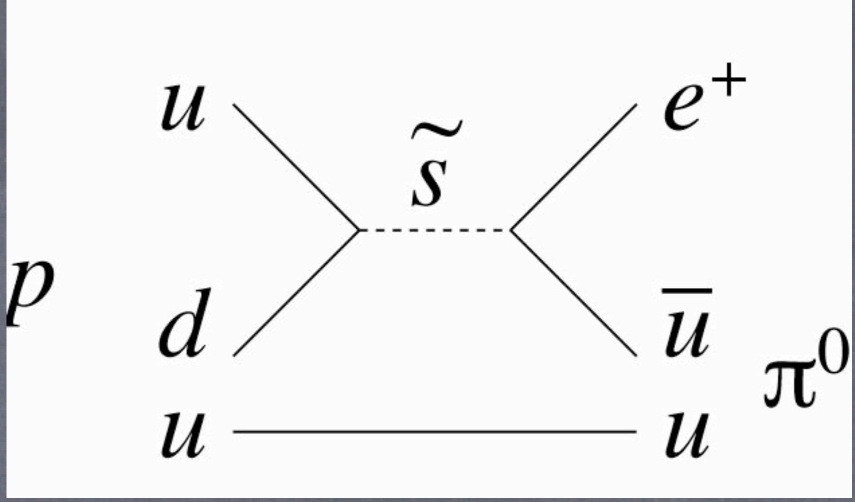
The paradigm



gauge coupling unification



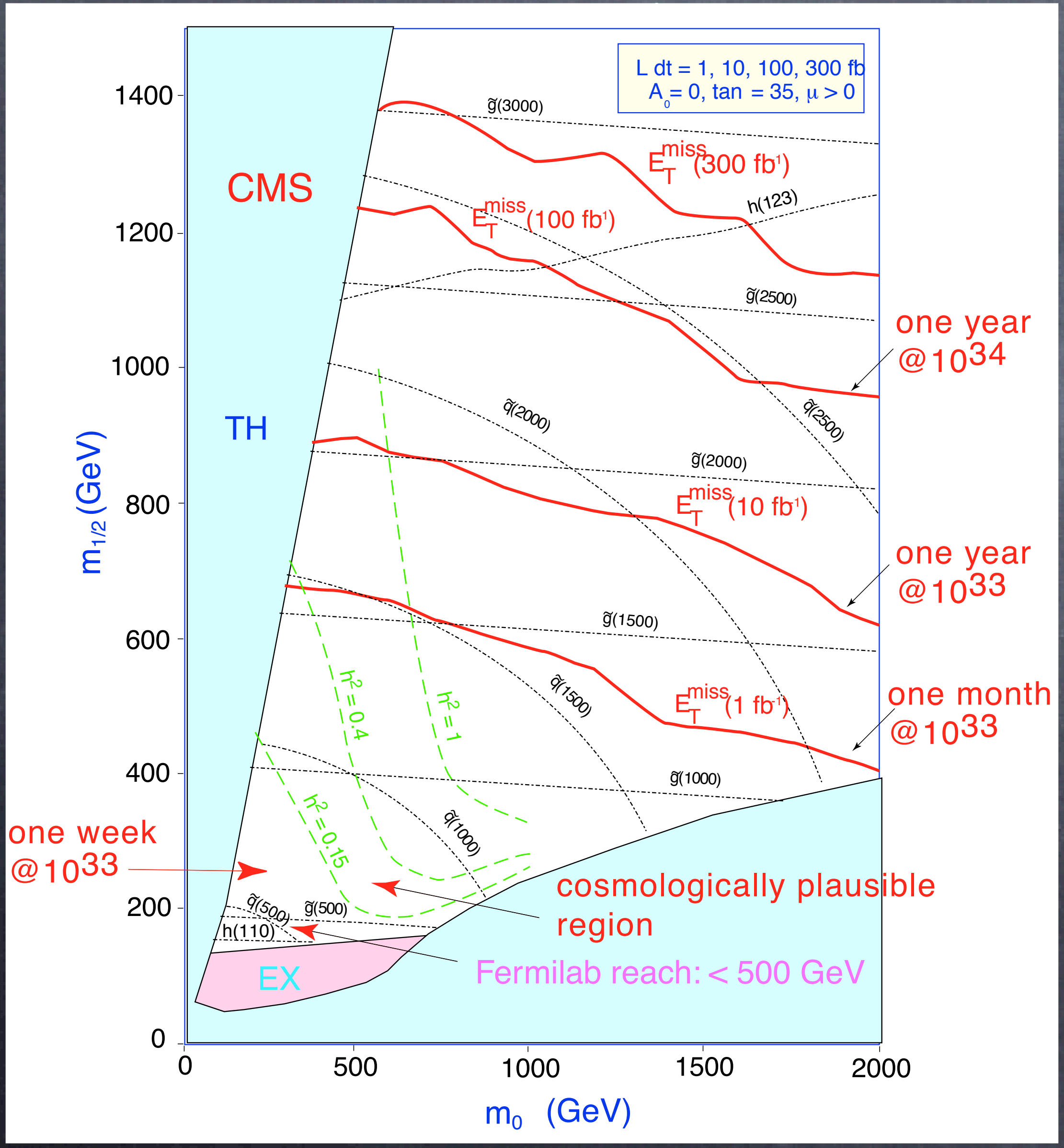
R-parity



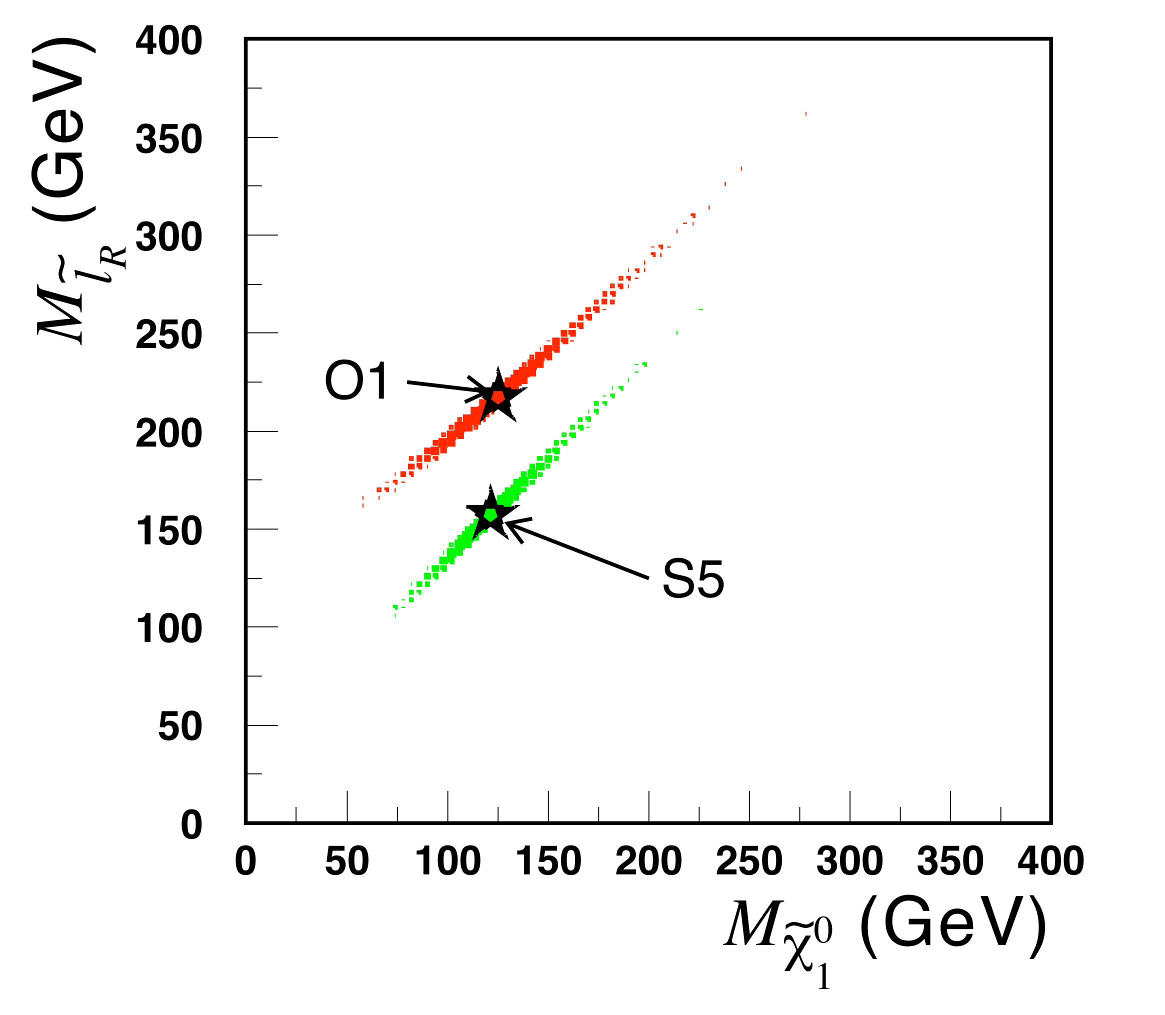
- B, L-conservation not automatic
- W=udd+QdL+LLe+LHu
- If they exist with $O(1)$ couplings:
- $\tau_p \sim m_{sq}^4/m_p^5 \sim 10^{-12}$ sec!
- Product of two couplings < 10^{-26}
- Impose R-parity = $(-1)^{3B+L+2S}$
- Forbids B and L number violation
- R-parity is non-anomalous; may be gauged
- Stable Lightest Supersymmetric Particle
  $\Rightarrow$ Cold Dark Matter
- SUSY particles always pair-produced and decay into the LSP: missing energy signal

Supersymmetry

Tevatron/LHC will discover



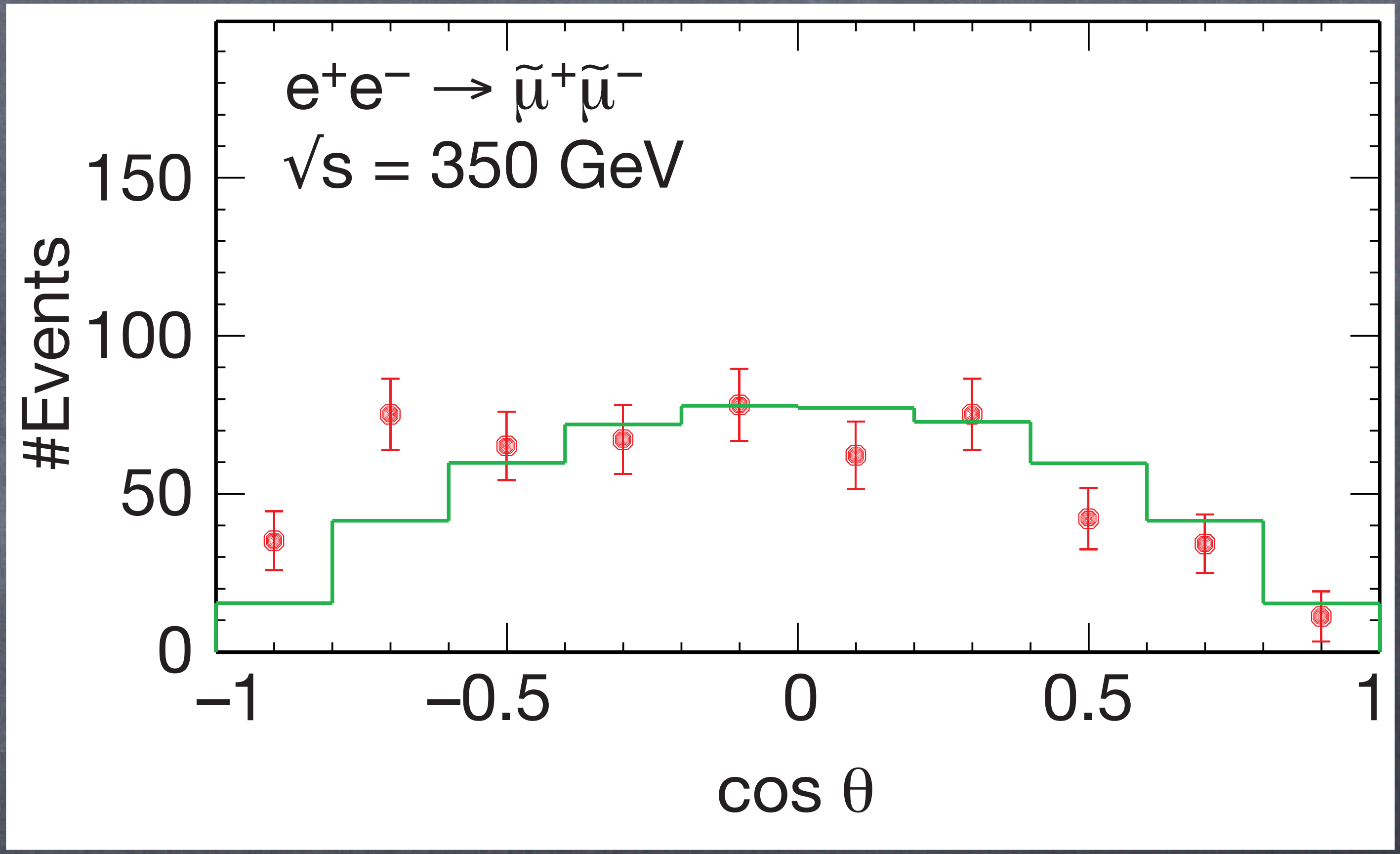
Can do many measurements at LHC



Prove Supersymmetry

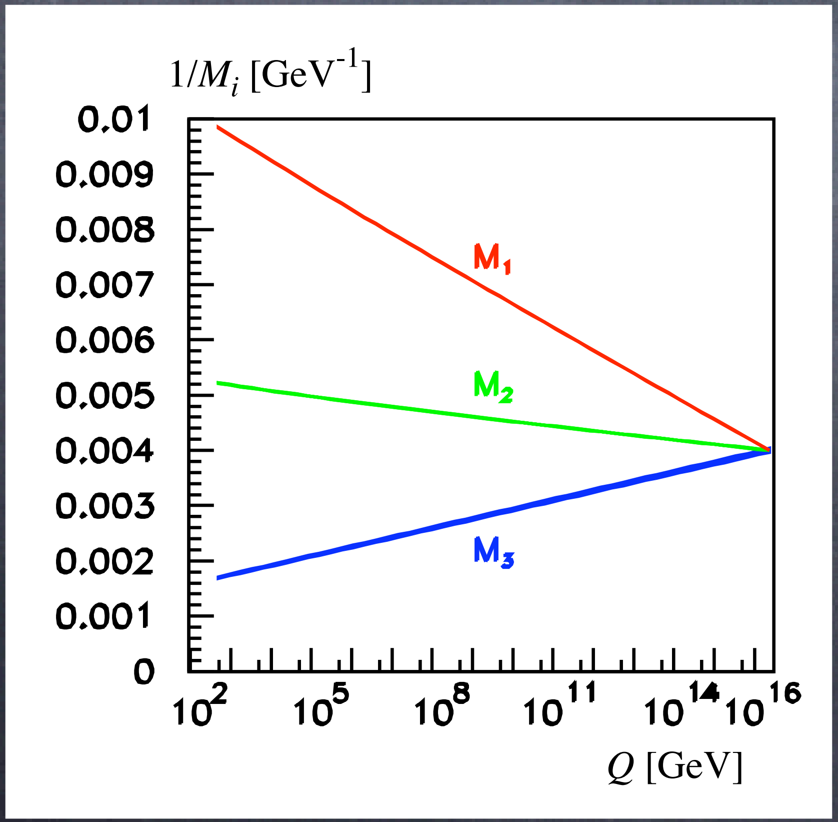
- Discovery at Tevatron Run II and/or LHC
- Test they are really superpartners
  - Spins differ by $1/2$
  - Same $SU(3)\times SU(2)\times U(1)$ quantum numbers
  - Supersymmetric couplings

Spin 0?



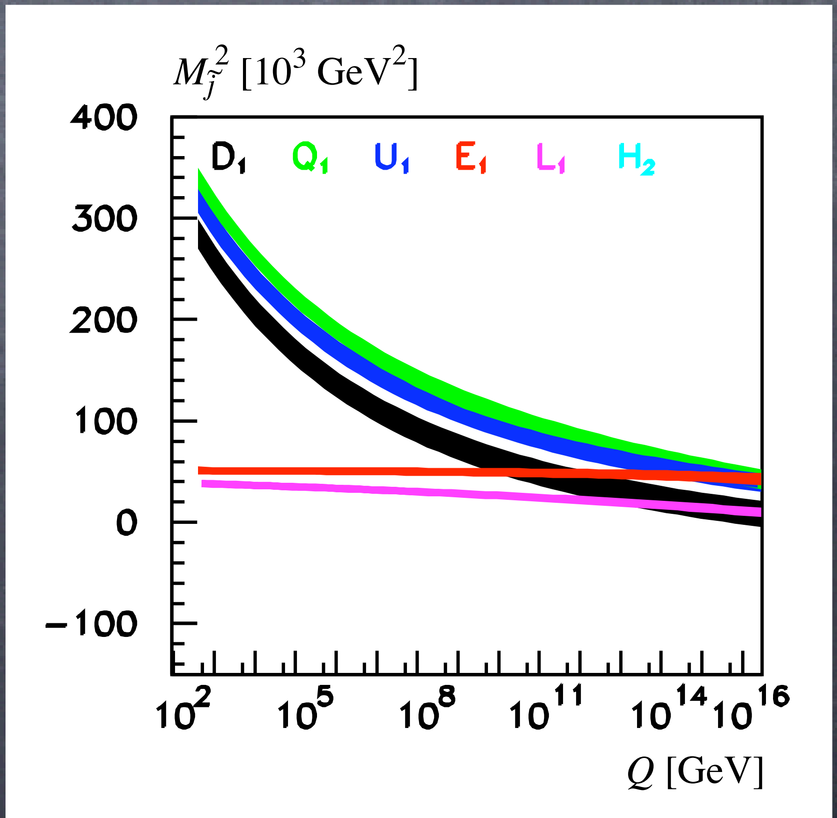
Gaugino and scalars

- Gaugino masses test unification itself independent of intermediate scales and extra complete SU(5)



- Scalar masses test beta functions at all scales, depend on the particle content

(Kawamura, HM, Yamaguchi)



Particle Dark Matter

- WIMP (Weakly Interacting Massive Particle) strongly favored
- Stable heavy particle produced in early Universe, left-over from near-complete annihilation

$$\Omega_M = \frac{0.756(n+1)x_f^{n+1}}{g^{1/2}\sigma_{ann}M_{Pl}^3}\frac{3s_0}{8\pi H_0^2} \approx \frac{\alpha^2/(\mathrm{TeV})^2}{\sigma_{ann}}$$

- TeV the correct energy scale
- We hope to produce DM directly at colliders

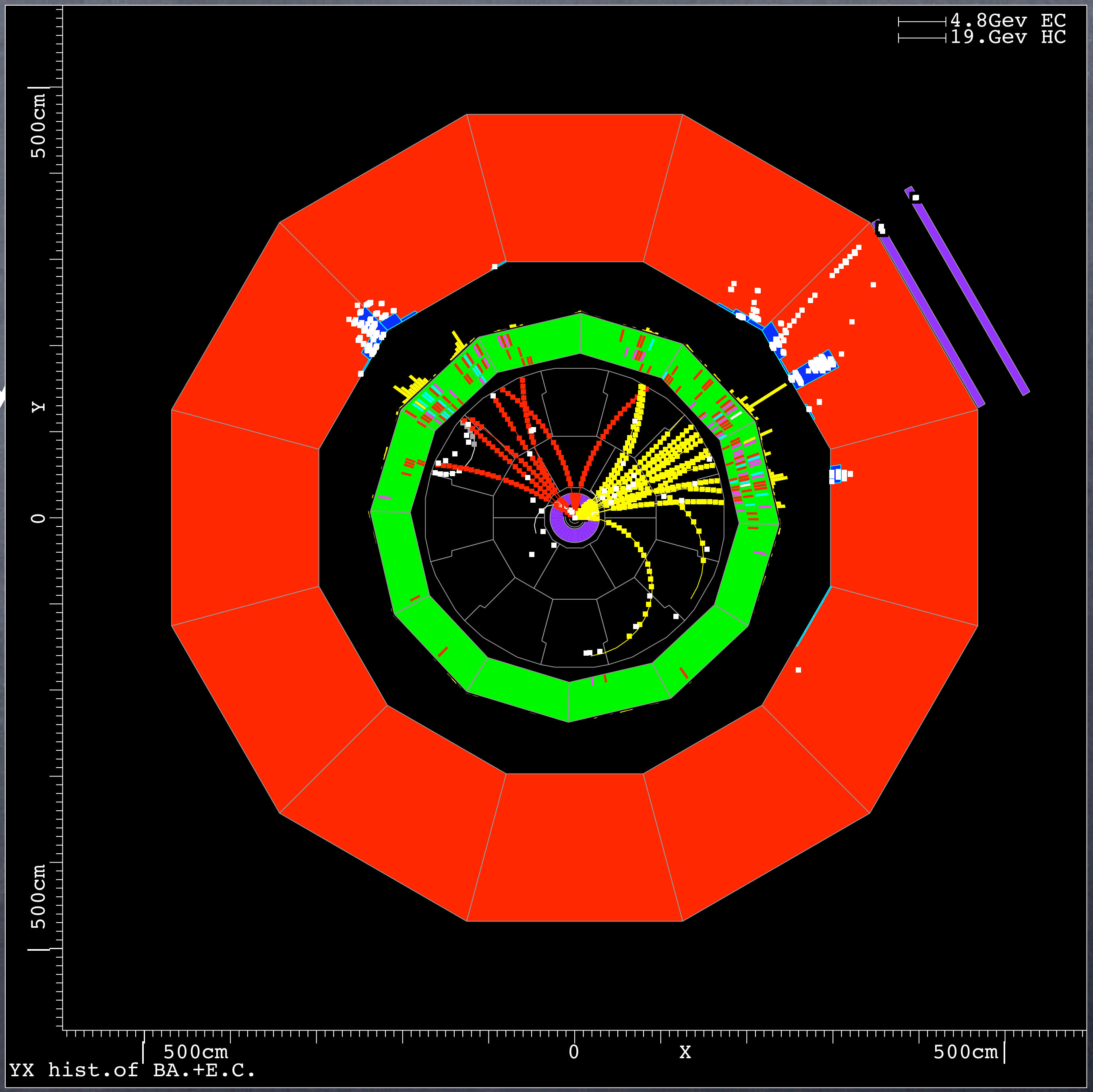
Producing Dark Matter in the laboratory

- Collision of high-energy particles mimic Big Bang
- We hope to create Dark Matter particles in the laboratory
- Look for events where energy and momenta are unbalanced

"missing energy" E_{miss}

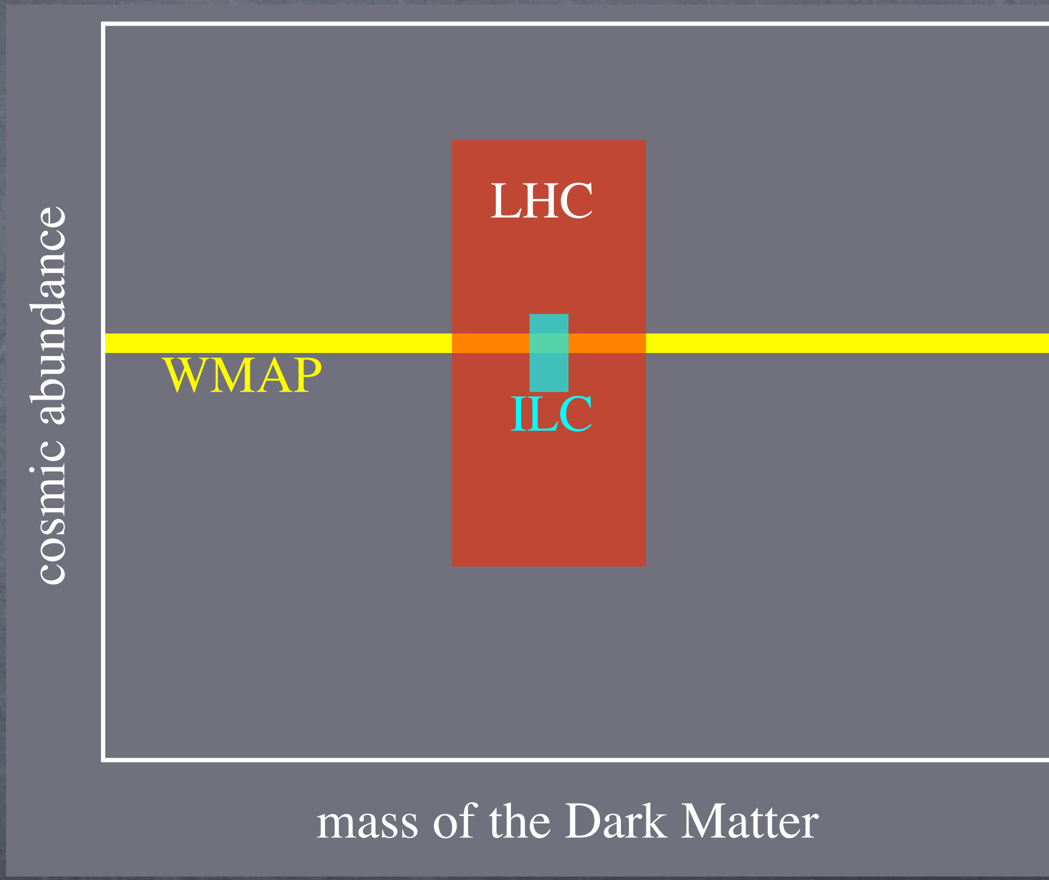
- Something is escaping the detector
- electrically neutral, weakly interacting

⇒Dark Matter!?



How do we know what Dark Matter *is*?

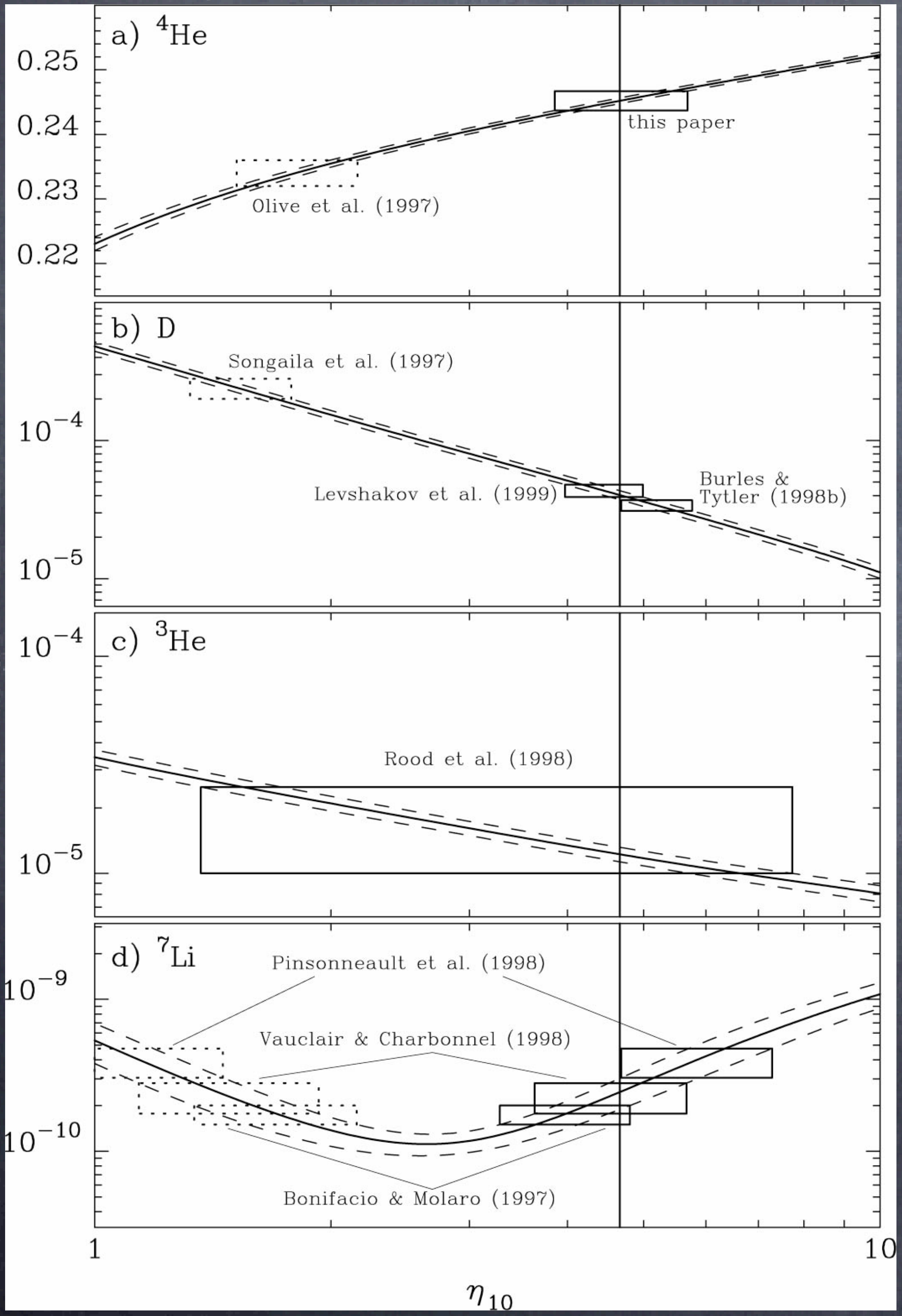
- cosmological measurement of dark matter
  - abundance $\propto \sigma_{ann}^{-1}$
- detection experiments
  - scattering cross section
- production at colliders
  - mass, couplings
  - can calculate cross sections
- If they agree with each other:



$\Rightarrow$ Will know what Dark Matter is

$\Rightarrow$ Will understand universe back to t~10^{-10}

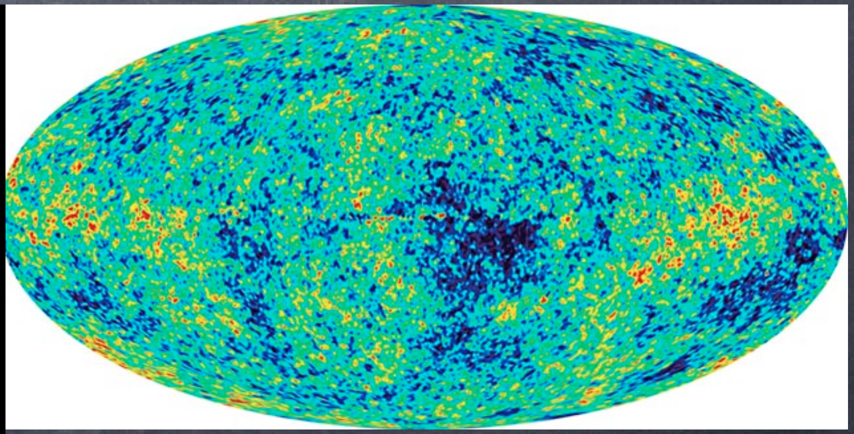
Big-Bang Nucleosynthesis
Cosmic Microwave Background

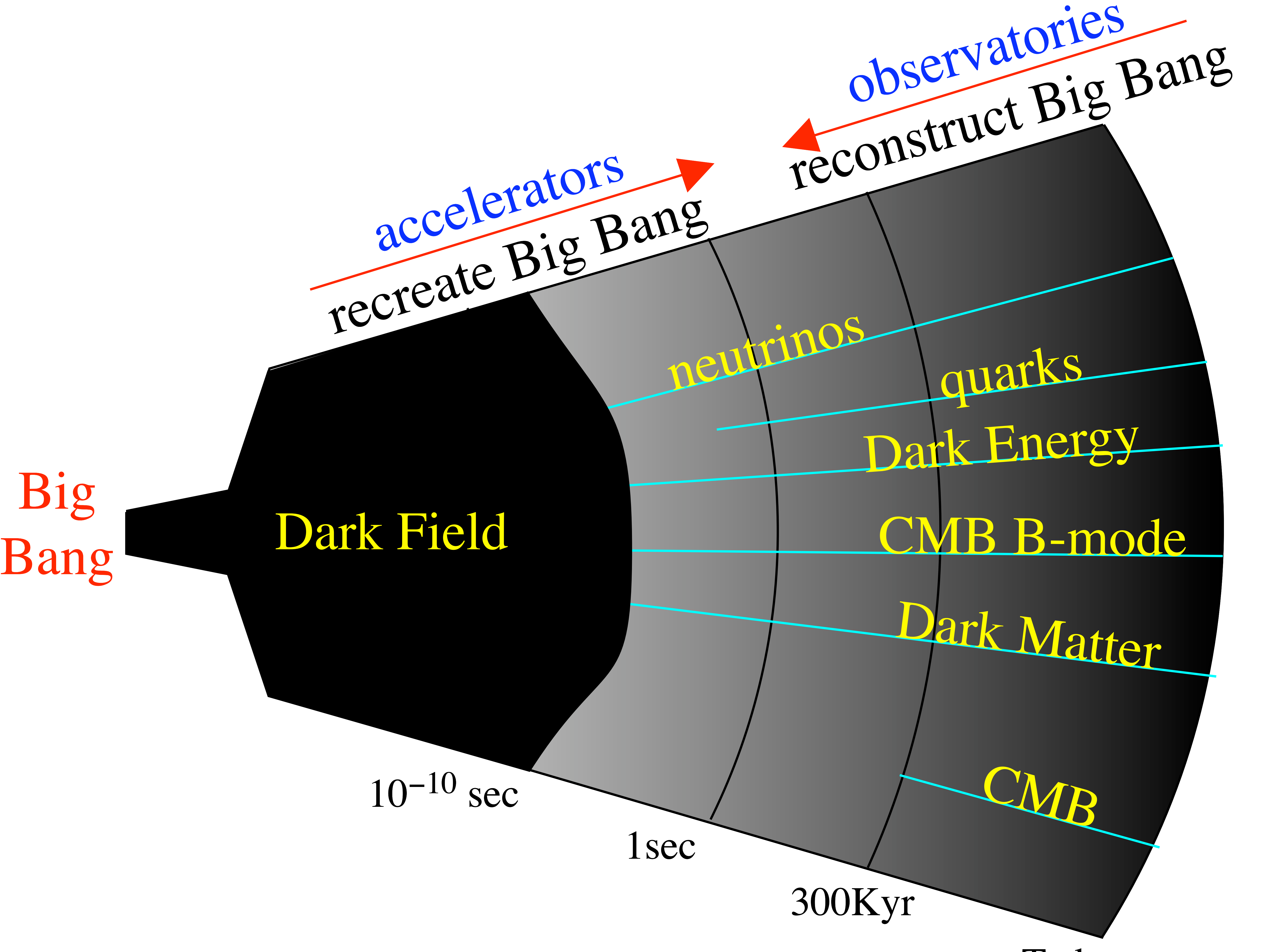


$$\eta = \frac{n_B}{n_\gamma} = \left(4.7^{+1.0}_{-0.8}\right) \times 10^{-10}$$ (Thuan, Izatov)

$$(5.0 \pm 0.5) \times 10^{-10}$$ (Burles, Nollett, Turner)

WMAP $(6.5^{+0.4}_{-0.3}) \times 10^{-10}$




accelerators
recreate Big Bang
observatories
reconstruct Big Bang
Big
Bang
Dark Field
neutrinos
quarks
Dark Energy
CMB B-mode
Dark Matter
CMB
10^{-10} sec
1sec
300Kyr