Introduction to the Standard Model

3. Precision studies of the Z boson

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In the previous lecture, we discussed the structure of the weak interactions and their description by the Standard Model gauge theory of SU(2) x U(1).

Now I would like to look at the weak interactions at a higher level of precision. During the 1990's, experiments at LEP and SLC made detailed measurements of the properties of the Z boson as a resonance in e+e- annihilation. Let's review what these experiments found.

The coupling of each chiral fermion flavor to the Z boson is proportional to the charge

$$Q_Z = I^3 - Q s_w^2$$

We can test the Standard Model description of the Z in a powerful way by measuring these charges individually and seeing whether they are accounted for by a single universal parameter s_w^2 .

In fact, the accuracy of the experiments was such that order-α radiative correction must be included to interpret the results. This brings in some additional issues that bear on the Standard Model and its extensions.

But, first, let's discuss the experiments.

To begin, let's make a table of the Q_Z for an illustrative value $s_w^2 = 0.231$

species	Q_{ZL}	Q_{ZR}	S_f	A_f
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	—	0.25	1
e, μ, τ	$-\frac{1}{2} + \sin^2\theta_w$	$\sin^2\theta_w$	0.126	0.15
u, c, t	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_w$	$-\frac{2}{3}\sin^2\theta_w$	0.144	0.67
d, s, b	$-\frac{1}{2} + \frac{1}{3}\sin^2\theta_w$	$\frac{1}{3}\sin^2\theta_w$	0.185	0.94

where

$$S_f = Q_{ZL}^2 + Q_{ZR}^2 \qquad A_f = \frac{Q_{ZL}^2 - Q_{ZR}^2}{Q_{ZL}^2 + Q_{ZR}^2}$$

The S_f give the total rate of Z decay to that species.

The A_f give the parity asymmetries in Z decays.

The partial width of the Z into a fermion species f is given by:

$$\Gamma(Z \to f\overline{f}) = \frac{\alpha m_Z}{6 s_w^2 c_w^2} \cdot S_f$$

times the factor $3(1+\alpha_s/\pi)$ = 3.04 for quarks.

This gives the following table of partial widths and branching ratios:

species	$\Gamma(Z \to f\overline{f})$	BR
ν_e, ν_μ, ν_τ	167 MeV	6.7%
e, μ, τ	84 MeV	3.4%
u, c	300 MeV	12.0%
d, s, b	383 MeV	15.3%

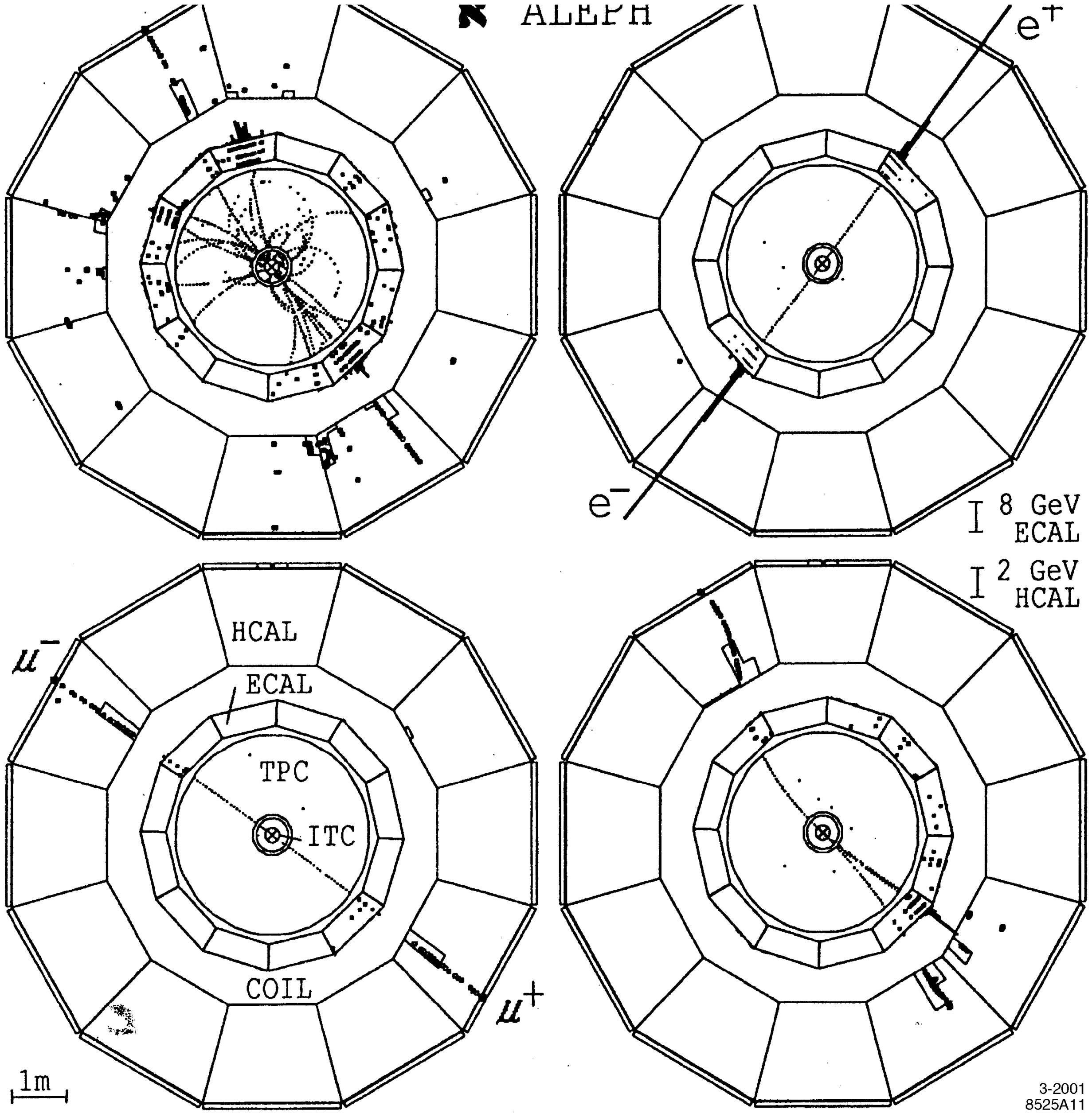
Including a small correction for the case of $\Gamma(Z \to b\overline{b})$, we find a total width

$$\Gamma_Z = 2.50 \text{ GeV}$$

To test these predictions, we first measure e+e- annihilation at the Z resonance and determine the relative branching ratios to hadrons and to visible leptons.

This involves sorting the Z events into the categories discussed in the first lecture.

Z decay event types, according to ALEPH



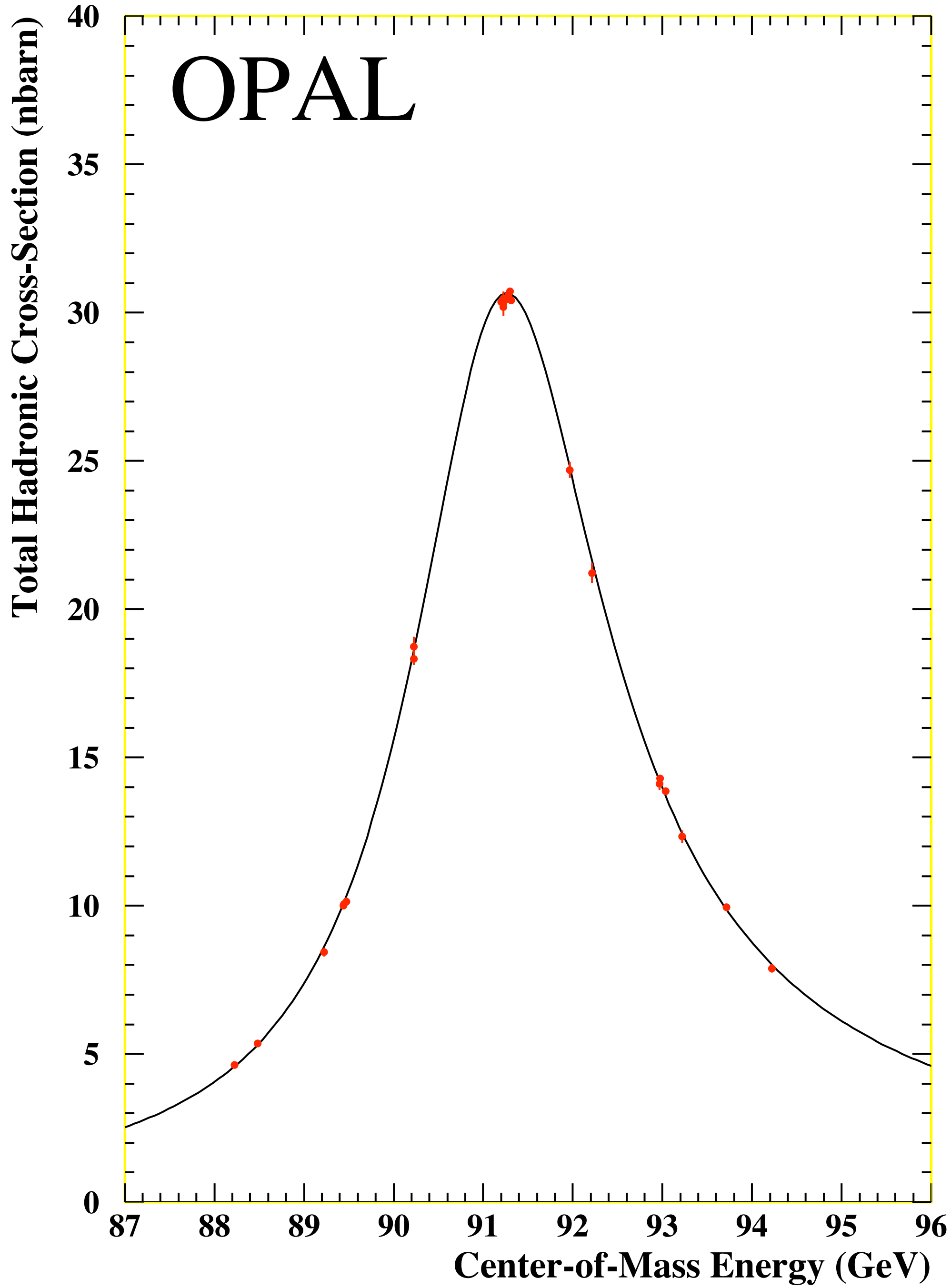
To test these predictions, we first measure e+e- annihilation at the Z resonance and measure the relative branching ratios to hadrons and to visible leptons.

Then we must determine the total width.

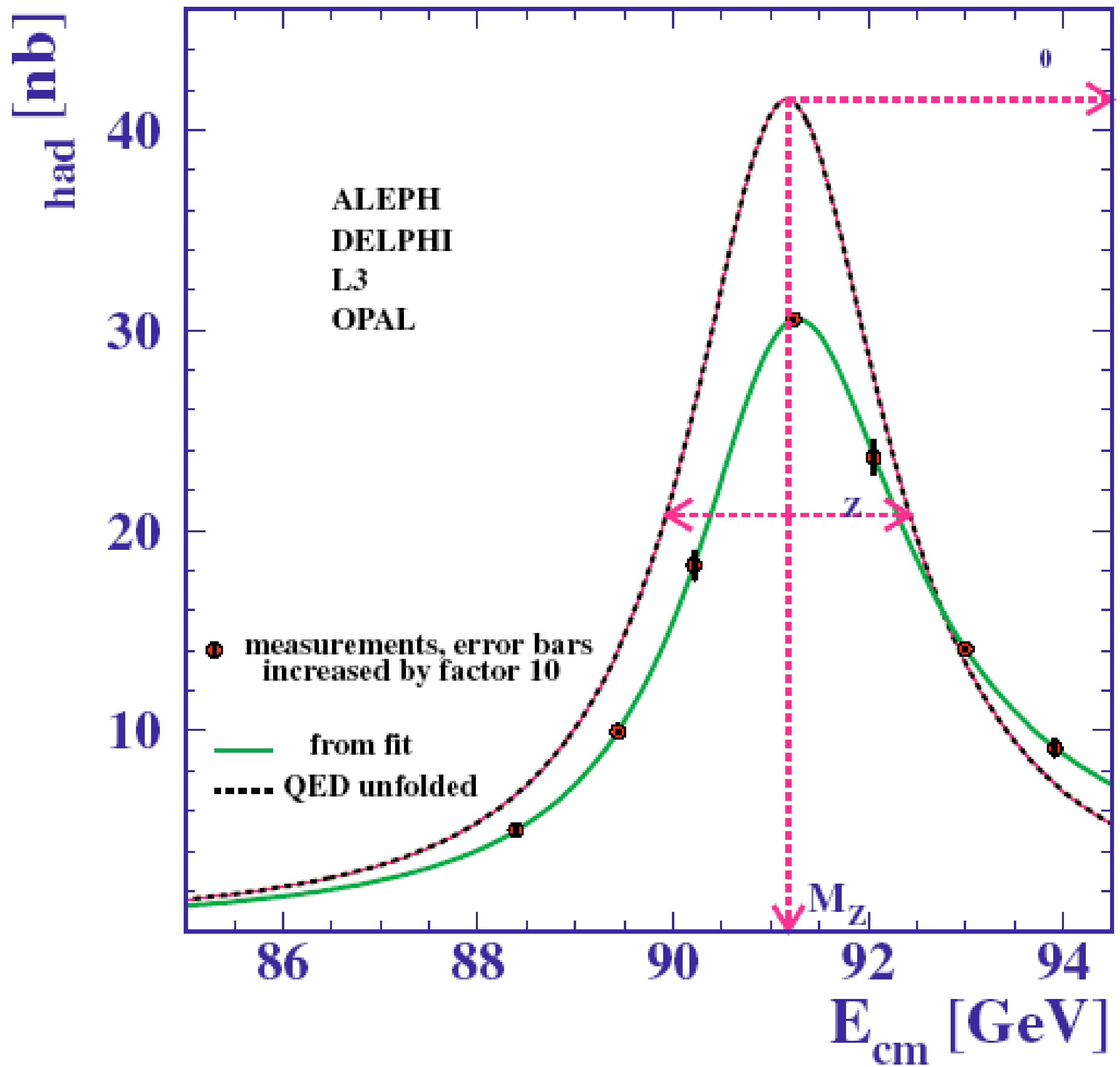
The shape of the resonance is distorted by initial-state photon radiation. Thus, it is necessary to measure the detailed shape of the resonance to extract Γ_Z.

It is amusing to note that all three of the Standard Model interactions - QED, QCD, and of course SU(2) x U(1) contribute to the Z line-shape.

The result is: $\Gamma_Z = 2.4952 \pm .0023 \text{ MeV}$

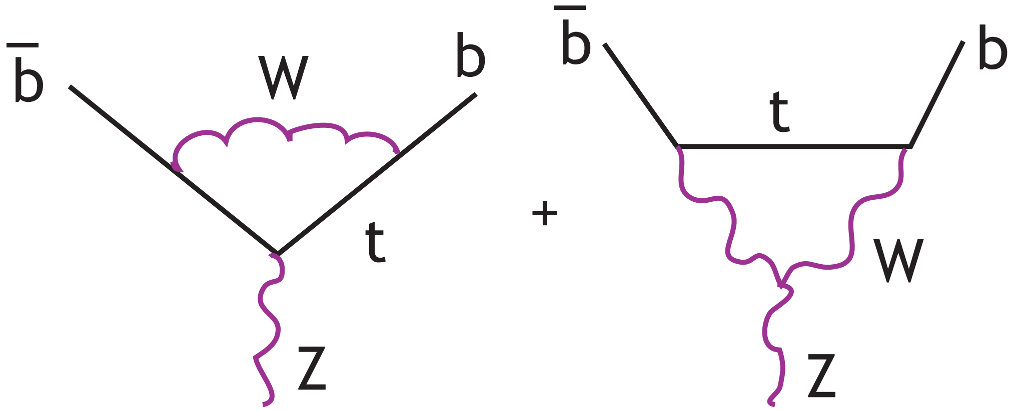


OPAL
Total Hadronic Cross-Section (nbarn)
Center-of-Mass Energy (GeV)
0
5
10
15
20
25
30
35
40
87
88
89
90
91
92
93
94
95
96



composite of the four LEP experiments, showing the effect of ISR

There is a special consideration for the b quark. The diagrams



contribute a correction to the b_L Z charge,

$$Q_{ZbL} = -(\frac{1}{2} - \frac{1}{3}s_w^2 - \frac{\alpha}{16\pi s_w^2}\frac{m_t^2}{m_W^2})$$

This is a -2% correction to the partial width. It is easier to measure the quantity

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \ \mathrm{hadrons})}$$

which is almost independent of s_w^2 and so directly tests the above correction.

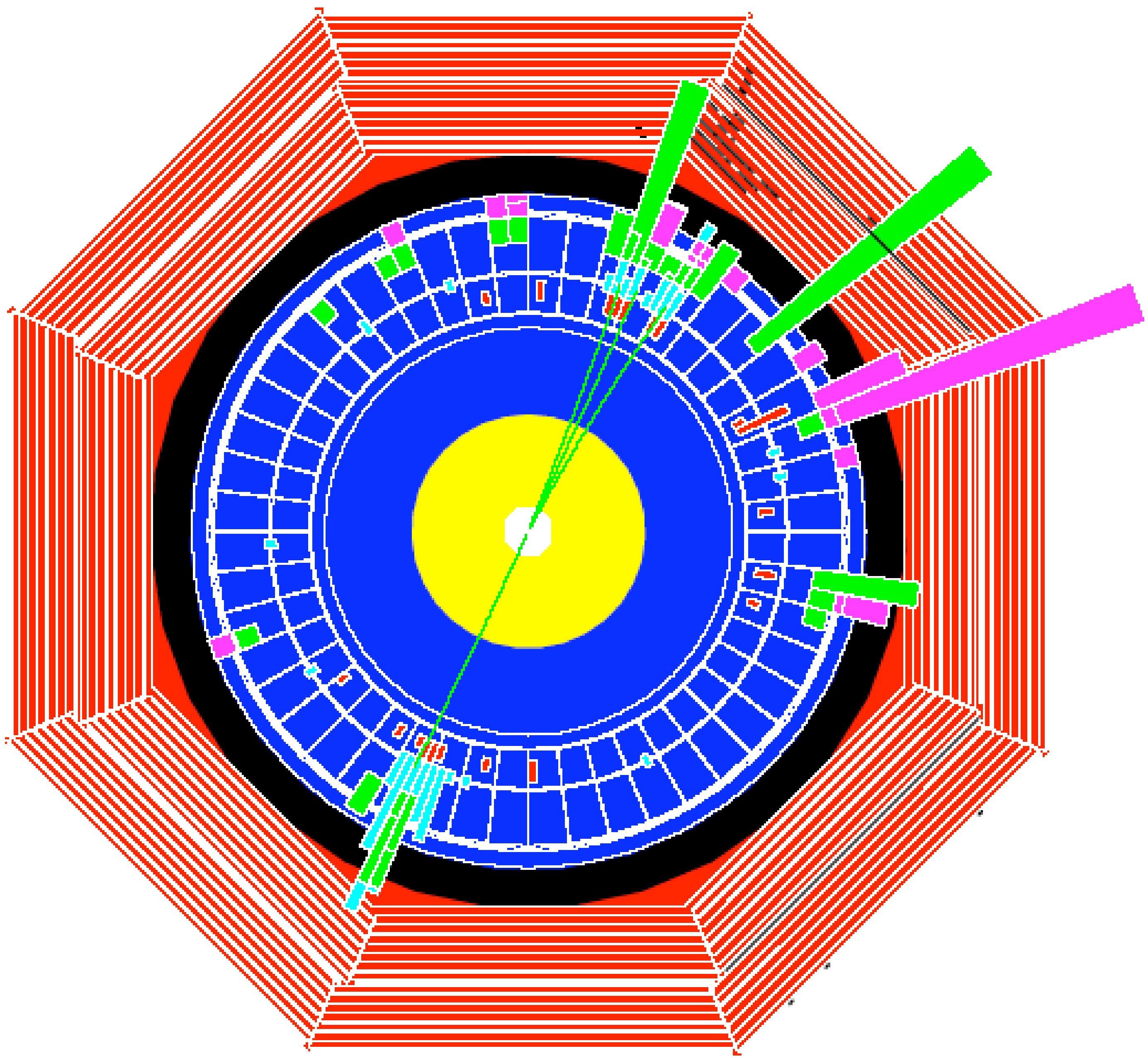
But how do we know which hadronic events contain b quarks ?

The three heavy fermions τ , c , b have weak-interaction decay times that are small but measurable. With a special-purpose device based on silicon strips or pixels, one can locate the decay vertices and identify the short-lived particles that they indicate.

	τ (ps)	$c\tau$ (mm)
τ	0.29	0.09
c (D^0)	0.41	0.12
b (B^0)	1.55	0.46

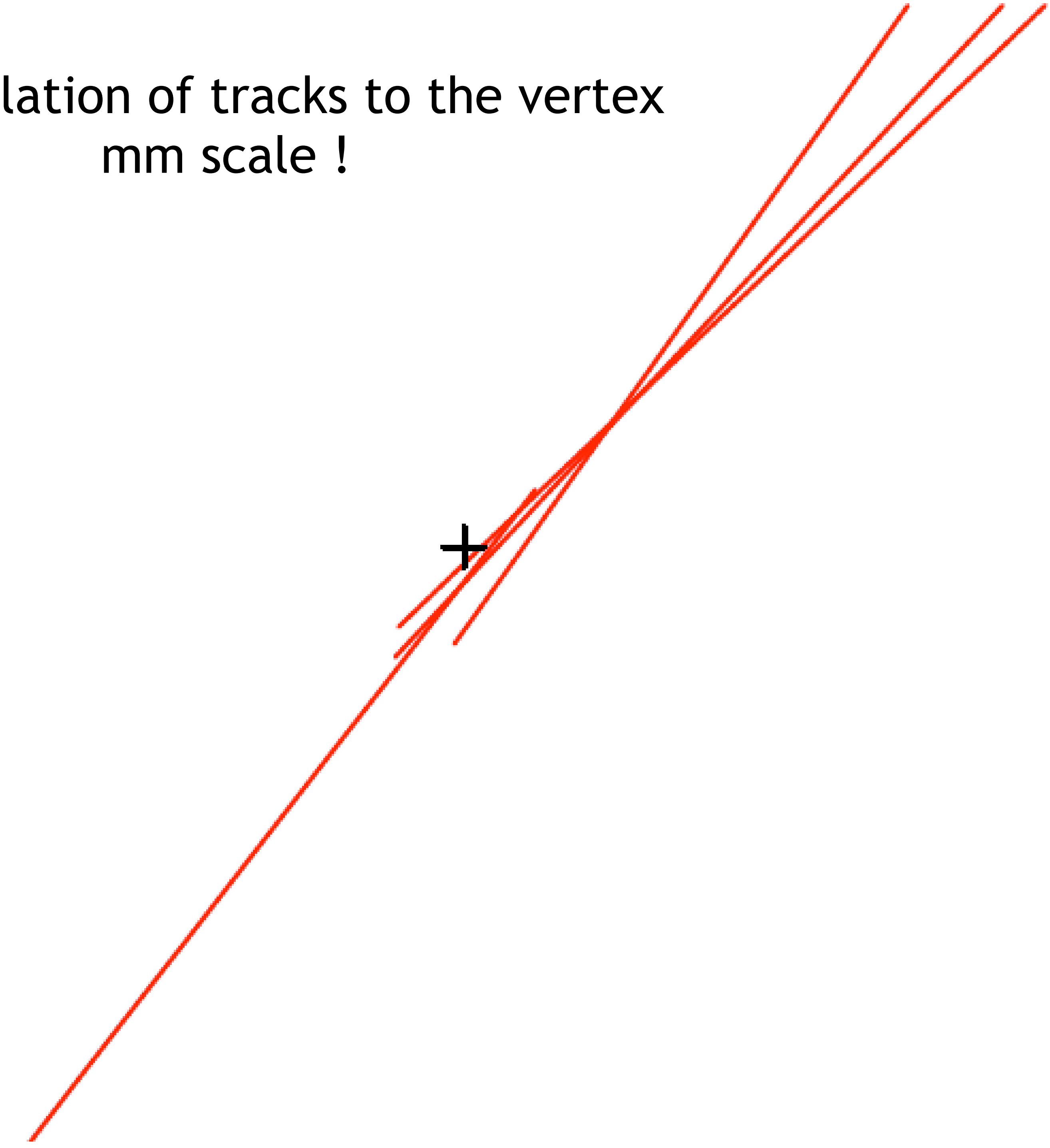
In analyses of this type, it is a challenge to the experimenters to choose a signal criterion that maximizes the efficiency of observing heavy quarks vertices while minimizing fakes.

$e^+e^- \rightarrow \tau^+\tau^-$

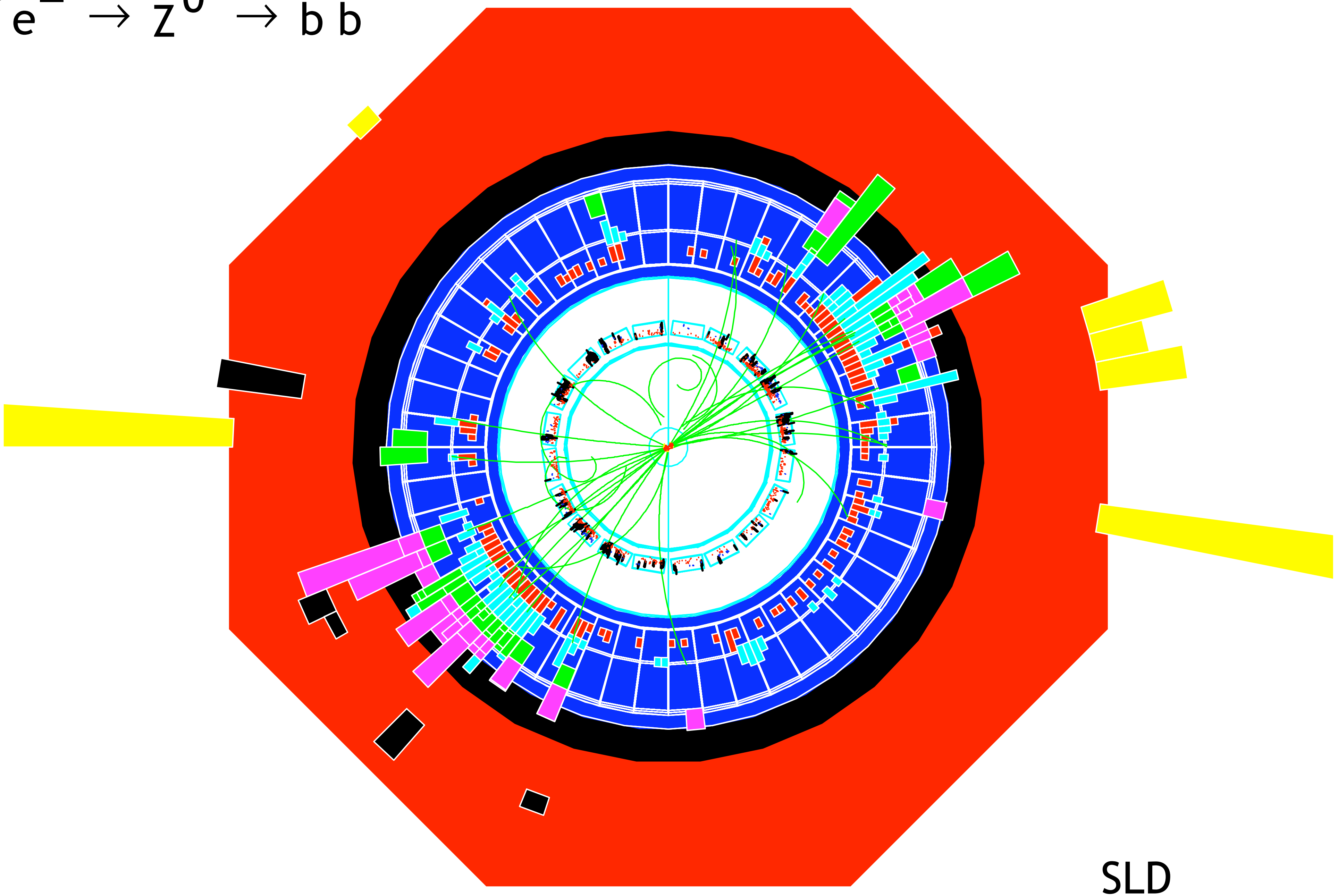


SLD

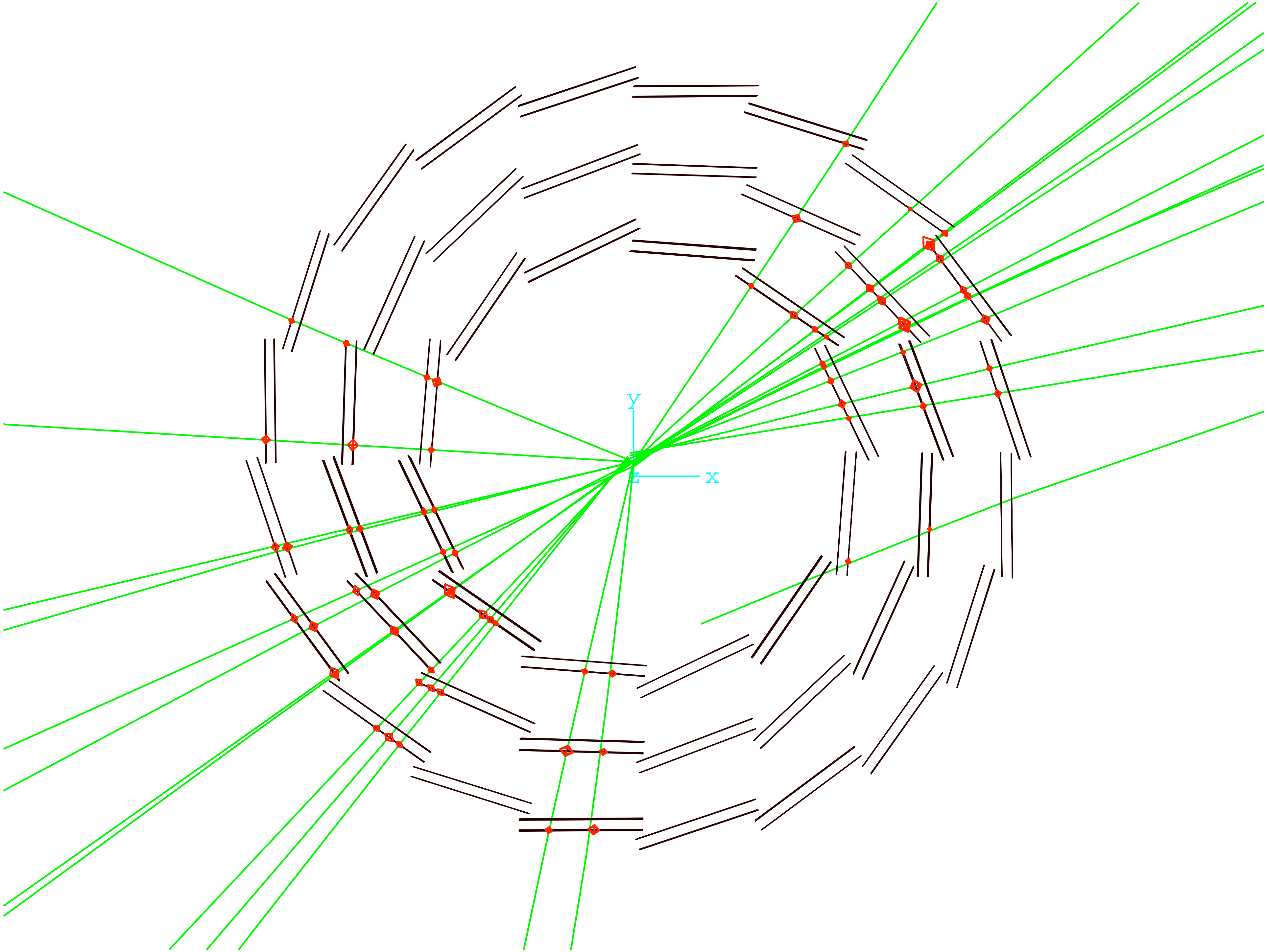
extrapolation of tracks to the vertex
mm scale !

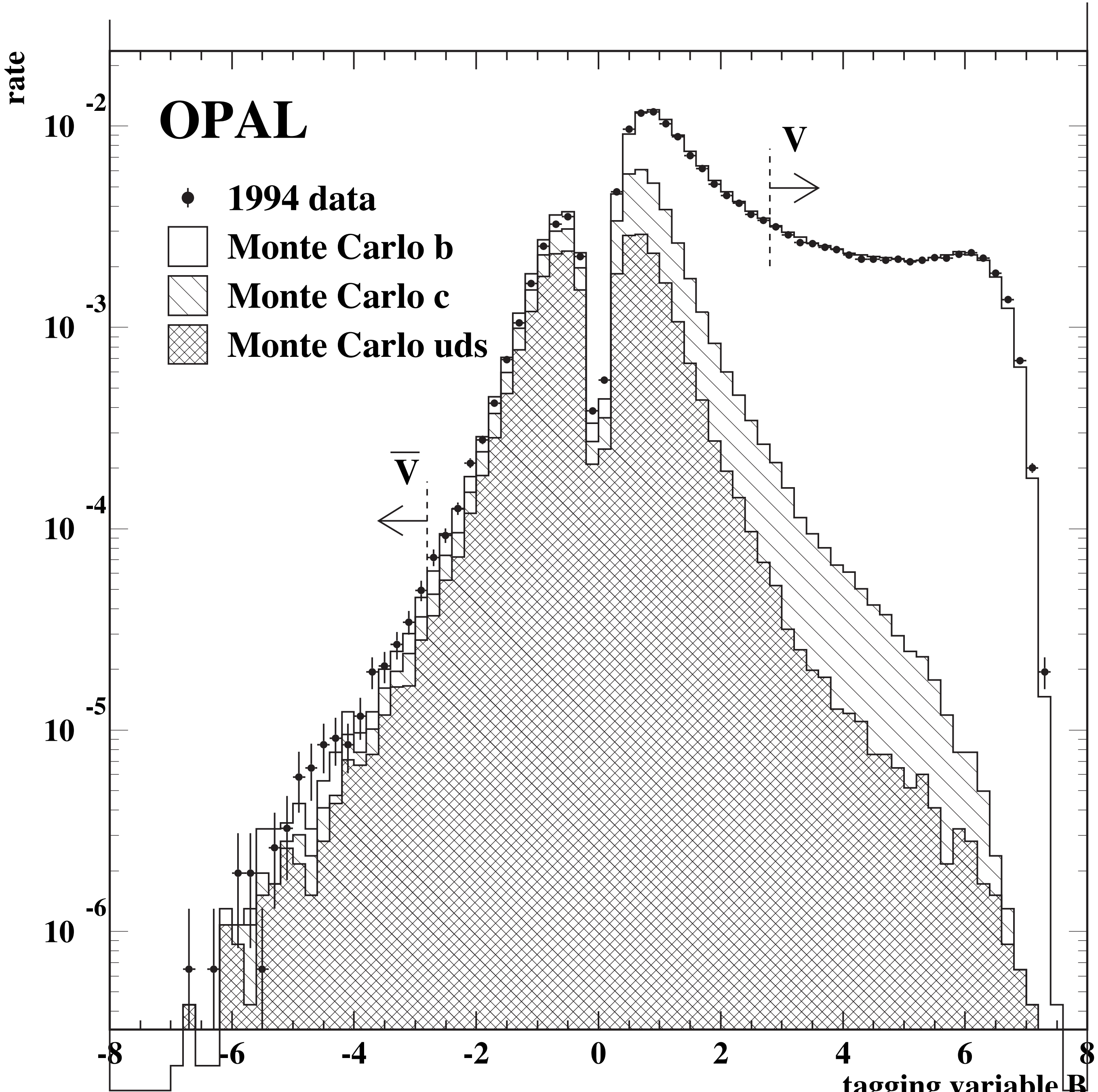


$e^+ e^- \rightarrow Z^0 \rightarrow b\,\bar{b}$

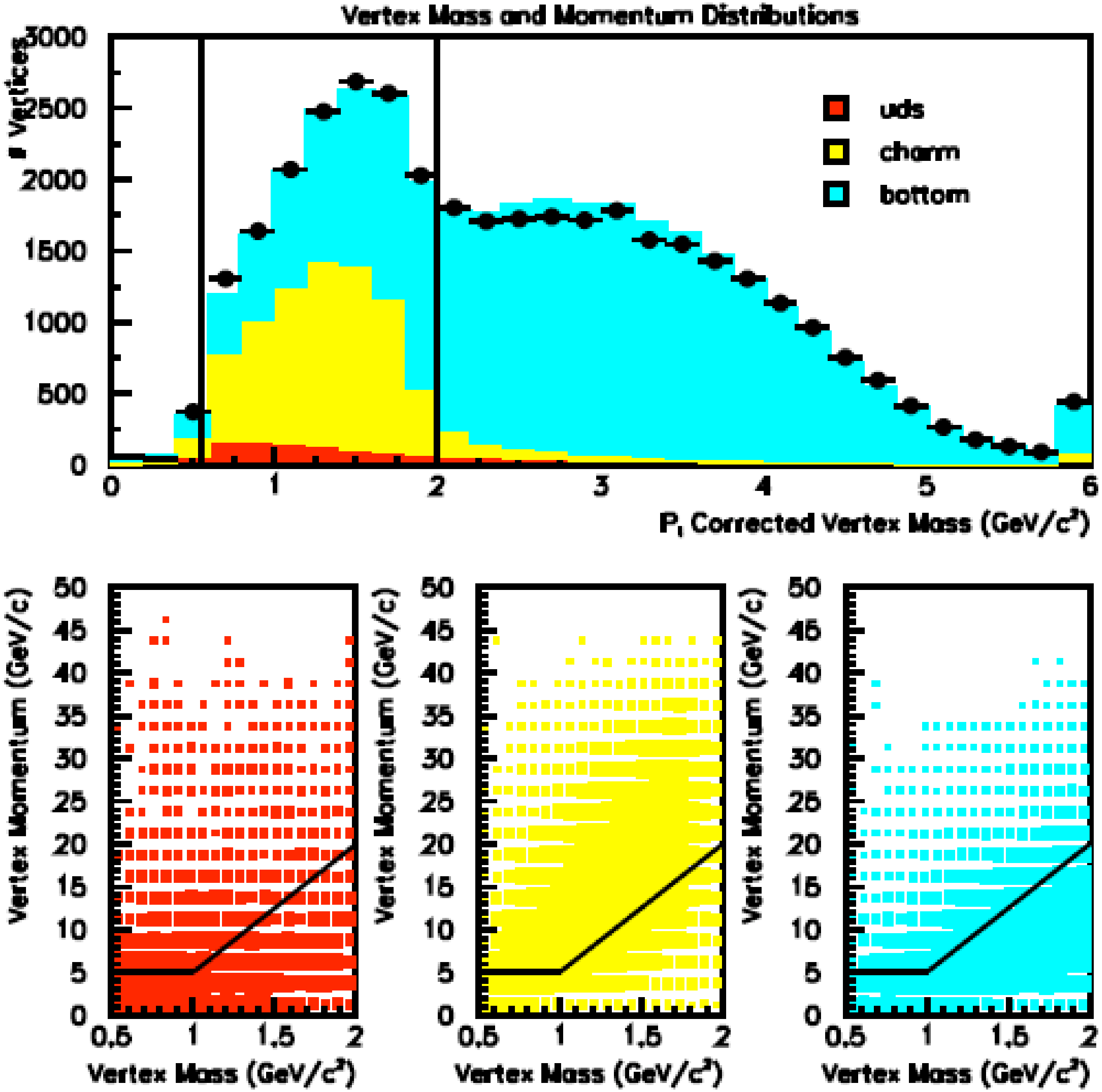


SLD


y
x
z



rate
OPAL
1994 data
Monte Carlo b
Monte Carlo c
Monte Carlo uds
V
$\overline{V}$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
-8
-6
-4
-2
0
2
4
6
8
tagging variable B


Vertex Mass and Momentum Distributions
Vertices
uds
charm
bottom
P_t Corrected Vertex Mass (GeV/c²)
Vertex Momentum (GeV/c)
Vertex Mass (GeV/c²)

SLD

The final result is:

$$R_b = 0.21643 \pm 0.00073$$

in excellent agreement with the Standard Model and confirming the -2% shift due to the t-W diagrams.

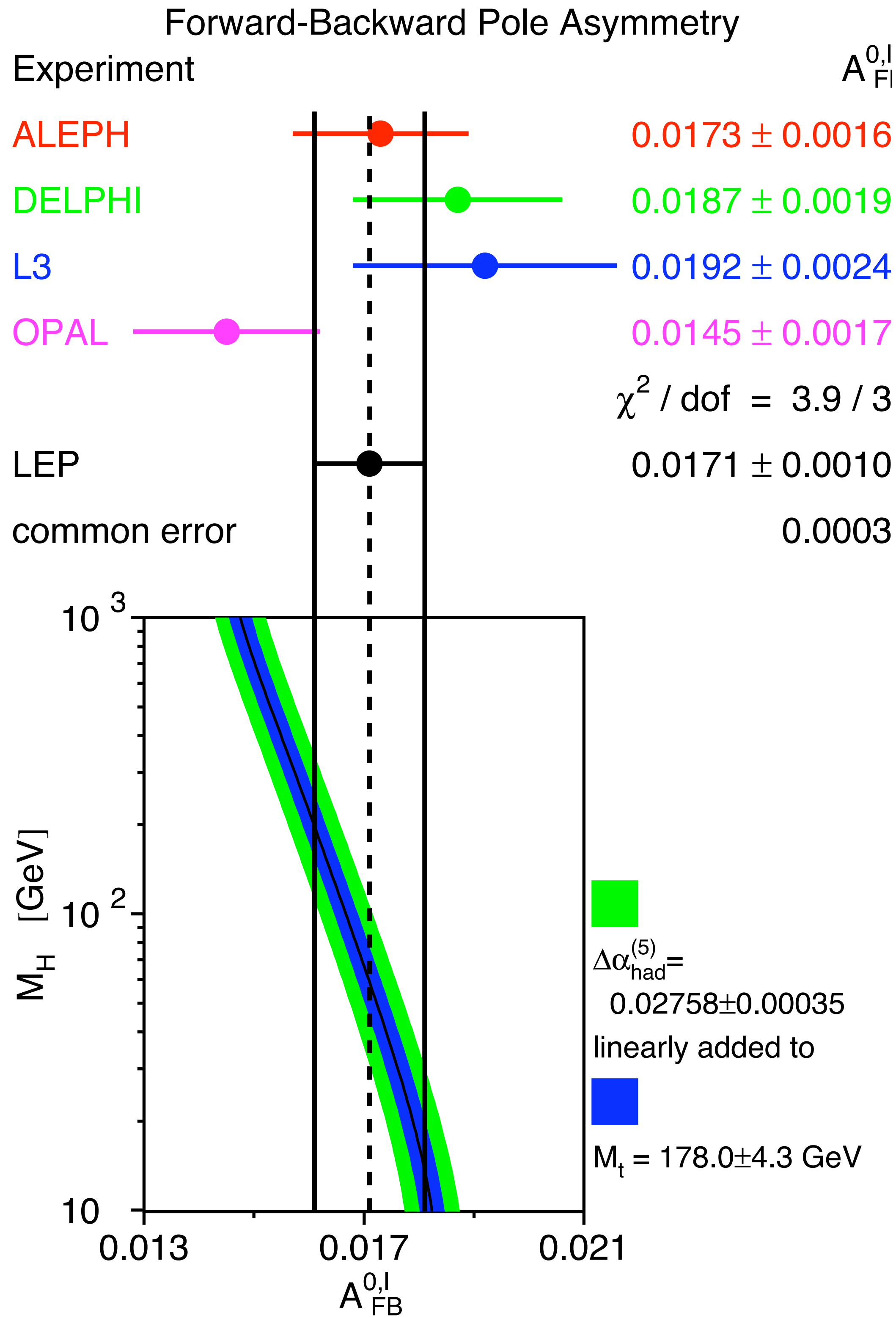
Next, consider the measurement of the A_f. There are three different techniques.

The first is to measure the unpolarized forward-backward asymmetry. For $e^+e^- \to f\overline{f}$ just at the Z resonance,

$$A_{FB} = \frac{3}{4} A_e A_f$$

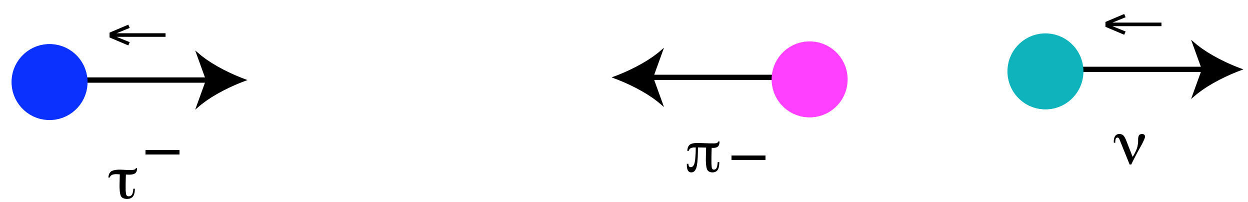
where the factor 3/4 was explained in the previous lecture.

Unfortunately, for leptonic final states, this is a 2% effect, reduced to 1% by ISR. Nevertheless, the effect can be observed and corrected for radiation.


Forward-Backward Pole Asymmetry
Experiment
$A_{FB}^{0,l}$
ALEPH
0.0173 ± 0.0016
DELPHI
0.0187 ± 0.0019
L3
0.0192 ± 0.0024
OPAL
0.0145 ± 0.0017
χ^2 / dof = 3.9 / 3
LEP
0.0171 ± 0.0010
common error
0.0003
10^3
10^2
10
M_H [GeV]
0.013
0.017
0.021
$A_{FB}^{0,l}$
$\Delta\alpha_{had}^{(5)}$=
0.02758±0.00035
linearly added to
M_t = 178.0±4.3 GeV

Second, one can directly measure the polarization of τ leptons.

When τ decays to $\nu_\tau \pi$, the pion goes dominantly in the spin direction:

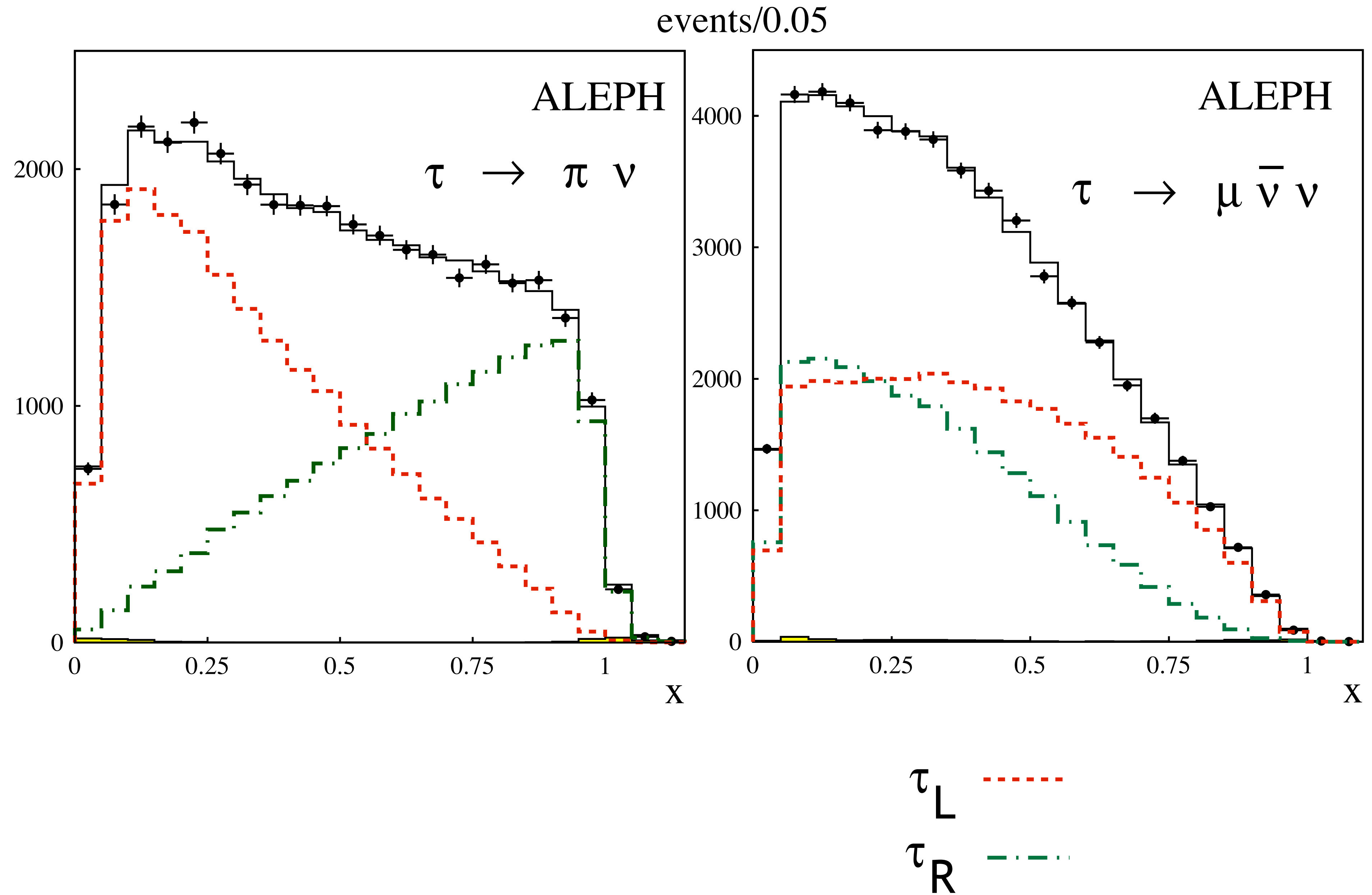


$$\frac{d\Gamma}{d\cos\theta} \sim (1 + \cos\theta)$$

Boosting to a relativistic τ,

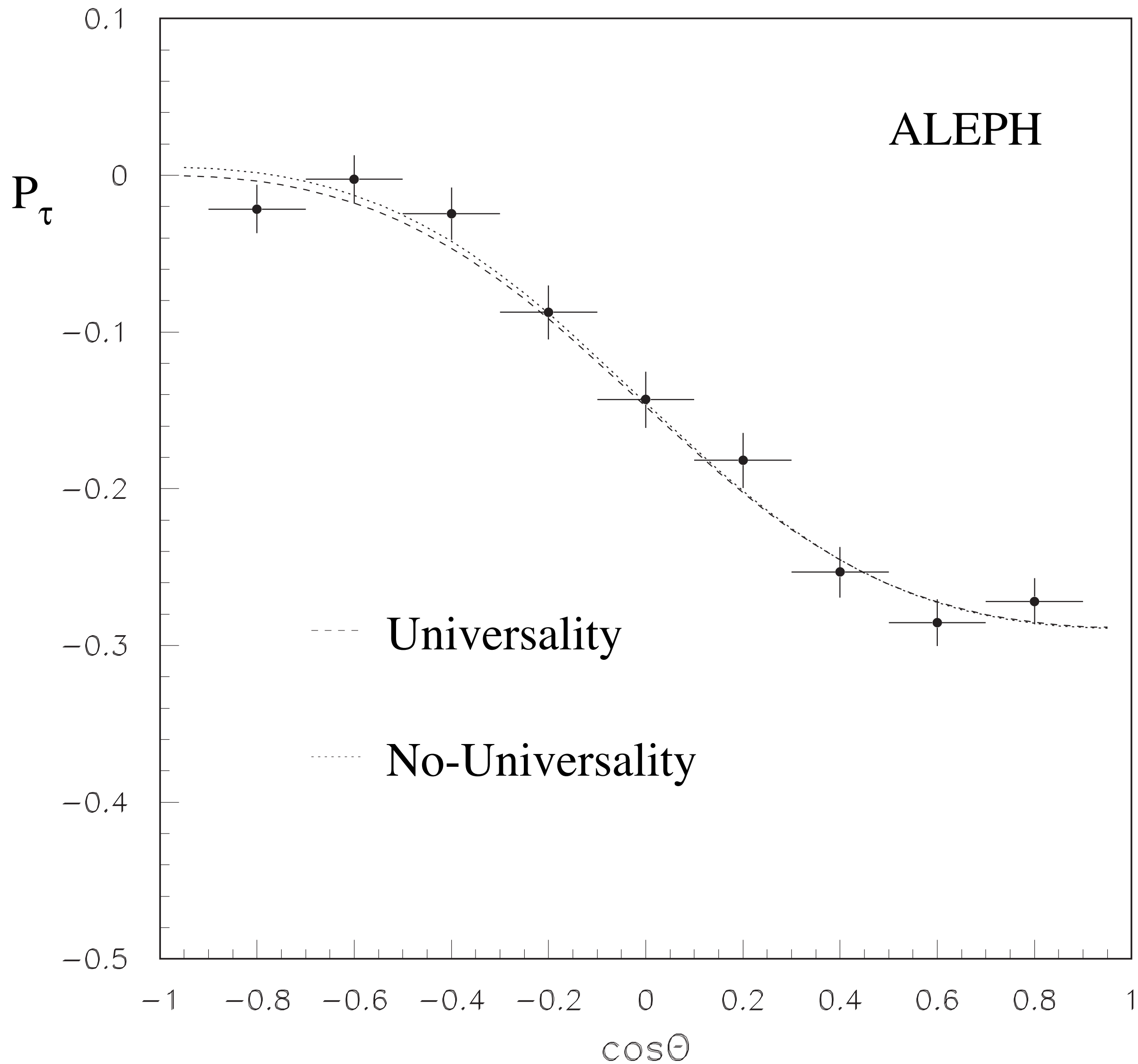
$$\frac{d\Gamma}{dE_\pi} \sim E_\pi$$

Similar effects are seen in other τ decay channels; for example, in $\tau \longrightarrow \nu_\tau \overline{\nu}_\mu \mu$, the muon goes in the direction opposite to the τ spin.



events/0.05
ALEPH
τ → π ν
2000
1000
0
0
0.25
0.5
0.75
1
x
ALEPH
τ → μ ν̄ ν
4000
3000
2000
1000
0
0
0.25
0.5
0.75
1
x
τL
τR

The measured polarization depends on $\cos\theta$; the effect is proportional to A_e .



Combining these effects, one obtains:

$$A_\ell = 0.1465 \pm 0.0033$$

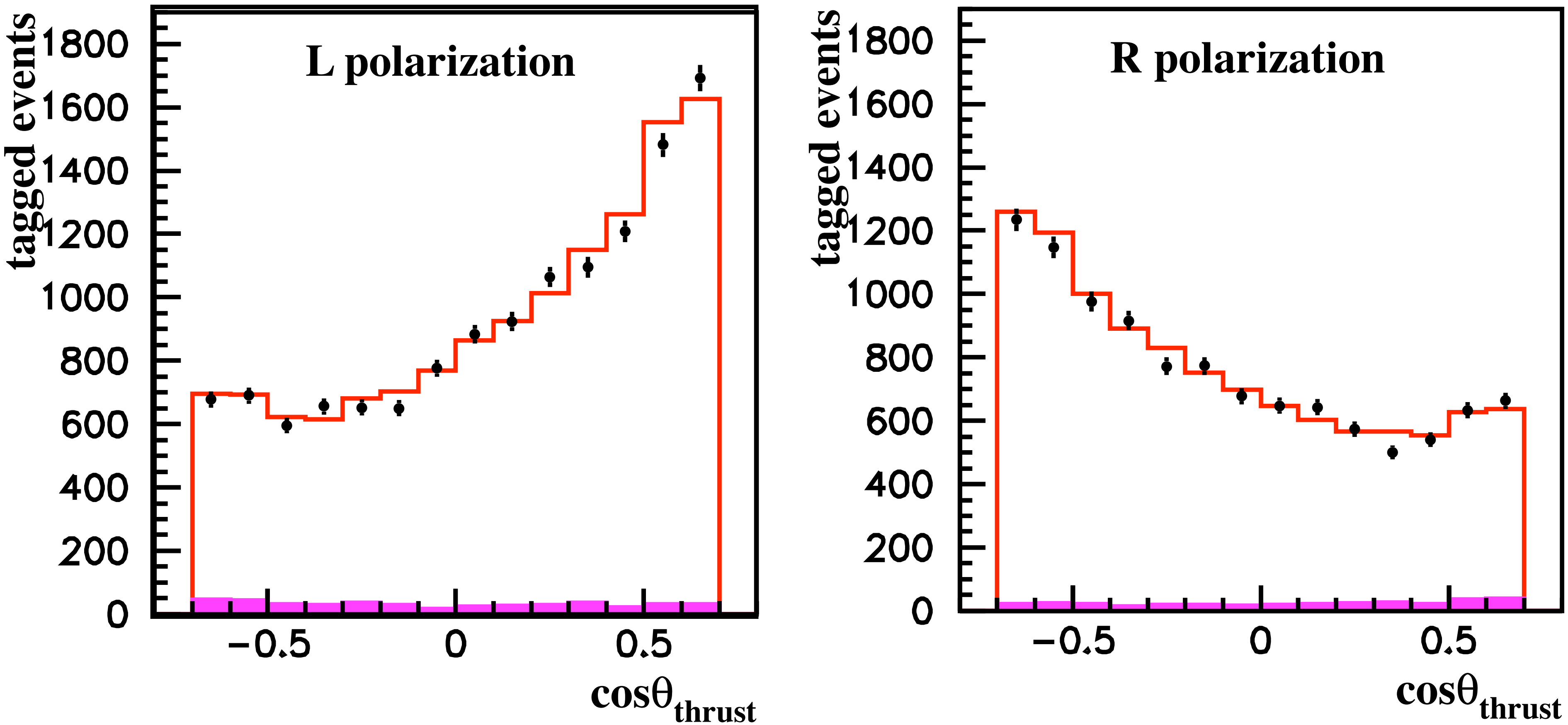
It was also possible at SLAC to polarize the electrons and measure A_e directly as an asymmetry in the total cross section on the Z resonances. This gives:

$$A_e = 0.1513 \pm 0.0021$$

From the sample of heavy quark events, we can measure A_b and A_c from the forward-backward asymmetries.

This depends on the ability to tell b from $\bar{b}$, c from $\bar{c}$, by the charges of leptons or K's from the weak decays.

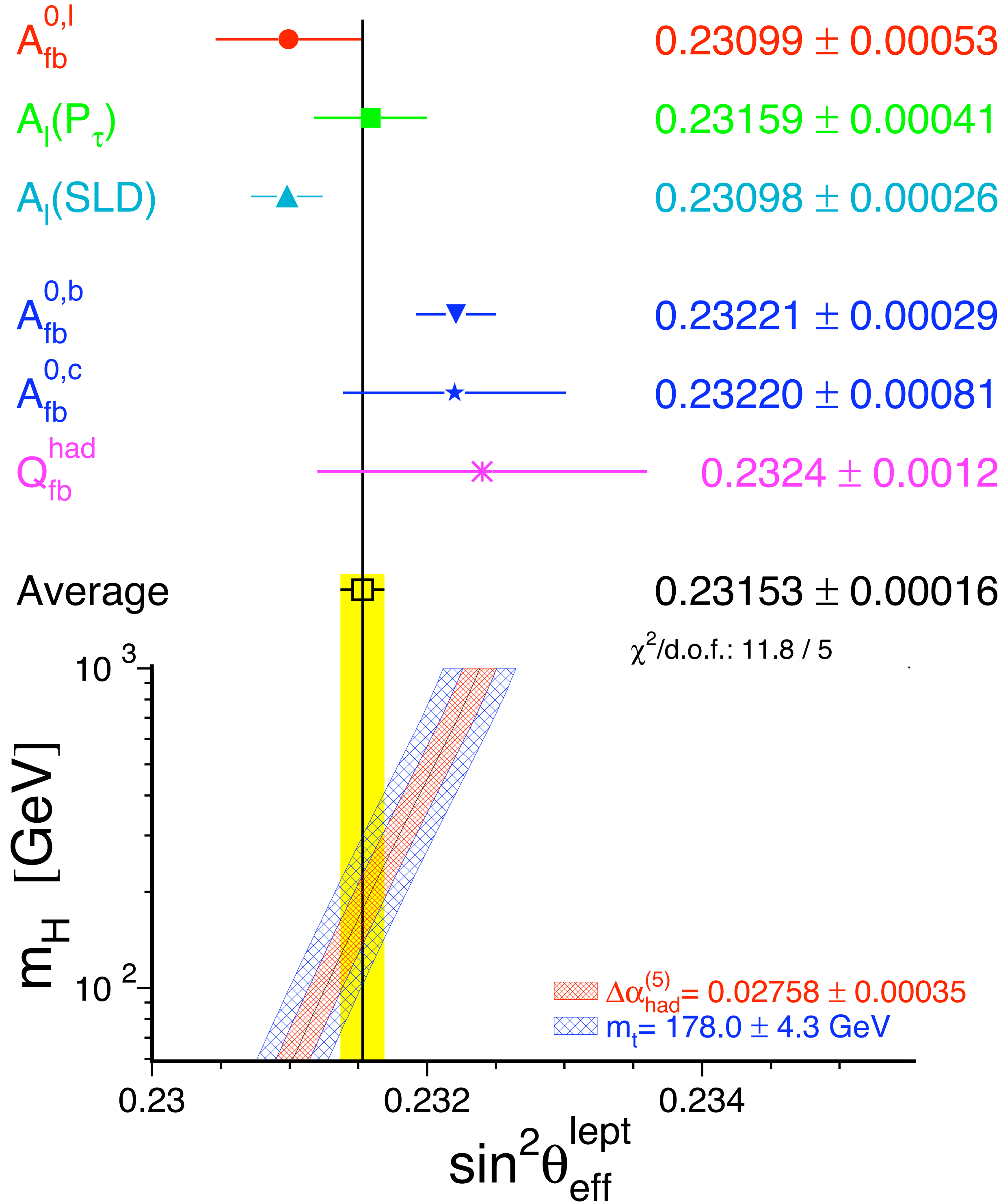
With polarized electron, there is a remarkable effect showing that A_b is almost maximal.



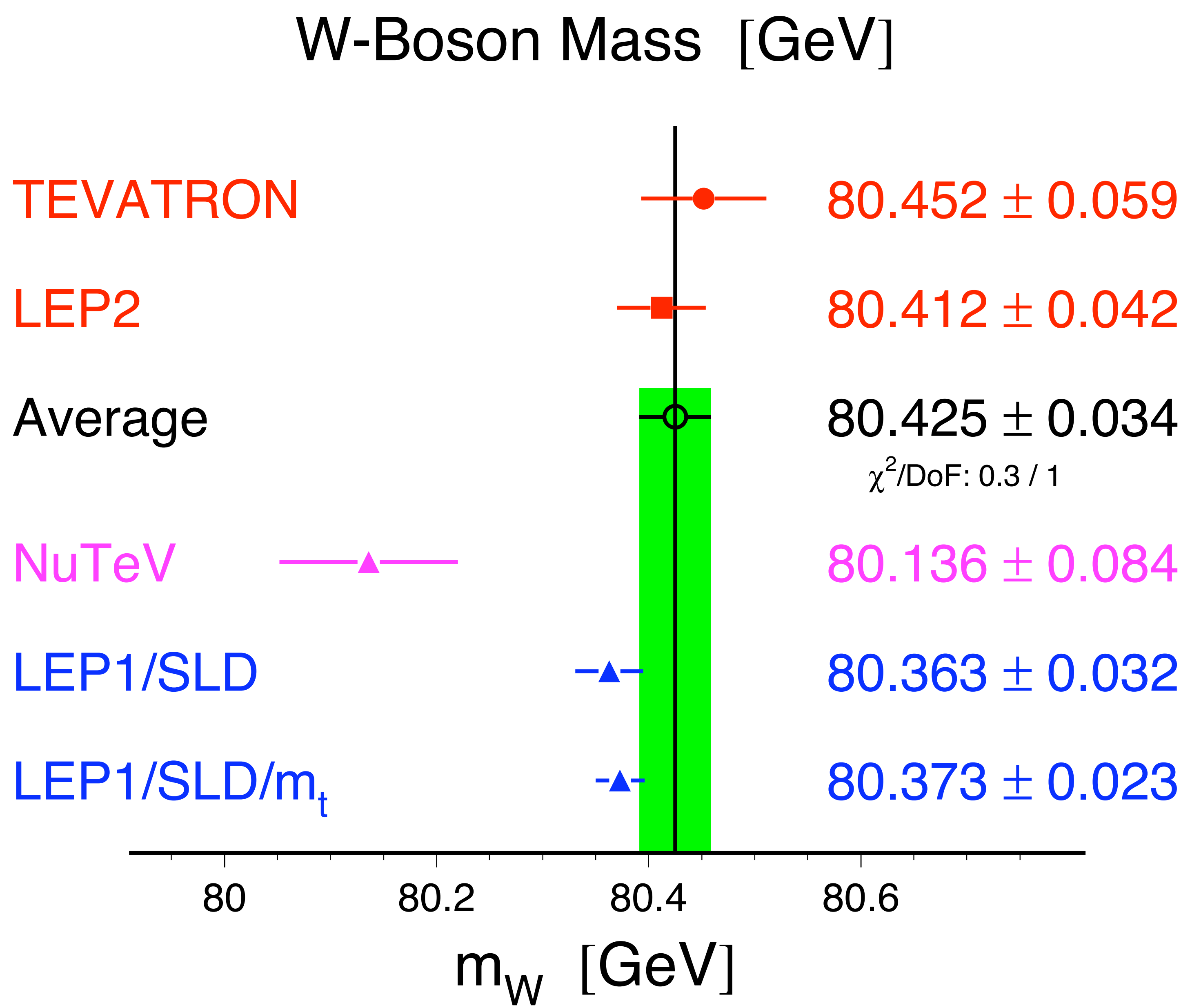
$A_b = 0.94$ at the Z^0

SLD

The various measurements are compatible, and they give a very precise value for s_w^2.



And recall from the previous lecture that we have also obtained a very accurate value of m_W.



Now, how can we use this information ?   I have already noted that the accuracy of the measurements requires taking into account 1-loop corrections from Standard Model particles. What about corrections from particles beyond the Standard Model ?

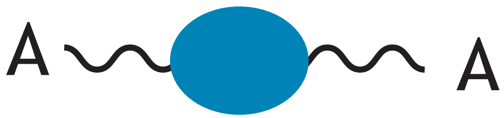
In general, we must analyze this model by model.

However, many heavy particles - especially particles associated with the Higgs sector - do not couple directly to light quarks and leptons. Then their effect on the Z precision observables comes only through their effects on the the electroweak bosons.

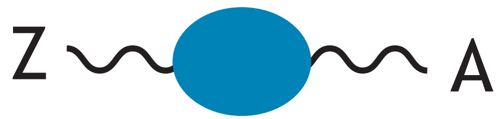
It is possible to make a general analysis of corrections of this type by parametrizing the vector boson vacuum polarization functions.

These vacuum polarization corrections - “oblique corrections” - are often the dominant effect of new physics, even in models such as supersymmetry that also have direct 1-loop corrections.

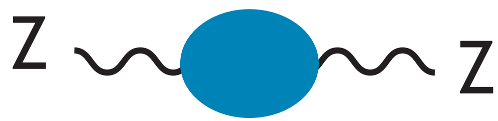
To begin this analysis, define



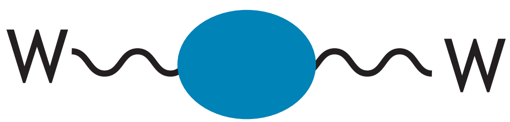
$$= ie^2\Pi_{11}g^{\mu\nu}$$



$$= i\frac{e^2}{s_w c_w}(\Pi_{3Q} - s_w^2\Pi_{QQ})g^{\mu\nu}$$



$$= i\frac{e^2}{s_w^2 c_w^2}(\Pi_{33} - 2s_w^2\Pi_{3Q} + s_w^4\Pi_{QQ})g^{\mu\nu}$$



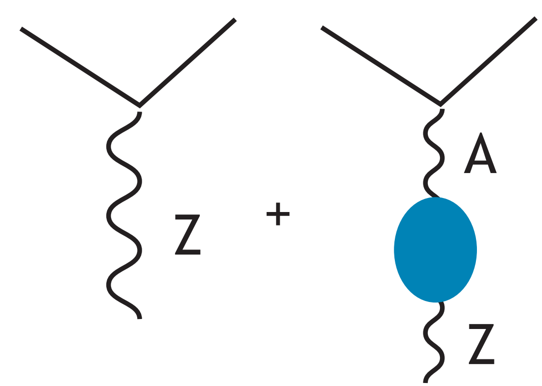
$$= i\frac{e^2}{s_w^2}\Pi_{11}g^{\mu\nu}$$

These amplitudes gives corrections to precision electroweak observables, for example

W mass $$m_W^2 = \frac{e^2v^2}{4s_w^2} + \frac{e^2}{s_W^2}\Pi_{11}(m_W^2)$$

value of s_w^2 obtained from Z asymmetries:

$$s_*^2 = s_w^2 - e^2(\Pi_{3Q} - s_w^2\Pi_{QQ})/m_Z^2$$



These formulae are not yet adequate, because the parameters

$$e^2, \; s_w^2, \; v^2$$

need to be determined from physical observables. A useful standard set of reference measurements is

$$\begin{aligned} \alpha^{-1}(m_Z^2) &= 128.95(5) \\ G_F &= 1.16637(1) \times 10^{-5} \ \mathrm{GeV}^{-2} \\ m_Z &= 91.1876(21) \ \mathrm{GeV} \end{aligned}$$

Notice that I have used the running α^{-1} at the Z rather than $\alpha^{-1} = 1/137.03599$; this avoids large corrections in the connection to weak processes.

If we define s_0^2 by $\qquad \sin^2 2\theta_0 = \dfrac{\pi\alpha(m_Z^2)}{\sqrt{2} G_F m_Z^2}$

then the differences, e.g. $\quad s_*^2 - s_0^2 \ , \qquad m_W/m_Z - c_0$

are well-defined and also - by renormalizability - UV finite.

Of course, the reference observables are also related to the parameters by formula that are shifted by vacuum polarization corrections.

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8m_W^2}\left(1 + \frac{e^2}{s_w^2 m_W^2}\Pi_{11}(0)\right)$$

This needs to be taken into account in computing the shifts defined on the previous slide.

At the end, a simple formalism results. The shifts depend on two combinations of vacuum polarization amplitudes

$$S = \frac{16\pi}{m_Z^2}(\Pi_{33}(m_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(m_Z^2))$$

$$T = \frac{4\pi}{s_w^2 m_W^2}(\Pi_{11}(0) - \Pi_{33}(0))$$

(and a third, U, which is typically very small).

In terms of these quantities, the shifts take simple forms.

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left(-\frac{1}{2}S + c^2 T\right)$$

$$s_*^2 - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\frac{1}{4}S - s^2 c^2 T\right)$$

The heavy particles of the Standard Model, top and Higgs, fit into the framework. We find contributions to S and T.

top: $$S = \frac{1}{6\pi} \log \frac{m_t^2}{m_Z^2} \qquad T = \frac{3}{16\pi s^2 c^2} \frac{m_t^2}{m_Z^2}$$

Higgs: $$S = \frac{1}{12\pi} \log \frac{m_h^2}{m_Z^2} \qquad T = -\frac{3}{16\pi c^2} \log \frac{m_h^2}{m_Z^2}$$

The dependence m_t^2/m_W^2 is a now-familiar result of Goldstone boson equivalence.

A doublet heavy fermions gives $S = \dfrac{1}{6\pi}$

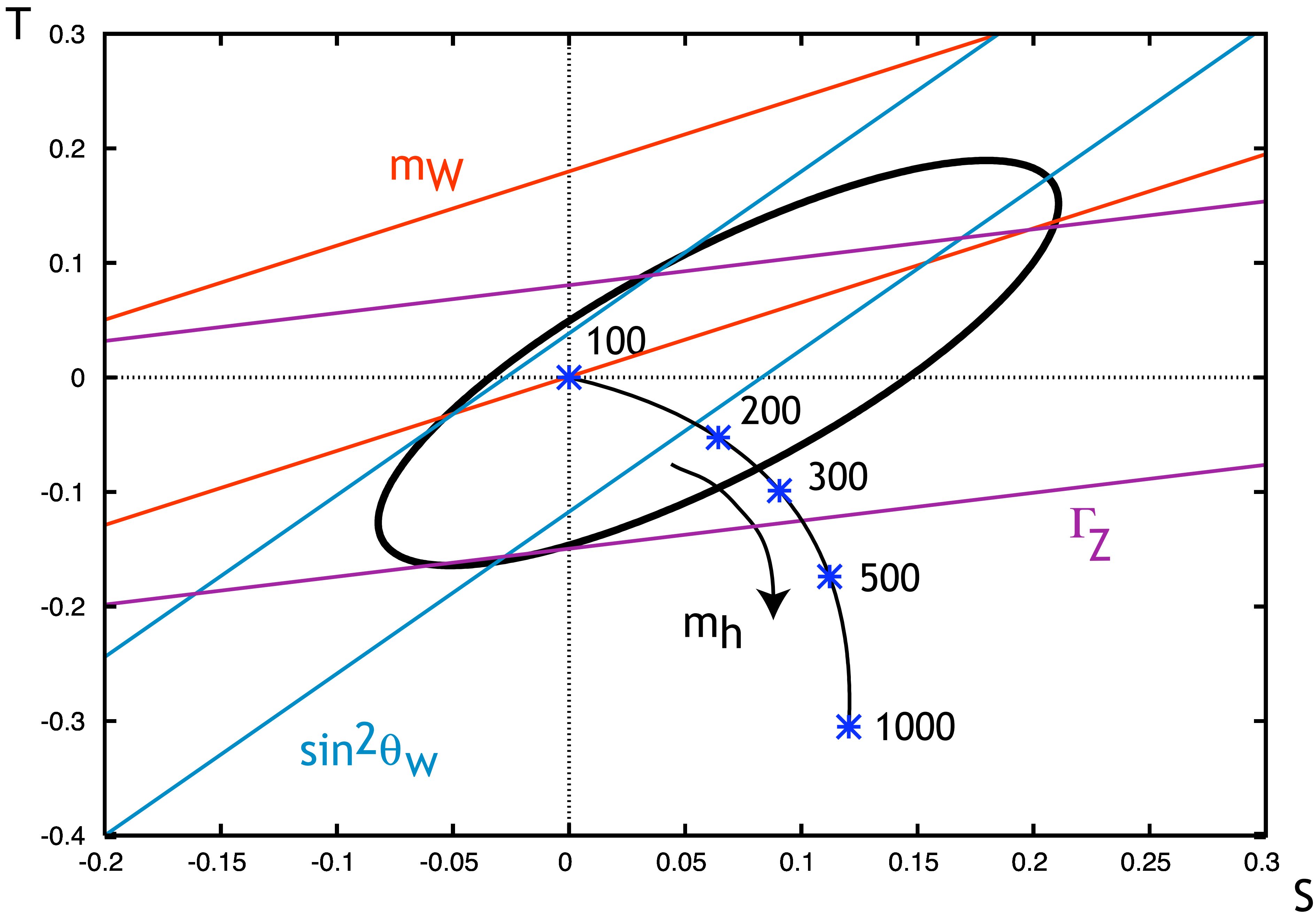
and a (potentially enormous) contribution to T proportional to

$$\frac{|m_U^2 - m_D^2|}{m_Z^2}$$

The large effect of the top quark is actually needed to explain the precision measurements. Fixing the top quark mass at its current value

$$m_t = 178 \pm 4 \text{ GeV}$$

we can fit for the shifts of S and T due to other heavy particles. This gives sensitivity to the Higgs boson mass, and to other possible new states.



In the SM, $m_h < 285\ \mathrm{GeV}$ (95% conf) (LEP EWWG)

What constraint do these measurements put on new physics ?

For supersymmetry, which requires a light Higgs boson and does not add new chirally-coupled fermions, the constraints are weak.

However, models with new strong interactions at the TeV scale typically contribute order-1 positive corrections to both S and T. Such models also typically correct R_b at the 5% level. All three effects must be avoided to be consistent with the precision measurements.

In these lectures, I have described the structure of the Standard Model of particle physics, and I have shown how this structure is support by experiment. The Standard Model has conceptual difficulties - which we will explore in this school - but it works extremely well at the energies we have explored up to now.

However particle physics is altered at high energies, the basic structures of the Standard Model and Yang-Mills theory will remain its foundation and its essential starting point.

I hope that this survey will help you understand this foundation and, hopefully, how to build on it.