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IAS - Lecture IV

Refs: Keith 0507205
0512102

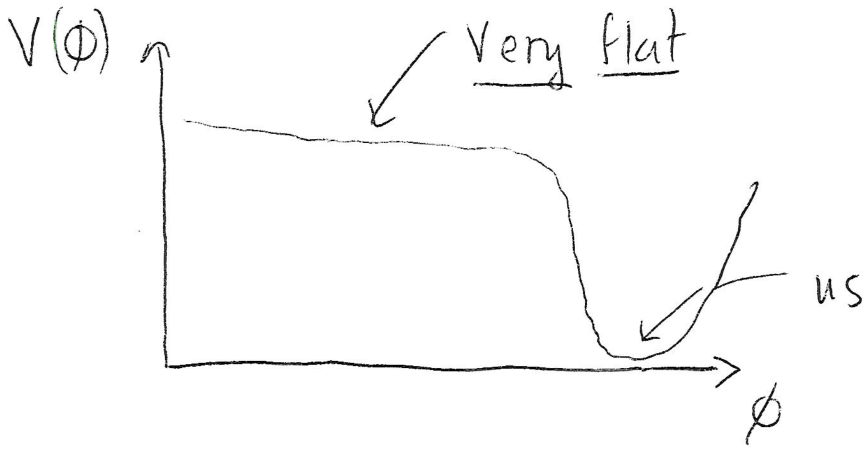
Today: the interface of inflation + string theory. I'll focus on issues about UV physics that could be raised by results of future experiments (Planck launch in < 2 years).

I. Inflationary wish list (standard slow roll)

To solve
- horizon
- flatness
- monopole

problems, inflationary theorists postulate a phase of dS-like exponential expansion in the early Universe. Need for a graceful exit →

②



- $V(\phi)$ must have very flat region to trick Universe into dS-like expansion

$$ds^2 = -dt^2 + a^2(t)\,[\,d\vec{x}^2\,]$$

Friedmann eqns for $a(t) \to$ need

- $\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

to get $a(t) \sim e^{Ht}$ $\quad (M_P^2 H^2 \cdot 3 = V)$

- $\eta = M_P^2 \, \frac{V''}{V} \ll 1$

to keep $a \sim e^{Ht}$ for enough "Hubble times" $\Delta t \sim \frac{1}{H}$ to solve horizon/flatness.

Current thinking says one needs

$$N_e \sim \frac{1}{M_p^2} \int_{\phi_0}^{\phi_F} \frac{V}{V'} \, d\phi \gtrsim 60$$

[assuming $V_{inf} \sim M_{GUT}$ -- comes down logarithmically for lower V].

<u>Bonus</u>: 'Quantum fluctuations' of inflaton can seed density perturbations!

$$\frac{\delta\rho}{\rho} \sim \frac{1}{\sqrt{75\pi}} \frac{1}{M_p^3} \frac{V^{3/2}}{V'} \qquad (\square)$$

$$\frac{\delta\rho}{\rho} \sim 10^{-5} \text{ (experiment)} \quad + \quad \varepsilon \sim O\left(\tfrac{1}{100}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{75\pi}} \times \frac{1}{\sqrt{2}} \frac{V^{1/2}}{M_p^2} \frac{1}{\sqrt{\varepsilon}} = 10^{-5}$$

$$\rightarrow \quad \frac{V^{1/2}}{M_p^2} \sim 10^{-5} \Rightarrow \boxed{(V)^{1/4} \sim M_{GUT}}$$

So we naively expect inflation happened at $H \sim 10^{14}$ GeV.

Results from eg WMAP are consistent w/ such a picture

$$n_s \lesssim 1 \rightarrow \epsilon, \eta \text{ small}$$

but PLANCK will pin down two numbers which are very sensitive to UV physics:

- "tensor to scalar ratio"

  $\underline{\underline{r(k)}}$ measures power in $\dfrac{\text{gravity waves}}{\text{scalar modes}}$:

$$r \sim .14 \frac{P_g}{\left(\frac{\delta\rho}{\rho}\right)}$$

In $\underline{\text{slow-roll}}$ inflation

$$r \sim 6.9 \, M_p^2 \left(\frac{V'}{V}\right)^2$$

and actually

$$V^{1/4} \simeq \left(\frac{r}{.07}\right)^{1/4} \times 2 \times 10^{16} \text{ GeV}$$

PLANCK will detect any $r \gtrsim .05$.

- Also, Non-Gaussianity f_{NL} -- see $\begin{pmatrix} 0404084 \\ 0605045 \end{pmatrix}$.

II. The Lyth bound

hep-ph/9606387

Notice that in slow roll models,

$$N_e \sim \frac{1}{M_p^2} \int d\phi \, \frac{V}{V'} \Rightarrow$$

$$\frac{1}{M_p} \frac{\Delta\phi}{\Delta N} \sim M_p \frac{V'}{V} \sim \left(\frac{r}{7}\right)^{1/2}$$

So even in the ~ 5 e-foldings nearest to our horizon, total $\Delta\phi$ was

$$\frac{\Delta\phi}{M_p} \simeq 5 \left(\frac{r}{6.9}\right)^{1/2} \sim .5 \left(\frac{r}{.07}\right)^{1/2}$$

For all 60, it seems then that <u>measurable</u> $\underline{r} \Rightarrow \frac{\Delta\phi}{M_p} > 1$, i.e. inflaton rolled over a super-Planckian distance. } (canonical eg $\underline{m^2\phi^2} \rightarrow \Delta\phi \gtrsim 15\, M_p$)

Efstathiou + Mack (astro-ph/0503360) argue (convincingly??) that $\frac{\Delta\phi}{M_p} > 6\, r^{1/4}$.

Why is this an <u>issue</u>?

For a "garden variety" modulus (of compact dims, of D-brane, ...)

$$V(\phi) = V_{renormalizable}(\phi) + \phi^4 \sum_{m \geq 1} C_m \left(\frac{\phi}{M_s}\right)^m \qquad (C_m \sim \mathcal{O}(1))$$

$\Delta\phi > M_P \gg M_s \Rightarrow$ how on earth can one keep V <u>flat</u> over such large $\Delta\phi$?

"<u>Functional</u> fine tune".

<u>Contrast</u> to hybrid inflation, where in almost all models $\Delta\phi \ll M_P$; there, $\eta \sim \frac{1}{100} \rightarrow$ usually tune of 1 coefficient.

All known D-brane inflation models are of this $\Delta\phi \ll M_P$ type.

Reasonable idea: Try to use a PNGB, e.g. an axion (cf "Natural inflation")

- shift symmetry forbids $\delta V \sim \frac{V}{M_P^2} a^2$ } spurion of shift breaking must enter
- Potential takes form

$$V(a) = \Lambda^4 \left[1 - \cos\left(a/f_a \right) \right]$$

- f_a = "axion decay constant"
- Λ = dynamical scale; eg in heterotic string theory, axions arise from soaking up forms in $H^2(M_6)$ into $B_{\mu\nu}$; and WS instantons break PQ symmetry, yield

$$\Lambda \sim M_s \, e^{-\text{Area (curve)}}$$

Can $V(a)$ give us a good inflaton candidate that has $\Delta a > M_P$ + could hence "explain" a measurement of r?

For <u>generic</u> initial value of a

- $\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \sim \left(\frac{M_P}{f_a}\right)^2$
- $\eta = M_P^2 \frac{V''}{V} \sim \left(\frac{M_P}{f_a}\right)^2$

Robustly getting inflation requires $\underline{\underline{f \gg M_P}}$.

But ...

- generic string theory axions have

$$f_a < \frac{M_P}{S_{inst}}$$

Svrcek, Witten

and $S \sim 20$ to give correct scale to V.

- Several authors have conjectured that having $f_a > M_P$ is impossible in string theory.

"Swampland" papers; Banks, Dine, Fox, Gorbatov

A simple (yet ugly) proposal to surmount this problem: "N-flation" hep-th/0507205.

- $N = h^2(M_6)$ axions in heterotic theory

Imagine $N \gg 1$.

<u>(rudest</u> toy model)

$$\mathcal{L} = \sum_{i=1}^{N} \left(\frac{1}{2}(\partial a_i)^2 - \Lambda^4 \times \left\{ 1 - \cos\left(\frac{a_i}{f}\right) \right\} \right)$$

- Obviously more realistic to have a metric on M & distinct Λ_i, f_i, properly diagonalize, etc -- see ref above & hep-th/0512102 for discussion, nothing important changes.

- N indep shift symmetries $\Rightarrow$ each axion has protection from receiving

$$\Delta V(a_i) \sim \frac{V}{M_p^2} a_i^2$$

(any symmetry breaking term with both a_i, a_j must have Λ_i^4, Λ_j^4 in it).

- So naively, each axion feels Hubble friction due to all N $\rightarrow$ for generic initial conditions

$$\left.\begin{aligned} \epsilon &\sim \left(\frac{M_P}{f}\right)^2 \frac{1}{N^2} \\ \eta &\sim \left(\frac{M_P}{f}\right)^2 \frac{1}{N} \end{aligned}\right\}$$

c.f. "assisted inflation," but note N axions crucial here (radiative stability)

For large enough N, it looks like one can make a working model with $(\Delta a)_{tot} \sim \sum_i (\Delta a_i)^2 > M_P$ (thanks to Pythagoras).

Is this true?

Leading N-dependent radiative correction:

$$\delta M_P^2 \sim \pm \frac{N}{16\pi^2} \Lambda_{uv}^2$$ "Species problem"

Then

$$\eta \sim \frac{1}{N}\left(\frac{M_P}{f}\right)^2 \left[1 \pm \frac{N \Lambda_{uv}^2}{16\pi^2 M_P^2}\right]$$

So we only trust our analysis up to

$$N_{max} \simeq 16\pi^2 \frac{M_P^2}{\Lambda_{uv}^2}$$

$$\Rightarrow \quad N_e \simeq N\left(\frac{f}{M_P}\right)^2 \lesssim 16\pi^2 \left(\frac{f}{M_P}\right)^2 \frac{M_P^2}{\Lambda_{uv}^2}$$

more careful → this is more like $\underline{\underline{4\pi^2}}$

This looks promising for $\Lambda_{uv} \ll M_P$, but is UV sensitive clearly. What does string theory give for Λ_{uv}? <u>Leading</u> correction:

$$\mathcal{L}_{10D} \supset M_*^8 \left[R_{10} + \zeta(3)(\alpha')^3 R_{10}^4 + \cdots \right]$$

→ using $\int_M R \wedge R \wedge R = \frac{\chi(M)}{(2\pi)^3}$, we get

$$\mathcal{L}_{4D} \supset M_P^2 \left(1 + \frac{\chi(M)}{8\pi^3} \frac{(\alpha')^3}{V_6} \zeta(3)\right) R_4$$

So

$$\delta M_P^2 = N \frac{\Lambda_{uv}^2}{16\pi^2} \underset{\text{string}}{=} \frac{\chi(M_6)\,\zeta(3)}{8\pi^3} \frac{(\alpha')^3}{V_6} \times M_P^2$$

So: $$\boxed{\Lambda_{uv}^2 = M_P^2 \times \frac{2\zeta(3)}{\pi} \left(\frac{\alpha'^3}{V_6}\right) \frac{\chi(M_6)}{N}}$$

Notice $\chi(M_6) = 2|N - \tilde{N}|$ for CY M_6

where: $N = \dim H_2$, $\tilde{N} = \dim H_3$.

Plugging into formula for N_e $\Rightarrow$

$$N_e \sim \frac{2\pi^3}{\zeta(3)} \frac{N}{|\chi(M)|}$$

-- can exceed 60 for small cancellation between N, $\tilde{N}$.

- Special case where we Taylor expand V around origin $\forall$ axions $\rightarrow$ <u>$m^2\phi^2$ chaotic infl.</u>

So, if we <u>see</u> $r \gtrsim .05$ (or maybe even $\gtrsim 10^{-2}$), it means:

- $\Delta\phi > M_P$ <u>challenge for strings</u>

  N-flation? Something prettier??

- Not slow roll inflation??? Hard to believe this... (earliest models <u>predicted</u> r...).

Finding plausible string models that $\rightarrow \underline{r}$ seems worthwhile.