IAS lecture I (Kachru)

There are many motivations for studying string compactifications

- to learn about "stringy geometry"
- to develop understanding of duality
- to generate examples of AdS / CFT or a similar structure for dS
- to make models of physics beyond SM
- to learn about possible cosmologies in string theory

We'll make occasional contact with these various goals, but our main activity will be to discuss classes of constructions that could a priori be useful for any of these purposes.

Rough Plan of lectures:

Lecture I: IIB flux vacua

Lecture II: Examples, Duality with IIA

Lecture III: Highly warped models and new ~~SUSY~~ states

Lecture IV: Inflation and string theory

I. Basic facts

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_s} \left\{ e^{-2\phi} \left[R_s + 4 (\nabla\phi)^2 \right] - \frac{F_1^2}{2} - \frac{1}{12} G_3 \cdot \overline{G_3} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right\}$$

$$+ \frac{1}{8 i \kappa_{10}^2} \int e^{\phi} \, C_4 \wedge G_3 \wedge \overline{G_3} + S_{loc}$$

Here

$$G_3 = F_3 - \tau H_3$$

$$\tau = C_0 + i e^{-\phi}$$

$$\tilde{F}_5 = F_5 - \tfrac{1}{2} C_2 \wedge H_3 + \tfrac{1}{2} B_2 \wedge F_3$$

And, must impose $\tilde{F}_5 = * \tilde{F}_5$ by hand on EOM.

To start with, we'll look for sol'ns with 4d Poincaré symmetry:

- $$ds_{10}^2 = e^{2A(y)} \eta_{\mu\nu} \, \overset{\overset{\mathbb{R}^{3,1}}{\downarrow}}{dx^{\mu}} dx^{\nu} + e^{-2A(y)} \tilde{g}_{mn}(y) \, \overset{\overset{M_6}{\downarrow}}{dy^m} dy^n$$

- $\tau = \tau(y)$
- $\tilde{F}_5 = (1+*)\left[d\alpha \wedge dx^0 \wedge \cdots \wedge dx^3 \right]$

  with $\alpha = \alpha(y)$.
- Only compact components of $G_3 \rightarrow$

$$F_3, \ H_3 \in H^3(M_6, \mathbb{Z})$$

One can show from the non-compact components of the Einstein equations that

$$\tilde{\nabla}^2 e^{4A} = e^{2A} \frac{G_{mnp} \bar{G}^{mnp}}{12 \operatorname{Im} \tau} + \qquad (\star)$$

$$e^{-6A} \left[\partial_m \alpha \, \partial^m \alpha + \partial_m e^{4A} \partial^m e^{4A} \right]$$

$$+ \frac{\kappa_{10}^2}{2} e^{2A} \left(T^m{}_m - T^\mu{}_\mu \right)^{loc}$$

where $T^{loc}_{MN} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{loc}}{\delta g^{MN}}$

<u>Notice</u>: 1^{st} two terms on RHS are ≥ 0, but $\int_{M_6} (\text{LHS}) = 0$. So in <u>absence</u> of localized sources $\exists$ no-go theorem: $G_3 = 0$, e^A = <u>const</u>.

Also, for warped soln's we'll need $(T^m{}_m - T^\mu{}_\mu)^{loc} < 0$.

Before finding soln's, we'll need one more fact.

Bianchi identity for $\tilde{F}_5$ $\Rightarrow$

local D3 charge density

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2\kappa_{10}^2 T_{D3} \rho_3^{loc} \quad (\because)$$

Integrating $\rightarrow$ Gauss' law

$$\frac{1}{2\kappa_{10}^2 T_{D3}} \int_{M_6} H_3 \wedge F_3 + Q_3^{loc} = 0$$

II. Solutions in 10d picture

In terms of the function $\alpha(y)$, can re-write

⑤

(☺) as

$$\tilde{\nabla}^2 \alpha = i e^{2A} \frac{G_{mnp} *_6 \bar{G}^{mnp}}{12 \, \mathrm{Im}\, \tau} +$$

$$2 e^{-6A} \partial_m \alpha \, \partial^m \alpha +$$

$$2 \kappa_{10}^2 e^{2A} T_3 \rho_3^{loc}$$

Subtracting <u>this</u> from the Einstein eqn ($\star$) $\Rightarrow$

$$\tilde{\nabla}^2 (e^{4A} - \alpha) = \frac{e^{2A}}{6 \, \mathrm{Im}\, \tau} \left| i G_3 - *_6 G_3 \right|^2 +$$

$$e^{-6A} \left| \partial (e^{4A} - \alpha) \right|^2 +$$

$$2 \kappa_{10}^2 e^{2A} \left[\frac{1}{4} (T^m{}_m - T^\mu{}_\mu)^{loc} - T_3 \rho_3^{loc} \right]$$

<u>If</u> we make the assumption

$$\frac{1}{4} (T^m{}_m - T^\mu{}_\mu)^{loc} \geq T_3 \rho_3^{loc}$$

[saturated by D3s, O3s, wrapped D7s; satisfied by $\overline{\text{D3}}$s;

violated by O5s, $\overline{\text{O3}}$s] then : (on compact M_6, in leading approx.)

- $G_{(3)}$ must be ISD

$$\boxed{*_6 G_3 = i G_3}$$

- Warp factor & C_4 are related

$$\boxed{e^{4A} = \alpha}$$

- The inequality is actually <u>saturated</u>.

We didn't yet us R_{mn} Einstein eqn, τ eqn :

$$\tilde{R}_{mn} = \kappa_{10}^2 \frac{\partial_m \tau \partial_n \bar{\tau} + \partial_m \bar{\tau} \partial_n \tau}{4 (\mathrm{Im}\, \tau)^2}$$

$$+ \text{ local}$$

$$\tilde{\nabla}^2 \tau = \frac{\tilde{\nabla} \tau \cdot \tilde{\nabla} \tau}{i \, \mathrm{Im}\, \tau} + \text{ local}$$

These eqns $\to$ solutions to F-theory in SUGRA approximation.

<u>Simplest</u> examples are perturbative D3/D7 type IIB orientifolds of CY 3-folds. We'll use this language. (Sen, hep-th/9702165 ⟹ <u>any</u> F-theory model has such a limit).

The sources of 'negative tension' that allow us to evade no-go: O3s and induced D3 charge on wrapped D7s ⟹ in general

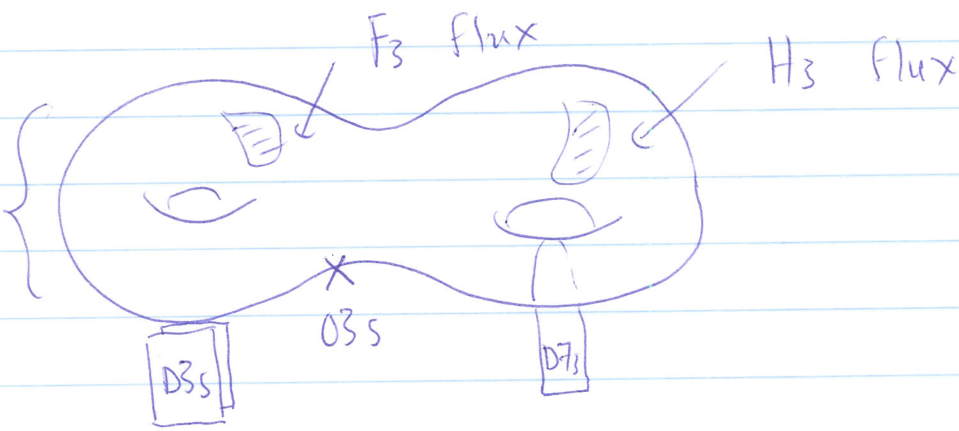
$$\int_{M_6} H_3 \wedge F_3 \leq L$$

$\mathcal{O}(20)$ in easy egs
$\mathcal{O}(10^3)$ in many CY_4s, where $L = \frac{\chi(X_4)}{24}$;
cf Sethi-Vafa-Witten

III. 4d EQFT

Our picture:

M_6
size R



D3/D7 type orientifold of CY $\Rightarrow$ expect a 4d $\mathcal{N}=1$ SUSY effective field theory below energy scale $\frac{1}{R}$.

Light Modes:

a) Closed string sector

- Complex str moduli of M_6 $\quad z^\alpha$
- Axio-dilaton $\quad \tau$
- Kähler moduli of M_6 $\quad \rho$ [generalization to >1 easy]

b) Open string sector

- D7 moduli
- D3 moduli

...

We'll focus on this for now; some comments on D7s/D3s later...

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<u>General expectation:</u>

Reduction of $\int_{M_6} d^6y \, -\frac{G_3 \cdot \overline{G_3}}{2 \cdot 3!} \Longrightarrow$ 4d

effective potential for moduli. Fixing $F_3, H_3 \in H^3(M_6, \mathbb{Z})$ [superselection sector], minima of potential occur at metrics s.t.

$$*_6 G_3 = i G_3 .$$

generically, fixes z_α + τ

4d $\mathcal{N}=1$ sugra $\rightarrow$

$$V = e^K \Big[\sum_{i,j} g^{i\bar{j}} \underbrace{D_i W}_{= \partial_i W + K_{,i} W} \overline{D_j W} - 3|W|^2 \Big]$$

So, we need to specify K & W. By clever argument (hep-th/9906070) or direct dimensional reduction (hep-th/0105097), you find

$$W_{flux} = \int_{M_6} G_3 \wedge \Omega$$

Ω ← hol (3,0) form on CY

The Kähler potential (computable from CY prepotential) is:

- $$K = K_{complex} + K_{k\ddot{a}hler} + K_\tau$$

$$K_{complex}(z^\alpha) = -\log\left[-i\int_{M_6}\Omega\wedge\bar{\Omega}\right]$$

$$K_{k\ddot{a}hler}(\rho) = -3\ln\left[-i(\rho-\bar{\rho})\right]$$

where $\rho \sim \int_{\Sigma_4} C_4 + i\int_{\Sigma_4} J\wedge J$

$\sim (\text{axion}) + i R^4$

$$K_\tau = -\log\left[-i(\tau-\bar{\tau})\right]$$

Notice that W_{flux} is ρ-independent. [By contrast, in IIA, W_{flux} can depend on all moduli -- see eg hep-th/0505160]. But

$$D_\rho W_{flux} = \frac{K_{,\rho}}{M_p^2} W_{flux} \Rightarrow$$

$$g^{\rho\bar{\rho}} D_\rho W \overline{D_\rho W} \equiv 3\frac{|W|^2}{M_p^2}$$ at leading order.

This no-scale structure $\Rightarrow$ V simplifies:

$$V = e^{K} \left(\sum_{i,j=\alpha,\tau} g^{i\bar{j}} D_i W \overline{D_j W} \right) \geq 0$$

- Vacua all arise at $V = 0$ ($V > 0 \rightarrow$ tadpole for e.g. ρ from e^K prefactor)
- $\cancel{\text{SUSY}}$ order parameter?

$$W_{flux}\Big|_{\text{vacuum for } z^\alpha, \tau} \equiv W_0$$

- If $W_0 \neq 0$, $D_\rho W \sim W_0 \neq 0 \rightarrow \cancel{\text{SUSY}}$
- If $W_0 = 0$, $DW = 0$ $\forall$ fields -- $\mathcal{N}=1$ SUSY is preserved.

$W_0 \neq 0 \rightarrow G_{(3)}$ has a $(0,3)$ piece.

$(1,2)$ + $(3,0)$ pieces forbidden by ISD / $D_\alpha W = D_\tau W = 0$. So $G_{(3)} \in \left[H^{(2,1)} \oplus H^{(0,3)} \right]$.

<u>Beyond no-scale</u>

α' + g_s corrections always break the no-scale structure, so $V=0$ w/ $\cancel{SUSY}$ at tree lvl is mis-leading.

- corrections to K (cf Becker^2 - Haack - Louis)
- non-pert corrections to $W \sim e^{i\rho}$ from gauge effects on D7s or 'stringy' D3 instantons [← Witten hep-th/9604030]

You'll work out 'model building' consequences of such corrections on HW.