IAS - Lecture II (Kachru)

Last time, we developed a 4d description of a class of IIB vacua:

Today, we'll:

- Describe how one computes the vacuum structure in very simple examples
- See that duality implies existence of strange new models
- Summarize some more "global" claims about {IIB flux vacua}

I. Examples

We'll discuss two very simple cases.

A) (Imaginary) "Rigid CY" [c.f. hep-th /0404116, 0411061]

Rigid → $h^{2,1}(M_6) = 0$ → no complex moduli Z_α

So $W = \int (F - \tau H) \wedge \Omega$

depends only on the axio-dilaton τ.

Suppose we have a symplectic basis for $H_3(M_6)$ A, B & dual cohomology α, β:

$$A \cap B = pt \quad \text{others } 0$$

$$\int_A \alpha = -\int_B \beta = 1 \quad \text{others } 0$$

$$\int_{M_6} \alpha \wedge \beta = 1$$

Expand

$$F_3 = f_1 \alpha + f_2 \beta \qquad f_i \in \mathbb{Z}$$

$$H_3 = h_1 \alpha + h_2 \beta \qquad h_i \in \mathbb{Z}$$

D3 brane charge in fluxes?

$$\int F \wedge H = f_1 h_2 - h_1 f_2 \equiv N_{flux}$$

③

Assuming Ω has periods

$$\int_A \Omega = i \qquad \left(\rightarrow \Omega = i\alpha - \beta \right)$$

$$\int_B \Omega = 1$$

the flux superpotential is

$$W = \int (F - \tau H) \wedge \Omega =$$

$$A\tau + B$$

$$A = -h_1 + i h_2 \, , \quad B = -(f_1 + i f_2)$$

Equation for no-scale vacua?

$$k = -\log\left[-i(\tau - \bar{\tau}) \right] \rightarrow$$

$$D_\tau W = \frac{\partial W}{\partial \tau} + \left(\frac{-1}{-i(\tau - \bar{\tau})} \cdot -i \right) W$$

$$= A + \frac{1}{\bar{\tau} - \tau} (A\tau + B) \Rightarrow$$

$$A\bar{\tau} + B = 0$$

Solving for τ given fluxes $\rightarrow$ $\boxed{\bar{\tau} = -\frac{B}{A}}$

$$\mathrm{Im}(\tau) > 0 \iff N_{flux} > 0.$$

Modular Group

For the rigid model, $G = SL(2, \mathbb{Z})$.

To avoid over-counting models, we should fix this symmetry group.

One approach: Count only $\tau \in \mathcal{F}$

$$\mathcal{F} : \{ \tau : -1/2 \leq \mathrm{Re}\, \tau \leq 1/2,\ |\tau| \geq 1,\ |\tau| \neq 1 \text{ for } \mathrm{Re}(\tau) < 0 \}$$

Another way to fix $SL(2, \mathbb{Z})$ is to take only a "canonical" form of the fluxes.

$SL(2, \mathbb{Z})$ acts on F_3, H_3 as

$$\begin{pmatrix} F \\ H \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F \\ H \end{pmatrix}$$

So we can also gauge fix by requiring:

$$h_1 = 0, \quad 0 \leq f_2 < h_2.$$

Then assuming that $\exists$ "L" units of D3 charge

$$N_{Flux} \leq L \Rightarrow f_1 h_2 \leq L$$

<u>Total # of vacua</u>

$$N_{vacua}(L) = \sum_{m=1}^{L} \sum_{k|m} k$$

↑ amount of D3 charge in flux

choice of h_2; then k choices of f_2...

$$= \sum_{m=1}^{L} \sigma(m) \sim \frac{\pi^2}{12} L^2 \qquad \text{(cf. Hardy / Wright)}$$

where $\sigma(m) \equiv$ sum of divisors of m.

Notice $N_{vacua} \sim L^{b_3}$. } This holds also for real models with $b_3 \gg 1$; cf Douglas et al.

<u>Distribution of vacua on $\mathcal{M}_\tau$?</u>

Let's consider the other gauge fixing, $\tau \in \mathcal{F}$. And, let's assume $L \gg 1$ --

(6)

"continuous flux approximation."

- The # of complex #s $z = z_1 + i z_2$, $z_{1,2} \in \mathbb{Z}$, of magnitude $|z|$, goes like $2\pi |z|\, d|z|$. For large $|z|$, the phases are uniformly distributed on S^1.

Thus: Can estimate # of pairs A, B giving rise to any particular $\tau \in \mathcal{F}$, as follows. Taking

$$A = |A| e^{i\theta}$$

$$B = |B|\, e^{i(\theta+\psi)} = e^{i\theta} \beta$$

we have

$$L \geq |A||B| \sin\psi$$

$$\mathrm{Im}\,\tau = \left|\frac{B}{A}\right| \sin\psi$$

$$\Rightarrow \quad |A| \leq \sqrt{\frac{L}{\mathrm{Im}\,\tau}}$$

So, distribution of vacua scales as

$$N_{vac}(L;\tau) \sim \int_0^{\sqrt{\frac{L}{\mathrm{Im}\,\tau}}} 2\pi |A|\, d|A| \quad \times$$

$$\int d^2\beta \;\; \delta^2\left(\tau - \frac{1}{|A|}\beta\right)$$

$$\sim \frac{L^2}{(\mathrm{Im}\,\tau)^2} \quad \Big\} \text{ Governed by volume form on } \mathcal{M}_\tau$$

On HW, you'll learn more about scaling + distribution of flux vacua on generic Calabi-Yau spaces. [Ashok-Douglas, hepTh/0307049 + ...]

<u>B)</u> Simple toroidal orientifold + its "mirror"

I have been (and will in lectures 3, 4) focusing on IIB CY flux vacua. Much is also known about IIA CY flux vacua. <u>This</u> example hints that the landscape of flux vacua is much richer than these sets.

<u>Warm-up</u>: The twisted torus (c.f. hepth/0211182)

Imagine starting with a square T^3 M, with metric

$$ds^2 = dx^2 + dy^2 + dz^2 , \quad \text{and}$$

$$\int_M H_3 = N .$$

$H = dB$ -- choose gauge $B_{yz} = Nx$.

[This background $\not\to$ to a <u>static</u> sol'n, but we'll use it as part of a SUSY vacuum momentarily].

Now, T-dualize along the z direction. Applying Buscher's T-duality rules, the result is a background with:

- $B = 0$
- $ds^2 = dx^2 + dy^2 + (dz + Nx\, dy)^2$

"Nilmanifold" M

$(x, y, z) \cong (x, y+1, z) \cong (x, y, z+1) \cong (x+1, y, z - Ny)$

$h^1(M) = 2$ -- distinct topology from T^3

T-dualizing again along $y \Rightarrow$

$$ds^2 = \frac{1}{1 + N^2 x^2} (dz^2 + dy^2) + dx^2$$

$$B_{yz} = \frac{Nx}{1 + N^2 x^2}$$

Not well defined as you go around x circle;

(g, B) <u>are</u> periodic up to $O(2,2,\mathbb{Z})$ element

which is <u>NOT</u> in $SL(2,\mathbb{Z})$.

"Non-geometrical" -- we'll avoid discussing such

things, but see eg. hep-th/0508133 (Shelton et al).

Now, consider the $T^6/\mathbb{Z}_2$ orientifold ($\mathbb{Z}_2$ acts on all 6 circles). For simplicity, take $(T^2)^3$ with complex moduli $\tau_{1,2,3}$:

$$dz^i = dx^i + \tau_i \, dy^i$$

$$\Omega = \prod dz^i$$

Flux vacua in this model were studied in eg hep-th (9908088, 0201028, 0201029). One example (from 0211082) suffices for us. Let

$$F_3 = 2 \{ dx^1 dx^2 dy^3 + dy^1 dy^2 dy^3 \}$$

$$H_3 = 2 \{ dx^1 dx^2 dx^3 + dy^1 dy^2 dx^3 \}$$

→ can easily read off

$$W = 2 \{ \tau_1 \tau_2 + 1 \} + 2\phi \{ \tau_1 \tau_2 \tau_3 + \tau_3 \}$$

↖ new name for dilaton (ϕ)

Then :

$$\frac{\partial W}{\partial \tau_1} \propto \tau_2 (1 + \phi \tau_3)$$

$$\frac{\partial W}{\partial \tau_2} \propto \tau_1 (1 + \phi \tau_3)$$

$$\frac{\partial W}{\partial \tau_3} \propto \phi (\tau_1 \tau_2 + 1)$$

$$\frac{\partial W}{\partial \phi} \propto \tau_3 (\tau_1 \tau_2 + 1)$$

$\Rightarrow$ $\exists$ moduli space of $W = 0$ vacua:

$$\mathcal{M}: \quad \phi \tau_3 = -1, \quad \tau_1 \tau_2 = -1$$

In appropriate regions of $\mathcal{M}$, the best description involves T-dualizing 1, 2 or 3 times. In gauge

$$B_{x^1 x^3} = 2x^2 \quad , \quad B_{y^1 x^3} = 2y^2$$

One T-duality: (Along x^1)

$\Rightarrow$ IIA model with

$$ds^2 = \frac{1}{R_{x^1}^2} (dx^1 + 2x^2 \, dx^3)^2 +$$

$$R_{x^2}^2 (dx^2)^2 + R_{x^3}^2 (dx^3)^2 + \sum_i R_{y^i}^2 (dy^i)^2$$

So $X^1 \cdot\cdot X^3$ sweep out a Nilmanifold over the y T^3. Also,

$$H_3 = 2\, dy^1\, dy^2\, dx^3 \qquad B_{y^1 x^3} = 2y^2$$

$$F_2 = 2\, dx^2 \wedge dy^3$$

$$F_4 = 2\,(dx^1 + 2x^2\, dx^3) \wedge dy^1 \wedge dy^2 \wedge dy^3$$

Note for this manifold M $h^1(M) = 5 \Rightarrow$ the space is Non-kähler.

Dualize again on $y^1 \Rightarrow$ IIB with

$$ds^2 = \tilde{R}_{x^1}^2 (dx^1 + 2x^2\, dx^3)^2 + R_{x^2}^2 (dx^2)^2 + R_{x^3}^2 (dx^3)^2$$
$$+ \frac{1}{R_{y^1}^2} (dy^1 + 2y^2\, dx^3)^2 + R_{y^2}^2 (dy^2)^2 + (R_{y^3})^2 (dy^3)^2$$

$$B = 0$$

$$F_3 = 2\,(dx^1 + 2x^2\, dx^3) \wedge dy^2 \wedge dy^3 +$$
$$2\,(dy^1 + 2y^2\, dx^3) \wedge dx^2 \wedge dy^3$$

This space is also Non-Kähler.

Third T-duality on e.g. y^3 → IIA "mirror" of original model (ala SYZ Mirror symmetry = T-duality on T^3 fibers of Calabi-Yau).

$$F_2 = 2\,(dx^1 + 2x^2\,dx^3) \wedge dy^2 + 2\,(dy^1 + 2\,y^2\,dx^3) \wedge dx^2$$

Not a CY space. The "new" metric info is encoded in windings:

- $x^1 x^3$ T^2 has $SL(2,\mathbb{Z})$ monodromy as $x^2 \to x^2 + 1$
- $y^1 x^3$ T^2 has $SL(2,\mathbb{Z})$ monodromy as $y^2 \to y^2 + 1$

This simple e.g. suggests IIB/IIA flux vacua based on CY spaces are just the tip of the iceberg.