## Introduction to the Standard Model

# 1. e+e- annihilation and QCD 

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In these lectures, I will describe the phenomenology of the Standard Model of particle physics.

I will discuss mainly processes seen in e+e- annihilation at CM energies from 10 to 200 GeV .

Later in the school, you will hear about proton-proton collisions at very high energy. But e+e-reactions are much simpler, so this is a good place to start.

Even at 10 GeV , the energy is high enough that we can ignore the masses of most of the quarks and leptons. Then we can see the Standard Model gauge interactions in a pure form and relate phenomena directly to the Standard Model Lagrangian.

You know the Standard Model as
a 4-d $\operatorname{SU}(3) \times S U(2) \times U(1)$ non-supersymmetric gauge theory that is chiral but anomaly-free
in which masses are forbidden by $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry
and therefore arise only by spontaneous symmetry breaking.

Fine. But, what do the particles of the Standard Model actually look like in the lab ?

Begin with the leptons: $e^{-}, \mu^{-}, \tau^{-}+\nu_{e}, \nu_{\mu}, \nu_{\tau}$
Begin, in fact, with the simplest reaction involving leptons

$$
e^{+} e^{-} \rightarrow \mu^{+} \mu^{-} \quad \text { in QED }
$$

At high energy, ignore all masses. Then, helicity

$$
h=\vec{S} \cdot \hat{P}
$$


is a Lorentz-invariant. Fermions are described as


The $e_{L}^{+}$is the antiparticle of the $e_{R}^{-}$and vice versa.

The amplitudes for $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$between states of definite helicity are very simple:

$$
\begin{aligned}
i \mathcal{M}\left(e_{L}^{-} e_{R}^{+}\right. & \left.\rightarrow \mu_{L}^{-} \mu_{R}^{+}\right) \\
& =2 i \frac{e^{2}}{q^{2}}\left[2 p_{1} \cdot k_{2} 2 p_{2} \cdot k_{1}\right]^{1 / 2} \\
& =i e^{2}(1+\cos \theta)
\end{aligned}
$$


that is,
$i \mathcal{M}\left(e_{L}^{-} e_{R}^{+} \rightarrow \mu_{L}^{-} \mu_{R}^{+}\right)=i \mathcal{M}\left(e_{R}^{-} e_{L}^{+} \rightarrow \mu_{R}^{-} \mu_{L}^{+}\right)=i e^{2}(1+\cos \theta)$
$i \mathcal{M}\left(e_{L}^{-} e_{R}^{+} \rightarrow \mu_{R}^{-} \mu_{L}^{+}\right)=i \mathcal{M}\left(e_{R}^{-} e_{L}^{+} \rightarrow \mu_{L}^{-} \mu_{R}^{+}\right)=i e^{2}(1-\cos \theta)$

These formulae lead to the regularities:

$$
\frac{d \sigma}{d \cos \theta} \sim\left(1+\cos ^{2} \theta\right) \quad \sigma=\frac{4 \pi \alpha^{2}}{3 s}=\frac{87 . \mathrm{fb}}{\left(E_{C M} \mathrm{TeV}\right)^{2}}
$$

with $s=q^{2}=\left(E_{C M}\right)^{2}$. The second formula sets the size of cross sections for all QED and electroweak processes.

Here are some examples of e+e- annihilation to leptons

$$
e^{+} e^{-} \rightarrow \mu^{+} \mu^{-} \quad e^{+} e^{-} \rightarrow \tau^{+} \tau^{-}
$$

and the related process of Bhabha scattering:


A relativistic muon is a "minimum ionizing" particle, giving a continuous small energy loss in matter:

$$
\frac{d E}{d x} \approx 1 \mathrm{GeV} / \mathrm{m}
$$

The muon is long-lived and essentially stable in high energy experiments.

$$
\tau=2.2 \times 10^{-6} \mathrm{sec} \quad c \tau=660 \mathrm{~m}
$$

Electrons emit hard gamma rays, and gammas convert to electron-positron pairs in a characteristic distance called a radiation length $X_{0}$. In heavy materials, $X_{0} \sim \mathrm{~cm}$

The result is an electromagnetic shower:


An electromagnetic calorimeter collects the resulting ionization, which gives a measure of the total energy of the original particle.

## SLD



$$
\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{z}^{0} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}
$$



$$
e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}
$$



$$
\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{z}^{0} \rightarrow \tau^{+} \tau^{-}
$$

For the $\tau$ lepton,

$$
\tau=2.9 \times 10^{-13} \mathrm{sec} \quad c \tau=0.087 \mathrm{~mm}
$$

The $\tau$ rapidly decays through weak interactions

$$
\begin{aligned}
\tau^{-} & \rightarrow \nu_{\tau} \bar{\nu}_{e} e^{-} \\
& \rightarrow \nu_{\tau} \bar{\nu}_{\mu} \mu^{-} \\
& \rightarrow \nu_{\tau} \bar{u} d, \quad \nu_{\tau} \bar{u} s \\
& \rightarrow \nu_{\tau} \pi^{-}, \quad \nu_{\tau} \rho^{-}, \\
& \nu_{\tau} a_{1}^{-}, \quad \cdots \\
& \quad / \\
& \pi^{-} \pi^{0} \\
& \\
& \\
& \\
&
\end{aligned}
$$

This produces a variety of $\tau$ events
$e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \rightarrow e \mu \nu \bar{\nu}$



Leptonic reactions are modified by radiative corrections.


I would like to examine in particular the diagrams giving initialstate radiation (ISR)

becomes singular when

$$
0 \leftarrow(p+q)^{2}=2 p \cdot q+p^{2} \sim-2 p_{\perp}^{2}
$$

In the limit in which the $e^{-}$and $\gamma$ are almost collinear, we can isolate the sigularity for $\gamma$ emission. Its coefficient has the form:

and this leads to the following approximate expression

$$
\begin{aligned}
\sigma\left(e^{-}\left(p_{1}\right) e^{+}\left(p_{2}\right) \rightarrow\right. & \left.\gamma+e^{-} e^{+} \rightarrow \gamma+X\right) \\
= & \int_{0}^{1} d x f(x) \sigma\left(e^{-}\left((1-x) p_{1}\right)+e^{+}\left(p_{2}\right) \rightarrow X\right) \\
& \quad+\int_{0}^{1} d x f(x) \sigma\left(e^{-}\left(p_{1}\right)+e^{+}\left((1-x) p_{2}\right) \rightarrow X\right)
\end{aligned}
$$

with

$$
f(x)=\frac{\alpha}{2 \pi} \int \frac{d p_{\perp}^{2}}{p_{\perp}^{2}} \frac{1+(1-x)^{2}}{x}=\frac{\alpha}{2 \pi} \cdot \log \frac{s}{m_{e}^{2}} \cdot \frac{1+(1-x)^{2}}{x}
$$

These singular terms give enhanced radiation in directions parallel to the momenta of the initial state particles (ISR) and also parallel to the momenta of the final state particles (FSR).


Here is a BaBar hadronic event with ISR


In addition to the leptonic processes, we also have

$$
e^{+} e^{-} \rightarrow q \bar{q}
$$

How do these reactions appear?

Quarks are not seen in isolation. If isolated quarks could be produced in e+e- annihilation, it would be obvious, because quarks carry fractional charge.

A property of the strong coupling phase of gauge theories is that the gauge charge is permanently confined into gauge singlet states.

A way to picture this is that the electric flux of the gauge field cannot spread into the vacuum. Since this flux is conserved, we have states such as


This object is a relativistic string. It has $E \sim L$
As the string becomes longer, it becomes energetically favorable to create an additional $q \bar{q}$ pair.

This leads to a first picture of the time development of a $q \bar{q}$ state created in e+e- annihilation:


Asymptotic freedom implies that all of these nonperturbative effects take place with no large momentum transfer.

Then, the final momenta are approximately collinear and parallel to the original quark directions.

Thus, we expect
the final states in $e^{+} e^{-} \rightarrow q \bar{q}$ should look like jets of mesons and baryons
the angular distribution of the axes of the jets should be

$$
\frac{d \sigma}{d \cos \theta} \sim\left(1+\cos ^{2} \theta\right)
$$

the cross section for such jet-like events should be

$$
\sigma=\frac{4 \pi \alpha^{2}}{3 s} \cdot 3 \cdot \sum_{f} Q_{f}^{2}
$$

$$
e^{+} e^{-} \rightarrow q \bar{q}
$$



BaBar





What do we see in such a jet ? The components are:
Long-lived charged hadrons: $\pi . K,{ }^{p} \quad c \tau\left(K^{+}\right)=3.7 \mathrm{~m}$ $\pi^{0{ }^{0}}{ }_{\mathrm{s}}: \quad \pi^{0} \rightarrow 2 \gamma$ with $c \tau=2.5 \times 10^{-6} \mathrm{~cm}$ Long-lived neutral hadrons: $n, K_{L}^{0}$
Short-lived hadrons with $c \tau \sim$ few $\mathrm{cm}: K_{S}^{0}, \quad \Lambda, \Sigma, \quad \Xi$
Charged particles appear as tracks.
$\gamma_{\mathrm{s}}$ appear as energy deposited in an electromagnetic calorimeter
Hadrons appear as energy deposition in a hadron calorimeter thick enough to allow many hadron interaction lengths

$$
\begin{array}{llll}
X_{0} & =0.56 \mathrm{~cm} \quad(\mathrm{~Pb}), & 1.76 \mathrm{~cm} & (\mathrm{Fe}), \\
\lambda_{I} & =17.1 \mathrm{~cm} \quad(\mathrm{~Pb}), & 16.8 \mathrm{~cm} & (\mathrm{Fe}), \\
\hline
\end{array}
$$

so by choosing different materials, it possible to separately measure the various components of the jet

Up to now we have ignored the asymptotically free QCD coupling

$$
\alpha_{s}=g_{s}^{2} / 4 \pi
$$

Let's now compute the effects of order $\alpha_{s}$ :
These include diagrams with virtual gluons



and diagrams with real gluon emission:

setting $\quad E_{q}=x_{q} \frac{\sqrt{s}}{2} \quad E_{\bar{q}}=x_{\bar{q}} \frac{\sqrt{s}}{2} \quad E_{g}=x_{g} \frac{\sqrt{s}}{2}$
the gluon emission diagrams predict:

$$
\frac{d \sigma}{d x_{q} d x_{\bar{q}}}=\sigma_{0} \cdot \frac{2 \alpha_{s}}{3 \pi} \cdot \frac{x_{q}^{2}+x_{\bar{q}}^{2}}{\left(1-x_{q}\right)\left(1-x_{\bar{q}}\right)}
$$

Notice that the gluon emission is singular as $x_{q}, x_{\bar{q}} \rightarrow 1$ These are configurations in which the gluon is collinear with the antiquark or quark.

It is possible integrate up the total cross section by working in

$$
d=4+\epsilon
$$

Then the real gluon emission gives

$$
\begin{aligned}
\sigma=\sigma_{0} \cdot \frac{2 \alpha_{s}}{3 \pi} \cdot & \left(\frac{8}{\epsilon^{2}}+\frac{1}{\epsilon}\left[4 \log \frac{s}{M^{2}}-6\right]\right. \\
& \left.+\log ^{2} \frac{s}{M^{2}}-3 \log \frac{s}{M^{2}}-\frac{7 \pi^{2}}{6}+\frac{57}{6}\right)
\end{aligned}
$$

The virtual diagrams give

$$
\sigma=\sigma_{0} \cdot\left\{1+\frac{2 \alpha_{s}}{3 \pi} \cdot\left(-\frac{8}{\epsilon^{2}}-\frac{1}{\epsilon}\left[4 \log \frac{s}{M^{2}}-6\right]\right.\right.
$$

in all

$$
\left.\left.-\log ^{2} \frac{s}{M^{2}}+3 \log \frac{s}{M^{2}}+\frac{7 \pi^{2}}{6}-8\right)\right\}
$$

$$
\sigma=\sigma_{0} \cdot\left(1+\frac{2 \alpha_{s}}{3 \pi} \cdot \frac{3}{2}\right)=\sigma_{0} \cdot\left(1+\frac{\alpha_{s}}{\pi}\right)
$$

The final result, and its generalization to higher orders, gives a small positive modification of the cross section for $e^{+} e^{-} \rightarrow q \bar{q}$

$$
\sigma=\sigma_{0} \cdot\left(1+\frac{\alpha_{s}(s)}{\pi}+1.441\left(\frac{\alpha_{s}(s)}{\pi}\right)^{2}-12.8\left(\frac{\alpha_{s}(s)}{\pi}\right)^{3}+\cdots\right)
$$

Gluon emission at wide angles gives
3-jet events at a rate proportional to $\alpha_{s}$
4-jet events at a rate proportional to $\alpha_{s}^{2}$ etc.


SLD 3-jet event


## SLD 4-jet event

- SLD - HERWIG 5.7




If a large fraction of $e^{+} e^{-} \rightarrow q \bar{q} \quad$ events contain gluon emissions, we need a systematic way to identify jets and to count the number of jets in an event.

Actually, since we only observe hadrons and not quarks, we need such a procedure anyway, to give a systematic way to compute, e.g., the orientation of the jet axis.

Among many possible ways to do this, a particularly useful one is based on thrust

$$
T=\max _{\hat{n}} \frac{\sum_{j}\left|\hat{n} \cdot \vec{p}_{j}\right|}{\sum_{j}\left|\vec{p}_{j}\right|}
$$

The axis that gives the maximum (the thrust axis) is a reasonable definition of the event axis.

QCD predicts in leading order

$$
\frac{d \sigma}{d T}=\sigma_{0} \cdot \frac{2 \alpha_{s}}{3 \pi}\left\{\frac{2\left(3 T^{2}-3 T+2\right)}{T(1-T)} \log \frac{2 T-1}{1-T}-\frac{3((3 T-2)(2-T)}{(1-T)}\right\}
$$




For multi-jet events, we need a way to partition the hadrons in the event into jets. Here is a particular method; later in the school, we will discuss other possible algorithms.
Let $y_{i j}=\frac{\left(p_{i}+p_{j}\right)^{2}}{s} \approx \frac{2 p_{i} \cdot p_{j}}{s} \approx \frac{1}{2} x_{i} x_{j}\left(1-\cos \theta_{i j}\right)$
Choose the 2 particles in the final state s.t. $\left(p_{i}+p_{j}\right)^{2}$ is minimal. Merge them and replace them with 1 particle with $p=\left(p_{i}+p_{j}\right)$. Continue until

$$
\frac{\left(p_{i}+p_{j}\right)^{2}}{s}>y_{c u t}
$$

Then the resulting $p_{i}$ can be taken as the operationally defined jet momenta.

Jets in QCD have a fractal structure, with more structure revealed as $y_{c u t}$ is decreased.



Thinking about how multiple gluons are emitted by a quark, we can arrive at a less naive model of how a $q \bar{q}$ state in e+e- annihilation evolves into a state of many hadrons.

Define the fragmentation function:
For a hadron $h$ in the final state of e+e- annihilation, let

$$
\text { longitudinal fraction } \quad z=E_{h} /\left(\frac{1}{2} \sqrt{s}\right)
$$

Then let

$$
f_{h}(z) d z
$$

be the probability of finding a hadron in the final state with longitudinal fraction $z$.


fragmentation functions measured by SLD

The variable $z$ gives the fraction of the original quark energy that ends up in a final handron. With QCD, we have studied the first stage of the degradation of the energy of the quark, due to gluon emission.

Let's look again at the formula for gluon radiation

$$
\frac{d \sigma}{d x_{q} d x_{\bar{q}}}=\sigma_{0} \cdot \frac{2 \alpha_{s}}{3 \pi} \cdot \frac{x_{q}^{2}+x_{\bar{q}}^{2}}{\left(1-x_{q}\right)\left(1-x_{\bar{q}}\right)}
$$

In the limit $\quad x_{\bar{q}} \rightarrow 1 \quad\left(\mathrm{q}+\mathrm{g}\right.$ collinear; $x_{q} \approx\left(1-x_{g}\right)$ ), integrating over $x_{\bar{q}}$, this becomes

$$
\frac{d \sigma}{d x_{g}} \approx \sigma_{0} \cdot \frac{4}{3} \frac{\alpha_{s}}{2 \pi} \cdot \log \frac{s}{m^{2}(q g)_{\min }} \cdot \frac{1+\left(1-x_{g}\right)^{2}}{x_{g}}
$$

This is the same radiation formula that we saw in QED.

This calculation describes the emission of one approximately collinear gluon. By repeating the process, we can describe multiple gluon emissions.

These multiple gluons build up the structure of a quark jet.


The quark first emits hard gluons. As we go to smaller $p_{\perp}$ or smaller virtualities, the quarks and gluons emit further quarks and gluons.


Each emission degrades the energy of the final hadrons. This effect of collinear gluon emission from the quark is described by the Gribov-Lipatov equation

$$
\frac{d f_{q}(x, Q)}{d \log Q}=\frac{\alpha_{s}}{\pi} \int_{0}^{1} \frac{d z}{z} P_{q \leftarrow q}(z) f_{q}\left(\frac{x}{z}, Q\right)
$$

Here $P(z)$ is the splitting function

$$
P_{q \leftarrow q}(z)=\frac{4}{3}\left(\frac{1+z^{2}}{(1-z)}-A \delta(1-z)\right)
$$

where $A$ is determined by the condition of quark number conservation

$$
\int_{0}^{1} d z P_{q \leftharpoondown q}(z)=0
$$

The singularity is normally treated in QCD by defining a distribution that agrees with $1 /(1-z)$ for $z<1$ such that

$$
\int_{0}^{1} d z \frac{1}{(1-z)_{+}}=0
$$

The full dynamics of QCD contains a number of possible collinear splitting processes. Each leads to a characteristic kernel, the splitting functions

$$
\begin{aligned}
& P_{q \leftarrow q}(z)=\frac{4}{3}\left(\frac{1+z^{2}}{(1-z)_{+}}+\frac{3}{2} \delta(1-z)\right) \\
& P_{g \leftarrow q}(z)=\frac{4}{3}\left(\frac{1+(1-z)^{2}}{z}\right) \\
& P_{q \leftarrow g}(z)=\frac{1}{2}\left(z^{2}+(1-z)^{2}\right) \\
& P_{g \leftarrow g}(z)=6\left(\frac{1-z}{z}+\frac{z}{(1-z)_{+}}+z(1-z)+\left(\frac{11}{12}-\frac{n_{f}}{18}\right) \delta(1-z)\right)
\end{aligned}
$$

Putting these effects together, we obtain the Altarelli-Parisi equations. For example

$$
\frac{d f_{h \leftarrow q}(x, Q)}{d \log Q}=\frac{\alpha_{s}}{\pi} \int_{0}^{1} \frac{d z}{z}\left\{P_{q \rightarrow q}(z) f_{h \leftarrow q}\left(\frac{x}{z}, Q\right)+P_{g \leftarrow q}(z) f_{h \leftarrow g}\left(\frac{x}{z}, Q\right)\right\}
$$

According to this equation, fragmentation functions evolve as a function of $s$. This effect is observed.



PDG (Biebel and Webber) compilation

Finally, we can ask about the value of $\alpha_{s}$ needed to explain the various QCD effects we have discussed in this lecture.

The hallmark of a gauge theory is that there is a single unified coupling constant. Is experiment consistent with this?
e+e- event shape variables only



We have now seen that the strong interactions at high energy show very directly the quark-gluon structure of an $\mathrm{SU}(3)$ gauge theory - QCD.

In the next lecture, we will examine the weak interactions at high energy. We will see that these also reflect the gauge stuctures of the Standard Model.

