Supersymmetry III

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PiTP 05, IAS

Plan

Mon: Non-technical Overview

what SUSY is supposed to give us

Tue: From formalism to the MSSM

Global SUSY formalism, Feynman rules, soft SUSY breaking, MSSM

Wed: SUSY breaking

how to break SUSY, mediation mechanisms

Thu: SUSY at colliders

basic reactions, signatures, and how do we know it is SUSY?

Fri: SUSY as a telescope

supersymmetry breaking, GUT, string

Breaking SUSY

Tree-level SUSY breaking

- O'Raifeartaigh model $W = \lambda X(Z^2 - v^2) + mYZ$

$$F_X^* = \frac{\partial W}{\partial X} = \lambda(Z^2 - v^2) = 0$$
$$F_Y^* = \frac{\partial W}{\partial Y} = mZ = 0$$

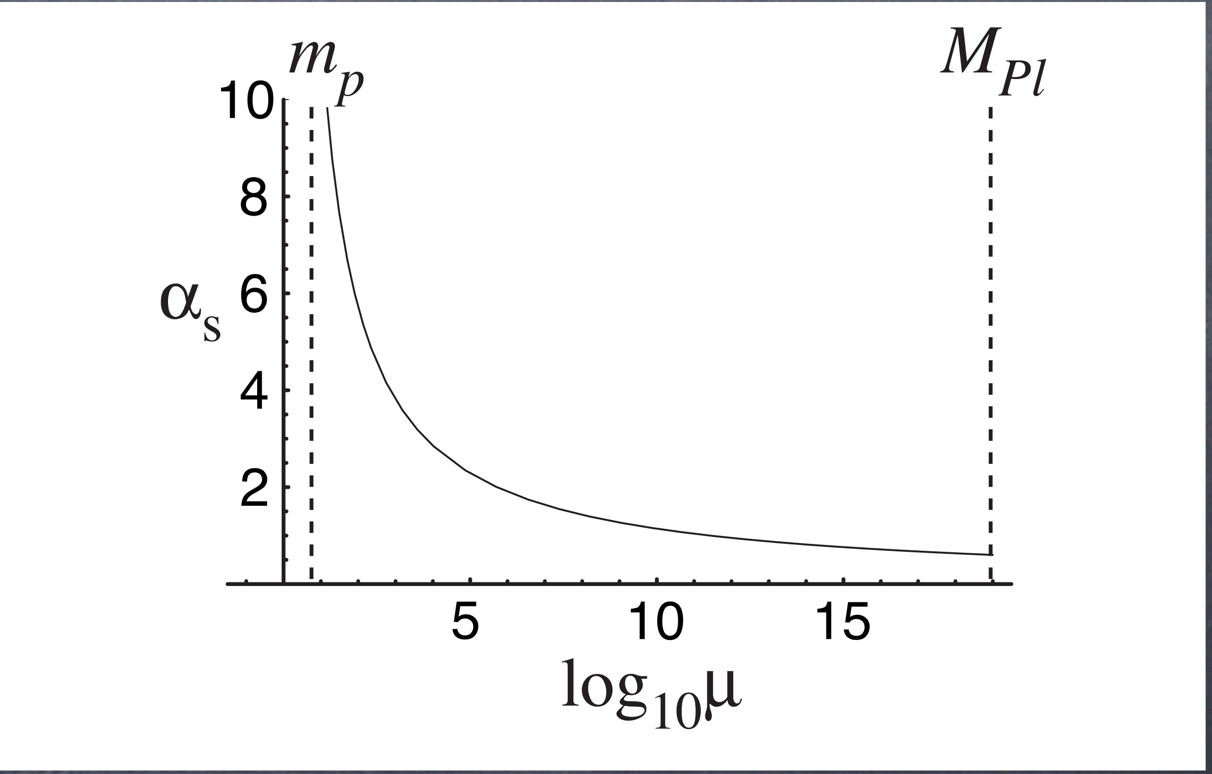
- Cannot be satisfied simultaneously
- Ground state at X=Y=Z=0
- $V=|F_X|^2=\lambda^2 v^4 \neq 0$
- ψ_Z: m^2
- A_Z: $m^2 \pm \lambda v^2$
- SUSY indeed broken
- However, the hierarchy $v \ll M_{Pl}$ put in by hand

Dynamical SUSY Breaking

- Nobody is worried why $m_p \ll M_{Pl}$
- If SUSY is broken also by strong gauge dynamics, hierarchy naturally understood
- If not broken at the tree-level, not broken at all orders in perturbation theory

broken non-perturbatively

$$m_p \approx M_{Pl} e^{-8\pi^2/g_s^2(M_{Pl})b_0}$$



Dynamical SUSY Breaking

- By now, quite a few models known that break SUSY dynamically
  - SO(10) with single 16
  - SU(5) with 10+5*
  - SU(3)xSU(2) with Q, u, d, L and W=QdL
  - SU(2) with 4 Q's and 6 singlets $W=S_{ij}Q_iQ_j$
- SUSY is broken with $V \approx \Lambda^4$

Cosmological constant?

- Once SUSY is broken, there is a large vacuum energy $V \approx \Lambda^4$
- supergravity allows fine-tuning of the cosmological constant
- massless goldstino eaten by gravitino
- Global SUSY: $V = \Sigma_i |\partial_i W|^2 \approx \Lambda^4$
- supergravity: $V = e^K(|D_i W|^2 - 3|W|^2/M_{Pl}{}^2)$
- can choose a constant term in the superpotential to cancel the vacuum energy
- gravitino mass $m_{3/2} = e^{K/2}|W| \approx \Lambda^2/M_{Pl}$

N=1 Supergravity on a slide

- start with conformal supergravity ($g_{\mu\nu}$, ψ^μ, b_μ, A_μ)
- remove unwanted components by integrating out Weyl compensator chiral superfield S

$$\int d^4\theta S\bar{S}(-3M_{Pl}^2 + \phi^*\phi + \cdots) + \int d^2\theta \left(S^3 W + f(\phi) W_\alpha W^\alpha\right)$$

- Weyl scale $S \to S/W^{1/3}$

$$\int d^4\theta S\bar{S}\frac{-e^{-3K}}{|W|^{2/3}} + \int d^2\theta \left(S^3 + f(\phi) W_\alpha W^\alpha\right)$$

- depends only on $G=K+\ln|W|^2$ $\quad K = -\frac{1}{3}\ln(3M_{Pl}^2 - \phi^*\phi - \cdots)$

$$V = e^G(G_i(G^i_j)^{-1}G^j - 3) = e^K\left(F_i^*(K^i_j)^{-1}F^j - 3|W|^2\right)$$

$$F_i = W_i + K_i W$$

- $\langle S\rangle = 1 + \theta^2\langle W\rangle$, $m_{3/2} = e^{K/2}|W|$

Phenomenological requirements on ~~SUSY~~

Soft SUSY breaking terms in the MSSM

- For each term in the superpotential

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

- we can have the "A-terms" and "B-term"

$$A_u^{ij} Y_u^{ij} Q_i u_j^c H_u + A_d^{ij} Y_d^{ij} Q_i d_j^c H_d + A_l^{ij} Y_l^{ij} L_i e_j^c H_d + B\mu H_u H_d$$

- scalar masses for all scalars

$$m_{Qij}^2 \tilde{Q}_i^* \tilde{Q}_j + m_{uij}^2 \tilde{u}_i^* \tilde{u}_j + m_{dij}^2 \tilde{d}_i^* \tilde{d}_j + m_{Lij}^2 \tilde{L}_i^* \tilde{L}_j + m_{eij}^2 \tilde{e}_i^* \tilde{e}_j + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2$$

- gaugino mass for all three gauge factors

$$M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}^a \tilde{W}^a + M_3 \tilde{g}^a \tilde{g}^a$$

- A(18x3)+B(2)+m(9x5+2)+M(2x3)+μ(2)=111

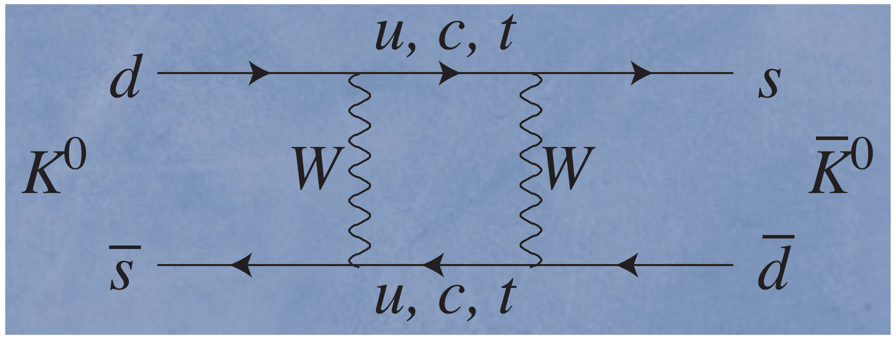
  $U(1)_R \times U(1)_{PQ}$ removes only two phases

  cf. SM has two params in the Higgs sector

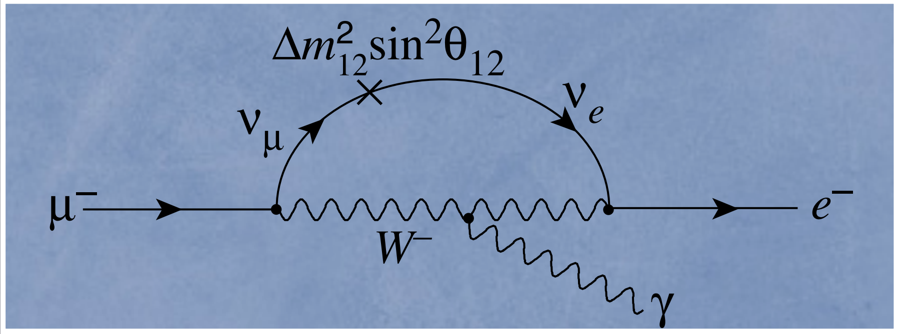
  107 more parameters than the SM!

Flavor-Changing Neutral Current

- There is no tree-level vertex such as $\bar{s}\gamma^{\mu} d Z_{\mu}$
- In the Standard Model, FCNC is highly suppressed
- e.g.,



$$\sim \frac{1}{16\pi^2} G_F^2 m_c^2 (V_{cd}^* V_{cs})^2$$



$$\sim \frac{e}{16\pi^2} G_F^2 m_{\mu} \Delta m_{12}^2 \sin^2\theta_{12}$$

SUSY flavor violation

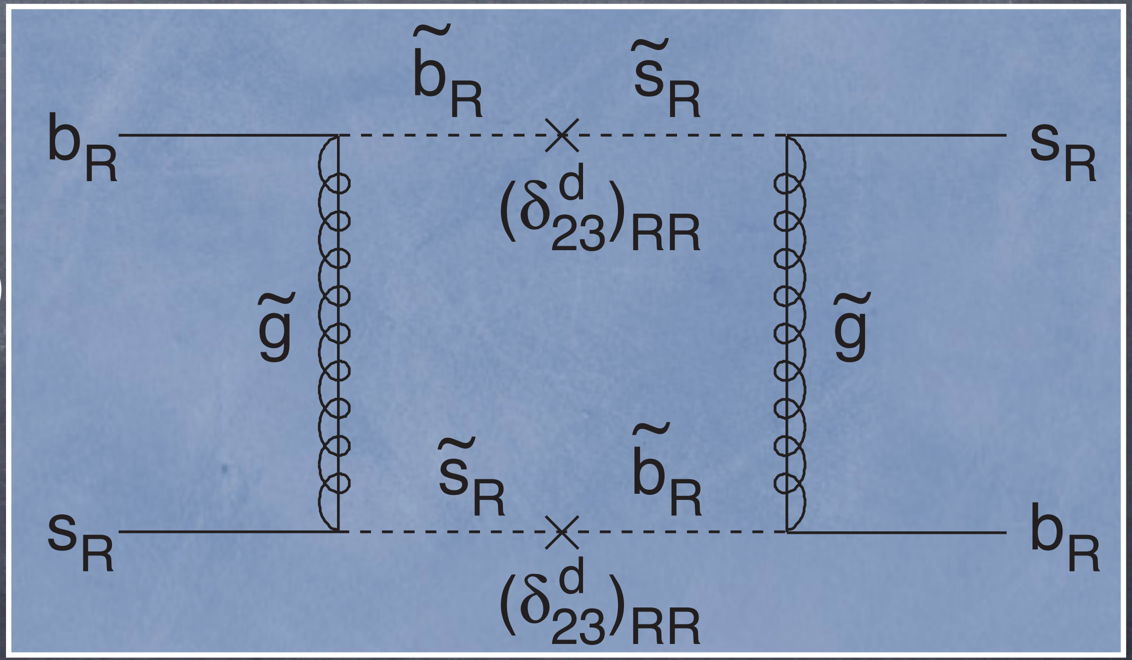
- soft SUSY breaking parameters can violate flavor

$$(\tilde{d},\, \tilde{s},\, \tilde{b}) \begin{pmatrix} m_{11}^2 m_{12}^2 m_{13}^2 \\ m_{21}^2 m_{22}^2 m_{23}^2 \\ m_{31}^2 m_{32}^2 m_{33}^2 \end{pmatrix} \begin{pmatrix} \tilde{d} \\ \tilde{s} \\ \tilde{b} \end{pmatrix}$$

$$(\delta_{12}^d)_{RR} \equiv \frac{m_{12}^2}{m_{11} m_{22}} < 0.04 \frac{m_{SUSY}}{500\text{GeV}}$$

$$\sqrt{(\delta_{12}^d)_{RR} (\delta_{12}^d)_{LL}} < 0.001 \frac{m_{SUSY}}{500\text{GeV}}$$

K^0



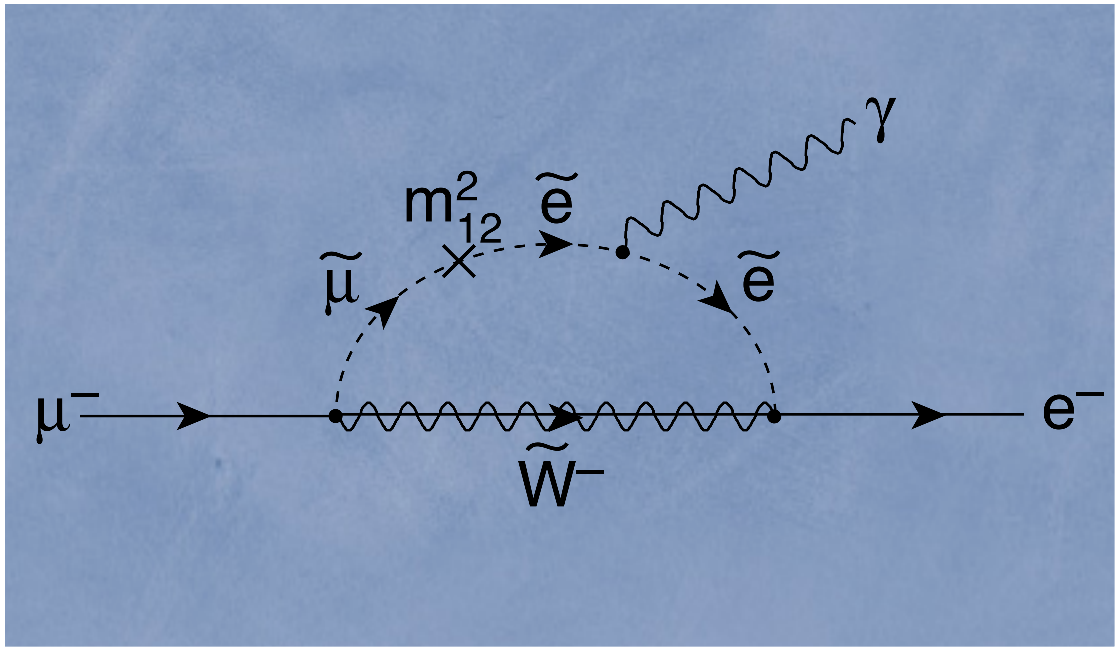
$\overline{K}^0$

SUSY flavor violation

- soft SUSY breaking parameters can violate flavor

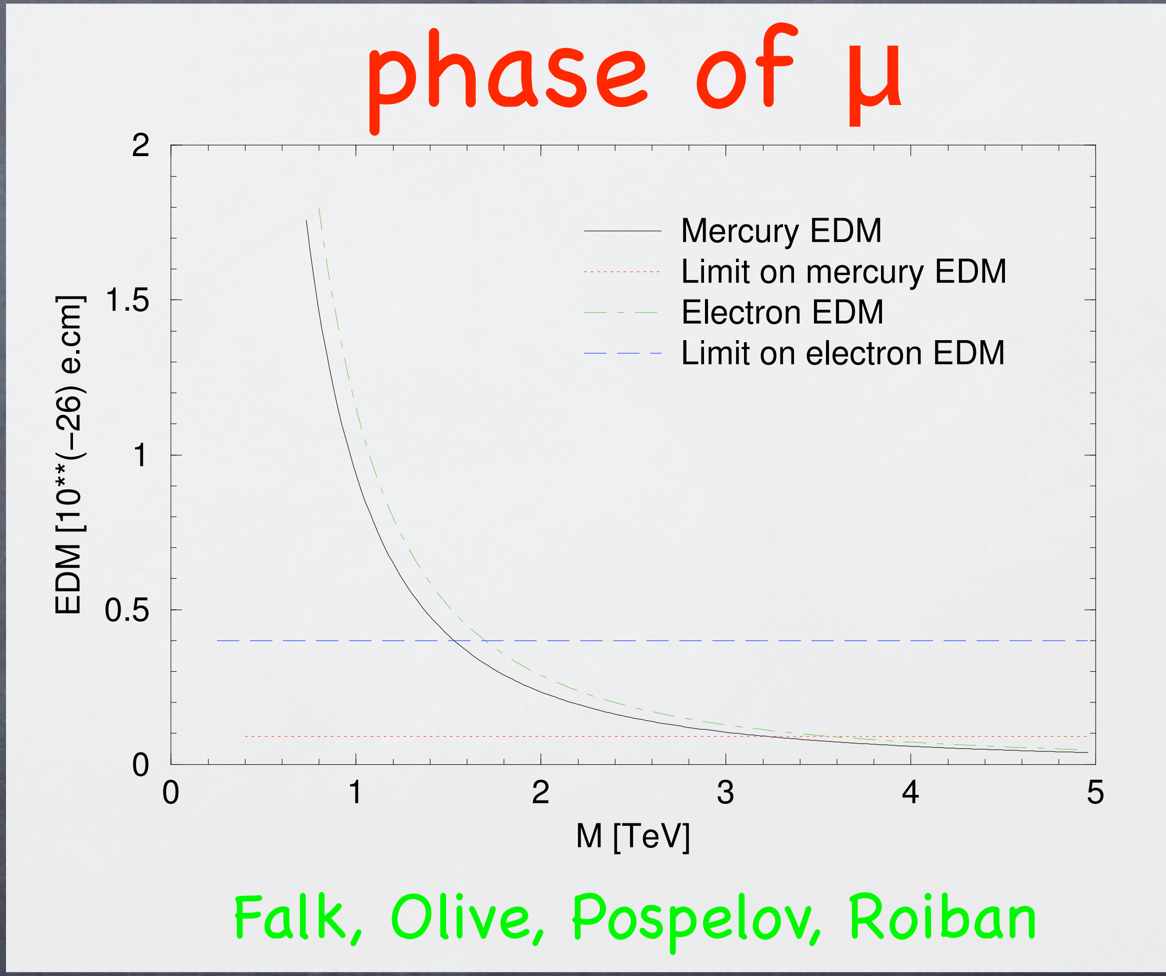
$$(\tilde{e}, \tilde{\mu}, \tilde{\tau}) \begin{pmatrix} m^2_{11} m^2_{12} m^2_{13} \\ m^2_{21} m^2_{22} m^2_{23} \\ m^2_{31} m^2_{32} m^2_{33} \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \tilde{\mu} \\ \tilde{\tau} \end{pmatrix}$$

$$(\delta^l_{12})_{RR} < 3.9 \times 10^{-3}$$



Supersymmetric CP problem

- The relative phases of μ and $M_{1,2,3}$ are physical
- induces electric dipole moments $H \propto \vec{s} \cdot \vec{E}$
- stringent limits on electron, neutron, and Hg atom
- either m_{SUSY}>TeV or phase~10^{-2}



Common simplifying assumptions

- soft SUSY breaking parameters all real
- "flavor-blind", namely, 3x3 sclar mass-squared matrices: $m_f^2 \propto I$
- gaugino masses unify: $M_1=M_2=M_3$ at M_{GUT}

Minimal SUGRA

(Hall, Lykken, Weinberg)

- Often, this problem is "solved" by assuming a very special Lagrangian called "minimal supergravity" $\int d^4\theta(-3M_{Pl}^2)\exp\left(\frac{-1}{3M_{Pl}^2}(\phi_i^*\phi^i + z_i^* z^i)\right)$
- Gives universal scalar mass: flavor-blind
- No theoretical justification for this very particular choice
- Just a convenient choice to obtain the minimal kinetic term with no Planck-suppressed corrections
- Not stable under renormalization

“minimal supergravity”

- At the GUT-scale 2×10^{16} GeV
- assume all scalar masses are equal m_0^2
- assume all gaugino massses are equal $M_{1/2}$
- assume all trilinear couplings are equal A_0
- in addition, B, $B\mu$
- calculate all SUSY breaking terms via RGE down from the GUT-scale
- fix m_Z: leaves four parameters (and sign(μ))

one-loop RGE

- GUT prediction of gaugino masses

$$\frac{d}{dt}\frac{M_i}{g_i^2} = 0$$

$$M_1 : M_2 : M_3 \approx 1 : 2 : 7 \text{ at } m_Z$$

- gauge interaction boosts scalar masses

$$\frac{d}{dt}m^2 = -\frac{1}{16\pi^2} 8 C_F g^2 M^2$$

- Yukawa interaction suppresses scalar masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2$$

$$16\pi^2 \frac{d}{dt} m_{\tilde{t}_R}^2 = 2X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2$$

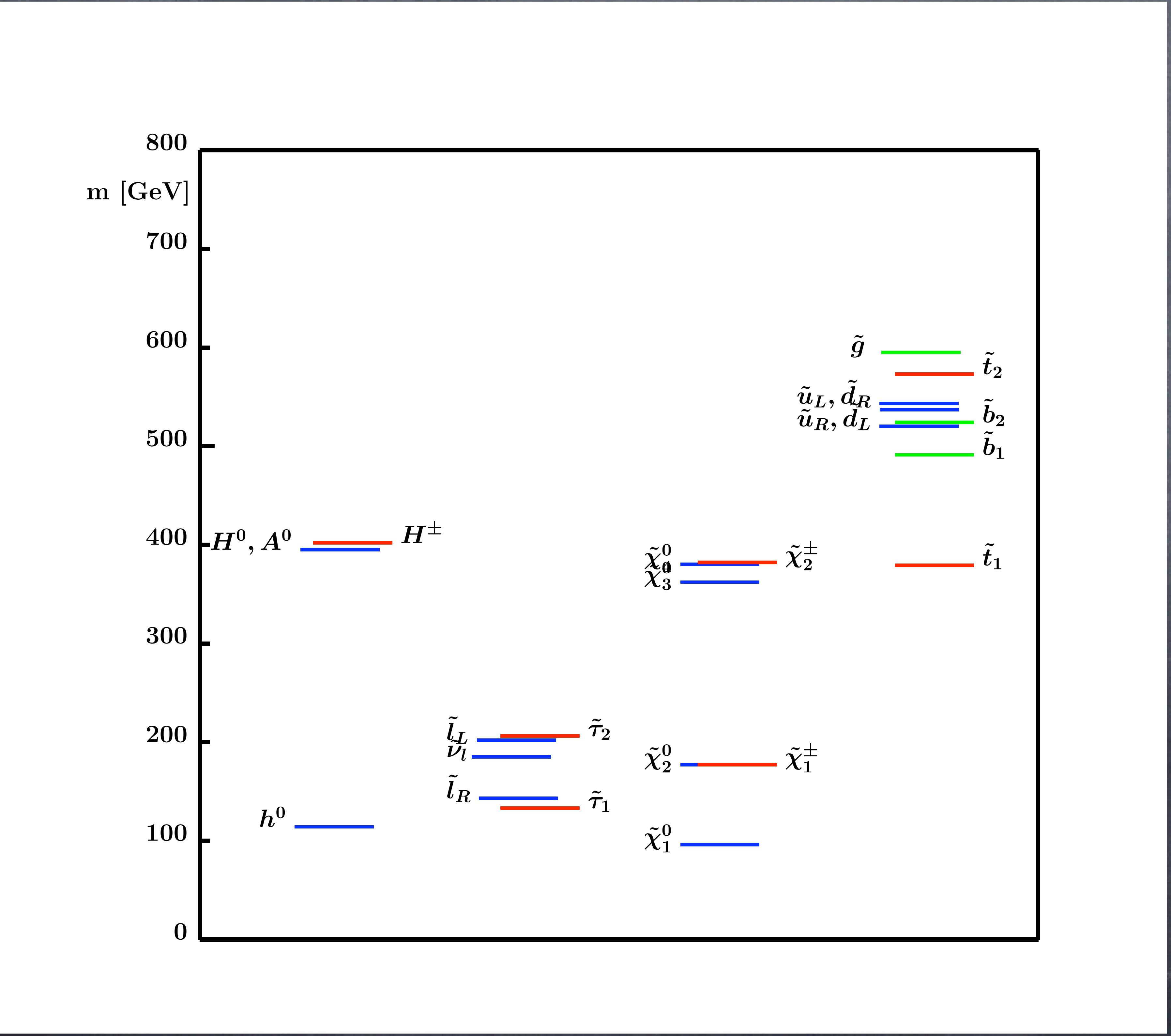
$$16\pi^2 \frac{d}{dt} m_{\tilde{t}_L}^2 = X_t - \frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2$$

$$X_t = 2 Y_t^2 (m_{H_u}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2 + A_t^2)$$

- H_u mass-squared most likely to get negative!

sample spectrum

$m_0 = 100,\ m_{1/2} = 250,\ A_0 = -100,\ \tan\beta = 10,\ \mu > 0$

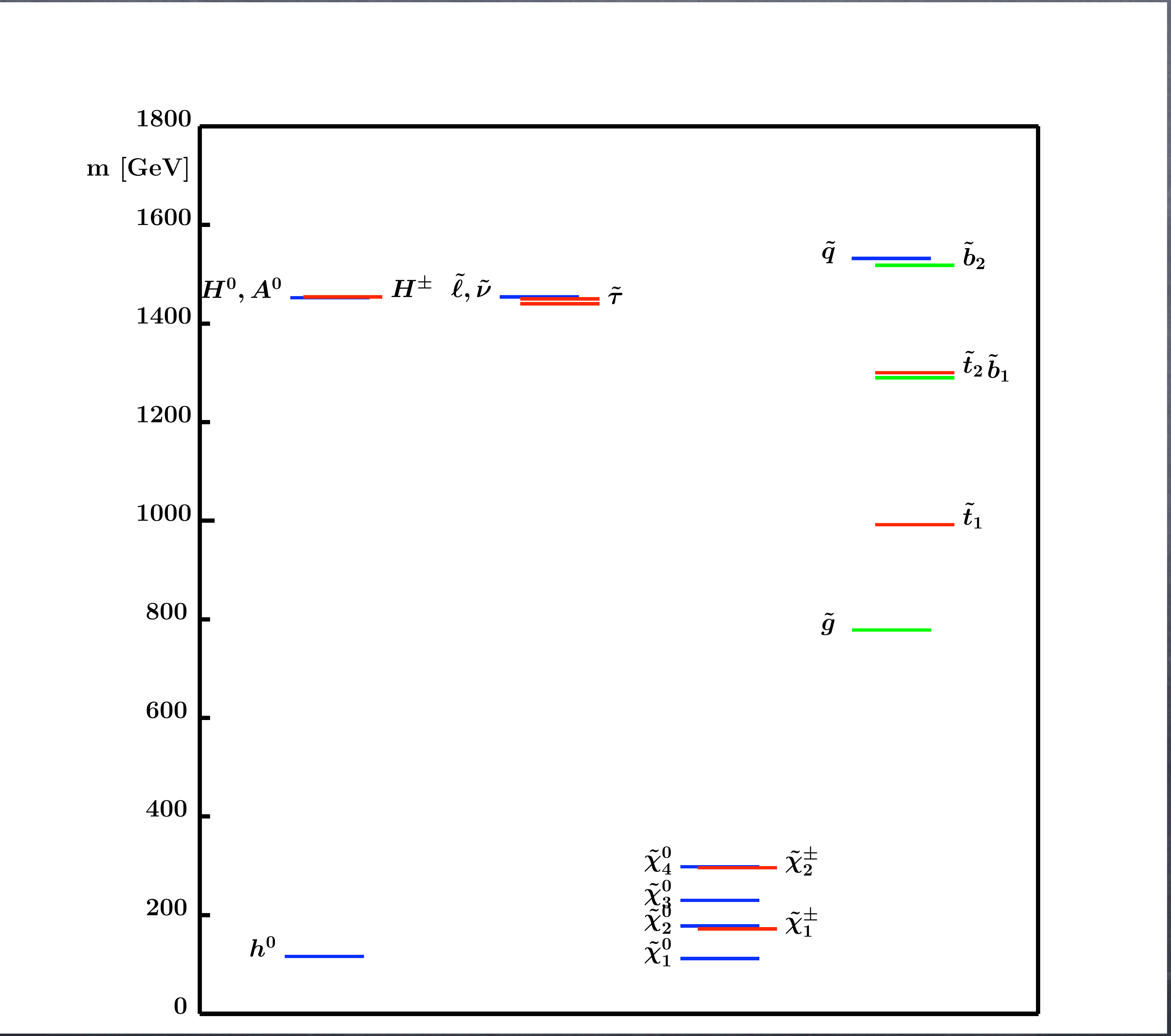


bulk region

SPS1a

sample spectrum

$m_0 = 1450,\ m_{1/2} = 300,\ A_0 = 0,\ \tan\beta = 10,\ \mu > 0$

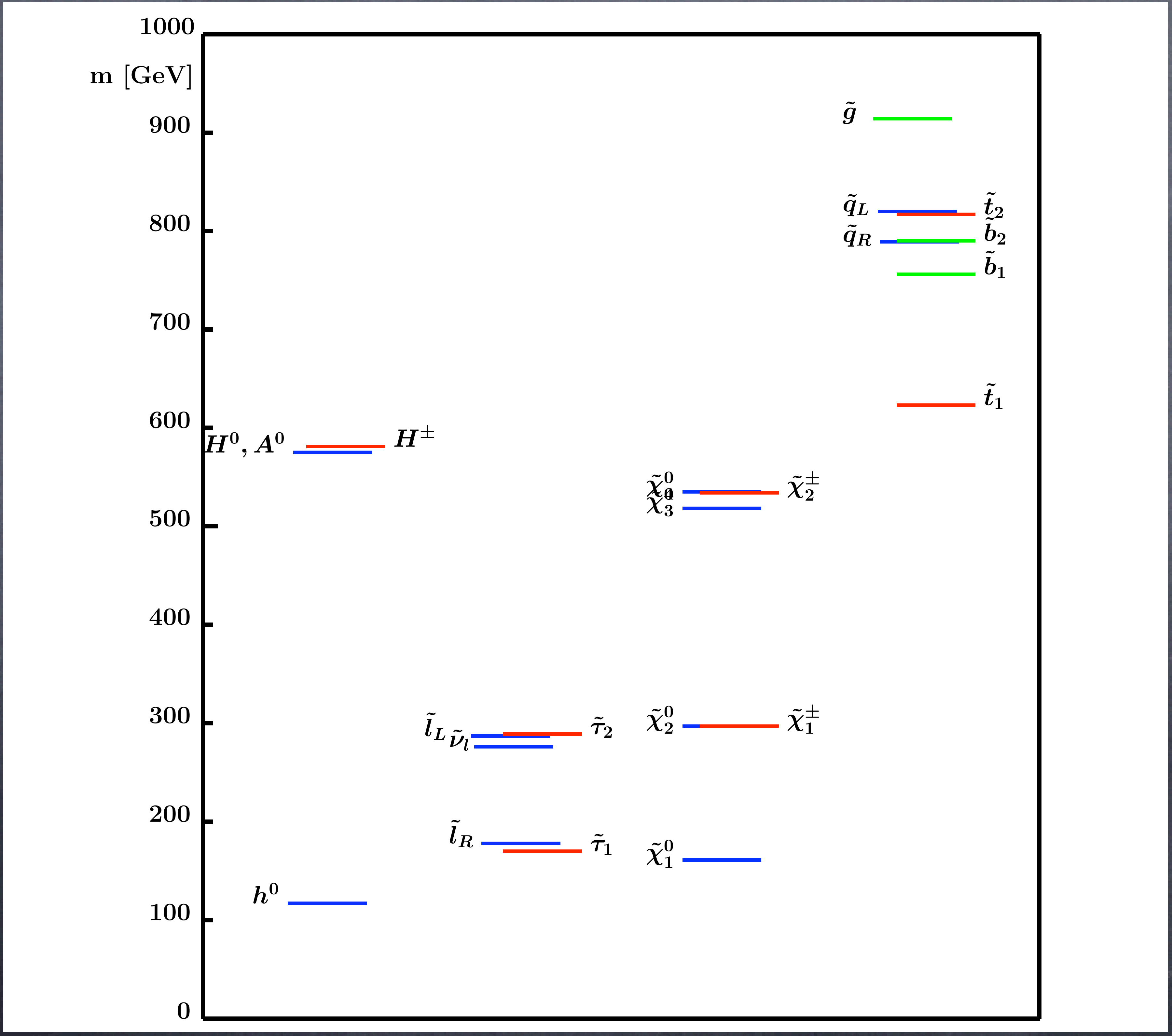


focus point region

SPS2

sample spectrum

$m_0 = 90,\ m_{1/2} = 400,\ A_0 = 0,\ \tan\beta = 10,\ \mu > 0$



coanni-
hilation
region

SPS3

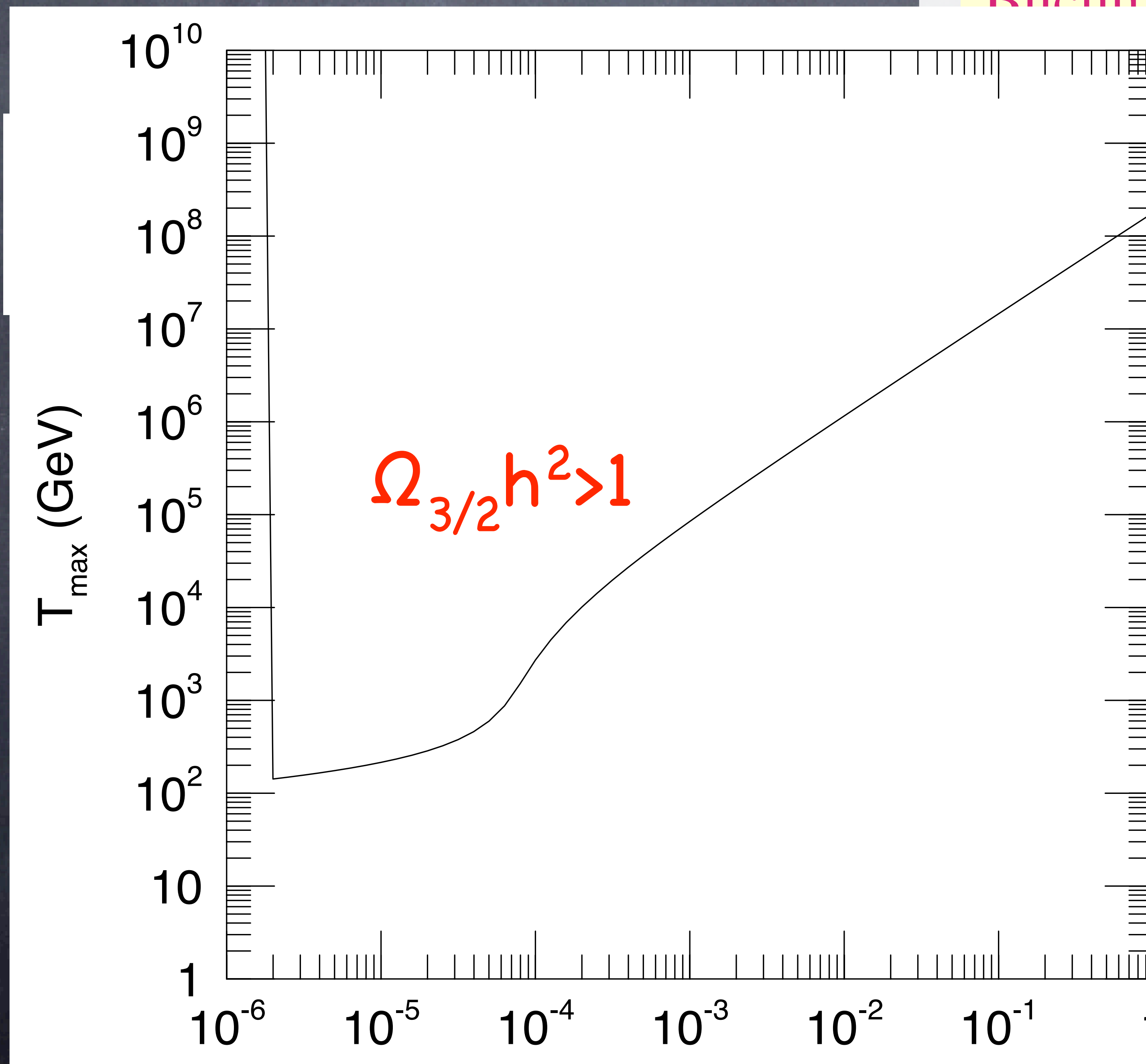
“Gravity” Mediation

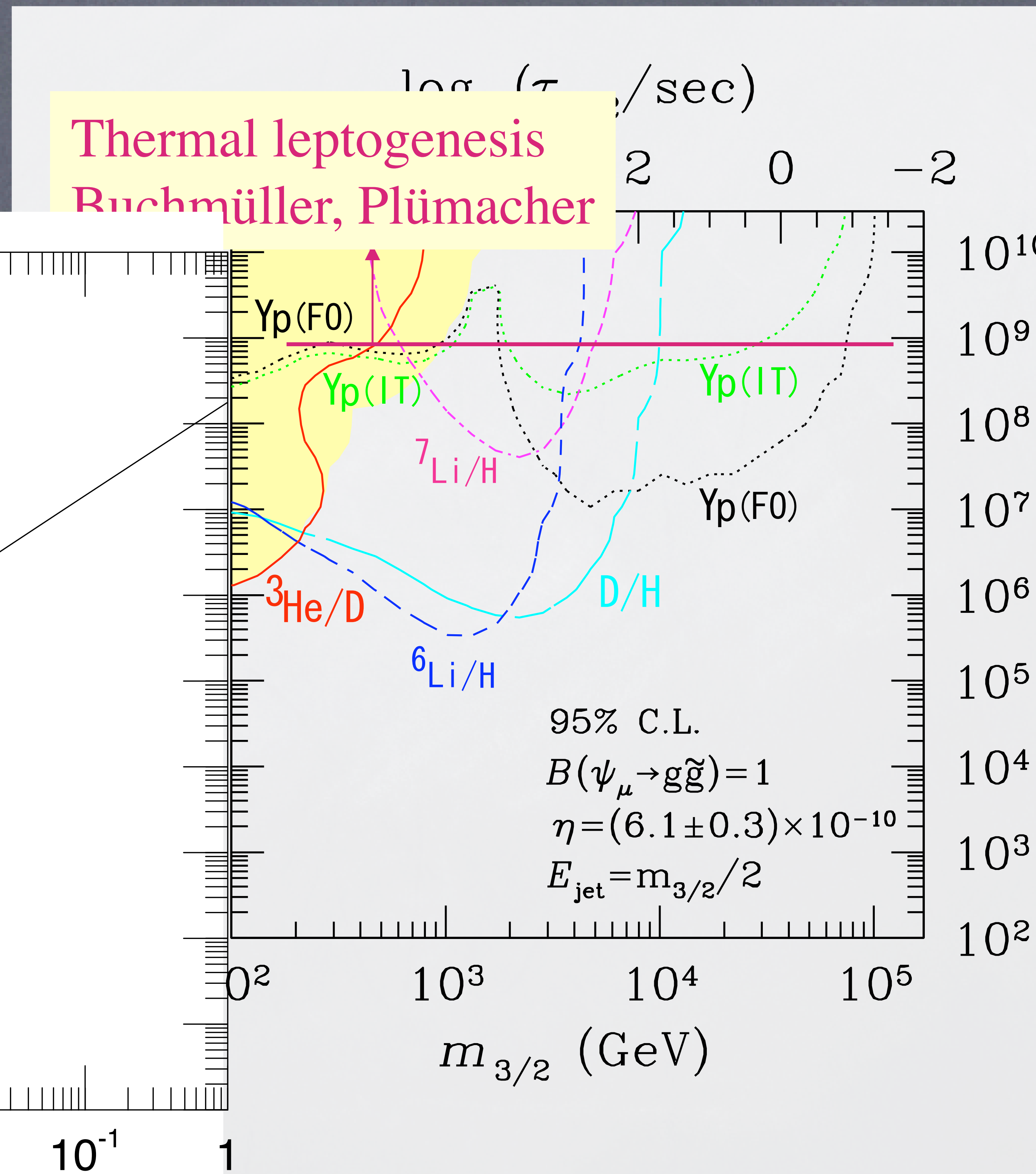
- People argued that the mediation of SUSY breaking by gravity is universal because the gravity couples universally
- But it is easy to see this is a big lie
- The minute you talk about gravity, we have a theory cutoff at the Planck-scale, and we can write arbitrary operators suppressed by the Planck scale w/o the knowledge of the fully consistent theory of quantum gravity

$$\int d^4\theta \lambda_{ij} \frac{z^* z}{M_{Pl}^2} \phi_i^* \phi_j \rightarrow m_{ij}^2 = \lambda_{ij} \left| \frac{F_z}{M_{Pl}} \right|^2 \qquad \int d^2\theta \lambda_i \frac{z}{M_{Pl}} W_\alpha^i W^{\alpha i} \rightarrow M_i = \lambda_i \frac{F_z}{M_{Pl}}$$

Gravitino Problem

- Gravitinos produced in





Moduli problem

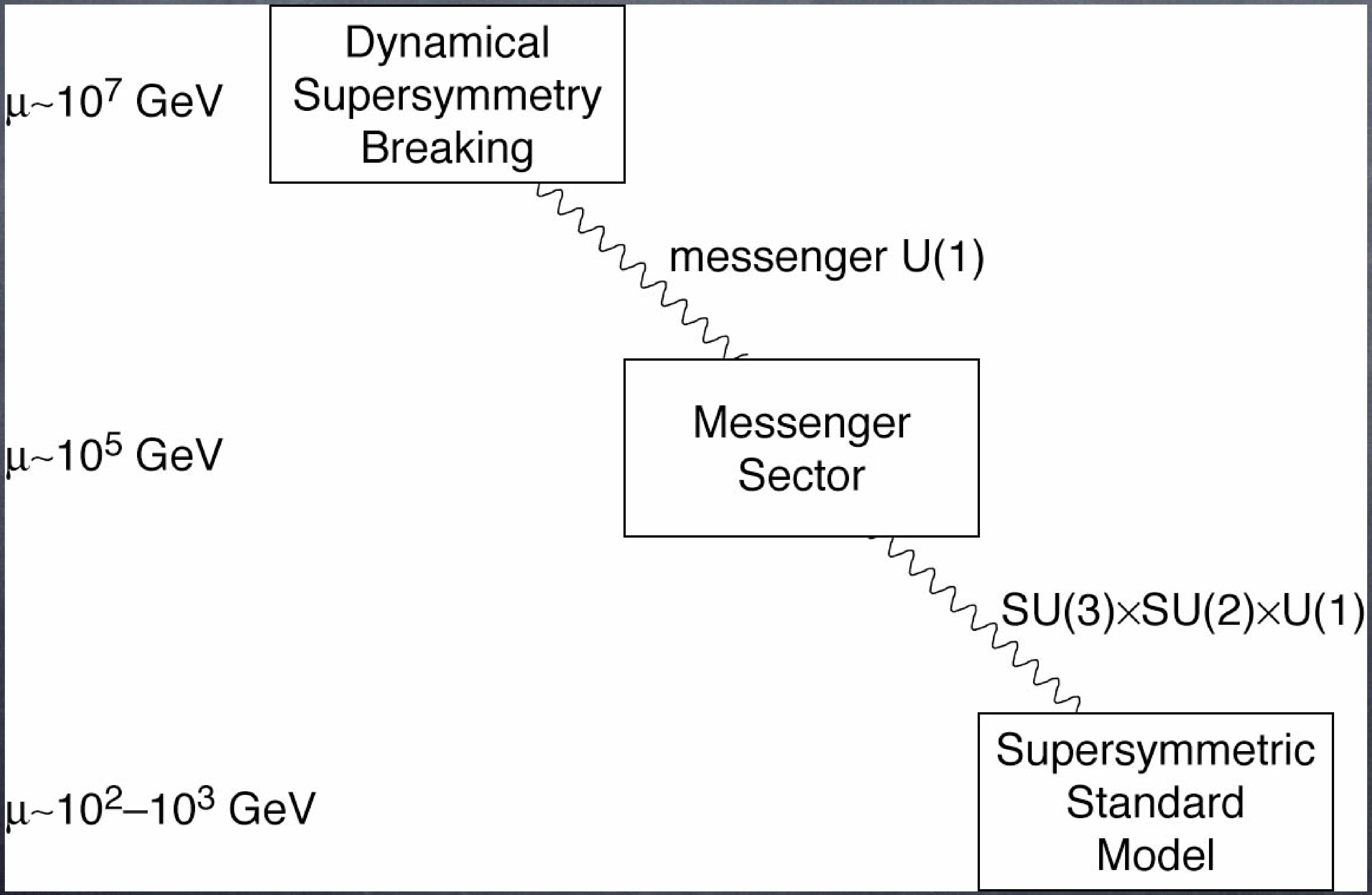
- In string theory, we need to compactify 6 (or 7) extra dimensions into a small size
- moduli fields parameterize the size and shape of the compactified space (⇒flux)
- they do not have any potential in the supersymmetric limit
- their mass is $O(m_{3/2})$, very flat potential
- in early universe, they had $O(M_{Pl})$ amplitudes
- oscillate around the minimum, dominate
- when it decays, dilutes entropy by $\sim m_{3/2}/M_{Pl}$
- If $m_{3/2}\sim$TeV, baryon asymmetry diluted by 10^{-15}!

Issue of mediation

- Many gauge theories that break SUSY dynamically known
- The main issue: how do we communicate the SUSY breaking effects to the MSSM? "mediation"
- If the mediation mechanism is *flavor-blind*, there is no problem with FCNC
  - Gauge mediation (direct & indirect)
  - Gaugino mediation
  - Anomaly mediation

Flavor-blind
Mediation Mechanisms

Gauge Mediation (GMSB)



Gauge Mediation (GMSB)

- Integrate out "messenger fields" N(5+5*) (i.e, d^c+L)

$$W = S f \bar{f}$$

$$\langle S \rangle = \langle A_S + \theta^2 F_S \rangle \neq 0$$

- integrate them out: changes the running of gauge coupling, wave function renormalizations

$$\frac{1}{g^2(\mu)} = \frac{1}{g_0^2} + \frac{b_0 + N}{8\pi^2} \ln \frac{\Lambda_{UV}}{S} + \frac{b_0}{8\pi^2} \ln \frac{S}{\mu}$$

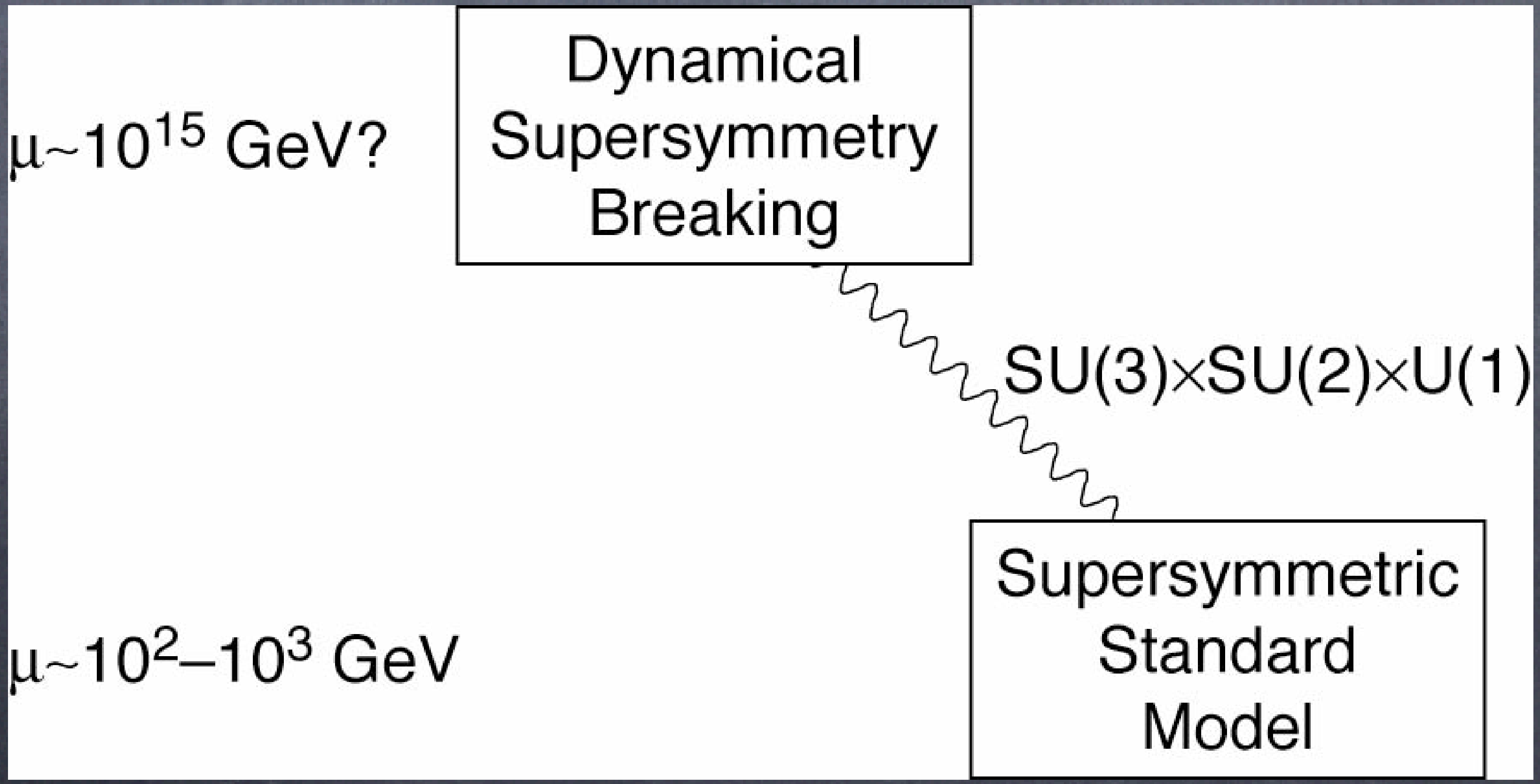
$$\frac{M}{g^2} = \frac{1}{g^2(\mu)}\bigg|_{\theta^2} = \frac{1}{8\pi^2} N \frac{F_S}{A_S}$$

$$Z_i(\mu) = Z_i(\Lambda_{UV}) \left(\frac{g^2(\Lambda_{UV})}{g^2(\sqrt{S^\dagger S})} \right)^{2C_F/b'} \left(\frac{g^2(\sqrt{S^\dagger S})}{g^2(\mu)} \right)^{2C_F/b}$$

$$m_i^2(\mu) = -\ln Z_i(\mu)|_{\theta^2\bar{\theta}^2} = 2C_F \frac{g^4}{(4\pi)^4} N \left(\frac{F_S}{A_S} \right)^2$$

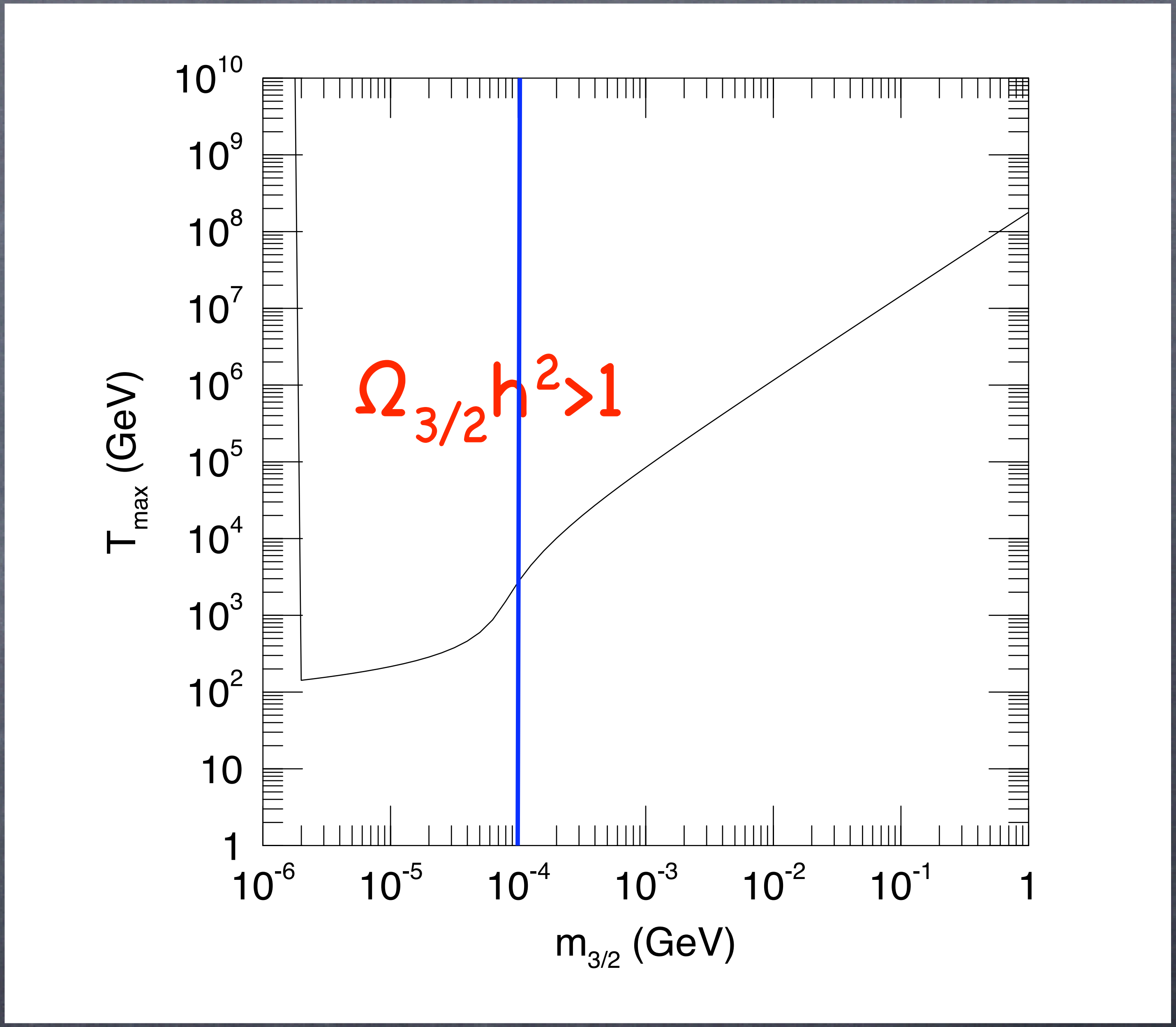
Direct Gauge Mediation

- Too many sectors to worry about!
- e.g., SU(2)xSU(2) with Σ(2,2), Q(2,1)x6, Q'(1,2)x6, embed 3x2x1 into 6 (Agashe)



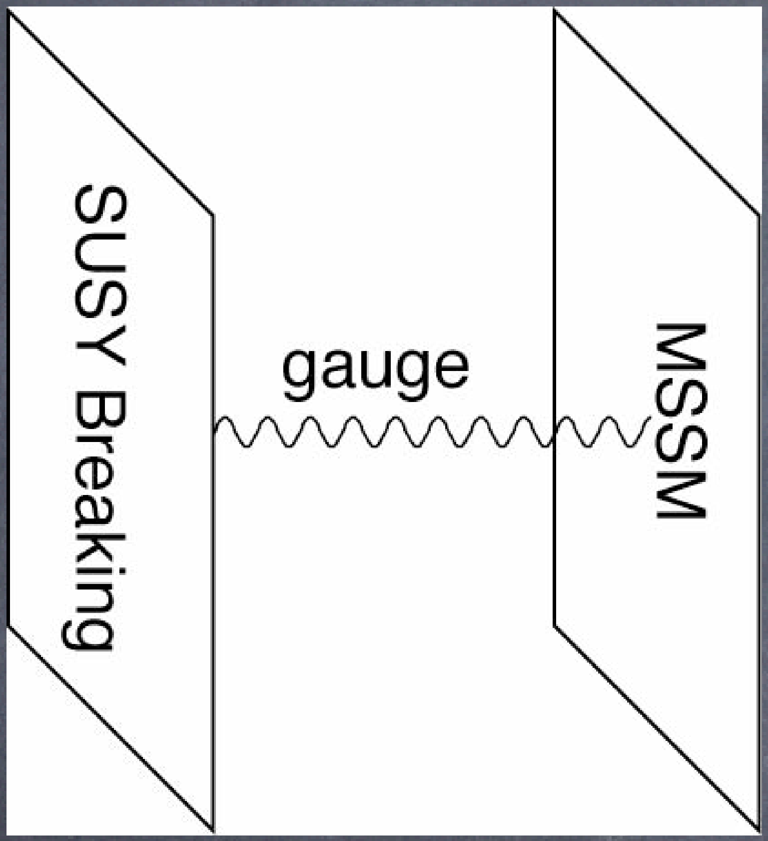
Gauge Mediation

- Assuming that the messenger scale is higher than ANY flavor physics, no FCNC
- $m_{3/2} \sim (10^7\ \text{GeV})^2/M_{Pl} \sim 100$ keV: the worst mass range
- there are models with m3/2<keV
- “LSP” (e.g., neutralino, stau) may decay inside detectors



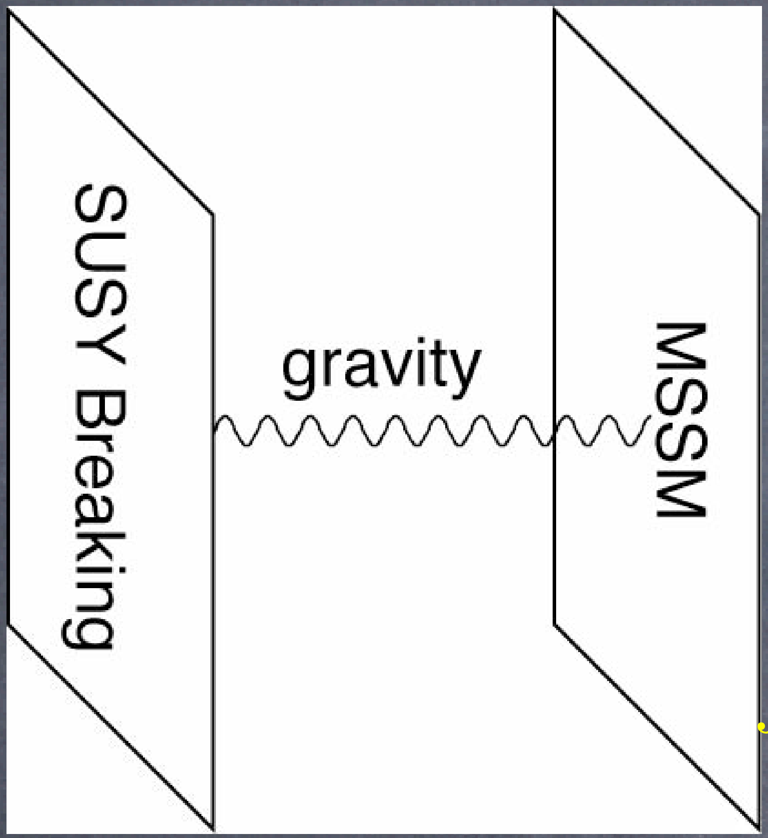
de Gouvêa, Moroi, HM

Gaugino Mediation (χMSB)



- DSB in another brane
- Gauge multiplet in the bulk
- Gauge multiplet learns SUSY breaking first, obtains gaugino mass
- MSSM at the compactification scale with gaugino mass only
- Scalar masses generated by RGE

Anomaly Mediation (AMSB)



- no direct coupling between two sectors
- Supersymmetry breaking in the chiral compensator <S>=1 $+\theta^2 m_{3/2}$

$$\int d^4\theta S\bar{S}\phi^*\phi + \int d^2\left(S^3\lambda_{ijk}\phi_i\phi_j\phi_k + \frac{1}{g^2}W_\alpha W^\alpha\right)$$

- can be scaled away $\phi \to \phi/S$
- but the UV cutoff acquires S: $\Lambda_{UV} \to \Lambda_{UV} S$
- SUSY breaking through cutoff dependence: superconformal anomaly

UV insensitivity

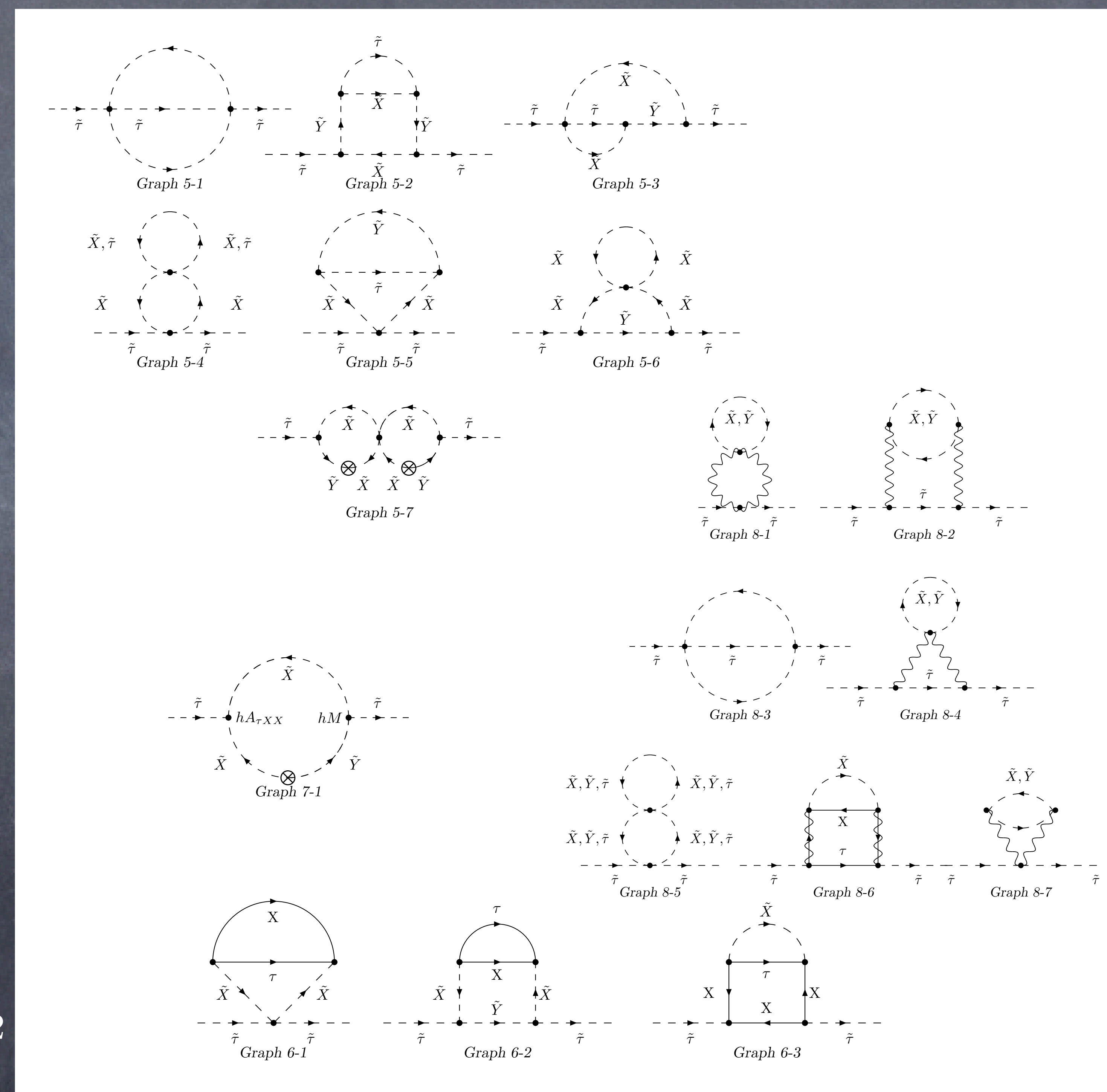
$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- Surprising result: depends only on physics at the energy scale of interest
- No matter how complicated the UV physics is, including flavor physics with O(1) generation-dependent couplings, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
  - Changes the beta function
  - one-loop threshold correction precisely account for the change in gaugino mass

UV insensitivity cont.

- decouple a massive matter field
- two-loop threshold correction precisely account for the change in the anomalous dimension and hence the scalar mass

$$
\begin{aligned}
m_i^2 &= -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \\
A_{ijk} &= -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}
\end{aligned}
$$

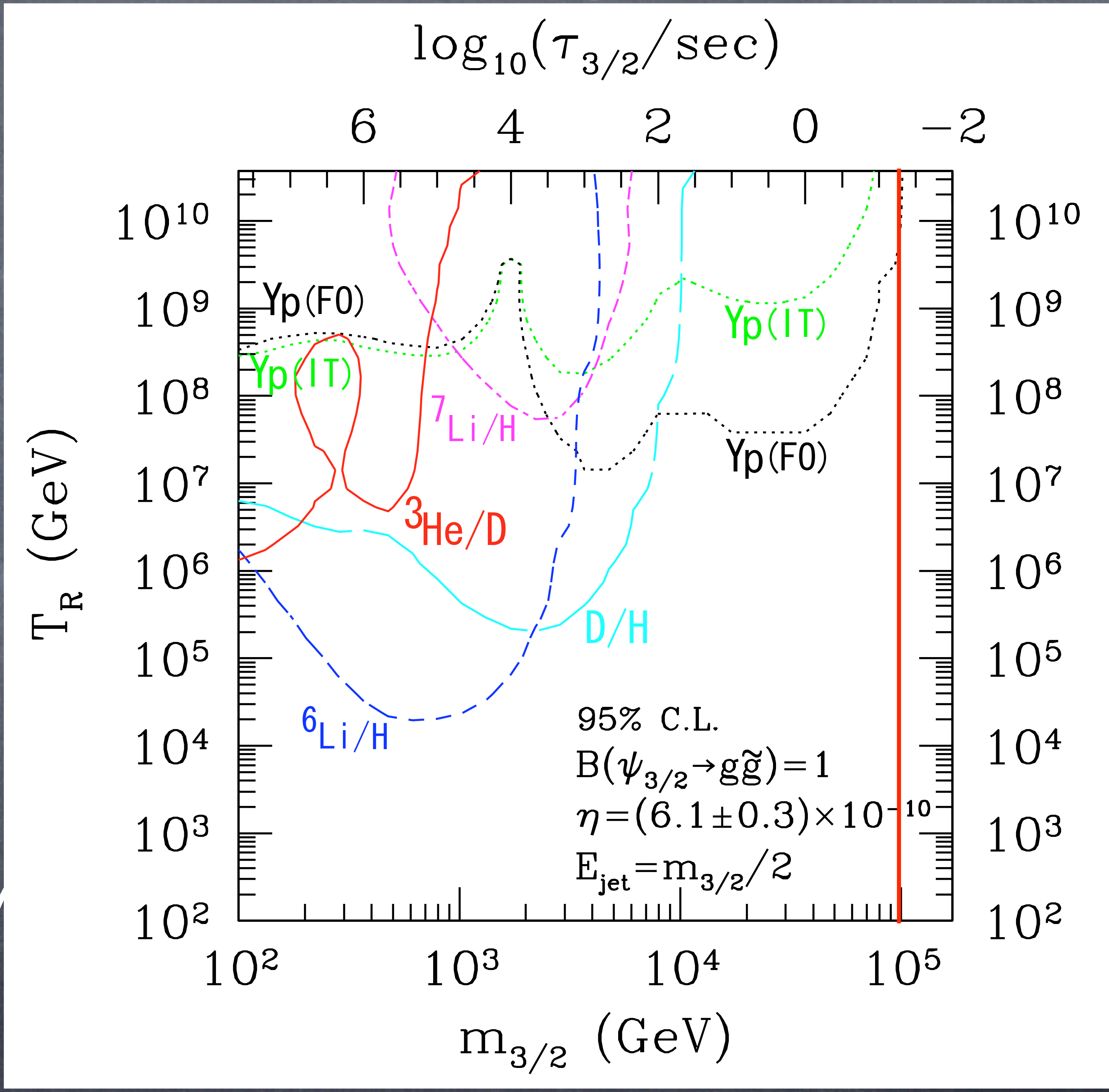


Gravitino OK

- Anomaly mediation with D-terms
- UV insensitive: solves flavor and CP problems no matter how complicated the UV physics is
- solves gravitino problem because

$m_{3/2} \sim (4\pi)^2 m_{SUSY} \sim 50$TeV

- moduli absent by definition



Kohri, Kawasaki, Moroi

What's the catch?

- Two problems
- negative slepton mass-squared
- can't have a light bulk moduli of $m \sim O(m_{3/2})$

cause additional terms of $O(m_{3/2}{}^2/m) \sim O(m_{3/2})$

- common fixes:
  - add $m_0{}^2$
  - add D_Y and D_{B-L}

$$
\begin{aligned}
m^2_{\tilde{l}} &= -0.344 M^2, \\
m^2_{\tilde{e}^c} &= -0.367 M^2, \\
m^2_{\tilde{q}} &= 11.6 M^2, \\
m^2_{\tilde{u}^c} &= 11.7 M^2, \\
m^2_{\tilde{d}^c} &= 11.8 M^2, \\
M &= \frac{m_{3/2}}{(4\pi)^2}
\end{aligned}
$$

fixing moduli

(Kachru, Kallosh, Linde, Trivedi)

- Use RR and NSNS anti-symmetric tensor fluxes on compactified space
- Fix complex structure moduli by fluxes
- Long throat in AdS (i.e. warped)
- Break SUSY with anti-D3 down the throat
- Kähler modulus with gaugino condensate?
- No SUSY breaking@tree-level (Camara, Ibañez, Uranga) in the "bulk"
- often Kähler moduli and anomaly mediated contribution comparable (Choi et al)
- can fix negative slepton mass-squared

SUSY spectra