Dynamical SUSY Breaking

Nathan Seiberg

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Throughout the talk we will not give references

1. Introduction

Hierarchy of solutions to the hierarchy problem:

1. In MSSM: explicit (soft) SUSY breaking
2. Since SUSY is gauged (SUGRA), need spontaneous SUSY breaking
3. More elegant and aesthetically natural – dynamical SUSY breaking

Since $m_{3/2} \ll M_P$, can use global SUSY – field theory

2. Spontaneous SUSY Breaking

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger) + \int d^2\theta W(\Phi) + \int d^2\bar{\theta}\bar{W}(\Phi^\dagger)$$

SUSY is unbroken iff $E = 0 \Leftrightarrow \partial W = 0$.

2.1. Simplest example

$$K = X^\dagger X \qquad ; \qquad W = M^2 X$$

$$V = |M|^4$$

Spectrum: massless fermion (Goldstone fermion) and a massless complex scalar X.
Flat direction: X
The spectrum looks SUSY, but if we couple the theory to SUGRA, $m_{3/2} \sim M^2/M_P$.

2.2. Second simplest example

$$K = K(X^\dagger, X) \qquad ; \qquad W = M^2 X$$

K includes higher dimension operators; e.g. from integrating out heavy field theory modes (not necessarily from gravity).

$$V = \frac{|M|^4}{\partial_X \partial_{X^\dagger} K}$$

This lifts the flat direction and the spectrum is in general not SUSY.

2.3. Renormalizable model - O'Raifeartaigh model

$$K = X^\dagger X + \Phi^\dagger \Phi + Y^\dagger Y \qquad ; \qquad W = X(\Phi^2 - M^2) + mY\Phi$$

$$V = |\Phi^2 - M^2|^2 + |m\Phi|^2 + |mY + 2X\Phi|^2$$

We cannot set all the terms to zero and hence SUSY is broken.

Flat direction, $Y = -\frac{2X\Phi}{m}$. Effective theory along flat direction is like our simplest example.

A nontrivial K is generated by radiative corrections. It leads to a minimum at the origin $X = \Phi = Y = 0$. This is the same as the second simplest example.

2.4. General comment

Using such models of SUSY breaking we can solve the technical hierarchy problem, but not the aesthetic problem of why $M_W \ll M_P$.

Witten suggested to solve this problem with DSB. The tree level theory has only dimensionless parameters and does not break SUSY. Because of the nonrenormalization theorems SUSY is not broken to all orders in perturbation theory. Nonperturbatively, because of dimensional transmutation, a low scale

$$\Lambda = M_P e^{-c/g^2(M_P)} \ll M_P$$

is generated. SUSY breaking at that scale solves the hierarchy problem.

In most known examples the low energy theory at energies Λ is similar to the previous examples.

3. Review of SUSY QCD

$SU(N_c)$ gauge theory with N_f flavors of quarks Q in $\mathbf{N_c}$ and anti-quarks $\tilde{Q}$ in $\bar{\mathbf{N}}_{\mathbf{c}}$. The classical potential is $\sum_a (D^a)^2 = \sum_a (Q^\dagger T^a Q - \tilde{Q}^\dagger T^a \tilde{Q})^2$.

3.1. Classical flat directions – moduli space of vacua

$$N_f < N_c$$

$$Q = \tilde{Q} \begin{pmatrix} v_1 & & & & \\ & v_2 & & & \\ & & v_3 & & \\ & & & \cdot & \\ & & & & v_{N_f} \end{pmatrix}$$

The $SU(N_c)$ gauge symmetry is broken to $SU(N_c - N_f)$. The moduli space of vacua can be parameterized by $M_a^{\tilde{a}} = \tilde{Q}_i^{\tilde{a}} Q_a^i$ where $i = 1, ..., N_c$ is a color index and $a, \tilde{a} = 1, ..., N_f$ are flavor indices.

$$N_f \geq N_c$$

$$Q = \begin{pmatrix} v_1 & & & & \\ & v_2 & & & \\ & & v_3 & & \\ & & & \cdot & \\ & & & & v_{N_c} \\ & & & & \\ & & & & \end{pmatrix} \quad ; \quad \tilde{Q} = \begin{pmatrix} \tilde{v}_1 & & & & \\ & \tilde{v}_2 & & & \\ & & \tilde{v}_3 & & \\ & & & \cdot & \\ & & & & \tilde{v}_{N_c} \\ & & & & \\ & & & & \end{pmatrix}$$

$$|v_i|^2 - |\tilde{v}_i|^2 \text{ independent of } i$$

The $SU(N_c)$ gauge symmetry is completely broken. The moduli space of vacua can be parameterized by $M_a^{\tilde{a}} = \tilde{Q}_i^{\tilde{a}} Q_a^i$ as well as

$$B_{a_1, a_2, \ldots, a_{N_c}} = \epsilon_{i_1, i_2, \ldots i_{N_c}} Q_{a_1}^{i_1} Q_{a_2}^{i_2} \ldots Q_{a_{N_c}}^{i_{N_c}}$$

$$\tilde{B}^{\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_{N_c}} = \epsilon^{i_1, i_2, \ldots i_{N_c}} \tilde{Q}_{i_1}^{\tilde{a}_1} Q_{i_2}^{\tilde{a}_2} \ldots Q_{i_{N_c}}^{\tilde{a}_{N_c}}$$

These fields are not independent. For example, for $N_f = N_c$ there is only one $B = \det Q$ and one $\tilde{B} = \det \tilde{Q}$ and they satisfy

$$\det M - B\tilde{B} = 0$$

This constraint is true at every point in the moduli space; i.e.

$$\mathcal{M} = \{M, B, \tilde{B} | \det M - B\tilde{B} = 0\}$$

3.2. Nonperturbative dynamics

$$N_f < N_c$$

Nonperturbative effects generate a superpotential along the moduli space of vacua

$$W_{dynamical} = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}}$$

It leads to a potential which slopes to zero and infinity and the system does not have a ground state.

$$N_f = N_c$$

The moduli space which is classically characterized by $\det M - B\tilde{B} = 0$ is deformed and the moduli space of quantum vacua is

$$\mathcal{M}_{quantum} = \{M, B, \tilde{B} | \det M - B\tilde{B} = \Lambda^{2N_c}\}$$

We can describe it by adding a Lagrange multiplier field X and the superpotential

$$W = X(\det M - B\tilde{B} - \Lambda^{2N_c})$$

This description is analogous to the linear sigma model as opposed to the nonlinear model for pions.

4. Dynamical SUSY Breaking

We'll discuss two characteristic examples

4.1. 3-2 model

The gauge group is $SU(3) \times SU(2)$ and we have chiral superfields: Q in $(\mathbf{3}, \mathbf{2})$, $\tilde{u}$ in $(\bar{\mathbf{3}}, \mathbf{1})$, $\tilde{d}$ in $(\bar{\mathbf{3}}, \mathbf{1})$, L in $(\mathbf{1}, \mathbf{2})$.

The classical flat directions are characterized by

$$X = Q\tilde{d}L \quad , \quad Y = Q\tilde{u}L \quad , \quad Z = QQ\tilde{u}\tilde{d}$$

As a first approximation we neglect the $SU(2)$ dynamics and then, as with $N_f = 2 < N_c = 3$ above the $SU(3)$ dynamics generates a superpotential

$$W_{dynamical} = \frac{\Lambda_3^7}{Z}$$

With this superpotential the theory does not have a ground state.

We add to the model a tree level superpotential

$$W_{tree} = \lambda Q\tilde{d}L = \lambda X$$

The combined model with $W_{dynamical}$ and W_{tree} breaks SUSY.

4.2. Modified moduli space example

Consider the $SU(N_c)$ theory with $N_f = N_c$ and add fields $S^a_{\tilde{a}}$, b and $\tilde{b}$ and a superpotential

$$W_{tree} = S^a_{\tilde{a}} \tilde{Q}^{\tilde{a}}_i Q^i_a + b \det \tilde{Q} + \tilde{b} \det Q$$

Classically $Q = \tilde{Q} = 0$. In the quantum theory we get the effective superpotential

$$W_{effective} = S^a_{\tilde{a}} M^{\tilde{a}}_a + b\tilde{B} + \tilde{b}B + X(\det M - B\tilde{B} - \Lambda^{2N_c})$$

which breaks SUSY

5. Conclusion

Dynamical SUSY breaking in four dimensions is possible.

6. Homework

Use DSB to construct a viable predictive SUSY model