Supersymmetry V

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PiTP 05, IAS

Plan

Mon: Non-technical Overview

what SUSY is supposed to give us

Tue: From formalism to the MSSM

Global SUSY formalism, Feynman rules, soft SUSY breaking, MSSM

Wed: SUSY breaking

how to break SUSY, mediation mechanisms

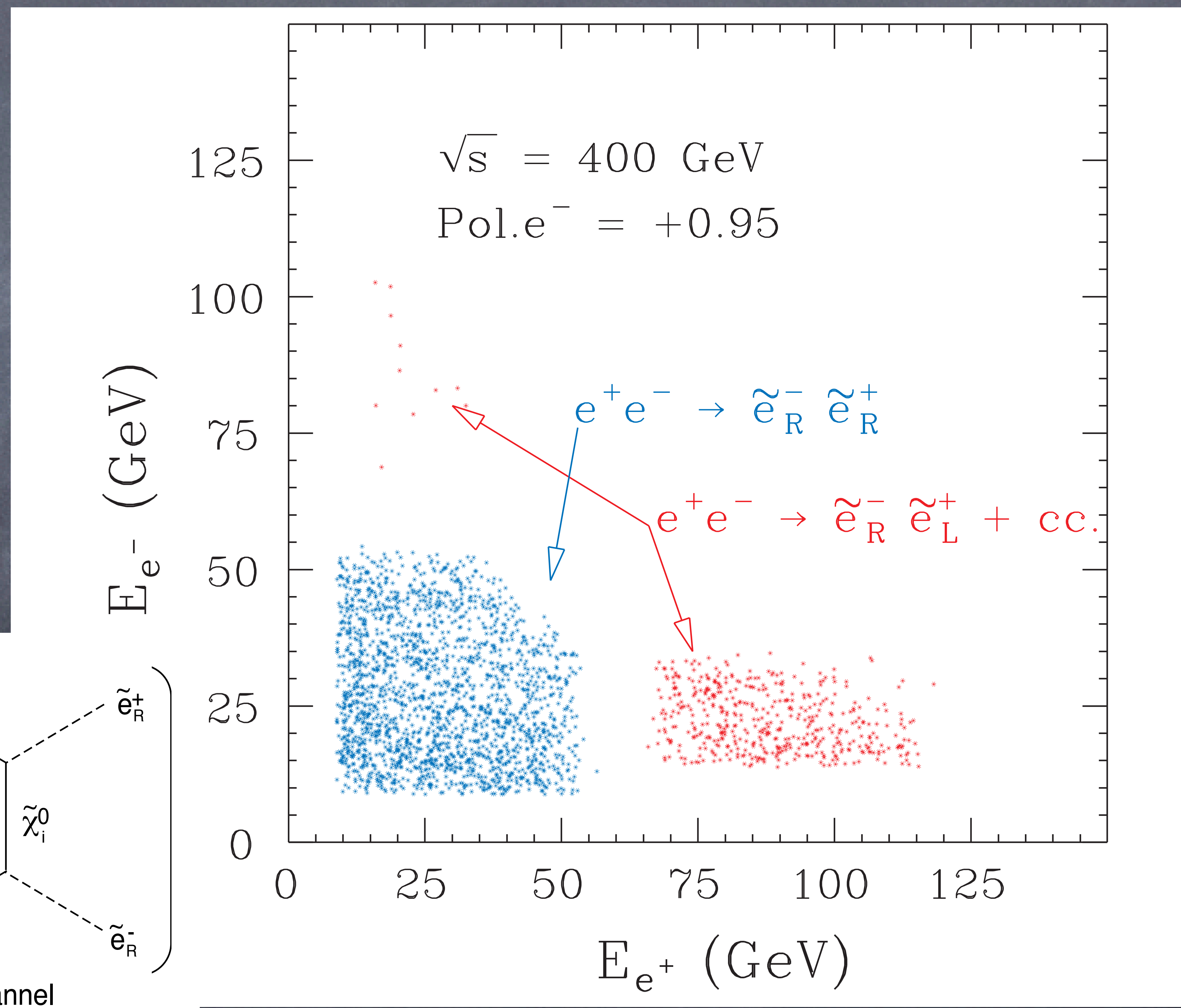
Thu: SUSY at colliders

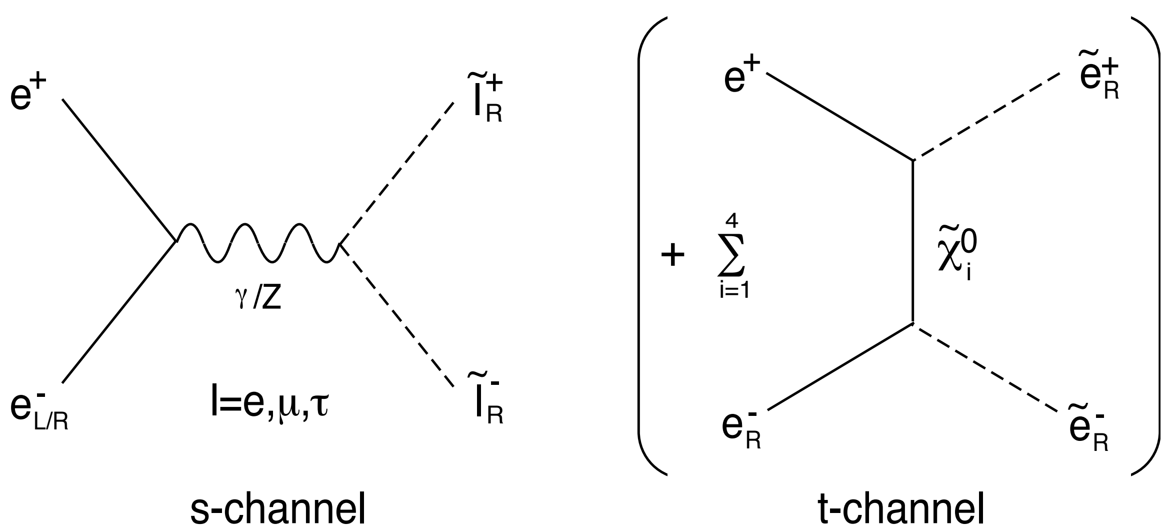
basic reactions, signatures, and how do we know it is SUSY?

Fri: SUSY as a telescope

supersymmetry breaking, GUT, string

testing chirality of scalar bosons

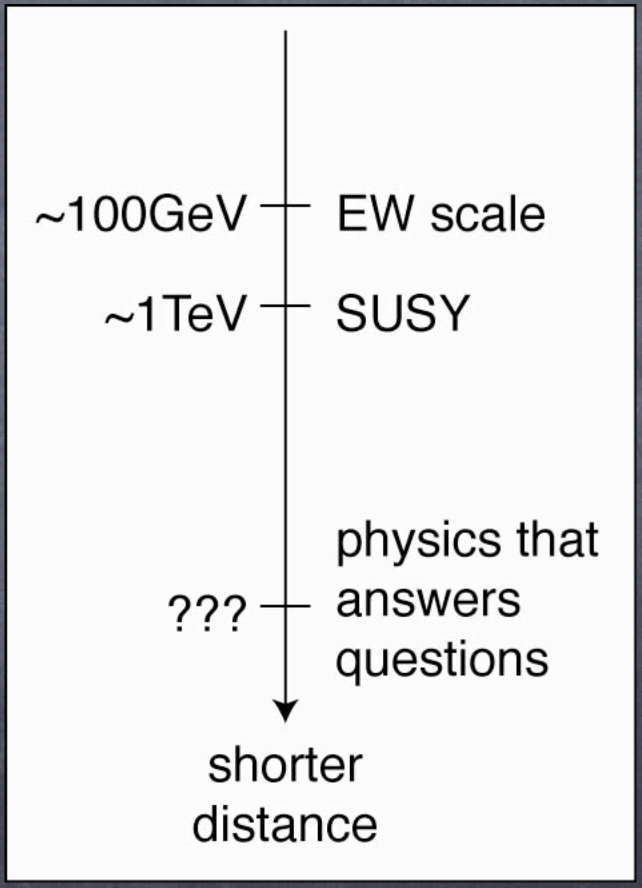


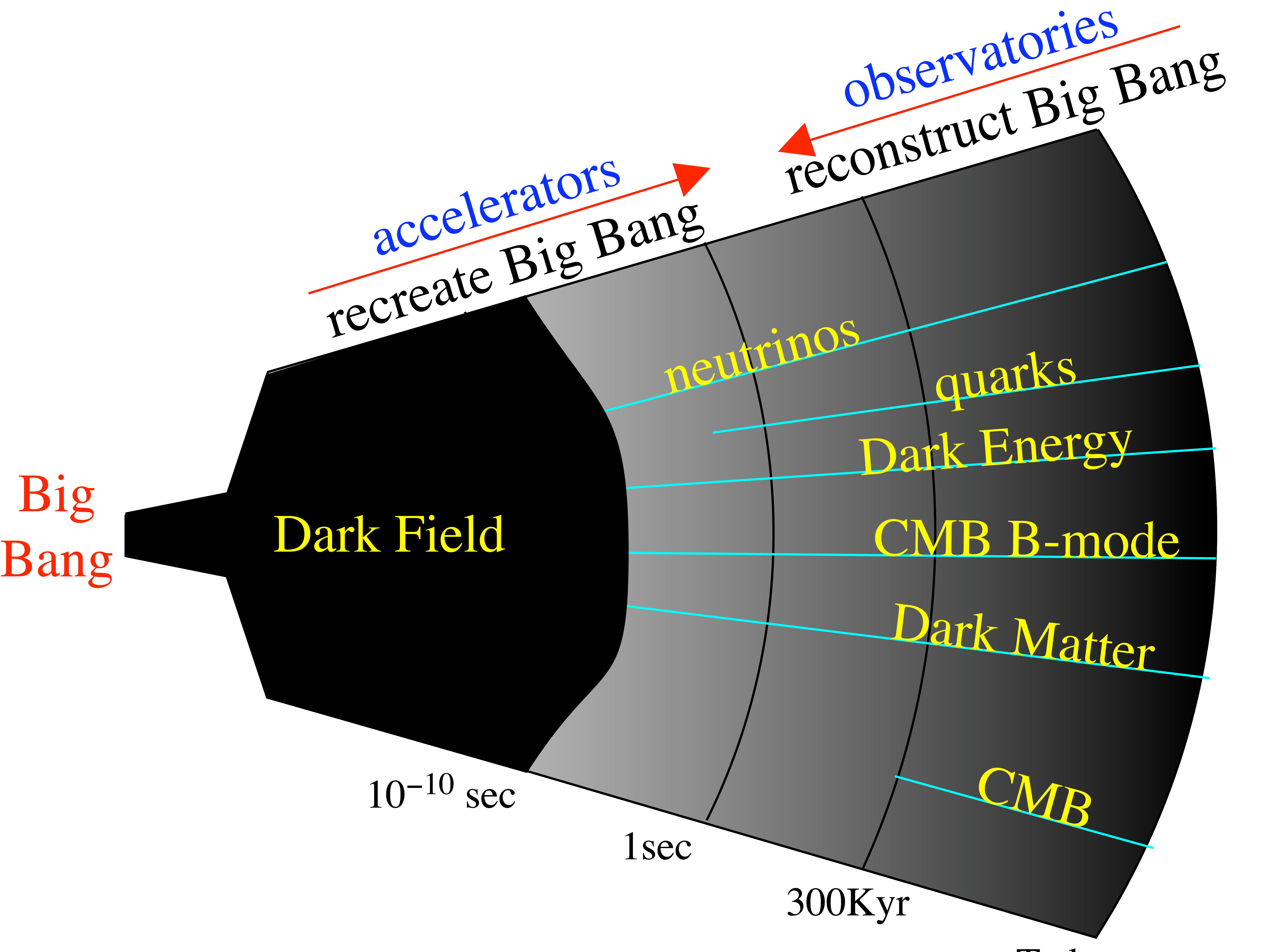


Opening the door

- Once the hierarchy problem solved, we can get started to discuss physics at shorter distances and earlier universe.
- It opens the door to the next level:

  Hope to answer big questions
- The solution to the hierarchy problem itself, e.g., SUSY, provides additional probe to physics at short distances




accelerators
recreate Big Bang
observatories
reconstruct Big Bang
Big
Bang
Dark Field
neutrinos
quarks
Dark Energy
CMB B-mode
Dark Matter
CMB
10^{-10} sec
1sec
300Kyr

Telescope to the Planck scale

- Imagine SUSY breaking originates from Planck-scale physics (but not anomaly mediation)
- Their low-energy values subject to all physics between the Planck and TeV scales
  - boundary conditions at the Planck scale
  - running due to extra particles above the GUT-scale
  - effects due to other particles below M_{GUT}

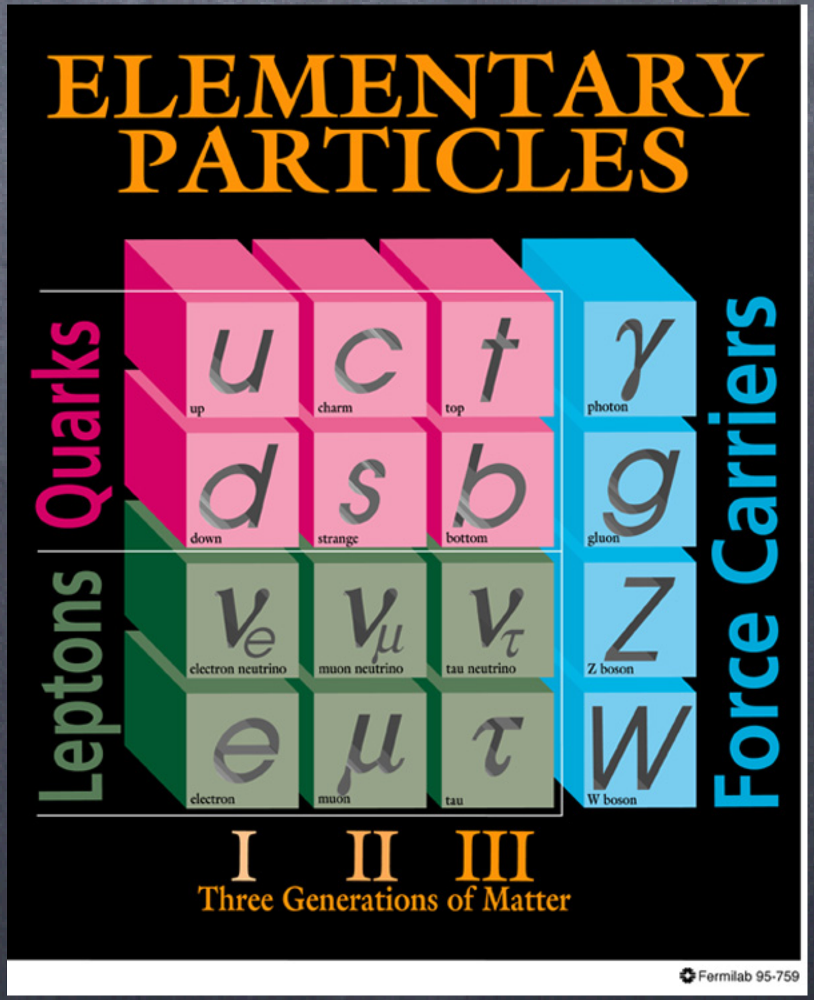
Grand Unification

Big Questions
-Vertical-

- Why are there three unrelated gauge forces?
- Why is strong interaction strong?
- Charge quantization
- anomaly cancellation
- quantum numbers
- Is there a unified description of all forces?
- Why is $m_W \ll M_{Pl}$?

  (Hierarchy Problem)

$$Q(\mathbf{3},\mathbf{2},+\frac{1}{6}),\quad u(\mathbf{3},\mathbf{1},+\frac{2}{3}),\quad d(\mathbf{3},\mathbf{1},-\frac{1}{3}),$$
$$L(\mathbf{1},\mathbf{2},-\frac{1}{2}),\quad e(\mathbf{1},\mathbf{1},-1)$$



Motivations for GUT

- Charge quantization, anomaly cancellation, bizarre hypercharge assignments in the Standard Model
- Three seemingly unrelated forces yet all gauge forces
- Einstein's dream towards a unified description of all forces
- Baryogenesis no longer a prime motivation

Quantum Numbers in the Standard Model

- I didn't become a physicist to memorize these weird numbers...

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,-\tfrac{1}{6}) \quad u_R(3,1,+\tfrac{2}{3}) \quad d_R(3,1,-\tfrac{1}{3})$$

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \quad l_R(1,1,-1)$$

Quantum Numbers in the Standard Model

- To treat them on equal footing, make all particles left-handed using CP

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,-\tfrac{1}{6}) \quad \bar{u}_L\,(3^*,1,-\tfrac{2}{3}) \quad \bar{d}_L\,(3^*,1,\tfrac{1}{3})$$

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \quad \bar{l}_L\,(1,1,1)$$

SU(5) GUT

- SU(3)×SU(2)×U(1)⊂SU(5)
- U(1) must be traceless: try <u>5*</u>: $\begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ \nu \\ l \end{pmatrix}$
- 5×5 matrices

SU(3) $\begin{pmatrix} -\frac{1}{2}\lambda^{a*} & 0 \\ 0 & 0 \end{pmatrix}$

SU(2 $\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\tau^{a} \end{pmatrix}$

U(1) $\begin{pmatrix} \frac{1}{3}I_3 & 0 \\ 0 & -\frac{1}{2}I_2 \end{pmatrix}$

SU(5) GUT

- Then the rest belongs to $\underline{10}$
- All quantum numbers work out this way

$$\begin{pmatrix} 0 & \bar{u} & -\bar{u} & d & -u \\ -\bar{u} & 0 & \bar{u} & d & -u \\ \bar{u} & -\bar{u} & 0 & d & -u \\ -d & -d & -d & 0 & \bar{l} \\ u & u & u & -\bar{l} & 0 \end{pmatrix}$$

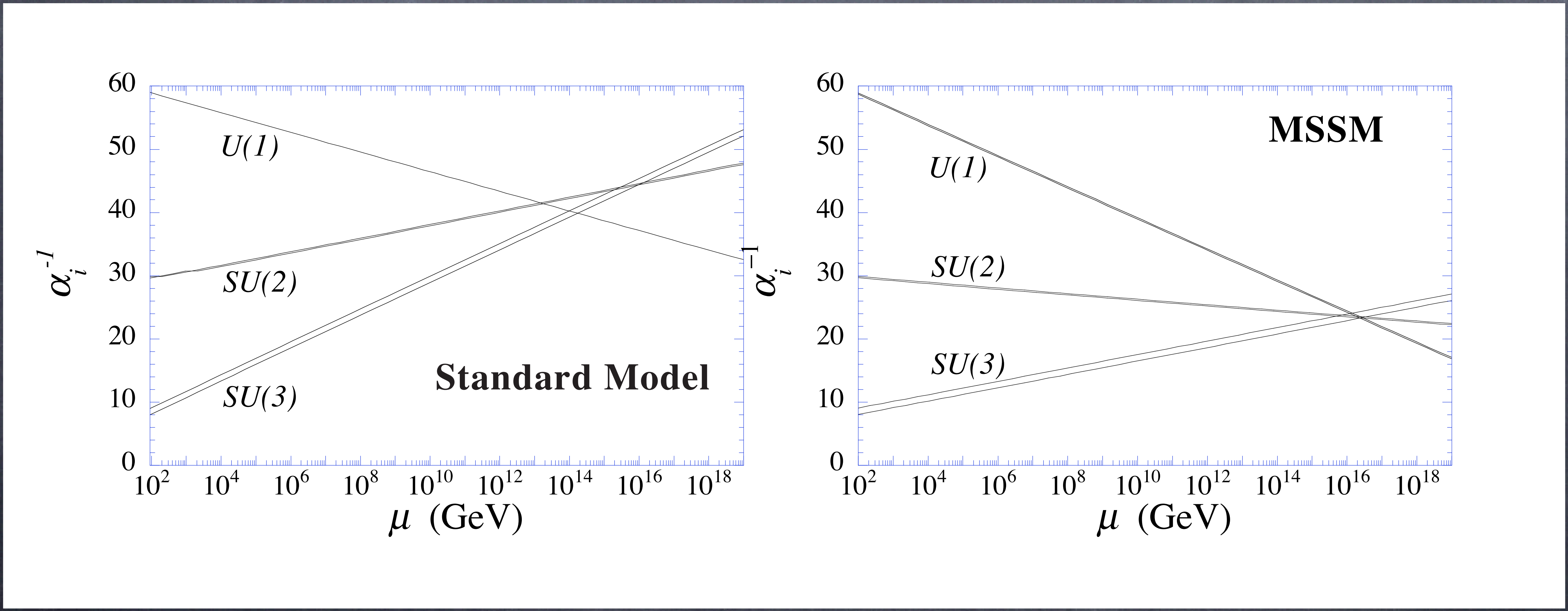
$$\begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,-\tfrac{1}{6}) \sim \left[\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \otimes \bar{d}_L (3^*,1,\tfrac{1}{3}) \right]^*$$

$$\bar{u}_L (3^*,1,-\tfrac{2}{3}) \sim \left[\bar{d}_L (3^*,1,\tfrac{1}{3}) \otimes \bar{d}_L (3^*,1,\tfrac{1}{3}) \right]^*$$

$$\bar{l}_L (1,1,1) \sim \left[\begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \otimes \begin{pmatrix} \nu \\ l \end{pmatrix}_L (1,2,-\tfrac{1}{2}) \right]^*$$

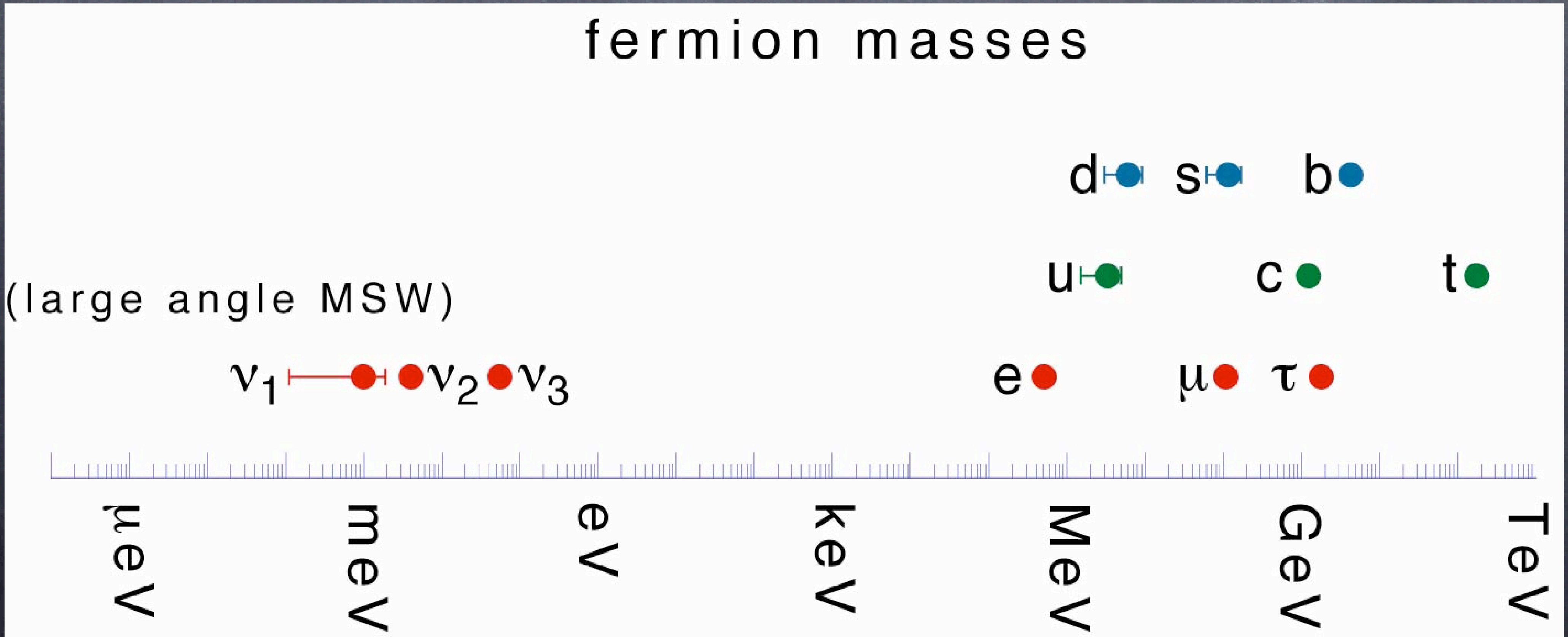
- Anomaly cancellation: $\# \underline{10} - \# \underline{5}^* = 0$

gauge coupling unification



Fermion Mass Relation

- Down- and lepton-Yukawa couplings come from the same SU(5) operator 10 5* H
- Fermion mass relation: $m_b = m_\tau,\ m_s = m_\mu,\ m_d = m_e$
- Reality: $m_b = m_\tau,\ 3m_s = m_\mu,\ m_d = 3m_e$
- Not bad!



SO(10) GUT

- SU(5)×U(1)⊂SO(10)

  $$16 = (10, +1) + (5^*, -3) + (1, +5)$$

  - Come with right-handed neutrinos!
- Certain uniqueness
  - anomaly-free for any multiplets
  - Smallest simple anomaly-free group with chiral fermions
  - Smallest chiral representation contains all standard model fermions

Seesaw meachanism

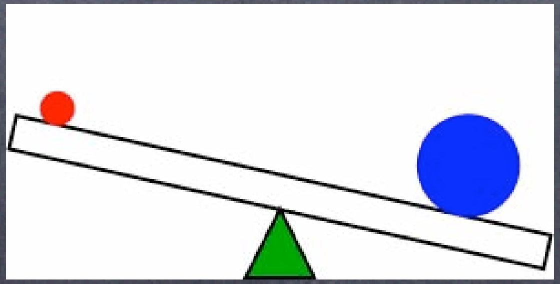
- Once SO(10) broken to the standard model, right-handed neutrino Majorana mass becomes allowed by the gauge invariance

$$M \sim h\, M_{\mathrm{GUT}}$$

Seesaw Mechanism

- Why is neutrino mass so small?
- Need right-handed neutrinos to generate neutrino mass , but ν_R SM neutral

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \qquad m_\nu = \frac{m_D^2}{M} << m_D$$

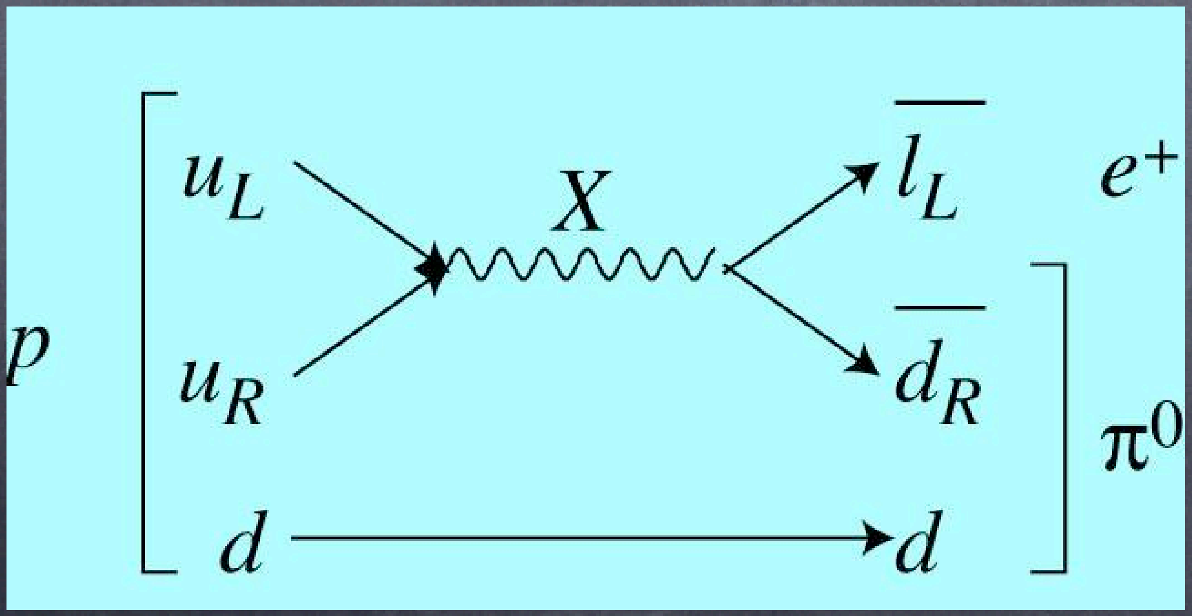


To obtain $m_3 \sim (\Delta m^2_{atm})^{1/2}$, $m_D \sim m_t$, $M_3 \sim 10^{15}$GeV (GUT!)

Neutrinos are Majorana

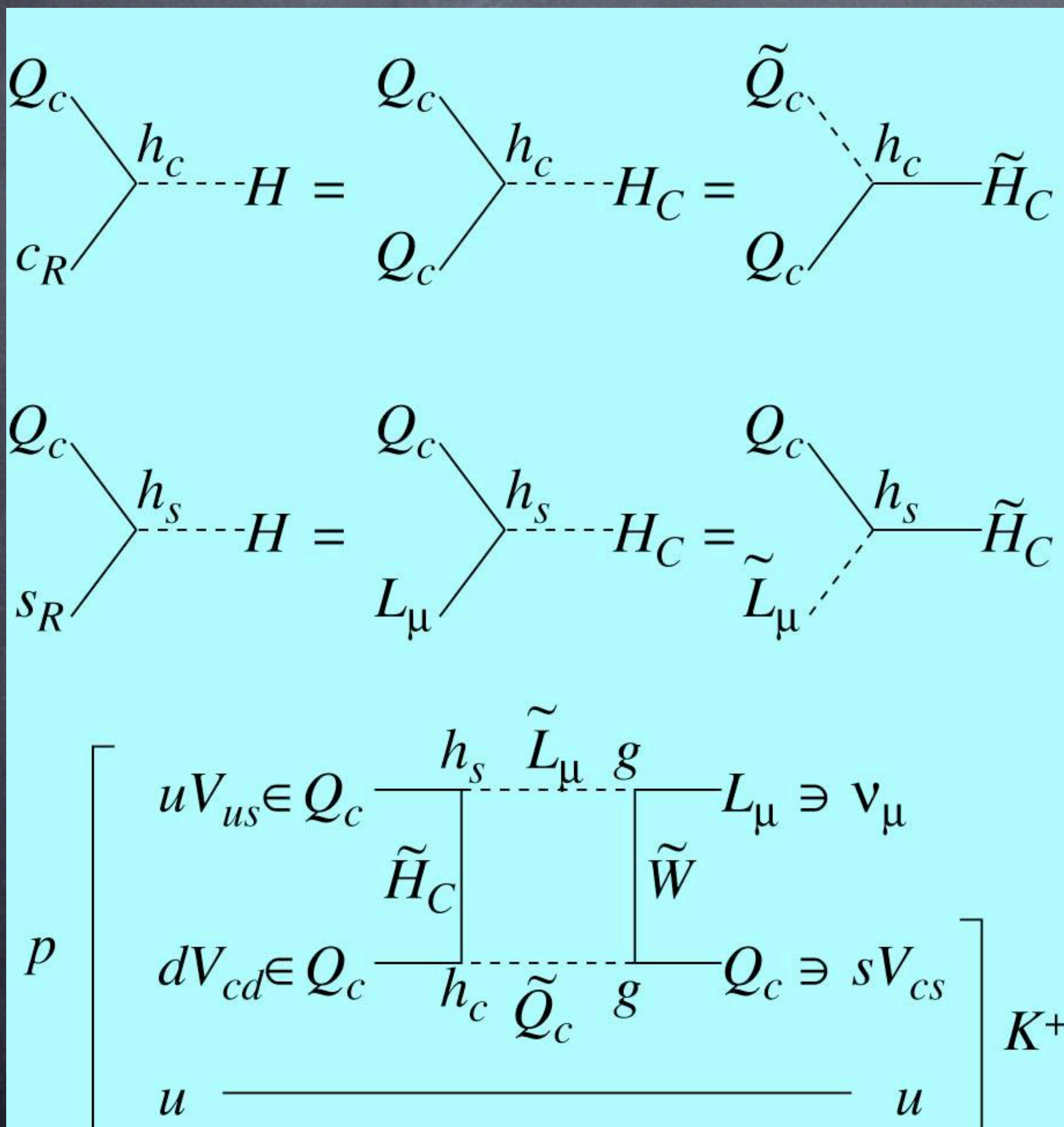
Proton Decay

- Quarks and leptons in the same multiplet
- Gauge bosons can convert q to l
- Cause proton decay!



$$\Gamma \propto \left(\frac{g^2}{M_X^2}\right)^2 m_p^5$$

Supersymmetric Proton Decay

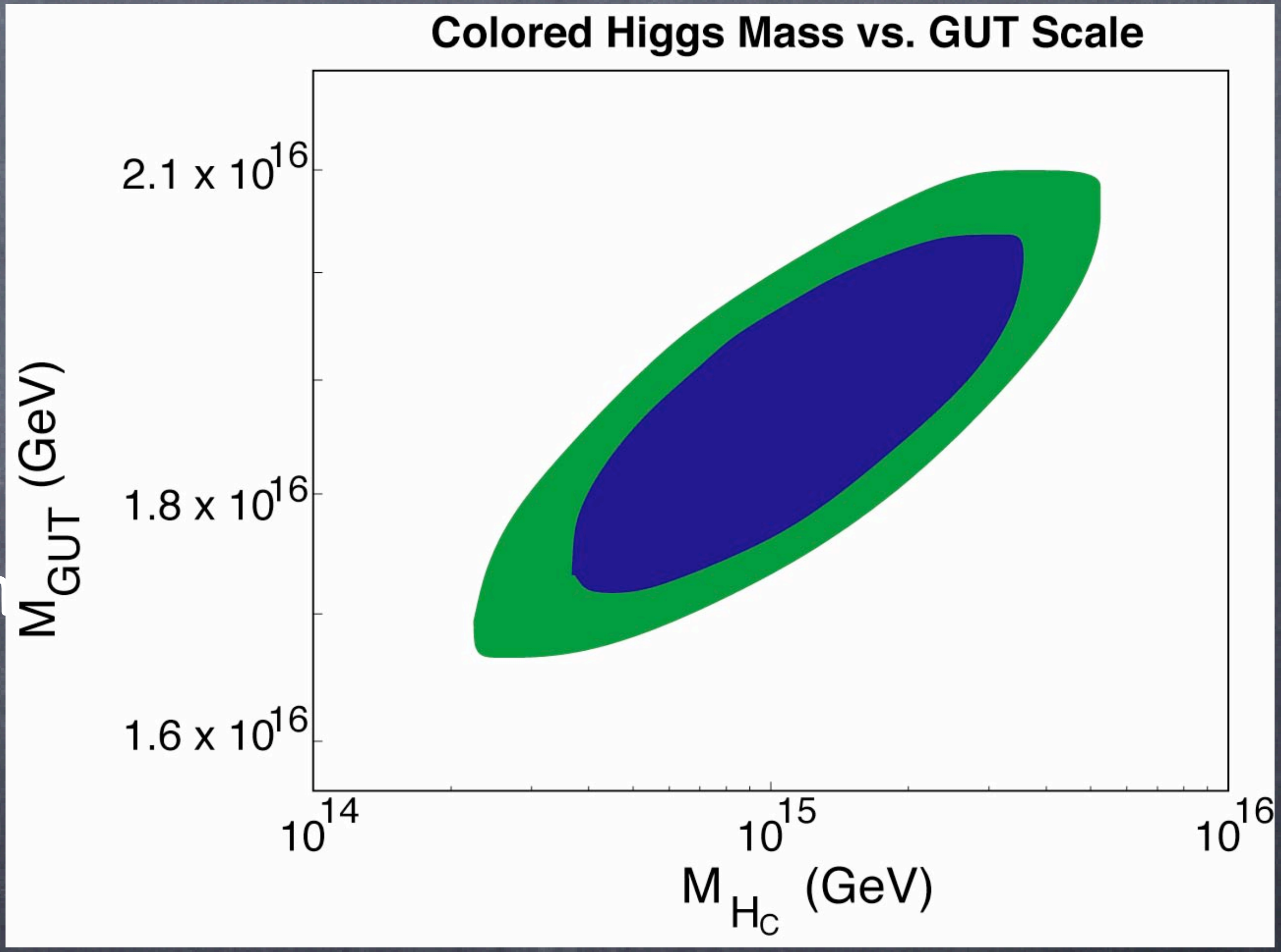


$$\Gamma \propto \left(\frac{g^2}{(4\pi)^2} \frac{h_s h_c \theta_C^2}{M_{H_C} m_{SUSY}} \right)^2 m_p^5$$

Suppressed only by the *second power* of GUT scale vs *fourth* in *X*-boson exchange

Rest In Peace
Minimal SUSY SU(5) GUT

- RGE analysis:
- use $\alpha_1(m_Z)$, $\alpha_2(m_Z)$, $\alpha_3(m_Z)$ to extract α_{GUT}, M_{Hc}, $M_{GUT}=(M_X{}^2 M_\Sigma)^{1/3}$
- SuperK limit
  $M_{Hc}>14\times10^{16}$ GeV
- Even if 1st, 2nd generation scalars "decoupled", 3rd generation contribution
  $M_{Hc}>11\times10^{16}$ GeV



Avoiding Proton Decay

- (Un)fortunately, proton decay rate/mode is highly model-dependent
  - more threshold corrections (HM, Pierce)
  - Some fine-tuning (Babu, Barr)
  - GUT breaking by orbifolds (Kawamura; Hall, Nomura)
  - Depends on the triplet-doublet splitting mechanism, Yukawa (non-)unification

Don't give up!

- Still, proton decay unique window to physics at $>10^{15}$ GeV
- Suppression by fine-tuning: $p \to K^+ \nu$ may be just around the corner
- Flipped SU(5): $p \to e^+ \pi^0$ possible
- Eventually with ~1000kt detector

$p \rightarrow e^+\pi^0$

- SuperK: $\tau(p \rightarrow e^+\pi^0) > 5.7 \times 10^{33}$ year (90% CL)
- Minimal SUSY GUT:

  $\tau(p \rightarrow e^+\pi^0) = 8 \times 10^{34}$ year $(M_V/10^{16}\text{GeV})^4$

  $M_V > 1.4 \times 10^{16}$ GeV
- Flipped SU(5):

  $\tau(p \rightarrow e^+\pi^0) = 4 \times 10^{35}$ year $(M_V/10^{16}\text{GeV})^4$

  $M_V > 2.6 \times 10^{15}$ GeV
- 5-D orbifold GUT: $\tau(p \rightarrow e^+\pi^0) \approx 10^{34}$ year

Any other tests of GUTs?

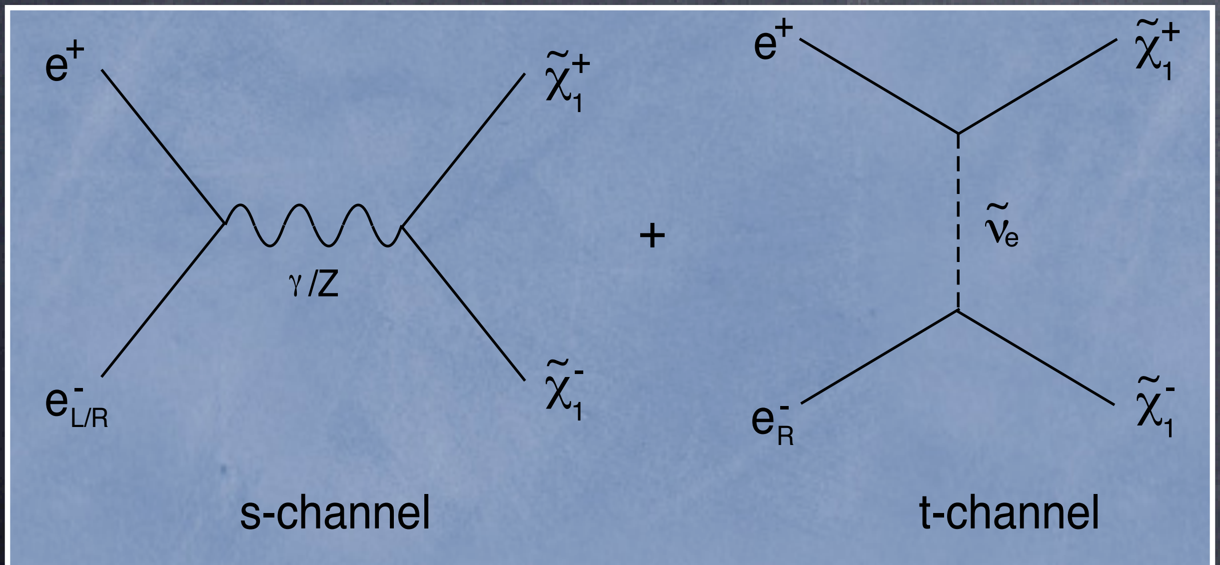
- Yes!
- Once you have superparticles, we can learn a great deal from them.

Model-independent parameter determination

- Chargino/neutralino mass matrices have four parameters M_1, M_2, μ, $\tan\beta$
- Can measure 2+4 masses
- can measure 10x2 neutralino cross sections

$$\sigma_{L,R}(e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0) \qquad \sigma_{L,R}(e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)$$

- can measure 3x2 chargino cross sections
- depend on masses of $\tilde{\nu}_e$, $\tilde{e}_L$, $\tilde{e}_R$



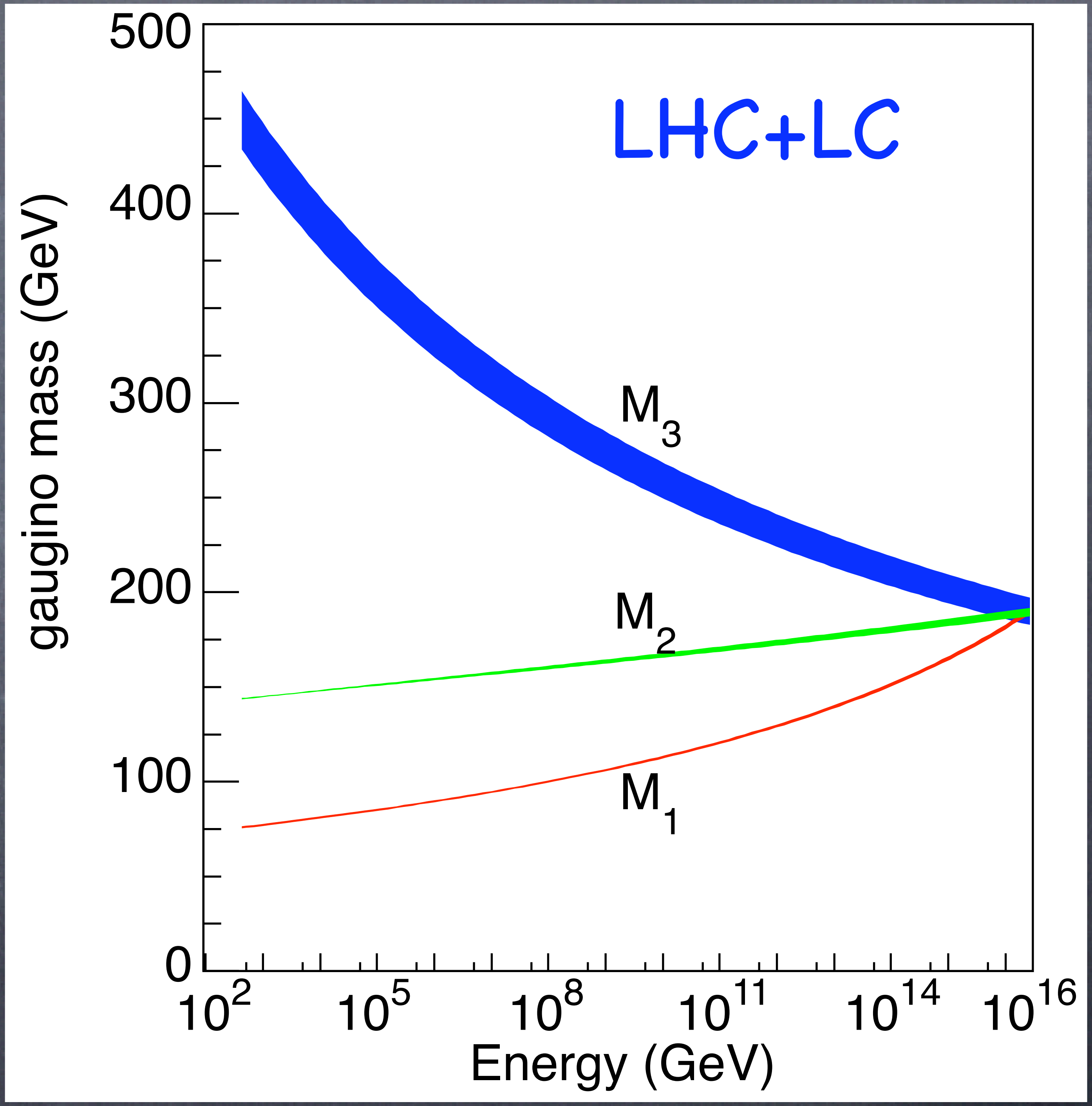
	input	fit
M_2	152 GeV	152 ±1.8 GeV
μ	316 GeV	316 ±0.9 GeV
$\tan\beta$	3	3 ±0.7
M_1	78.7 GeV	78.7 ±0.7 GeV

Superpartners as probe

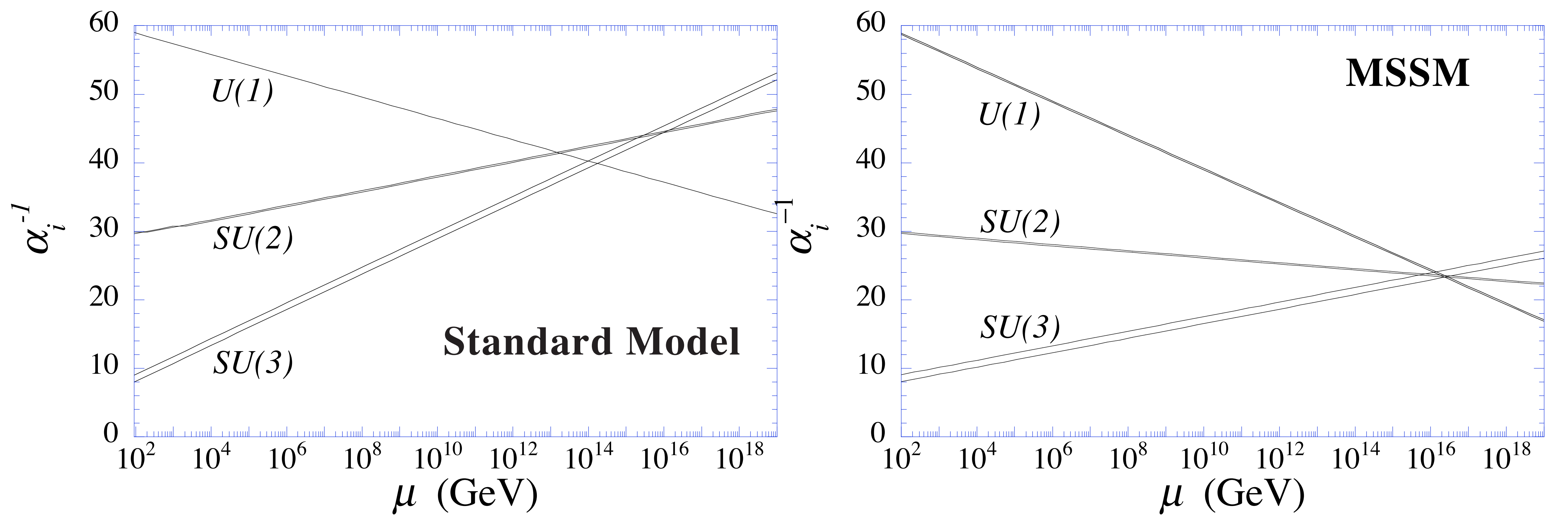
- Most exciting thing about superpartners beyond existence:

They carry information of small-distance physics to something we can measure

"Are forces unified?"

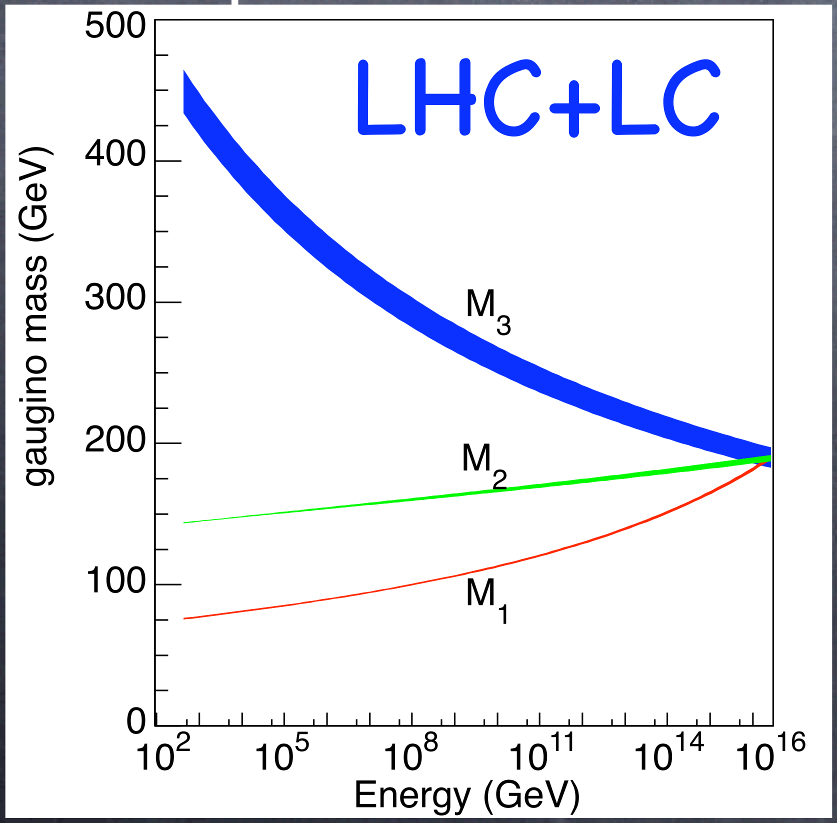


cf. gauge coupling unification



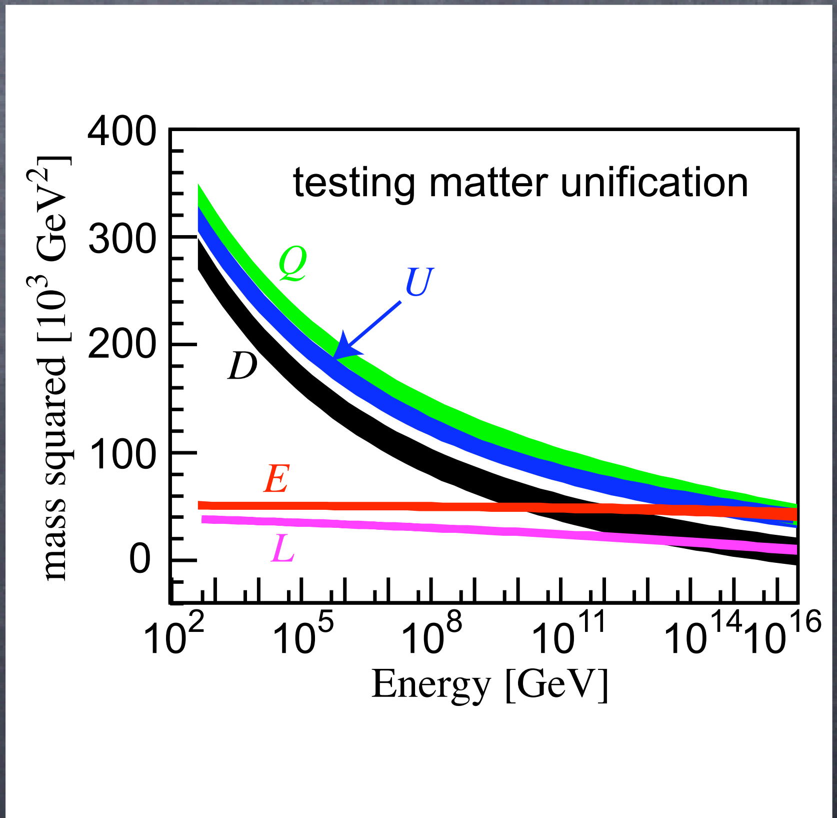
Gaugino and scalars

- Gaugino masses test unification itself independent of intermediate scales and extra complete SU(5) multiplets, also GMSB



- Scalar masses test beta functions at all scales, depend on the particle content

(Kawamura, HM, Yamaguchi)



grand desert

- LHC finds SUSY, LC establishes SUSY
- no more particles beyond the MSSM at TeV scale
- Gaugino masses unify (two more coincidences)
- Scalar masses unify for 1st, 2nd generations (two for 10, one for 5*, times two)
- Scalar masses unify for the 3rd generation 10 (two more coincidences)

⇒ strong hint that there are no additional particles beyond the MSSM below M_{GUT} except for gauge singlets.

seesaw mechanism

- $0\nu\beta\beta$ seen, neutrinos are Majorana
- lepton-flavor violation ($\mu\to e$ conversion, $\tau\to\mu\gamma$) seen at the "reasonable" level expected in SUSY seesaw (even though I don't believe mSUGRA) LBL oscillation finds θ_{13} soon just below the CHOOZ limit
- Scalar masses unify for the 3rd generation 5* up to the neutrino Yukawa coupling y_3~1 above $M_3=y_3^2v^2/m_3$

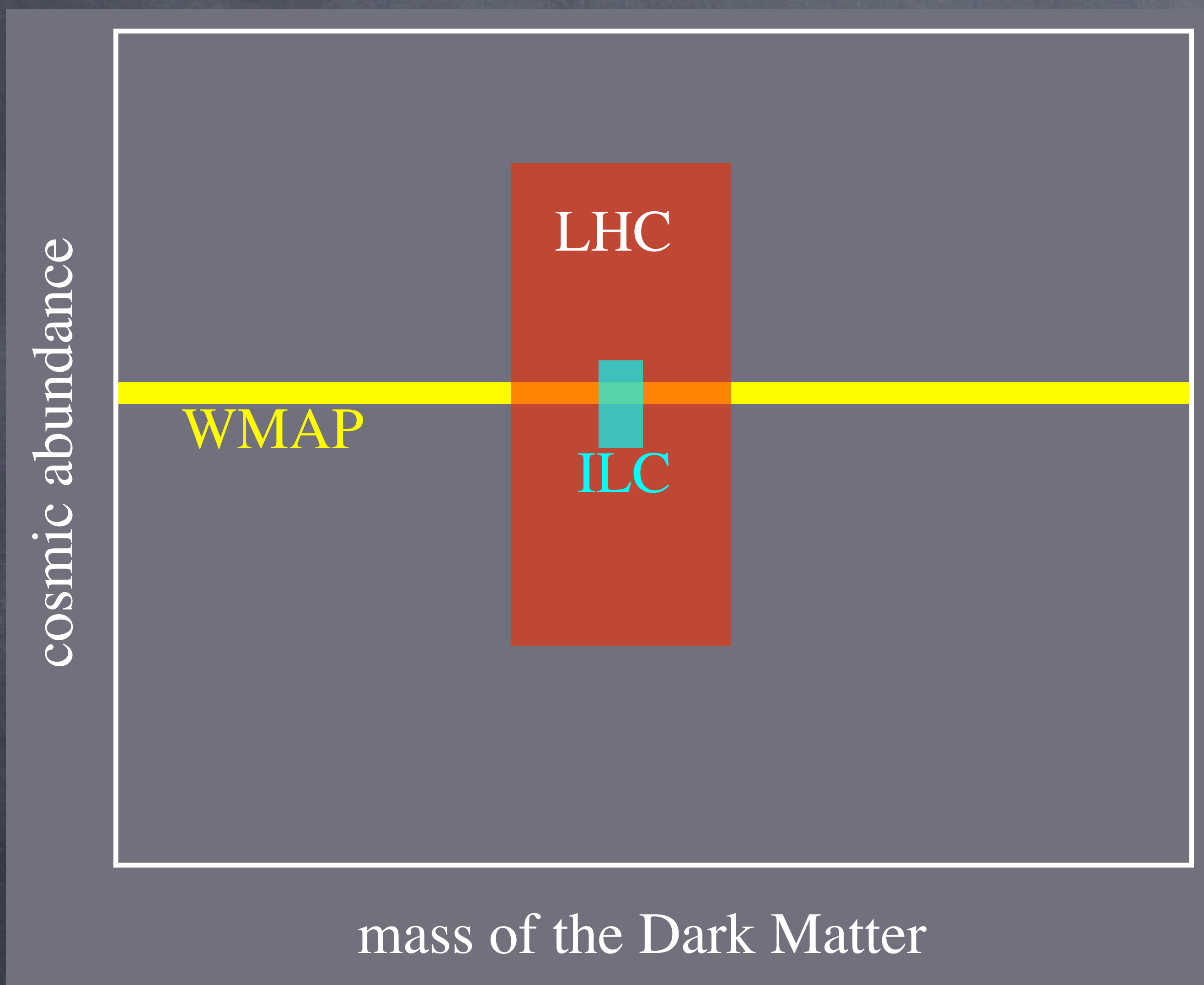
⇒ pretty much proves seesaw

cosmology

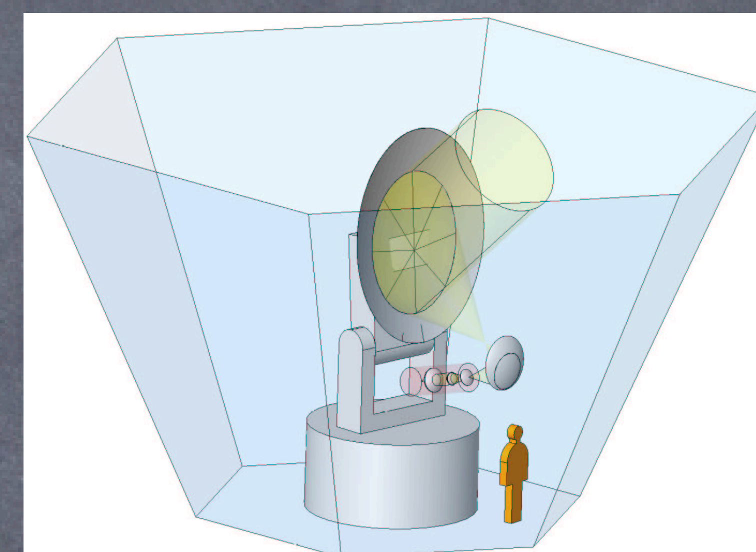
- The neutralino mass and its coupling to other SUSY particles are measured
- Calculate the neutralino annihilation cross section, agrees with the $\Omega_M h^2$=0.14
- Calculate the neutralino scattering cross section, agrees with the direct detection
- B-mode fluctuation in CMB is detected, with a reasonable inflationary scale

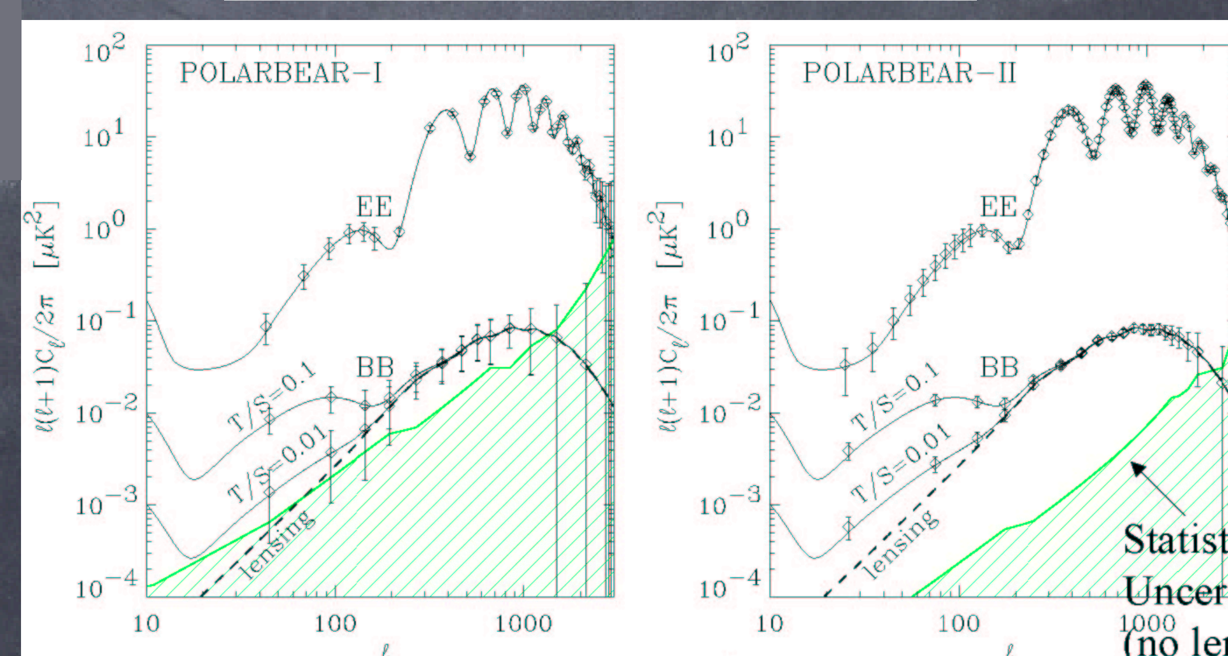
$\Rightarrow$ strong hint that the cosmology has been 'normal' since inflation (no extra D etc)

“Normal” cosmology



B-mode fluctuation





$$\Omega_M = \frac{0.756(n+1)x_f^{n+1}}{g^{1/2}\sigma_{ann}M_{Pl}^3}\frac{3s_0}{8\pi H_0^2} \approx \frac{\alpha^2/(TeV)^2}{\sigma_{ann}}$$

Annihilation cross section

Large θ_{23} and quarks

- Large mixing between ν_τ and ν_μ
- Make it SU(5) GUT
- Then a large mixing between s_R and b_R
- Mixing among right-handed fields drop out from CKM matrix
- But mixing among superpartners physical

$$\begin{pmatrix} \tilde{s}_R \\ \tilde{s}_R \\ \tilde{s}_R \\ \tilde{\nu}_\mu \\ \tilde{\mu} \end{pmatrix} \longleftrightarrow \begin{pmatrix} \tilde{b}_R \\ \tilde{b}_R \\ \tilde{b}_R \\ \tilde{\nu}_\tau \\ \tilde{\tau} \end{pmatrix}$$

- O(1) effects on b→s transition possible

  (Chang, Masiero, HM)
- Expect CP violation in neutrino sector especially if leptogenesis

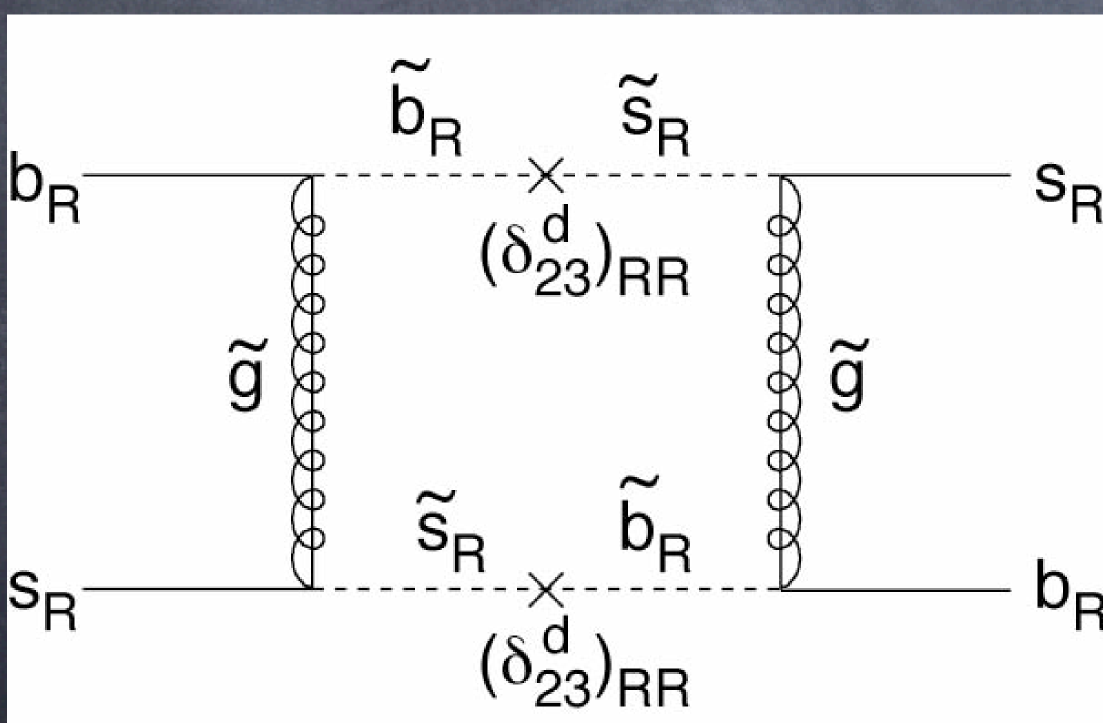
more indirect evidence

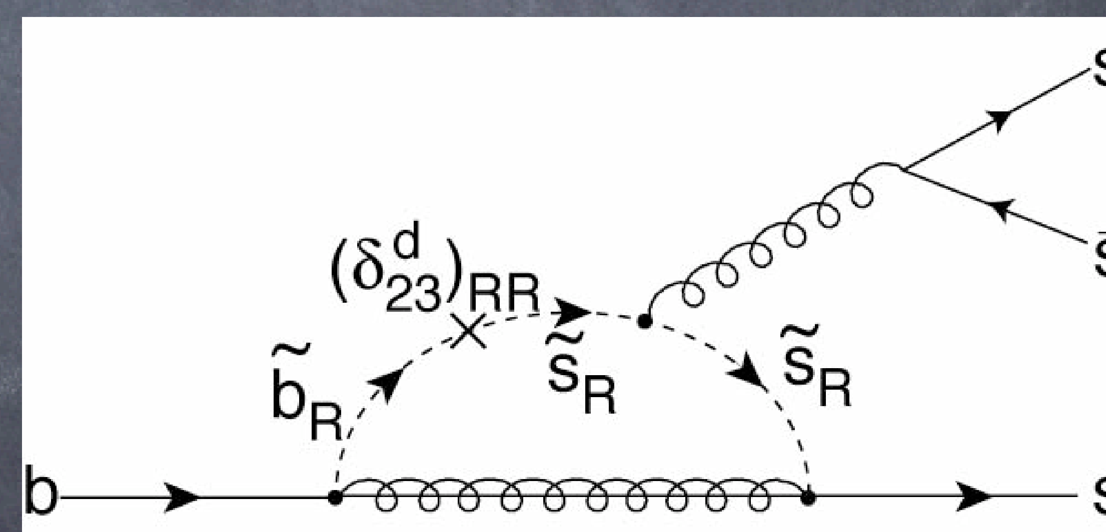
Possible additional evidence, *e.g.*,:

- $B_d \to \phi\ K_S$ shows deviation from the SM consistent with large b_R-s_R mixing above M_{GUT}
- LBL oscillation finds θ_{13} soon just below the CHOOZ limit
- determines the normal hierarchy and finds CP violation
- Isocurvature fluctuation seen suggestive of N_1 coherent oscillation (curvaton), avoiding the gravitino problem

Consequences in B physics

- CP violation in B_s mixing ($B_s \rightarrow J/\psi\ \phi$)
- Addt'l CP violation in penguin $b \rightarrow s$ ($B_d \rightarrow \phi\ K_s$)





Indirect evidence for lepton-quark unification

Testing string theory?

Dirty Little Secret about Supersymmetry

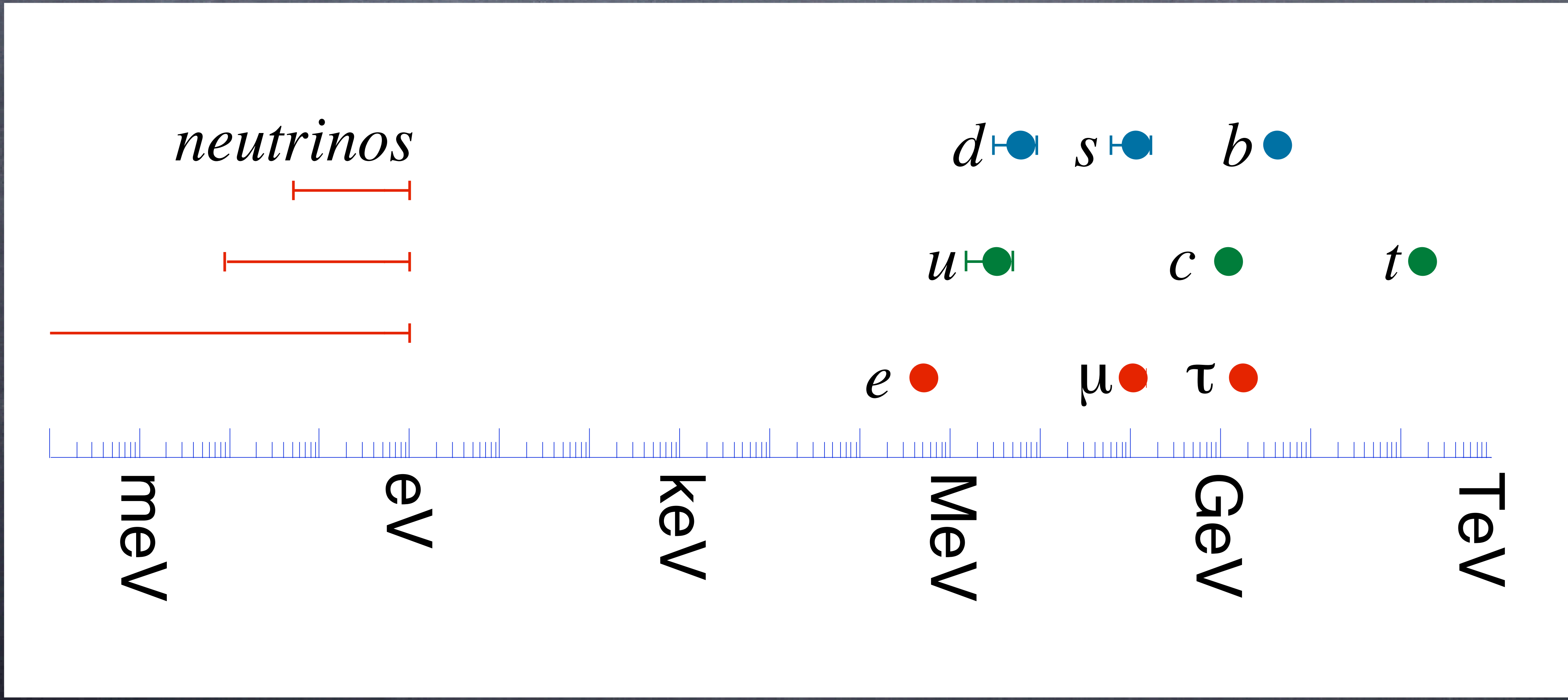
- R-parity is not enough to ensure longevity of matter!
- Once supersymmetry is there, with or without grand unification, *Planck-scale physics can cause too-rapid proton decay*
- Dangerous operators:

$$\frac{h}{M_{Pl}} Q_1 Q_1 Q_2 L_i \qquad \frac{h}{M_{Pl}} Q_1 Q_2 Q_2 L_i$$

- Typically, $h < 4\times10^{-8}$, 10^{-7}, respectively

But there are small numbers

- But remember that we actually do see small numbers in our daily life.



But there are small numbers

- But remember that we actually do see small numbers in our daily life.
- Yukawa couplings for 1st, 2nd generations are pretty small. Using $\lambda \sim \theta_C \sim 0.22$,
  - $h_u/h_t \sim \lambda^8$, $h_d/h_b \sim \lambda^4$, $h_e/h_t \sim \lambda^5$
- Aren't they unnatural?
  - Yes, of course!

Broken Flavor Symmetry

- Flavor quantum numbers ($SU(5)$-like):
  - 10(Q, u_R, e_R) (+4, +2, 0)
  - 5*(L, d_R) (+2, +2, +2)
- Flavor symmetry broken by a VEV $\langle\lambda\rangle$ ~0.22

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, M_d \sim \begin{pmatrix} \lambda^6 & \lambda^6 & \lambda^6 \\ \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix}, M_l \sim \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^6 & \lambda^4 & \lambda^2 \end{pmatrix}$$

  - $m_u:m_c:m_t \sim m_d^2:m_s^2:m_b^2 \sim m_e^2:m_\mu^2:m_\tau^2 \sim \lambda^8: \lambda^4 :1$
  - Neutrinos are anarchy (Hall, HM, Weiner; Haba, HM; de Gouvêa, HM)

Flavor Symmetry Suppresses Proton Decay, too!

- Once the quarks and leptons carry a new charge, it would forbid the dangerous proton decay operators.
- Proton decay may be suppressed because of the same reason why 1st and 2nd generation particles are light. (HM, D.B. Kaplan)

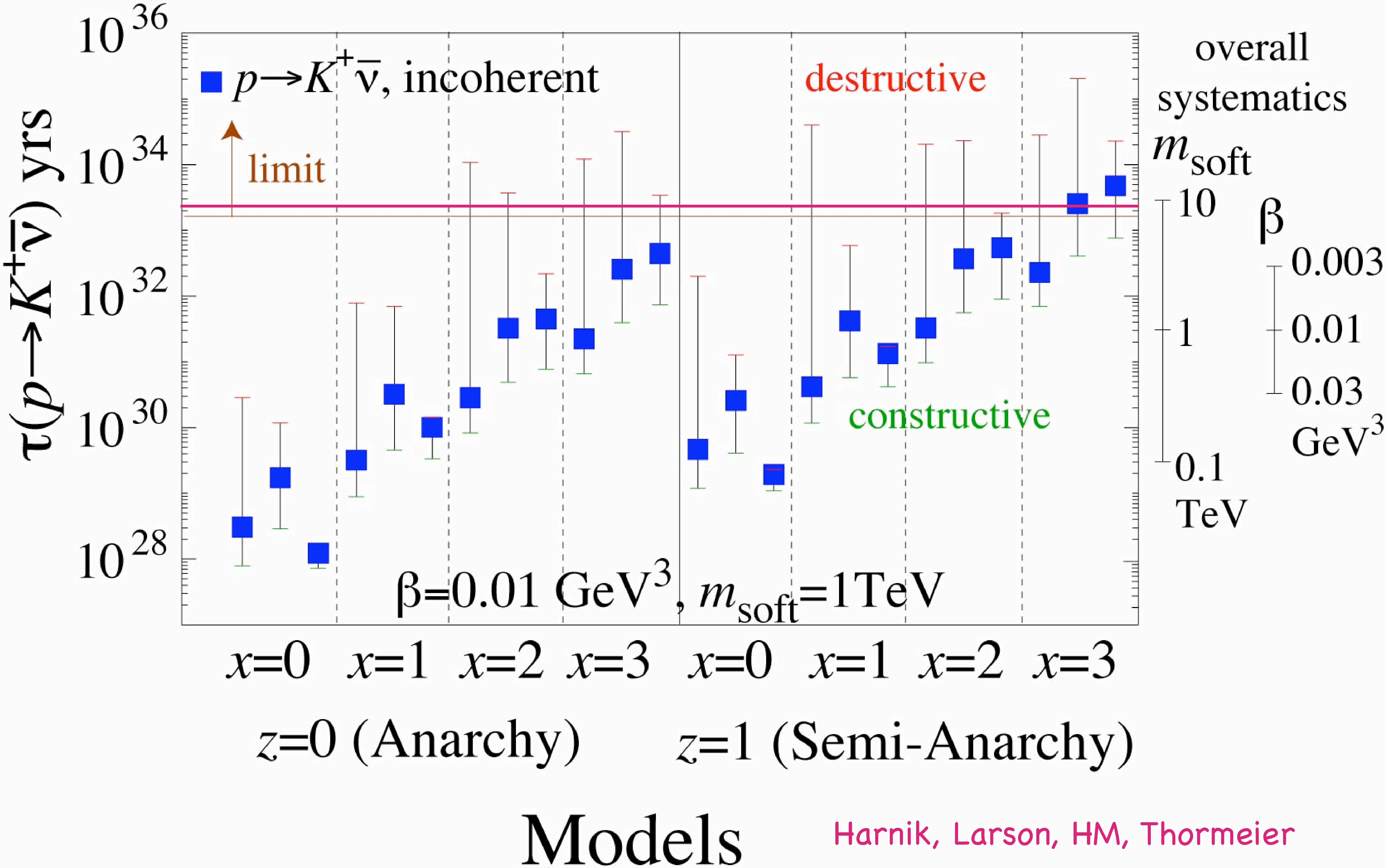
$$\frac{h}{M_{Pl}} Q_1 Q_1 Q_2 L_i \qquad \frac{h}{M_{Pl}} Q_1 Q_2 Q_2 L_i$$

A Very Ambitious Model

Use string-inspired anomalous U(1) for *everything*

- The only symmetry beyond $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Only two right-handed neutrinos
- No new mass scales except for M_{Pl} and m_{SUSY}
- Quark masses and CKM matrix
- Lepton masses
- Right-handed neutrino masses (no GUT-scale)
- Left-handed neutrino masses and MNS matrix
- R-parity as an unbroken subgroup of U(1)
- Adequate suppression of proton decay?

(Dreiner, HM, Thormeier)



Harnik, Larson, HM, Thormeier

dilaton domination

(Kaplunovsky, Louis; Brignole, Ibañez, Muñoz)

- In string theory, there are many moduli fields
- Once SUSY broken, they tend to acquire SUSY breaking F-terms
- If they couple universally to all three generations, you get universal scalar mass
- dilaton (but no model exists)
- overall modulus if all generations have the same modular weight (but then how do we understand the fermion mass hierarchy?)

parametrization

(Binétruy, Gaillard, Nelson)

- Assume SUSY breaking due to F-terms of the dilaton and the overall modulus
- threshold corrections to the gauge couplings:

$$\alpha_i^{-1}(M_U) = \alpha^{-1}(M_{String}) + \Delta\alpha_i^{-1}$$

$$\Delta\alpha_i^{-1} = \frac{1}{4\pi}(b'_i - b_{GS}) \log|\eta(t)|^4$$

$$b'_3 = 9 + \sum_{i=1}^{3}(2n_{Q_i} + n_{U_i} + n_{D_i})$$

$$b'_2 = 15 + \sum_{i=1}^{3}(3n_{Q_i} + n_{L_i}) + n_{H_1} + n_{H_2}$$

$$b'_1 = \frac{99}{5} + \frac{1}{5}\sum_{i=1}^{3}(n_{Q_i} + 8n_{U_i} + 2n_{D_i} + 3n_{L_i} + 6n_{E_i}) + \frac{3}{5}(n_{H_1} + n_{H_2})$$

parametrization

(Binétruy, Gaillard, Nelson)

- Assume SUSY breaking due to F-terms of the dilaton and the overall modulus

$$M_i = -g_i^2 m_{3/2} s\sqrt{3}\sin\theta + \Delta M_i$$

$$\Delta M_i = -g_i^2 m_{3/2}\left\{ b_i + s\sqrt{3}\sin\theta g_s^2\left(C_i - \sum_j C_i^j\right)\right.$$

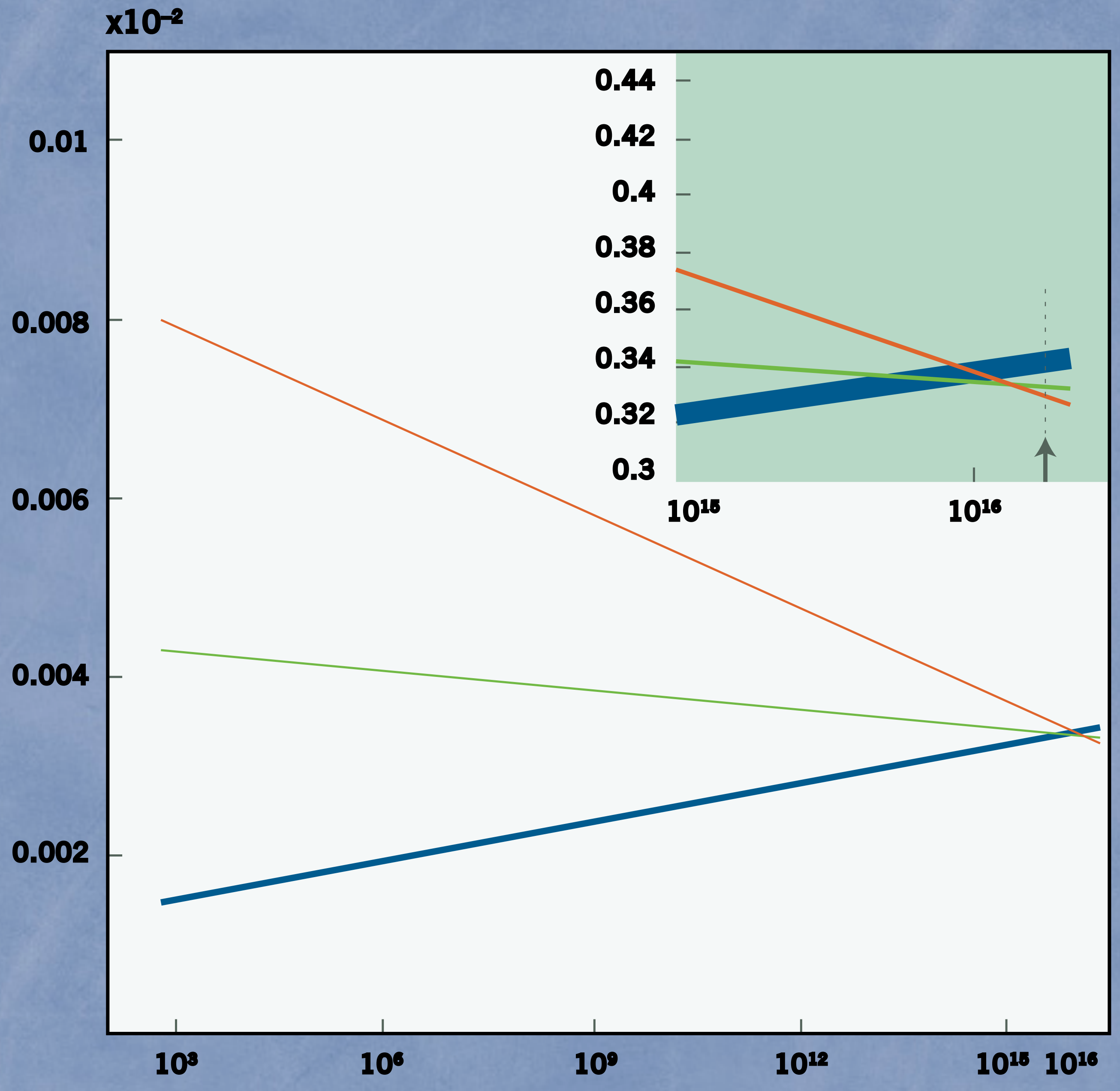
$$\left. + 2t\cos\theta\, G_2(t)\left[\delta_{GS} + b_i - 2\sum_j C_i^j(1+n_j)\right]\right\}/16\pi^2$$

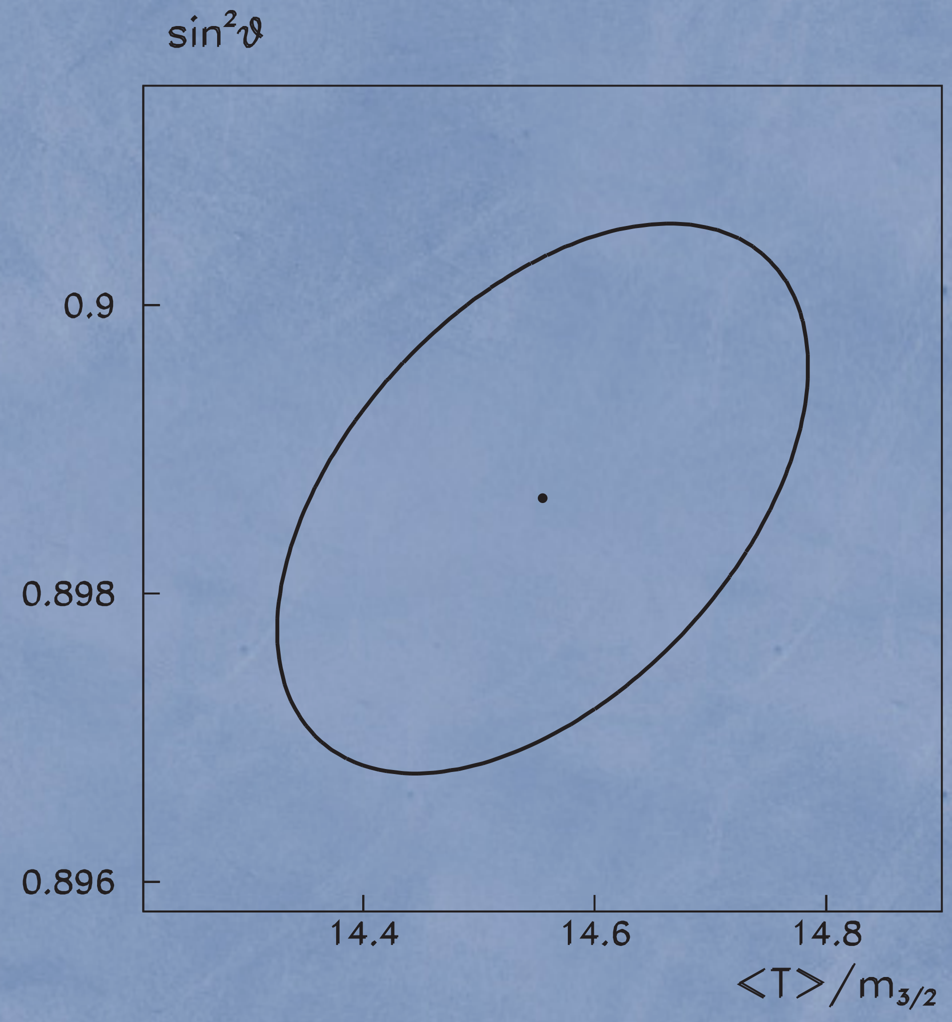
$$M_{\tilde{j}}^2 = m_{3/2}^2\left(1 + n_j\cos^2\theta\right) + \Delta M_{\tilde{j}}^2$$

$$\Delta M_{\tilde{j}}^2 = m_{3/2}^2\left\{\gamma_j + 2t\cos\theta\, G_2(t)\sum_{km}\gamma_j^{km}(n_j + n_k + n_m + 3)\right.$$

$$\left. + 2\sqrt{3}s\sin\theta\left[\sum_i \gamma_j^i g_i^2 - \frac{1}{2s}\sum_{km}\gamma_j^{km}\right]\right\}$$

extracting string parameters





"Data"

- Precision measurements of supersymmetry parameters at LHC/ILC
- fit the data to the string predictions (Blair, Porod, Zerwas)

Parameter	Ideal	Reconstructed
$m_{3/2}$	180	179.9 ± 0.4
$\langle S \rangle$	2	1.998 ± 0.006
$\langle T \rangle$	14	14.6 ± 0.2
$\sin^2\theta$	0.9	0.899 ± 0.002
g_s^2	0.5	0.501 ± 0.002
δ_{GS}	0	0.1 ± 0.4
n_L	-3	-2.94 ± 0.04
n_E	-1	-1.00 ± 0.05
n_Q	0	0.02 ± 0.02
n_U	-2	-2.01 ± 0.02
n_D	+1	0.80 ± 0.04
n_{H_1}	-1	-0.96 ± 0.06
n_{H_2}	-1	-1.00 ± 0.02
$\tan\beta$	10	10.00 ± 0.13

Conclusion

- Supersymmetry has been motivated as a way to stabilize (and explain) the hierarchy
- If it is true, we expect exciting time at the near-future collider experiments
- Once seen and studied, they may be our telescope to physics at GUT and Planck/ string scales