Supersymmetry II

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Plan

Mon: Non-technical Overview

what SUSY is supposed to give us

Tue: From formalism to the MSSM

Global SUSY formalism, Feynman rules, soft SUSY breaking, MSSM

Wed: SUSY breaking

how to break SUSY, mediation mechanisms

Thu: SUSY at colliders

basic reactions, signatures, and how do we know it is SUSY?

Fri: SUSY as a telescope

supersymmetry breaking, GUT, string

Global SUSY

SUSY algebra

- In addition to the momentum P_μ and Lorentz generators $M_{\mu\nu}$, add spinorial charges $Q_\alpha{}^i$, i=1, .., N

$$[P_\mu, Q^i_\alpha] = 0$$
$$[M^{\mu\nu}, Q^i_\alpha] = \frac{1}{2}(\sigma^{\mu\nu})_\alpha{}^\beta Q^i_\beta$$
$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot\beta}\delta^{ij}$$
$$\{Q^i_\alpha, Q^j_\beta\} = \varepsilon_{\alpha\beta}C^{ij}$$

- $C^{ij}=-C^{ji}$ central charges, possible only if N>1

Massless particle

- Specialize to P_μ=E(1,0,0,1)
- helicity: h = ϵ_{0ijk} P^i $M^{jk}/2P^0$ = M^{12}

$$P^\mu = E(1,0,0,1)$$

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot\beta}\delta^{ij} = 2E\begin{pmatrix} 2 & \\ & 0 \end{pmatrix}\delta^{ij}$$

- creation/annihilation operators (C^{ij}=0)

$$b^i = Q^i_1/\sqrt{4E}, \quad b^{i\dagger} = \bar{Q}^i_1/\sqrt{4E}$$

$$\{b^i, b^{j\dagger}\} = \delta^{ij}$$

$$\{h, b^i\} = -\frac{1}{2}b^i$$

$$\{h, b^{i\dagger}\} = \frac{1}{2}b^{i\dagger}$$

N=1 multiplets

- Restrict $|h| \leq 1$ for renormalizability
- Multiplet structure: $\{|0\rangle, b^\dagger|0\rangle\}$
- chiral multiplet: pick $h|0\rangle=0$ or $-1/2$
  - $h=\{0, 1/2\}$ (anti-chiral) or $\{-1/2, 0\}$ (chiral)
  - i.e., a Weyl fermion and a complex scalar
- vector multiplet: pick $h|0\rangle=1/2|0\rangle$ or -1
  - $h=\{1/2, 1\}$ or $\{-1, -1/2\}$
  - i.e, a gauge field and a Weyl (Majorana) fermion
  - must be in adjoint (real) rep $\Rightarrow$ non-chiral

N=2 multiplets

- Multiplet: $\{|0\rangle, b_1^\dagger|0\rangle, b_2^\dagger|0\rangle, b_1^\dagger b_2^\dagger|0\rangle\}$
- hypermultiplet: pick $h|0\rangle=-1/2$
  - $h=\{-1/2, 0, 0, 1/2\}$
  - under N=1, sum of chiral and anti-chiral
  - a Dirac fermion and two complex scalars
- vector multiplet: pick $h|0\rangle=0|0\rangle$
  - $h=\{0, 1/2, 1/2, 1\}$
  - under N=1, sum of chiral and vector
  - a gauge field and two Weyl (Majorana) fermions, and a real scalar
- Neither multiplet chiral: can't be used to supersymmetrize the Standard Model

N=1 superspace chiral superfield

- introduce Grassman-odd coordinates θ_α
- superspace: $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$

$$P_\mu = i\partial_\mu$$
$$M^{\mu\nu} = i(x^\mu \partial^\nu - x^\nu \partial^\mu)$$
$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\bar{\theta}^{\dot{\beta}} \sigma^\mu_{\dot{\beta}\alpha} \partial_\mu$$

- Covariant derivative

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\bar{\theta}^{\dot{\beta}} \sigma^\mu_{\dot{\beta}\alpha} \partial_\mu$$
$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = 0$$
$$\{D_\alpha, \bar{D}_{\dot{\beta}}\} = -2i\partial_\mu (\sigma^\mu)_{\alpha\dot{\beta}}$$

- Can place a constraint $\bar{D}_{\dot{\alpha}} \phi(x, \theta, \bar{\theta}) = 0$
- Note $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}, \quad \bar{D}_{\dot{\alpha}} y^\mu = 0$
- solution: $\phi(x, \theta, \bar{\theta}) = \phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$

vector superfield

- introduce Grassman-odd coordinates θ_α
- real (vector) superfield:

$$V(x,\theta,\bar\theta) = C + \theta\chi + \bar\theta\bar\chi + \theta^2 M + \bar\theta^2 M^* + \theta\sigma^\mu\bar\theta A_\mu + \theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\lambda + \theta^2\bar\theta^2 D$$

- gauge transformation: $V \to V + i\Lambda - i\bar\Lambda$

Λ is a chiral superfield

Can eliminate C, χ, M: Wess-Zumino gauge

- remaining dof: A_μ, λ, D
- Field strength chiral superfield:

$$W_\alpha = \bar D^2 D_\alpha V = \lambda_\alpha(y) + \theta^\beta \sigma^{\mu\nu}_{\beta\alpha} F_{\mu\nu} + \theta_\alpha D$$

Non-abelian gauge symmetry

- Generalization to non-abelian case

$$
\begin{aligned}
&V = V^a T^a, \quad \Lambda = \Lambda^a T^a \\
&e^V \to e^{-i\bar{\Lambda}} e^V e^{i\Lambda} \\
&\phi \to e^{-i\Lambda}\phi \\
&\phi^\dagger e^V \phi : \ \text{invariant}
\end{aligned}
$$

- Field strength chiral superfield

$$
\begin{aligned}
&W_\alpha = \bar{D}^2 e^{-V} D_\alpha e^V \\
&W_\alpha \to e^{-i\Lambda} W_\alpha e^{i\Lambda}
\end{aligned}
$$

Kähler and superpotentials

- Two ways to construct invariants (up to total derivatives)
- full superspace integral of general superfield "Kähler potential"

$$\int d^4\theta\phi^*\phi = |\partial_\mu A|^2 + \bar{\psi} i\sigma^\mu\partial_\mu\psi + F^*F$$

- chiral superspace integral of chiral superfield "superpotential"

$$\int d^2\theta W(\phi) = \frac{\partial W}{\partial\phi^i}F^i + \frac{1}{2}\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\psi^i\psi^j$$

- Renormalizable theory is fixed by W

gauge theory (Wess-Zumino gauge)

matter kinetic term

$$\int d^4\theta \phi^\dagger e^V \phi = |D_\mu A|^2 + \bar{\psi} i \not{D} \psi + F^\dagger F + D^a A^\dagger T^a A + (\sqrt{2} A^\dagger \lambda^a T^a \psi + h.c.)$$

gauge kinetic term

$$\int d^2\theta \frac{1}{g^2} W^a_\alpha W^{a\alpha} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu a} + \frac{1}{g^2} \bar{\lambda}^a i \not{D} \lambda^a + \frac{1}{2g^2} D^a D^a$$

superpotential

$$\int d^2\theta W(\phi) = \frac{\partial W}{\partial \phi^i} F^i + \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j$$

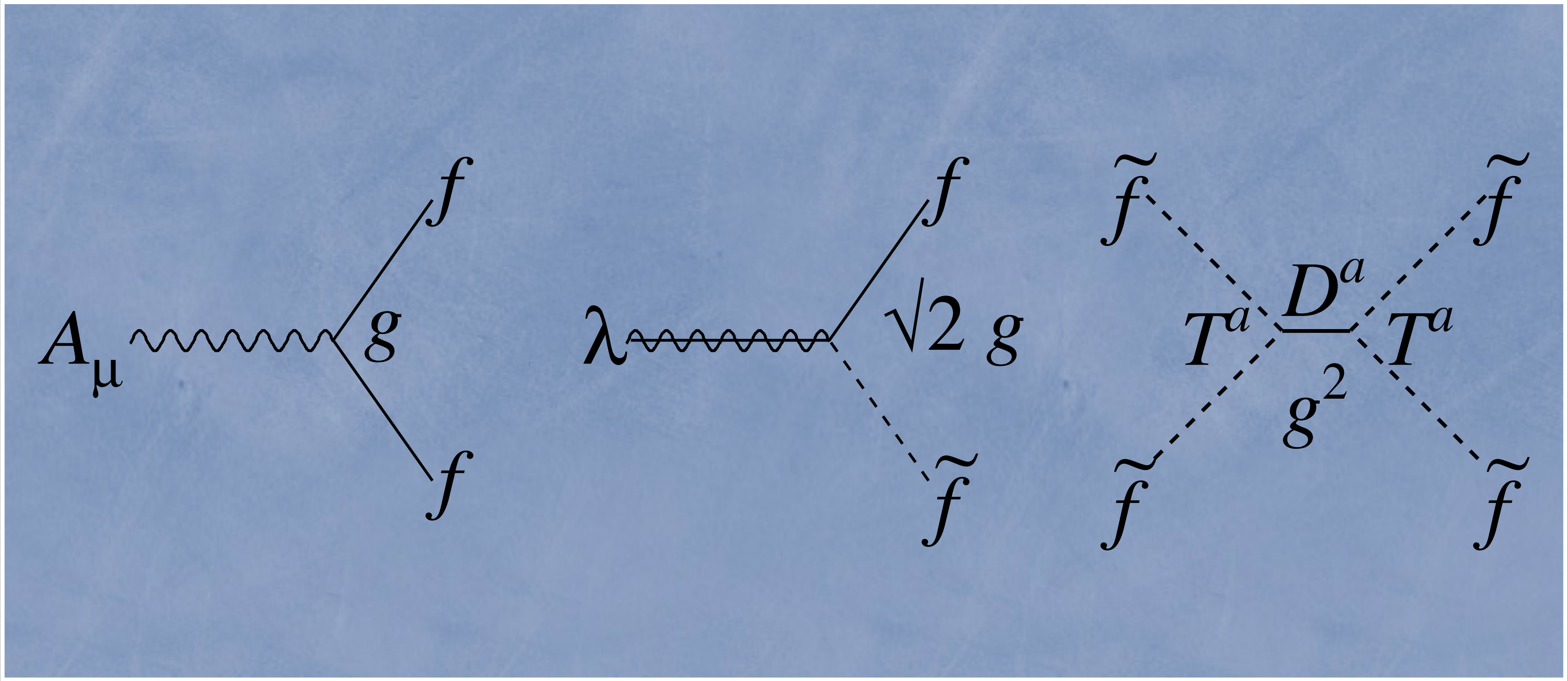
Solve for auxiliary fields

$$D^a = g^2 A^\dagger T^a A, \quad F^{i*} = \frac{\partial W}{\partial \phi^i}$$

$$V = \frac{1}{2g^2} D^a D^a + |F^i|^2 = \frac{g^2}{2} (A^\dagger T^a A)^2 + \left| \frac{\partial W}{\partial \phi^i} \right|^2$$

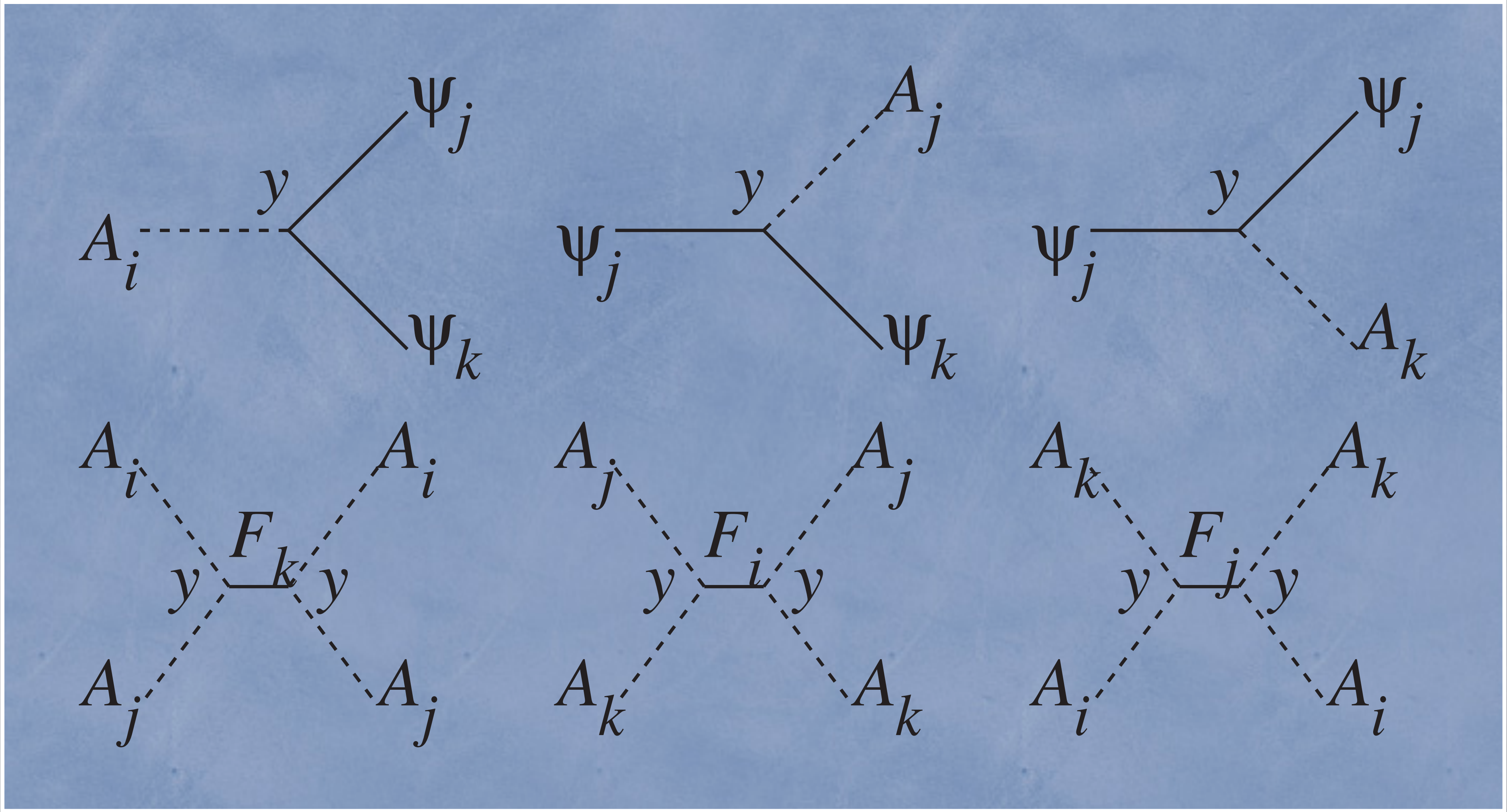
Feynman rules

- Single gauge coupling constant gives all of these Feynman vertices



Feynman rules

- Single Yukawa coupling constant gives all of these Feynman vertices



Fayet-Illiopoulos D-term

- Only for U(1) gauge factors, there is another possible term

$$\int d^4\theta V \to \int d^4\theta (V + i\Lambda - i\bar{\Lambda}) = \int d^4\theta V + \text{surface terms}$$

- constant term: ξD
- changes the D-term potential: $\frac{g^2}{2}\left(A^{i\dagger}Q_i A^i - \frac{\xi}{g^2}\right)^2$
- not consistent with supergravity by itself unless $U(1)_R$ is gauged or Green-Schwarz mechanism is employed
- I will not discuss it any further

Renormalization

- Start with the Wilsonian action at scale μ

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\frac{1}{g_0^2} W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- non-renormalization theorem, holomorphy, and transivity says at scale $\mu' = \mu e^{-t}$,

$$\int d^4\theta \sum_i Z_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t \right) W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

$$b_0 = 3C_A - \Sigma_i T_F^i$$

- to identify coupling constants, need to rescale fields to canonical normalization
- However, rescaling fields yield anomalous Jacobians

Renormalization

- Konishi anomaly $\int \mathcal{D}\phi_i = \int \mathcal{D}(e^{\sigma}\phi_i)e^{-\int d^2\theta T_F^i \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$
- rescaling anomaly $\int \mathcal{D}V = \int \mathcal{D}(e^{\sigma}V)e^{+\int d^2\theta C_A \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$
- first rescale matter fields

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i \right) W_\alpha W^\alpha + Z_i^{-1/2} Z_j^{-1/2} Z_k^{-1/2} \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- then rescale the gauge field $V \to g_c V$

$$\frac{1}{g_c^2} = \frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i - C_A \frac{1}{8\pi^2} \ln g_c^2$$

$$\int d^2\theta \frac{1}{g_c^2} (\bar{D}^2 e^{-g_c V} D_\alpha e^{g_c V})^2 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots$$

The Minimal Supersymmetric Standard Model (MSSM)

supersymmetrize it

- All quarks and leptons are Weyl fermions
- chiral superfields have left-handed Weyl fermions
- Use charge conjugation to make them all left-handed: Q, L, u^c, d^c, e^c
- Promote them to chiral superfields, namely add their scalar partners
- Naming convention: add "s" as a prefix, which stands for supersymmetry or scalar

  terrible convention!
- e.g.: selectron, smuon, stop, sup, sstrange

supersymmetrize it

- All gauge fields are promoted to vector multiplets
- namely add massless Weyl=Majorana fermions “gauginos”
- Naming convention: add “ino” as a suffix, which doesn’t mean “small” in any sense

  terrible convention!
- e.g.: gluino, wino, photino, zino, bino

supersymmetrize it

- Minimal Standard Model has only one Higgs doublet
- It gives mass to both up- and down-type fields $\mathcal{L}_{Yukawa} = Y_u^{ij}\bar{Q}_i u_j \tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i e_j H, \quad \tilde{H} = i\sigma_2 H^*$
- Promote it to a chiral superfield, namely add a "Higgsino"
- But having only one higgsino makes the SU(2)xU(1) anomalous
- Also complex conjugation not allowed in a superpotential
- solution: introduce two Higgs doublets

  $H_u(1,2,+1/2)$, $H_d(1,2,-1/2)$

The MSSM

- $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory

	Q	d^c	u^c	L	e^c	H_u	H_d	g	B	W
$SU(3)_C$	3	3^*	3^*	1	1	1	1	8	1	1
$SU(2)_L$	2	1	1	2	1	2	1	1	1	3
$U(1)_Y$	+1/6	+1/3	−2/3	−1/2	−1	+1/2	−1/2	0	0	0
mltplt	χ	χ	χ	χ	χ	χ	χ	V	V	V
flavor	3	3	3	3	3	1	1	1	1	1
Z_2	−	−	−	−	−	+	+	+	+	+

The superpotential

- The terms we want

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

- The terms we don't want (violates B or L)

$$W_{\not{R}_p} = \lambda_{ijk} u_i^c d_j^c d_k^c + \lambda'_{ijk} Q_i d_j^c L_k + \lambda''_{ijk} L_i L_j e_k^c + \mu_i L_i H_u$$

- Impose Z_2 symmetry ("matter parity") that all matter chiral superfields are odd, Higgs even
- combined with 2π rotation of space $(-1)^{2S}$ (is equivalent to $\theta \to -\theta$), it gives R-parity

The Higgs potential

Without supersymmetry breaking effects, the superpotential

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

gives the potential for the Higgs field

$$V = \mu^2 (H_u^\dagger H_u + H_d^\dagger H_d) + \frac{g^2}{8} (H_u^\dagger \vec{\tau} H_u + H_d^\dagger \vec{\tau} H_d)^2 + \frac{g'^2}{8} (H_u^\dagger H_u - H_d^\dagger H_d)^2$$

which has only one ground state

$$\langle H_u \rangle = \langle H_d \rangle = 0$$

Namely the electroweak SU(2)xU(1) is unbroken unless supersymmetry is broken

Breaking Supersymmetry

Auxiliary fields

- SUSY is broken if the auxiliary component of a superfield has an expectation value
- F-term breaking: $\langle z \rangle = \theta^2 f$, $f \neq 0$
- D-term breaking: $\langle W_\alpha \rangle = \theta_\alpha d$, $d \neq 0$
- Irrespective of dynamics that breaks supersymmetry, its effect can be parameterized in terms of these order parameters (“spurion”)
- assume f and d dimension 1 for this purpose
- A spurion does not change the UV behavior of the theory, i.e. reintroduce quadratic divergences

Soft SUSY breaking

- Take $W = \lambda\phi^3 + \mu\phi^2 + m^2\phi$
- Using the spurion <z>=θ²f, we can write the most general SUSY breaking terms in Kähler

$$\int d^4\theta\,(\alpha\langle z\rangle\phi^*\phi + h.c. + \beta\langle z^*z\rangle\phi^*\phi) = (\alpha f\phi^*F + \alpha^* f^*\phi F^*) + \beta f^* f\phi^*\phi$$

- Solving for the auxiliary component

$$F^*F + (W'F + \alpha f\phi^*F + h.c.) = -|W' + \alpha f\phi^*|^2$$
$$= -|W'|^2 - (\alpha f\phi W' + h.c.) - |\alpha f|^2\phi^*\phi$$
$$= -|W'|^2 - \alpha f(3\lambda\phi^2 + 2\mu\phi^2 + m^2\phi + h.c.) - |\alpha f|^2\phi^*\phi$$

- From the superpotential

$$\int d^2\theta\langle z\rangle(a\lambda\phi^3 + b\mu\phi^2 + cm^2\phi) = af\lambda\phi^3 + bf\mu\phi^2 + cfm^2\phi$$

- bottomline: any terms of type

$$-(A\lambda\phi^3 + B\mu\phi^2 + Cm^2\phi + h.c.) - m_\phi^2\phi^*\phi$$

Soft SUSY breaking

- Similarly for the gauge multiplet,

$$\int d^2\theta \langle z \rangle W_\alpha W^\alpha = f\lambda\lambda$$

namely gaugino masses

- If there is a chiral superfield in the adjoint rept, another possible term with D-term spurion is $$\int d^2\theta \phi W_\alpha \langle W^\alpha \rangle = d\psi\lambda$$

- Now the complete set of soft SUSY breaking

$$A\lambda\phi^3,\ B\mu\phi^2,\ Cm^2\phi,\ m_\phi^2\phi^*\phi,\ m_\lambda\lambda\lambda,\ m_d\lambda\psi$$

Sofly broken MSSM

Soft SUSY breaking terms in the MSSM

- For each term in the superpotential

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

- we can have the "A-terms" and "B-term"

$$A_u^{ij} Y_u^{ij} Q_i u_j^c H_u + A_d^{ij} Y_d^{ij} Q_i d_j^c H_d + A_l^{ij} Y_l^{ij} L_i e_j^c H_d + B\mu H_u H_d$$

- scalar masses for all scalars

$$m_{Qij}^2 \tilde{Q}_i^* \tilde{Q}_j + m_{uij}^2 \tilde{u}_i^* \tilde{u}_j + m_{dij}^2 \tilde{d}_i^* \tilde{d}_j + m_{Lij}^2 \tilde{L}_i^* \tilde{L}_j + m_{eij}^2 \tilde{e}_i^* \tilde{e}_j + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2$$

- gaugino mass for all three gauge factors

$$M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}^a \tilde{W}^a + M_3 \tilde{g}^a \tilde{g}^a$$

- A(18x3)+B(2)+m(9x5+2)+M(2x3)+μ(2)=111

  $U(1)_R \times U(1)_{PQ}$ removes only two phases

  cf. SM has two params in the Higgs sector

  107 more parameters than the SM!

Higgs potential

The Higgs potential in the MSSM is

$$V = m_1^2 H_d^\dagger H_d + m_2^2 H_u^\dagger H_u - m_3^2 (H_u H_d + c.c.) + \frac{g^2}{8}(H_d^\dagger \vec{\tau} H_d + H_u^\dagger \vec{\tau} H_u)^2 + \frac{g'^2}{8}(H_d^\dagger H_d - H_u^\dagger H_u)^2$$

where

$$m_1^2 = m_{H_d}^2 + \mu^2, \quad m_2^2 = m_{H_u}^2 + \mu^2, \quad m_3^2 = B\mu$$

Leaving on the neutral components

$$V = (H_d^{0*}\ H_u^{0*}) \begin{pmatrix} m_1^2 & m_3^2 \\ m_3^2 & m_2^2 \end{pmatrix} \begin{pmatrix} H_d^0 \\ H_u^0 \end{pmatrix} + \frac{g^2 + g'^2}{8}(|H_d^0|^2 - |H_u^0|^2)^2$$

breaks SU(2)xU(1) if $m_1^2 m_2^2 < m_3^4$

stable along the D-flat direction $H_d^0 = H_u^0$ if $m_1^2 + m_2^2 > 2m_3^2$

i.e., $m_1^2 m_2^2 < m_3^4 < (m_1^2 + m_2^2)/2$ which is possible

Higgs particles

$$V = (H_d^{0*}\; H_u^{0*}) \begin{pmatrix} m_1^2 & m_3^2 \\ m_3^2 & m_2^2 \end{pmatrix} \begin{pmatrix} H_d^0 \\ H_u^0 \end{pmatrix} + \frac{g^2 + g'^2}{8}(|H_d^0|^2 - |H_u^0|^2)^2$$

- Two Higgs doublets = 8 Klein-Gordon fields
- 3 of them eaten by the Higgs mechanism
- 8-3=5 physical degrees of freedom

  h^0, H^0, A^0, H^+, H^-
- 3 parameters m_1^2, m_2^2, m_3^2
- one of them fixed by $\langle H_d \rangle^2 + \langle H_u \rangle^2 = (174\text{GeV})^2$
- two remaining parameters

  $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$ and m_A

Higgs particles

$$V = (H_d^{0*}\ H_u^{0*}) \begin{pmatrix} m_1^2 & m_3^2 \\ m_3^2 & m_2^2 \end{pmatrix} \begin{pmatrix} H_d^0 \\ H_u^0 \end{pmatrix} + \frac{g^2 + g'^2}{8}(|H_d^0|^2 - |H_u^0|^2)^2$$

- Solve for mass eigenvalues of physical states

$$m_{A^0}^2 = 2m_3^2/\sin 2\beta, \qquad m_{H^\pm}^2 = m_A^2 + m_W^2$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2 m_Z^2 \cos^2 2\beta}\right), \qquad m_h \leq m_Z!$$

- Lightest Higgs below m_Z: excluded!
- However, the quartic coupling evolves differently from the gauge coupling below the scalar top threshold

$$\Delta m_{h^0}^2 \simeq \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\ln\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$$

- Need stop > 500GeV to evade limits

gauginos, higgsinos

- charged ones “charginos”

$$(\tilde{W}^- \; \tilde{H}_d^-) \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix}$$

- neutral ones “neutralinos”

$$(\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0) \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}^0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix}$$

Sfermions

- e.g., stop

$$(\tilde{t}_L^* \; \tilde{t}_R^*) \begin{pmatrix} m_{\tilde{Q}_3}^2 + m_t^2 & (A_t - \mu^* \cot\beta) m_t \\ (A_t^* - \mu \cot\beta) m_t & m_{\tilde{t}}^2 + m_t^2 \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

one-loop RGE

- GUT prediction of gaugino masses

$$\frac{d}{dt}\frac{M_i}{g_i^2} = 0$$

$$M_1 : M_2 : M_3 \approx 1:2:7 \text{ at } m_Z$$

- gauge interaction boosts scalar masses

$$\frac{d}{dt}m^2 = -\frac{1}{16\pi^2}8C_F g^2 M^2$$

- Yukawa interaction suppresses scalar masses

$$16\pi^2\frac{d}{dt}m_{H_u}^2 = 3X_t - 6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2$$

$$16\pi^2\frac{d}{dt}m_{\tilde{t}_R}^2 = 2X_t - \frac{32}{3}g_3^2M_3^2 - \frac{32}{15}g_1^2M_1^2$$

$$16\pi^2\frac{d}{dt}m_{\tilde{t}_L}^2 = X_t - \frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - \frac{2}{15}g_1^2M_1^2$$

$$X_t = 2Y_t^2(m_{H_u}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2 + A_t^2)$$

- H_u mass-squared most likely to get negative!

RGE

- running of soft SUSY breaking parameters can be inferred from the RGE of coupling constants

$$Z_i \to \mathcal{Z}_i = Z_i(1-\theta^2 A_i)(1-\bar{\theta}^2 A_i^*)(1-\theta^2\bar{\theta}^2 m_i^2)$$

$$\frac{1}{g^2} \to \mathcal{S} = \frac{1}{g^2}\left(1-\theta^2 m_\lambda\right)$$

$$Z_i(t) = Z_i(0) - \frac{g^2 C_F^i}{4\pi^2} t + \sum_{j,k} \frac{\lambda_{ijk}^* \lambda_{ijk}}{16\pi^2} t$$

$$\mathcal{Z}_i(t) = \mathcal{Z}_i(0) - \frac{C_F^i}{4\pi^2} \frac{2}{\mathcal{S}+\bar{\mathcal{S}}} t + \sum_{j,k} \frac{\lambda_{ijk}^{0\,*} \lambda_{ijk}^0}{16\pi^2} \mathcal{Z}_i^{-1} \mathcal{Z}_j^{-1} \mathcal{Z}_k^{-1} t$$