## Introduction to the Standard Model

# 2. Weak interactions at high energy 

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In the previous lecture, we discussed the description of the strong interactions by an SU(3) Yang-Mills theory.

Now I would like to discuss the description of the weak interactions by an $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge theory -the Glashow-Salam-Weinberg model.

Before detailing the theory, I would like to present the evidence for its key component:

Weak interactions violate parity. But, more completely,
Only left-handed quarks and leptons couple to charge-changing weak interactions. ( "V-A coupling" )

This is the insight of Feynman, Gell-Mann, Marshak, and Sudarshan. At the time that these authors proposed this theory, the experimental situation seemed to contradict it. In the face of that situation, the original papers display a remarkable insistence on the power of this simple theoretical picture.

However historically inaccurate it may be, I recommend the account in "Surely You're Joking, Mr. Feynman":

Telegdi sent us a letter, which wasn't exactly scathing, but ... At the end, he wrote, "The F-G (Feynman-Gell-Mann) theory of beta decay is no F-G."

Murray says, "What should we do about this? You know, Telegdi's pretty good."

I say, "We just wait."
Two days later, there's another letter from Telegdi. He's a complete convert...

From today's perspective, there are many sharp tests of V-A.
For example,
The electron polarization in beta decay is given by

$$
P\left(e^{-}\right)=-\frac{v_{e}}{c}
$$

The distributions of $y=E_{\text {hadrons }} / E_{\nu} \quad$ in deep-inelastic neutrino scattering have characteristic shapes.


CDHS expt.

The sharpest tests, however, come from the precision study of muon decay: $\mu^{-} \rightarrow e^{-} \bar{\nu}_{e} \nu_{\mu}$

From

$$
\Delta \mathcal{L}=\frac{4 G_{F}}{\sqrt{2}} \bar{u}_{L}\left(\nu_{\mu}\right) \gamma^{\mu} u_{L}(\mu) \bar{u}_{L}\left(e^{-}\right) \gamma_{\mu} v_{L}\left(\bar{\nu}_{e}\right)
$$

we have

$$
|\mathcal{M}|^{2}=2\left(\frac{4 G_{F}}{\sqrt{2}}\right)^{2}\left(2 p_{\mu} \cdot p_{\bar{\nu}_{e}}\right)\left(2 p_{e} \cdot p_{\nu_{\mu}}\right)
$$

Introduce the variables $\quad x_{i}=\frac{E_{i}}{\left(m_{\mu} / 2\right)} \quad$; the

$$
\begin{aligned}
2 p_{\mu} \cdot p_{\bar{\nu}_{e}}=2 m_{\mu}^{2} x_{\bar{\nu}} \quad\left(2 p_{e} \cdot p_{\nu_{\mu}}\right) & =\left(p_{\mu}-p_{\bar{\nu}_{e}}\right)^{2}=p_{\mu}^{2}-22 p_{\mu} \cdot p_{\bar{\nu}_{e}} \\
& =2 m_{\mu}^{2}\left(1-x_{\bar{\nu}}\right)
\end{aligned}
$$

Integrating over the phase space, $x_{e}+x_{\nu}+x_{\bar{\nu}}=2 \quad x_{i} \leq 1$
we find:

$$
\frac{d \Gamma}{d x_{e}} \sim \frac{1}{2} x_{e}^{2}\left(1-\frac{2}{3} x_{e}\right)
$$


M.Bardon, et al.

At the endpoint (the maximum e- energy), the e-should be produced only in the opposite direction from the muon spin.


This has been tested in the following experiment, done at TRIUMF:


B. Balke et al, Berkeley-TRIUMF

Now complete this model to a spontaneously broken non-Abelian gauge theory:

$$
\mathcal{L}=-\frac{1}{4}\left(F_{\mu \nu}^{a}\right)^{2}-\frac{1}{4}\left(G_{\mu \nu}\right)^{2}+\left|D_{\mu} \phi\right|^{2}-V(|\phi|)
$$

$\phi$ is a complex scalar field, the Higgs field, with $I=\frac{1}{2}, \quad Y=\frac{1}{2}$ This is just the simplest of many possible choices for the mechanism of symmetry breaking. From now on I will just assume that $V(\phi)$ has a symmetry-breaking form.

Higgs himself would not approve of this attitude.
Rotate in SU(2) so that $\quad\langle\phi\rangle=\frac{1}{\sqrt{2}}\binom{0}{v}$


Then

$$
\begin{aligned}
\left|D_{\mu} \phi\right|^{2} \rightarrow & \phi^{*}\left|-i g A_{\mu}^{a} \tau^{a}-i g^{\prime} \frac{1}{2} B_{\mu}\right|^{2} \phi \\
& =\frac{1}{2}\left(A_{\mu}^{a}, B_{\mu}\right) \mathbf{m}^{2}\binom{A_{\mu}^{a}}{B_{\mu}}
\end{aligned}
$$

The mass matrix is

$$
\mathbf{m}^{2}=\frac{v^{2}}{4}\left(\begin{array}{llll}
g^{2} & & \\
& g^{2} & & \\
& & g^{2} & -g g^{\prime} \\
& & & -g g^{\prime} \\
& g^{\prime 2}
\end{array}\right)
$$

Its eigenstates are

$$
\begin{aligned}
W^{ \pm} & =\frac{1}{\sqrt{2}}\left(A^{1} \mp i A^{2}\right) & & \\
A & =s_{w} A^{3}+c_{w} B & & s_{w}=\sin \theta_{w} \\
Z & =c_{w} A^{3}-s_{w} B & & c_{w}=\cos \theta_{w}
\end{aligned}
$$

The gauge covariant derivative takes the form
$D_{\mu}=\partial_{\mu}-i \frac{g}{\sqrt{2}}\left(W_{\mu}^{+} \sigma^{+}+W_{\mu}^{-} \sigma^{-}\right)-i e Q A_{\mu}-i \frac{e}{s_{w} c_{w}}\left(I^{3}-s_{w}^{2} Q\right) Z_{\mu}$
where $I^{3}=\frac{\sigma^{3}}{2} \quad$ and $\sigma$ acts only on left-handed fermions.

These couplings lead to modifications of the cross sections for $e^{-} e^{+} \rightarrow f \bar{f}$

$i \mathcal{M}\left(e_{L}^{-} e_{R}^{+} \rightarrow f_{L} \bar{f}_{R}\right)=i e^{2}(1+\cos \theta) \cdot s$

$$
\cdot\left[-\frac{Q}{s}+\frac{\left(-\frac{1}{2}+s_{w}^{2}\right)\left(I^{3}-Q s_{w}^{2}\right)}{s_{w}^{2} c_{w}^{2}} \frac{1}{s-m_{Z}^{2}}\right]
$$

$i \mathcal{M}\left(e_{L}^{-} e_{R}^{+} \rightarrow f_{R} \bar{f}_{L}\right)=i e^{2}(1-\cos \theta) \cdot s$

$$
\left[-\frac{Q}{s}+\frac{\left(-\frac{1}{2}+s_{w}^{2}\right)\left(-Q s_{w}^{2}\right)}{s_{w}^{2} c_{w}^{2}} \frac{1}{s-m_{Z}^{2}}\right]
$$

Notice that these formulae lead to all possible P, C violating effects - dependence on e- polarization, dependence of f polarization, forward-backward asymmetry in $e^{-} e^{+} \rightarrow f \bar{f}$

It is worth pausing to give a precise definition of asymmetries. Take the forward-backward asymmetry as an example

$$
A_{F B}=\frac{N(\cos \theta>0)-N(\cos \theta<0)}{N(\cos \theta>0)+N(\cos \theta<0)}
$$

for pure $e_{L}^{-} e_{R}^{+} \rightarrow f_{L} \bar{f}_{R} \quad \frac{d \sigma}{d \cos \theta} \sim(1+\cos \theta)^{2}$

$$
\begin{gathered}
N(\cos \theta>0) \sim \int_{0}^{1} d \cos \theta(1+\cos \theta)^{2}=\frac{7}{3} \\
N(\cos \theta<0) \sim \int_{-1}^{0} d \cos \theta(1+\cos \theta)^{2}=\frac{1}{3} \\
A_{F B}=\frac{7-1}{7+1}=\frac{3}{4}
\end{gathered}
$$

This corresponds to $88 \%$ forward, $12 \%$ backward.


Mark-J at 34.6 GeV

DELPHI



In the comparison to data, for CM energies up to 200 GeV at LEP, there is an important effect that must be taken into account
$e^{+} e^{-} \rightarrow \gamma+Z$


This cross section is strongly enhanced when the $\gamma$ is emitted collinear to the initial electron or positron

$$
\begin{aligned}
& \sigma \approx 2 \int_{0}^{1} d x \frac{\alpha}{2 \pi} \log \frac{s}{m_{e}^{2}} \frac{1+(1-x)^{2}}{x} \cdot \sigma\left(e^{+} e^{-} \rightarrow Z ; \hat{s}=(1-x) s\right) \\
& \sigma\left(e^{+} e^{-} \rightarrow Z ; \hat{s}\right)=\frac{2 \pi^{2} \alpha}{3 s_{w}^{2} c_{w}^{2}}\left(\frac{1}{4}-s_{w}^{2}+2 s_{w}^{4}\right) \delta\left(\hat{s}-m_{Z}^{2}\right)
\end{aligned}
$$

so $x=1-m_{Z}^{2} / s$ is picked out. The final cross section is larger than the direct cross section for $e^{+} e^{-} \rightarrow f \bar{f}$


In the limit of very high energies
$Q_{e}=-1 \quad I_{e}^{3}=-\frac{1}{2}, 0$

$$
\begin{aligned}
e^{2} & {\left[\frac{Q_{e} Q_{f}}{s}+\frac{\left(I_{e}^{3}-Q_{e} s_{w}^{2}\right)\left(I_{f}^{3}-Q_{f} s_{w}^{2}\right)}{s_{w}^{2} c_{w}^{2}} \frac{1}{s-m_{Z}^{2}}\right] } \\
& \rightarrow \frac{e^{2}}{s}\left[\frac{I_{e}^{3} I_{f}^{3}}{s_{w}^{2}}+\frac{I_{e}^{3} I_{f}^{3}}{c_{w}^{2}}-\frac{Q_{e} I_{f}^{3}+I_{e}^{3} Q_{f}}{c_{w}^{2}}+Q_{e} Q_{f}\left(1+\frac{s_{w}^{2}}{c_{w}^{2}}\right)\right] \\
& =\frac{e^{2}}{s}\left[\frac{I_{e}^{3} I_{f}^{3}}{s_{w}^{2}}+\frac{\left(I_{e}^{3}-Q_{3}\right)\left(I_{f}^{3}-Q_{f}\right)}{c_{w}^{2}}\right] \\
& =\frac{1}{s}\left[g^{2} I_{e}^{3} I_{f}^{3}+g^{\prime 2} Y_{e} Y_{f}\right]
\end{aligned}
$$

that is, $S U(2) \times U(1)$ is restored at high energies.

data compilation by Hildreth

Next, look at the W boson. What do W's look like ?
$W^{+}$decays to $d_{L}+\bar{u}_{R}$ in each (light) $\operatorname{SU}(2)$ doublet of the SM

$$
\begin{aligned}
i \mathcal{M}\left(W^{+} \rightarrow \nu_{e} e^{+}\right) & =i \frac{g}{\sqrt{2}} \bar{u}_{L}\left(\nu_{e}\right) \gamma_{\mu} v_{L}\left(e^{+}\right) \epsilon^{\mu}\left(W^{+}\right) \\
\Gamma\left(W^{+} \rightarrow \nu_{e} e^{+}\right) & =\frac{g^{2}}{48 \pi} m_{W}=220 \mathrm{MeV}
\end{aligned}
$$

the partial widths to $\nu_{\mu} \mu^{+}, \nu_{\tau} \tau^{+}$are the same. the partial widths to quarks are multiplied by $3 \cdot\left(1+\alpha_{s} / \pi\right)$

Then

$$
\begin{aligned}
B R\left(W^{+} \rightarrow \nu_{e} e^{+}\right) & =11 \% \\
B R\left(W^{+} \rightarrow u \bar{d}\right) & =34 \%
\end{aligned}
$$

This leads to an interesting variety of $e^{+} e^{-} \rightarrow W^{+} W^{-}$events

$$
\begin{aligned}
\mathrm{e}^{+} \mathrm{e}^{-} & \rightarrow \mathrm{W}^{+} \mathrm{W}^{-} \\
& \rightarrow \mathrm{e}^{-} \overline{\mathrm{v}} \mu \mathrm{v}
\end{aligned}
$$



$$
e^{+} e^{-} \rightarrow W^{+} W^{-} \rightarrow q \bar{q} \tau \nu
$$




These events can be used to make a precision measurement of the W boson mass.

For example, look at $e^{+} e^{-} \rightarrow W^{+} W^{-} \rightarrow(q \bar{q})(\ell \nu)$
This leads to final states with 2 jets, an isolated lepton, and missing (unseen) momentum.


Determine the axes of the jets, leaving the energies unkown.

$$
\vec{p}_{1}=E_{1} \hat{n}_{1} \quad \vec{p}_{2}=E_{2} \hat{n}_{2} \quad \vec{p}_{\ell}=E_{\ell} \hat{n}_{\ell} \quad \vec{p}_{\nu}=E_{\nu} \hat{n}_{\nu}
$$

Fix the jet energies and the neutrino parameters from EM conservation and

$$
\left(p_{1}+p_{2}\right)^{2}=\left(p_{\ell}+p_{\nu}\right)^{2}=m_{W}^{2}
$$

There are 5 parameters, 5 unknowns. ISR, quark fragmentation, give small, estimable, corrections.

## OPAL $\sqrt{s}=189 \mathbf{G e V}$



Finally, study the cross section $\sigma\left(e^{+} e^{-} \rightarrow W^{+} W^{-}\right)$
Immediately, there is a problem

$$
\sim \bar{v} \gamma^{\mu} u \frac{1}{s}\left(k_{+}-k_{-}\right)_{\mu} \epsilon_{+}^{*} \cdot \epsilon_{-}^{*}
$$

$W^{+}$has 3 polarization states. In the rest frame $\epsilon^{\mu}=(0, \hat{n})^{\mu}$
but for a W in motion

$$
p^{\mu}=\left(E_{W}, 0,0, k_{W}\right)
$$

$$
\begin{aligned}
\epsilon_{R} & =\frac{1}{\sqrt{2}}(0,1, i, 0) \\
\epsilon_{L} & =\frac{1}{\sqrt{2}}(0,1,-i, 0) \\
\epsilon_{0} & =\frac{1}{m_{W}}\left(k_{W}, 0,0, E_{W}\right) \approx p^{\mu} / m_{W}
\end{aligned}
$$

Notice that

$$
\epsilon_{+0}^{*} \cdot \epsilon_{-0}^{*}=\frac{E_{W}^{2}+k_{W}^{2}}{m_{W}^{2}} \approx \frac{s}{2 m_{W}^{2}}
$$

This is trouble; unitarity requires $\left|i \mathcal{M}\left(e^{+} e^{-} \rightarrow W^{+} W^{-}\right)\right|<$const in each partial wave.

Similar issues affect the study of another SM object that I should introduce you to, the top quark.

$$
m_{t} \approx 175 \mathrm{GeV} \quad m_{b} \approx 4 \mathrm{GeV}
$$

so $t$ decays, not by a Fermi weak interaction, but rather by the direct 2-body decay process

$$
t \rightarrow W^{+} b
$$

One would expect $\quad \Gamma_{t} \sim \frac{\alpha}{6 s_{w}^{2}} m_{t} \approx 1 \mathrm{GeV}$
In fact, $\Gamma_{t}$ is slightly larger. Consider $\quad t \rightarrow W_{0}^{+} b$

$$
\begin{aligned}
i \mathcal{M} & =i \frac{g}{\sqrt{2}} \bar{u}_{L}(b) \gamma^{\mu} \epsilon_{\mu}^{*} u_{R}(t) \\
& \approx i \frac{g}{\sqrt{2}} \bar{u}_{L}(b) \frac{\not x_{+}}{m_{W}} u_{R}(t)=i \frac{g}{\sqrt{2}} \bar{u}_{L}(b) \frac{\not p_{t}-\not p_{b}}{m_{W}} u_{R}(t) \\
& \approx i \frac{g}{\sqrt{2}} \frac{m_{t}}{m_{W}} \bar{u}_{L}(b) u_{R}(t)
\end{aligned}
$$

This leads to

$$
\Gamma_{t}=\frac{g^{2}}{64 \pi} \frac{m_{t}^{3}}{m_{W}^{2}}\left(1-\frac{m_{W}^{2}}{m_{t}^{2}}\right)^{2}\left(1+2 \frac{m_{W}^{2}}{m_{t}^{2}}\right)
$$

which gives

$$
\Gamma_{t} \approx 1.7 \mathrm{GeV}
$$

and

$$
P_{0}=\frac{\Gamma_{t}\left(\rightarrow W_{0}^{+}\right)}{\Gamma_{t}}=70 \%
$$

The final $t$ decay products are $t \rightarrow b q \bar{q}, \quad t \rightarrow b \ell \nu$ with the same fractions as for W decay.

Does the dependence $m_{t}^{3} / m_{W}^{2} \quad$ make sense ?
Think about the unbroken gauge theory of $\mathrm{SU}(2) \times \mathrm{U}(1)$. In this theory, the Higgs doublet $\phi$ couples to t . Its Yukawa coupling is large,

$$
\lambda_{t}=\frac{\sqrt{2} m_{t}}{v}=\frac{g}{\sqrt{2}} \frac{m_{t}}{m_{W}}
$$


where $\pi^{+}$is the Goldstone boson eaten by the $W^{+}$in the Higgs mechanism. The matrix element is

$$
i \mathcal{M}=i \lambda_{t} \bar{u}_{L}(b) u_{R}(t)=i \frac{g}{\sqrt{2}} \frac{m_{t}}{m_{W}} \bar{u}_{L}(b) u_{R}(t)
$$

which is exactly what we found for

$$
t \rightarrow W_{0}^{+} b
$$

This illustrates Goldstone boson equivalence: a $W_{0}^{+}$at high energy has the couplings of the Goldstone boson that it ate to obtain mass.

In $e^{+} e^{-} \rightarrow W^{+} W^{-}$, Goldstone boson equivalence implies

$$
i \mathcal{M}\left(e_{L}^{-} e_{R}^{+} \rightarrow W_{0}^{+} W_{0}^{-}\right)=
$$



$$
=i e^{2}\left[\frac{1}{4 c_{w}^{2}}+\frac{1}{4 s_{w}^{2}}\right] \bar{v} \gamma^{\mu} u \frac{1}{s}\left(k_{+}-k_{-}\right)_{\mu}
$$

This requires a delicate cancellation among the diagrams


You can check that this cancellation occurs in the $\mathrm{SU}(2) \mathrm{xU}(1)$ gauge theory. It takes place only if the form of the 3-boson vertex is exactly that given by Yang-Mills theory.

What does experiment have to say about this ?


We have now seen that the Standard Model gives an excellent qualitative description of the behavior of weak interactions at high energy.

In the next lecture, we will see how this theory works when tested quantitatively at a higher level of precision.

