D-Branes on Cones and Gauge/String Dualities

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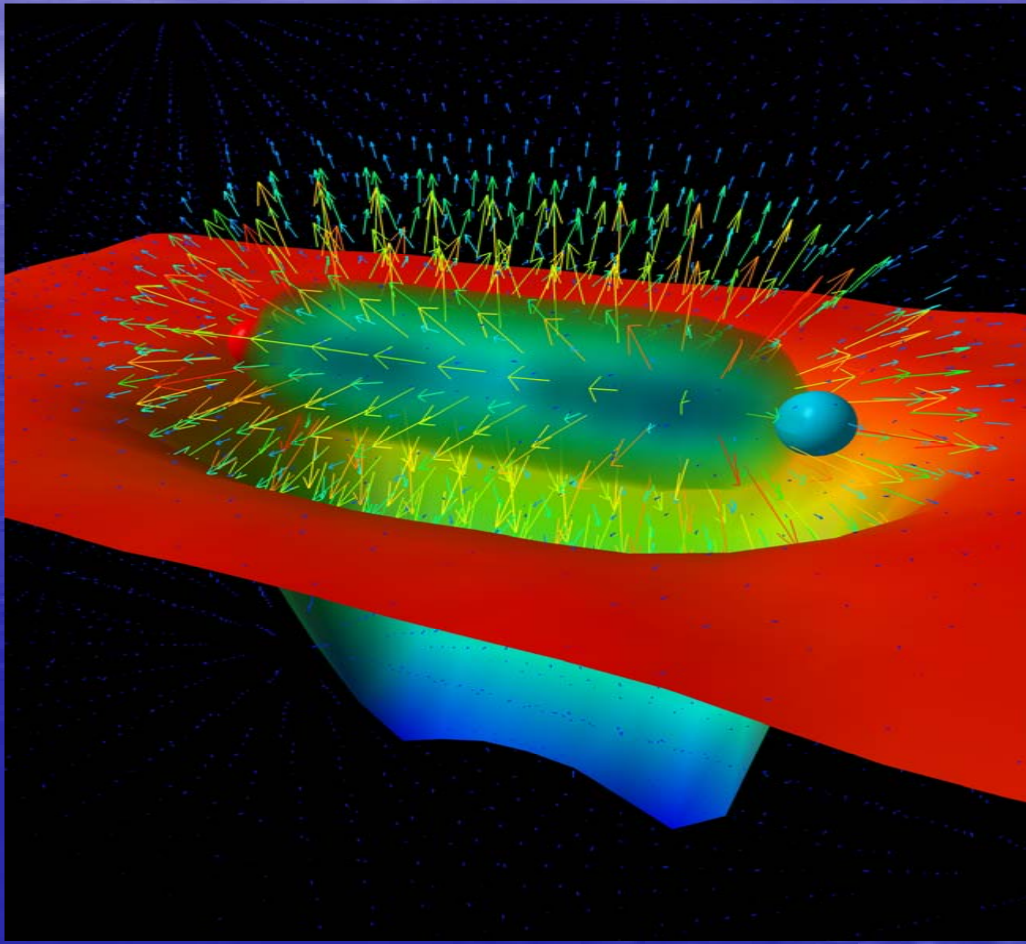
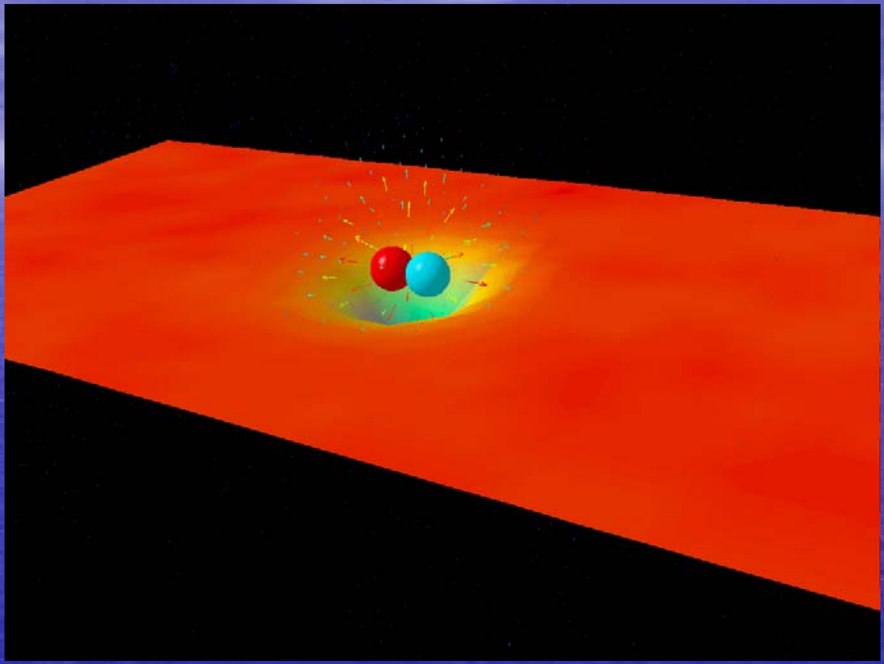
QCD and String Theory

- At short distances, must smaller than 1 fermi, the quark-antiquark potential is approximately Coulombic, due to the Asymptotic Freedom.
- At large distances the potential should be linear (Wilson) due to formation of confining flux tubes.

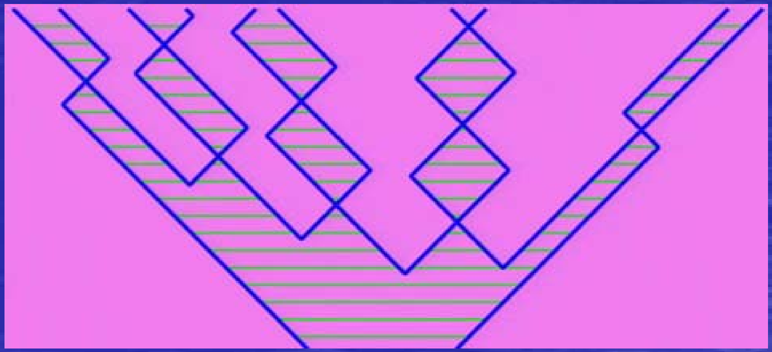


Flux Tubes in QCD

- These objects may be approximately described by the Nambu strings
  (animation from lattice work by D. Leinweber et al, Univ. of Adelaide)



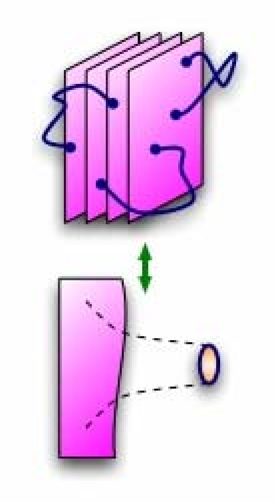
- The tubes are widely used, for example, in jet hadronization algorithms (the Lund String Model) where they snap through quark-antiquark creation.



Large N Gauge Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N$ fixed.
- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.
- Yet, the planar diagrams needed in the large N limit are very difficult to sum explicitly.

D-Branes vs. Geometry



- Dirichlet branes (Polchinski) led string theory back to gauge theory in the mid-90's.
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}=4$ supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

- Successful matching of graviton absorption by D3-branes, related to 2-point function of stress-energy tensor in the SYM theory, with a gravity calculation in the 3-brane metric (IK; Gubser, IK, Tseytlin) was a precursor of the AdS/CFT correspondence.

Conformal Invariance

- In the $\mathcal{N}$=4 SU(N) SYM theory theory there are 3 adjoint chiral superfields Z^i coupled to the $\mathcal{N}$=1 SU(N) SYM theory with superpotential Tr Z^1 [Z^2,Z^3].
- The Asymptotic Freedom is canceled by the extra fields; the beta function is exactly zero! Hence, the theory is invariant under scale transformations $x^\mu \to \lambda x^\mu$. It is also invariant under space-time inversions.
- Such a theory is called a Conformal Field Theory (CFT).
- The $\mathcal{N}$=4 SU(N) SYM is also invariant under the SU(4) R-symmetry. Its full super-conformal symmetry is SU(2,2|4).

The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-sphere realizing the SU(4) R-symmetry.
- The SO(2,4) geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The d-dimensional AdS space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \ .$$

- Its metric is

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2}(dx^i)^2\right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\mathrm{YM}}^2 N}$
- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator dimension is determined by the mass of the dual field; e.g. for scalar operators

$$\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$$

- Correlation functions are calculated from the dependence of string theory path integral on boundary conditions ϕ_0 in AdS_5, imposed near z=0:

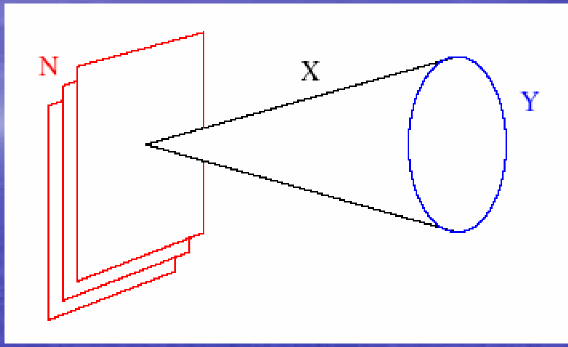
$$\langle \exp \int d^4x \phi_0 \mathcal{O} \rangle = Z_{\mathrm{string}}[\phi_0]$$

- In the large N limit the path integral is found from the classical string action: $Z_{\mathrm{string}}[\phi_0] \sim \exp(-I[\phi_0])$

Conebrane Dualities

- To reduce the number of supersymmetries in AdS/CFT, we may place the stack of N D3-branes at the tip of a 6-d Ricci-flat cone X whose base is a 5-d Einstein space Y:

$$ds_X^2 = dr^2 + r^2 ds_Y^2$$



- Taking the near-horizon limit of the background created by the N D3-branes, we find the space AdS_5 x Y, with N units of RR 5-form flux, whose radius is given by

$$L^4 = \frac{\sqrt{\pi}\kappa N}{2\,\mathrm{Vol}(Y)} = 4\pi g_s N \alpha'^2 \frac{\pi^3}{\mathrm{Vol}(Y)}$$

- This type IIB background is conjectured to be dual to the IR limit of the gauge theory on N D3-branes at the tip of the cone X.

Trace Anomaly

- In a 4-d CFT there are two trace anomaly coefficients, a and c: $\langle T^\alpha_\alpha \rangle = -aE_4 - cI_4$

$$E_4 = \frac{1}{16\pi^2}\left(R_{ijkl}^2 - 4R_{ij}^2 + R^2\right)$$
$$I_4 = -\frac{1}{16\pi^2}\left(R_{ijkl}^2 - 2R_{ij}^2 + \tfrac{1}{3}R^2\right)$$

- Calculations on AdS_5 x Y give their leading large N values Henningson, Skenderis; Gubser

$$a = c = \frac{\pi^3 N^2}{4\,\mathrm{Vol}(Y)}$$

- In super-conformal theories the anomalies are related to the spectrum of R-charges of the chiral fermions: Anselmi, Freedman, Grisaru, Johansen

$$a = \frac{3}{32}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)\ ; \quad c = \frac{1}{32}(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R)$$

- This provides basic checks of the dualities.
- For the $\mathcal{N}=4$ SYM theory the gauginos have R=1, while the fermion fields from the Z^i chiral multiplets have R=-1/3.
- Since Tr R=0, we find a=c, and

$$a = c = \frac{9}{32}\mathrm{Tr}R^3 = (N^2-1)\frac{9}{32}\left(1+3(-1/3)^3\right) = \frac{N^2-1}{4}$$

- On the gravity side, the volume of S^5 is π^3 (the radius of Y is fixed so that $R_{ij} = 4g_{ij}$)
- For large N the two calculations of anomaly coefficients agree.

Orbifold Cones

Kachru, Silverstein; Lawrence, Nekrasov, Vafa

- The simplest set of examples is provided by cones that are orbifolds R^6/Γ , where Γ is a subgroup of the rotation group SU(4).
- For abelian orbifolds, all group elements can be brought to the form

$$\begin{pmatrix} e^{2\pi i x_1} & 0 & 0 & 0 \\ 0 & e^{2\pi i x_2} & 0 & 0 \\ 0 & 0 & e^{2\pi i x_3} & 0 \\ 0 & 0 & 0 & e^{-2\pi i(x_1+x_2+x_3)} \end{pmatrix}$$

- For Z_k orbifolds, the n-th group element is specified by three integers m_i defined mod k: $x_i=nm_i/k$.
- If none of the eigenvalues of the generator = 1, then all SUSY is broken; if one of the eigenvalues = 1, then $\mathcal{N}$=1 SUSY is preserved; if two of the eigenvalues = 1, then $\mathcal{N}$=2 SUSY is preserved.

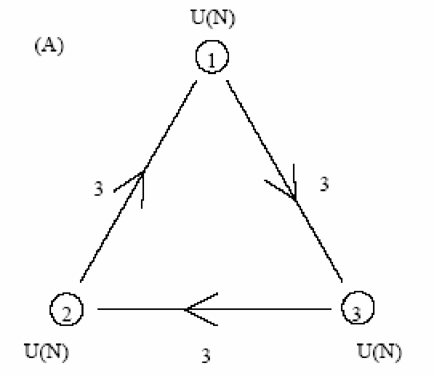
- The action in the n-th twisted sector on 3 complex coordinates of C^3, $Z^1 = X^1 + iX^2\,, \quad Z^2 = X^3 + iX^4\,, \quad Z^3 = X^5 + iX^6$

  and their complex conjugates, is

$$R(g_n) = \mathrm{diag}(\omega_k^{n(m_1+m_2)}, \omega_k^{n(m_1+m_3)}, \omega_k^{n(m_2+m_3)}, \omega_k^{-n(m_1+m_2)}, \omega_k^{-n(m_1+m_3)}, \omega_k^{-n(m_2+m_3)})$$

  where $\omega_k = e^{2\pi i/k}$

- If none of these phases = 1, then the orbifold acts freely on S^5/Γ. (The tip of the cone is a fixed point that is removed in the basic near-horizon limit.)
- A well-known example of a freely-acting orbifold is Z_3 with m_i=1. Since one of the eigenvalues of the generator = 1, i.e. $\Gamma \subset SU(3)$, this orbifold preserves $\mathcal{N}$=1 SUSY.



Construction of the quiver gauge theories Douglas, Moore

- Gauge theory on N D3-branes at the tip of R^6/Γ is found by applying projections to the U(Nk) gauge theory on the covering space. Retain only the fields invariant under the orbifold action combined with conjugation by a U(Nk) matrix γ acting on the gauge indices: $\gamma = \mathrm{diag}(I_N, e^{2\pi i/k} I_N, e^{4\pi i/k} I_N, \ldots, e^{-2\pi i/k} I_N)$

$$\psi^1 \to e^{2\pi i m_1/k} \gamma \psi^1 \gamma^{-1} , \qquad \psi^2 \to e^{2\pi i m_2/k} \gamma \psi^2 \gamma^{-1} , \ \ldots$$

$$Z^1 \to e^{2\pi i (m_1+m_2)/k} \gamma Z^1 \gamma^{-1} , \qquad Z^2 \to e^{2\pi i (m_1+m_3)/k} \gamma Z^2 \gamma^{-1} , \ \ldots$$

- In the supersymmetric examples, such as C^3/Z_3 or the conifold, the conebrane dualities have been tested almost as thoroughly as in the maximally supersymmetric case.
- But when all SUSY is broken, problems may arise.
- When the orbifold Γ breaks all SUSY and is not freely acting, then the weakly curved background $AdS_5 \times S^5/\Gamma$ is unstable due to the presence of tachyons that have $(mL)^2 < -4$, and therefore violate the BF bound.
- But for freely acting orbifolds, the negative zero-point energy is compensated by the large stretching of the twisted sector closed strings in the compact space. Hence, at large radius, there are no `bad tachyons.' This makes freely acting orbifolds particularly interesting from AdS/CFT point of view.
- Yet, before the formal decoupling limit, the non-SUSY freely acting orbifolds have closed-string tachyons localized at the tip of the cone.

Closed String Zero-Point Energy

- In light-Cone Green-Schwarz, there are 4 complex world sheet bosons and fermions.
- In the n-th twisted sector the boundary conditions on the fermions are $$b^l(\sigma + 2\pi) = e^{2\pi i n m_l/k} b^l(\sigma)$$ and on the bosons
$$X^1(\sigma + 2\pi) = X^1(\sigma) \ ,$$
$$X^2(\sigma + 2\pi) = e^{2\pi i n(m_1+m_2)/k} X^2(\sigma)$$
$$X^3(\sigma + 2\pi) = e^{2\pi i n(m_1+m_3)/k} X^3(\sigma)$$
$$X^4(\sigma + 2\pi) = e^{2\pi i n(m_2+m_3)/k} X^4(\sigma)$$
- The total zero-point energy of these modes is

$$\begin{aligned} 8E_0(x_1, x_2, x_3) \;\; = \;\; & -1 - (2\{x_1 + x_2\} - 1)^2 - (2\{x_1 + x_3\} - 1)^2 - (2\{x_2 + x_3\} - 1)^2 \\ & + (2\{x_1 + x_2 + x_3\} - 1)^2 + \sum_{i=1}^{3} (2\{x_i\} - 1)^2 \end{aligned}$$

Weak coupling analysis of non-SUSY quivers

- My recent work with Dymarsky and Roiban (hep-th/0505099 and 0509132) reconsiders quiver gauge theory on a stack of D3-branes at the tip of a cone R^6/Γ where the orbifold group Γ breaks all the supersymmetry.
- At first sight, the gauge theory seems conformal because the planar beta functions for all single-trace operators vanish. The candidate string dual is $AdS_5 \times S^5/\Gamma$. Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson
- However, dimension 4 double-trace operators made out of twisted single-trace ones, $f\, O_n\, O_{-n}$, are induced at one-loop. Their planar beta-functions have the form

$$\beta_f = a\,\lambda^2 + 2\,\gamma\, f\,\lambda + f^2$$

$$\beta_\lambda = 0$$

A Note on Normalizations

- The VEV of a single trace operator is of order N.
- The standard Yang-Mills action $S = -\int d^4x \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu}^2$ is of order N^2 in the `t Hooft limit.
- The double-trace operators f O_n O_{-n} make contributions of the same order (for the coupling constant of order 1). They cannot be ignored in the leading large N limit.
- In fact, the tree-level potential of the $SU(N)^k$ quiver theories (with the interacting U(1)'s decoupled) contains such double-trace terms.

Problem 1

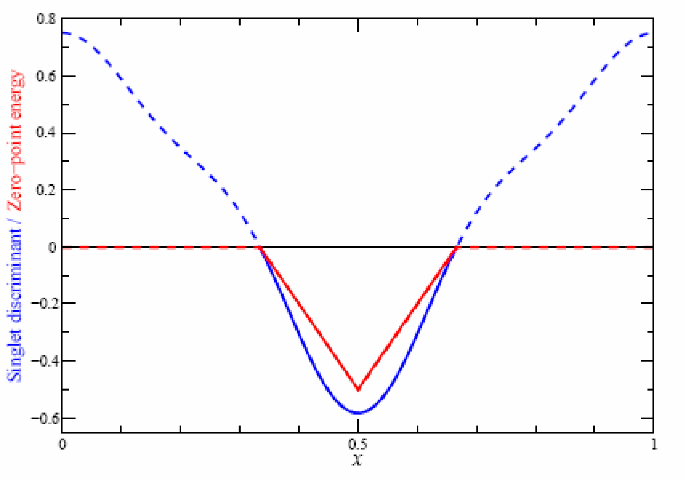
a) Derive the Z_2 quiver gauge theory obtained by the projection on the $U(2N)$ $\mathcal{N} = 4$ SYM where the Z_2 is generated by $-I$ in $SU(4)$ accompanied by conjugation with $\gamma = \mathrm{diag}(I_N, -I_N)$. What is the gauge group and field content? Is this a freely acting orbifold?

b) In this gauge theory, calculate the one-loop Coleman-Weinberg potential as a function of the eigenvalues of the adjoint scalars. What are the operators that pick up one-loop beta functions?

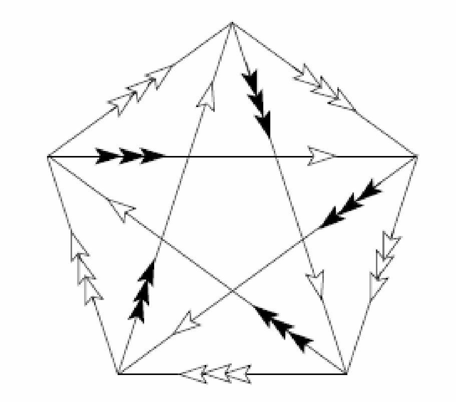
- If $D=\gamma^2 - a < 0$, then there is no real fixed point for f.
- A class of Z_k orbifolds with global SU(3) symmetry, that are freely acting on the 5-sphere, has the group action in the fundamental of SU(4)

$$r(g^n) = \mathrm{diag}(\omega_k^n, \omega_k^n, \omega_k^n, \omega_k^{-3n})$$

$$\omega_k = e^{i\alpha_k}\ , \qquad \alpha_k = \frac{2\pi}{k}$$



- Here is a plot of a one-loop $SU(N)^k$ gauge theory discriminant, D, and of the ground state closed string m^2 on the cone without the D-branes. n=1, ..., k-1 labels the twisted sector, and x=n/k.



- The simplest freely acting non-susy example is Z_5 where there are four induced double-trace couplings

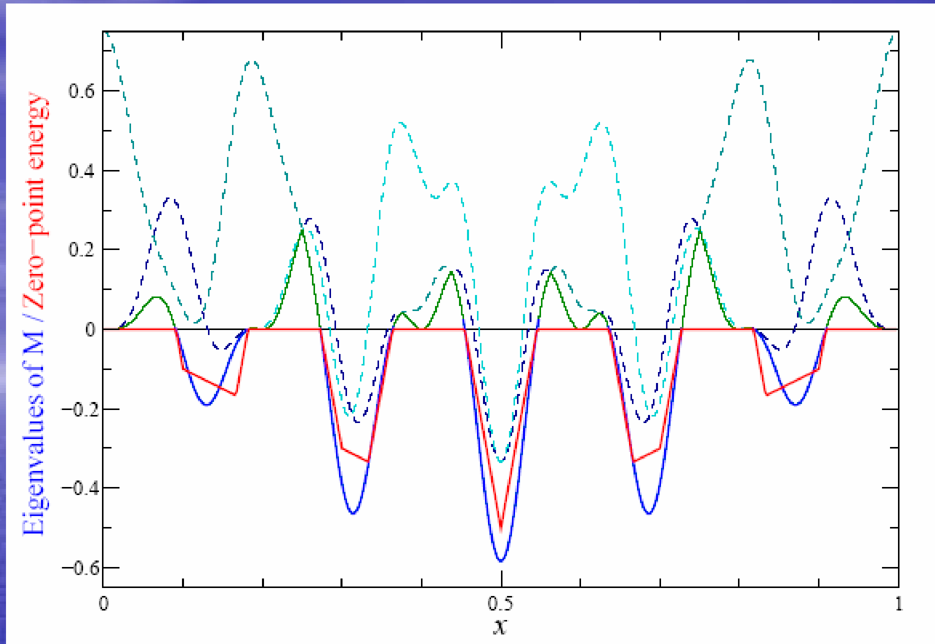
$$\delta_{2\ \mathrm{trace}}\mathcal{L} = f_{8,1} O_1^{\langle i\bar{j}\rangle} O_{-1}^{\langle j\bar{i}\rangle} + f_{8,2} O_2^{\langle i\bar{j}\rangle} O_{-2}^{\langle j\bar{i}\rangle} + f_{1,1} O_1 O_{-1} + f_{1,2} O_2 O_{-2}$$

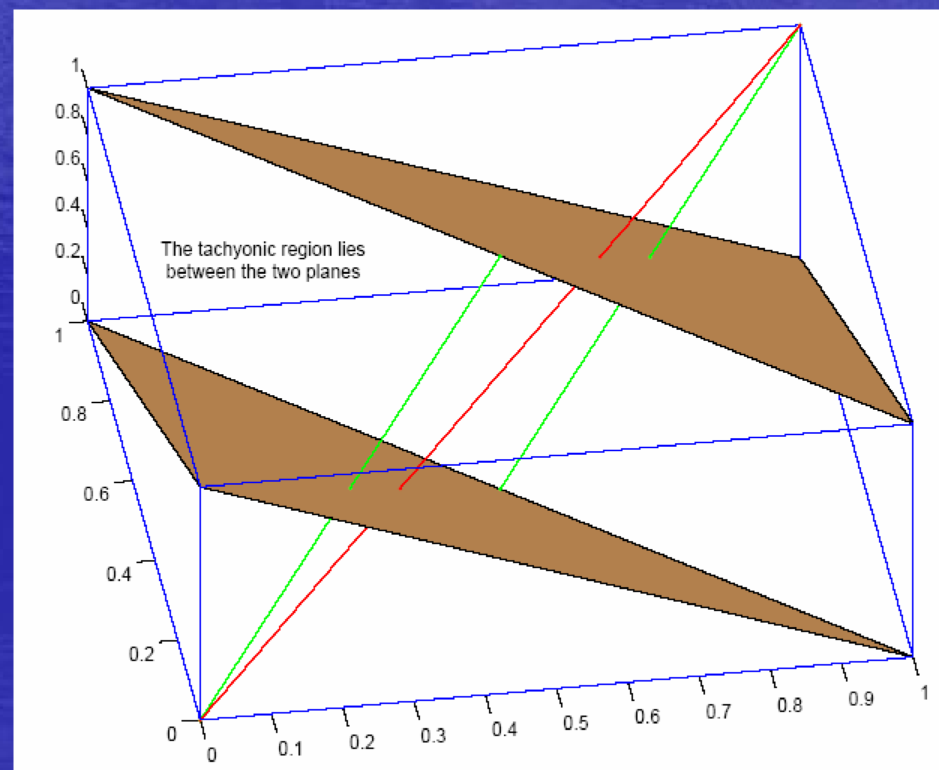
- For example, the SU(3) adjoints

$O_n^{\langle i\bar{j}\rangle}$ are $$\sum_{k=1}^{5}\left(\Phi^i_{k,k+2}\Phi^{\bar{j}}_{k+2,k} - \frac{1}{3}\eta^{i\bar{j}}\Phi^l_{k,k+2}\Phi^{\bar{l}}_{k+2,k}\right) e^{in\alpha(k-1)}$$

($\alpha=2\pi/5$)

- For more complicated orbifolds, crossing of eigenvalues of the discriminant matrix becomes important. The agreement with closed strings continues to hold.
- Generally, there are three twist angles x_i that define a cube. The stability/instability regions agree between one-loop gauge theory and string theory.





- Any non-SUSY abelian orbifold contains unstable operators. This appears to remove all such orbifold quivers from a list of large N perturbatively conformal gauge theories.
- The one-loop beta functions destroy the conformal invariance precisely in those twisted sectors where there exist closed-string tachyons localized at the tip of R^6/Γ. Thus, a very simple correspondence emerges between perturbative gauge theory and free closed string on an orbifold. Why? Perhaps, in the presence of tachyons, the standard AdS/CFT decoupling argument may fail.
- The $AdS_5 \times S^5/\Gamma$ background is tachyon-free at large radius. Could it have some instabilities? If not, then there is a transition from instability to stability as λ is increased.

- What is the end-point of the RG flow?
- Condensation of localized tachyon smoothes out the tip of the cone. Adams, Polchinski, Silverstein
- The gauge theory on D3-branes at a smooth point is $\mathcal{N}$=4 SYM. Hence, a natural conjecture is that the gauge theory flows from the non-SUSY $SU(N)^k$ quiver gauge theory to the $\mathcal{N}$=4 SU(N) SYM. Dymarsky, Franco, Roiban, IK (work in progress)

D3-branes on the Conifold

- The conifold is a Calabi-Yau 3-fold cone X described by the constraint $\sum_{a=1}^{4} z_a^2 = 0$ on 4 complex variables.
- Its base Y is a coset $T^{1,1}$ which has symmetry $SU(2)_A x SU(2)_B$ that rotates the z's, and also $U(1)_R$: $z_a \to e^{i\theta} z_a$
- The Sasaki-Einstein metric on $T^{1,1}$ is

$$\mathrm{d}s^2_{T^{1,1}} = \frac{1}{9}\left(\mathrm{d}\psi + \sum_{i=1}^{2} \cos\theta_i \mathrm{d}\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(\mathrm{d}\theta_i^2 + \sin^2\theta_i \mathrm{d}\phi_i^2\right)$$

  where $\theta_i \in [0,\pi], \phi_i \in [0,2\pi], \psi \in [0,4\pi]$
- The topology of $T^{1,1}$ is S^2 x S^3.

- The $\mathcal{N}$=1 SCFT on the D3-branes at the apex of the conifold has gauge group SU(N)xSU(N) coupled to bifundamental chiral superfields A_1, A_2, in $(\overline{\mathbf{N}}, \mathbf{N})$, and B_1, B_2 in $(\mathbf{N}, \overline{\mathbf{N}})$. IK, Witten
- The R-charge of each fields is ½. This insures $U(1)_R$ anomaly cancellation.
- The unique $SU(2)_A \times SU(2)_B$ invariant, exactly marginal quartic superpotential is added:

$$W = \epsilon^{ij}\epsilon^{kl}\,\mathrm{tr}\,A_i B_k A_j B_l$$

- This theory also has a baryonic U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$, and a Z_2 symmetry which interchanges the A's with the B's and implements charge conjugation.

Comparison with a Z_2 Orbifold Quiver

- The simplest $\mathcal{N}=2$ SUSY quiver has k=2; $m_1=m_2=1$, $m_3=0$. The gauge group is again SU(N)xSU(N), but in addition to the bifundamentals A_i, B_j, there is one adjoint chiral superfield for each gauge group, with superpotential $g\mathrm{Tr}\Phi(A_1B_1 - A_2B_2) + g\mathrm{Tr}\tilde{\Phi}(B_1A_1 - B_2A_2)$
- Adding a Z_2 odd mass term $\frac{m}{2}(\mathrm{Tr}\Phi^2 - \mathrm{Tr}\tilde{\Phi}^2)$

and integrating out the adjoints, we obtain the superpotential of the conifold theory,

$$-\frac{g^2}{m}\left[\mathrm{Tr}(A_1B_1A_2B_2) - \mathrm{Tr}(B_1A_1B_2A_2)\right]$$

Problem 2

In a supersymmetric field theory, the trace anomaly coefficients a and c are given by the formulae

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - 3\mathrm{Tr}R\right) \ , \qquad c = \frac{1}{32}\left(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R\right) \ ,$$

where R refers to the $U(1)_R$ charges, and the trace is over all the chiral fermion fields.

a) Calculate a and c in the following two gauge theories: the $\mathcal{N}=2$ supersymmetric Z_2 orbifold quiver, and in the $\mathcal{N}=1$ SCFT on N D3-branes at the conifold.

b) For $AdS_5 \times Y$ with N units of RR 5-form flux, it was found at leading order in N that

$$a = c = \frac{N^2\pi^3}{4\mathrm{vol}(Y)} \ ,$$

where the radius of Y is normalized so that $R_{ij} = 4g_{ij}$ on Y. Compare this formula with the gauge theory results of part a).

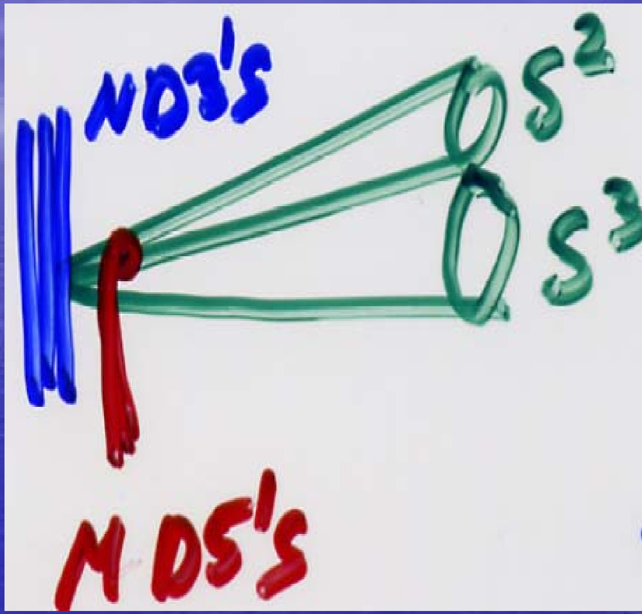
Breaking the Conformal Symmetry

- A useful tool is to add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (KS, Strassler)

$$ds^2_{10} = h^{-1/2}(t)\left(-(dx^0)^2 + (dx^i)^2\right) + h^{1/2}(t)ds^2_6$$

- ds^2_6 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

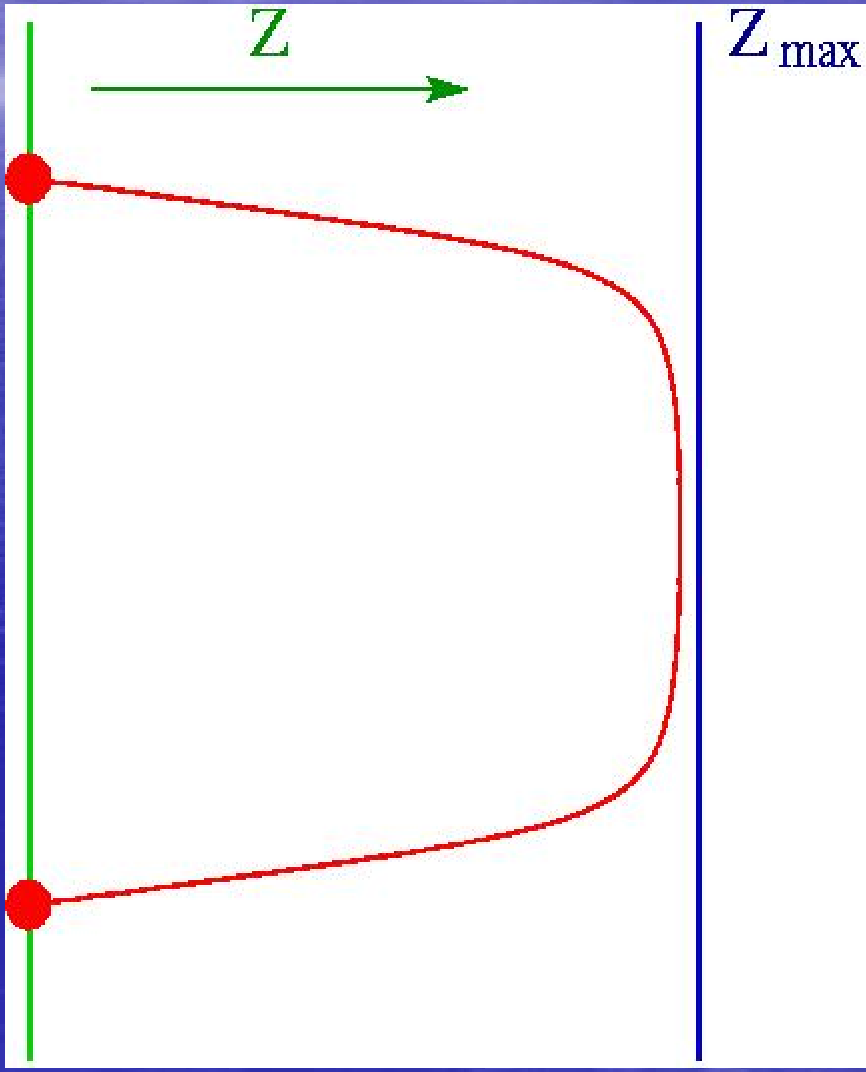


String Theoretic Approach to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- A "cartoon" of the necessary metric is

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\max})}{2\pi\alpha'}$



- The warp factor is finite at the `end of space' t=0, as required for the confinement: $h(t)= 2^{-8/3}\gamma\, I(t)$

$$I(t) = \int_t^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh 2x - 2x)^{1/3} \ , \qquad \gamma = 2^{10/3} (g_s M \alpha')^2 \varepsilon^{-8/3}$$

- The standard warp factor a^2 , which measures the string tension, is identified with $h(t)^{-1/2}$ and is minimized at t=0. It blows up at large t (near the boundary).
- The dilaton is exactly constant due to the self-duality of the 3-form background

$$\star_6 G_3 = i G_3 \ , \qquad G_3 = F_3 - \frac{i}{g_s} H_3$$

- The radius-squared of the S^3 at $t=0$ is g_sM in string units.
- When g_sM is large, the curvatures are small everywhere, and the SUGRA solution is reliable in `solving’ this confining gauge theory.

Anomalously light glueballs

- The confining string tension is $$T_s = \frac{1}{2^{4/3} a_0^{1/2} \pi} \frac{\varepsilon^{4/3}}{(\alpha')^2 g_s M}$$
- The glueballs are the normalizable modes localized near at small t. In the supergravity limit (at large g_s M) their mass scales are

$$m_{glueball} \sim m_{KK} \sim \frac{\varepsilon^{2/3}}{g_s M \alpha'}$$

$$T_s \sim g_s M (m_{glueball})^2$$

- In order to eliminate the anomalously light bound states, we need a small g_s M, which requires a departure from the SUGRA limit.
- Even for small g_s M, SUGRA becomes reliable in the UV (at large t).

Log running of couplings

- The large radius asymptotic solution is characterized by logarithmic deviations from $AdS_5 \times T^{1,1}$ IK, Tseytlin
- The near-AdS radial coordinate is $r \sim \varepsilon^{2/3} e^{t/3}$
- The NS-NS and R-R 2-form potentials:

$$F_3 = \frac{M\alpha'}{2}\omega_3 \ , \qquad B_2 = \frac{3g_s M\alpha'}{2}\omega_2 \ln(r/r_0)$$

$$\omega_2 = \frac{1}{2}(g^1 \wedge g^2 + g^3 \wedge g^4) = \frac{1}{2}(\sin\theta_1 d\theta_1 \wedge d\phi_1 - \sin\theta_2 d\theta_2 \wedge d\phi_2)$$

$$\omega_3 = \frac{1}{2} g^5 \wedge (g^1 \wedge g^2 + g^3 \wedge g^4)$$

- This translates into log running of the gauge couplings through

$$\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s e^\Phi} \ ,$$

$$\left[\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2}\right] g_s e^\Phi = \frac{1}{2\pi\alpha'}\left(\int_{S^2} B_2\right) - \pi$$

- The warp factor deviates from the M=0 solution logarithmically.

$$h(r) = \frac{27\pi(\alpha')^2[g_s N + a(g_s M)^2 \ln(r/r_0) + a(g_s M)^2/4]}{4r^4}$$

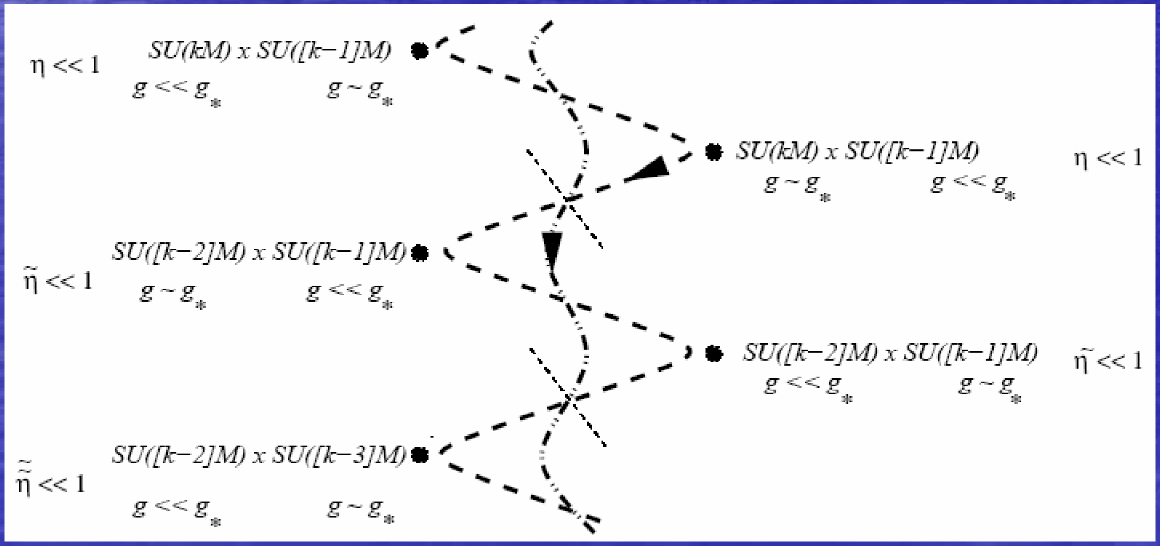
- Remarkably, the 5-form flux, dual to the number of colors, also changes logarithmically with the RG scale.

$$\tilde{F}_5 = \mathcal{F}_5 + \star\mathcal{F}_5 \ , \qquad \mathcal{F}_5 = 27\pi\alpha'^2 N_{eff}(r)\mathrm{vol}(\mathrm{T}^{1,1})$$

$$N_{eff}(r) = N + \frac{3}{2\pi} g_s M^2 \ln(r/r_0)$$

- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a **duality cascade**, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler

(diagram of RG flows from a review by M. Strassler)



$Y^{p,q}$ Dualities

- Cascading behavior is not limited to the conifold. Recently, an infinite family of new CY cones over Sasaki-Einstein spaces $Y^{p,q}$ of topology $S^2 \times S^3$ have been constructed (p and q are co-prime integers). Gauntlett, Martelli, Sparks, Waldram

$$d\Omega^2_{Y^{p,q}} = (e^\theta)^2 + (e^\phi)^2 + (e^y)^2 + (e^\beta)^2 + (e^\psi)^2$$

$$e^\theta = \sqrt{\frac{1-y}{6}}d\theta\,, \quad e^\phi = \sqrt{\frac{1-y}{6}}\sin\theta d\phi$$

$$e^y = \frac{1}{\sqrt{wv}}dy\,, \quad e^\beta = \frac{\sqrt{wv}}{6}(d\beta + \cos\theta d\phi)$$

$$e^\psi = \frac{1}{3}(d\psi - \cos\theta d\phi + y(d\beta + \cos\theta d\phi))$$

$$\begin{aligned} w(y) &= \frac{2(b-y^2)}{1-y}\,, \\ v(y) &= \frac{b-3y^2+2y^3}{b-y^2} \end{aligned}$$

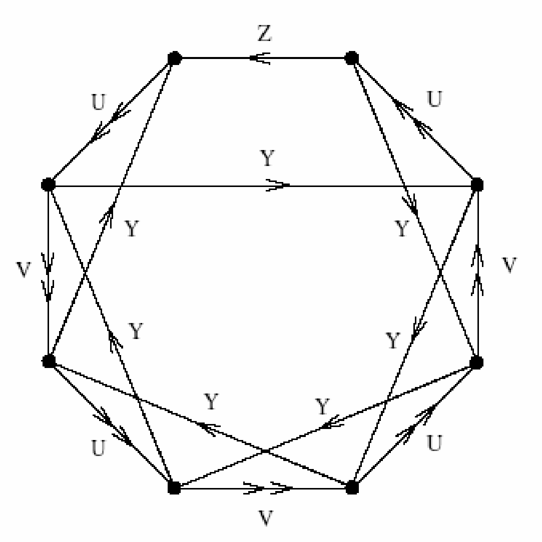
$$b = \frac{1}{2} - \frac{p^2-3q^2}{4p^3}\sqrt{4p^2-3q^2}$$

- y ranges between two smaller roots of v(y):

$$y_1 = \frac{1}{4p}\left(2p - 3q - \sqrt{4p^2 - 3q^2}\right)$$
$$y_2 = \frac{1}{4p}\left(2p + 3q - \sqrt{4p^2 - 3q^2}\right)$$

$$\mathrm{Vol}(Y^{p,q}) = \frac{q^2\left(2p + \sqrt{4p^2 - 3q^2}\right)}{3p^2\left(3q^2 - 2p^2 + p\sqrt{4p^2 - 3q^2}\right)}\pi^3$$

- The $SU(N)^{2p}$ SCFT's on N D3-branes at the tip of the cones have also been constructed.
Benvenuti, Franco, Hanany, Martelli, Sparks
- For example, here is the quiver diagram for the SCFT dual to $AdS_5 \times Y^{4,3}$



R-charges from a-maximization

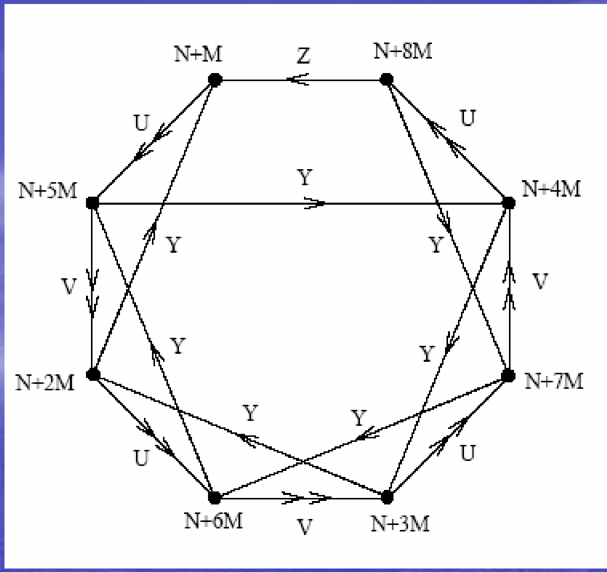
- The conformal invariance conditions do not fully determine the R-charges. Let $R_Z=x$, $R_Y=y$, $R_U=1-(x+y)/2$, $R_V=1+(x-y)/2$
- The technique of a-maximization Intriligator, Wecht gives

$$x = \frac{1}{3q^2}\left(-4p^2+2pq+3q^2+(2p-q)\sqrt{4p^2-3q^2}\right)$$
$$y = \frac{1}{3q^2}\left(-4p^2-2pq+3q^2+(2p+q)\sqrt{4p^2-3q^2}\right)$$

- Remarkably, this gives the trace anomaly agreeing with the AdS/CFT $a(Y^{p,q}) = \frac{\pi^3 N^2}{4\,\mathrm{Vol}(Y^{p,q})}$

Benvenuti et al; Bertolini, Bigazzi, Cotrone

- Addition of M D5-branes wrapped over the S^2 modifies the quiver diagram to
- Performing the Seiberg duality on the biggest gauge group gives the same quiver with N -> N-M. This fact is necessary for the existence of a self-similar cascading RG flow.
- The gravity duals of these cascades include the ISD 3-form field strength. Herzog, Ejaz, IK



$$\Omega_{2,1} = K\left(\frac{dr}{r} + ie^{\psi}\right) \wedge \omega$$

$$\omega = F(y)(e^{\theta} \wedge e^{\phi} - e^{y} \wedge e^{\beta})$$

$$F(y) = \frac{1}{(1-y)^2}$$

- The metric and 5-form are determined by a single warp factor, which however depends on 2 coordinates.

$$ds^2 = h^{-1/2} dx_4^2 + h^{1/2}(dr^2 + r^2 d\Omega^2_{Y^{p,q}})$$

$$g_s F_5 = d(h^{-1}) \wedge d^4x + *[d(h^{-1}) \wedge d^4x]$$

- Luckily, the PDE for h(r,y) is exactly solvable

$$h(r,y) = \frac{A \ln(r/r_0) + s(y)}{r^4}$$

$$s(y) = -\frac{C}{4(b-1)}\left[\frac{1}{1-y} + \frac{(1+2y_1)(1+2y_2)\ln(y_3 - y)}{2(b-1)}\right]$$

- The log behavior characteristic of the cascade produces a naked singularity in the IR. Is there a smooth solution with the above large r behavior?

IR Behavior of the Conifold Cascade

- Here the dynamical deformation of the conifold renders the solution smooth, and explains the IR dynamics of the gauge theory.
- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: Z_{2M} -> Z_2
- Yet, the IR gauge theory is somewhat more complicated.

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- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f = N_c$.
- The baryon and anti-baryon operators Seiberg

$$\mathcal{A} = \epsilon^{i_1 \ldots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \ldots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$

$$\mathcal{B} = \epsilon_{i_1 \ldots i_{N_c}} B^{i_1}_{\dot{\alpha}_1 a_1} \ldots B^{i_{N_c}}_{\dot{\alpha}_{N_c} a_{N_c}}$$

acquire expectation values and break the U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner. There exists a baryonic branch of the moduli space

$$\mathcal{A} = i\Lambda_1^{2M}\zeta \ , \qquad \mathcal{B} = i\Lambda_1^{2M}/\zeta$$

- The KS solution is part of a moduli space of confining SUGRA backgrounds, resolved warped deformed conifolds. Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- To look for them we need to use the PT ansatz:

$$ds_{10}^2 = H^{-1/2} dx_m dx_m + e^x ds_6^2,$$

$$ds_6^2 = (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} \sum_{i=1}^{2} \left(\epsilon_i^2 - 2a e_i \epsilon_i\right) + v^{-1}(\tilde{\epsilon}_3^2 + dt^2)$$

- H, x, g, a, v, and the dilaton are functions of the radial variable t.
- Additional radial functions enter into the ansatz for the 3-form field strengths. The PT ansatz preserves the SO(4) but breaks a Z_2 charge conjugation symetry, except at the KS point.

- BGMPZ used the method of SU(3) structures to derive the complete set of coupled first-order equations.
- A result of their integration is that the warp factor and the dilaton are related:

$$H(t) = \tilde{H}\left(e^{-2\phi(t)} - 1\right)$$

Dymarsky, IK, Seiberg

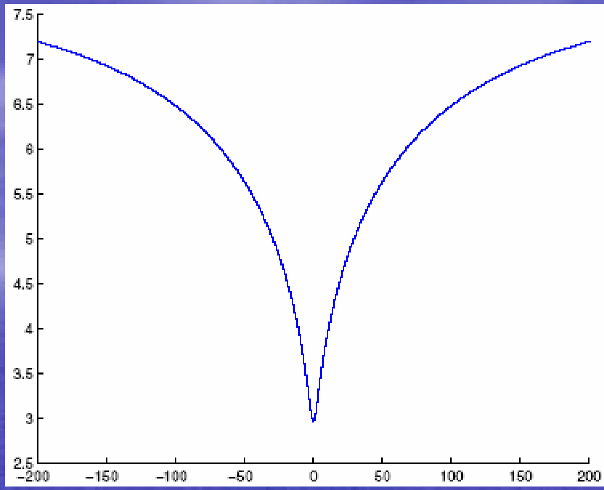
- The integration constant determines the `modulus' U: $\tilde{H} = \gamma U^{-2}$ where $\gamma = 2^{10/3}(g_s M \alpha')^2 \varepsilon^{-8/3}$
- At large t the solution approaches the KT `cascade asymptotics': $a(t) = -2e^{-t} + U e^{-5t/3}(-t+1) + \ldots$

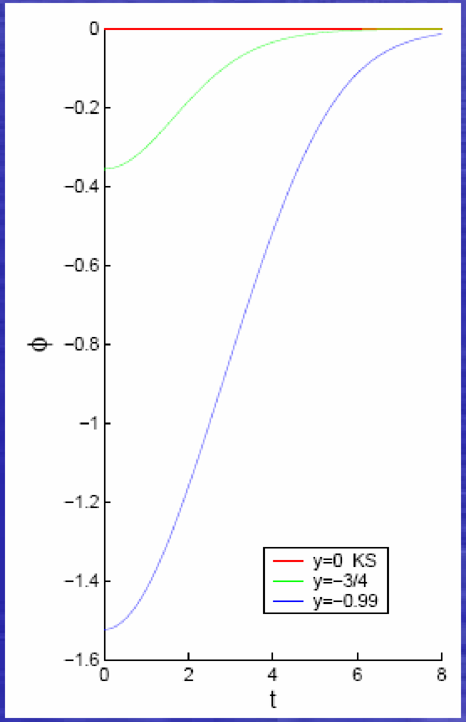
$$\gamma^{-1}H(t) = \frac{3}{32}e^{-4t/3}(4t-1) - \frac{3}{32\cdot 512}U^2(256t^3 - 864t^2 + 1752t - 847)e^{-8t/3} + O\left(e^{-10t/3}\right)$$

- The resolution parameter U is proportional to the VEV of the operator

$$\mathcal{U} = \mathrm{Tr}\left(\sum_{\alpha} A_\alpha A_\alpha^\dagger - \sum_{\dot\alpha} B_{\dot\alpha}^\dagger B_{\dot\alpha}\right)$$

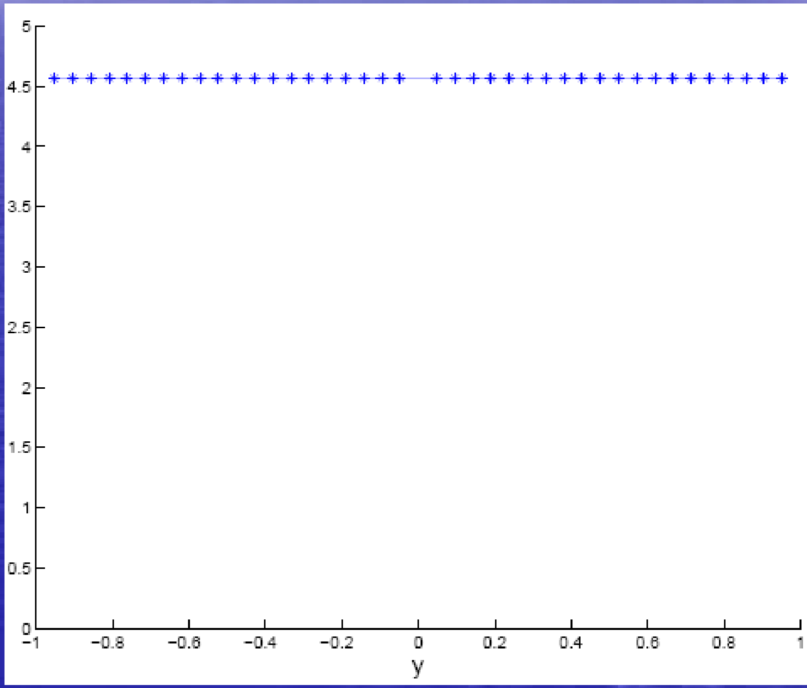
- This family of resolved warped deformed conifolds is dual to the `baryonic branch' in the gauge theory (the quantum deformed moduli space).
- At large U the IR part of the solution approaches that of the Maldacena-Nunez solution. But we always have the `cascade' asymptotics at large t.
- Here are plots of the string tension (a fundamental string at the bottom of the throat is dual to an `emergent' chromo-electric flux tube) and of the dilaton profiles as a function of the modulus U=ln $|\zeta|$. Dymarsky, IK, Seiberg





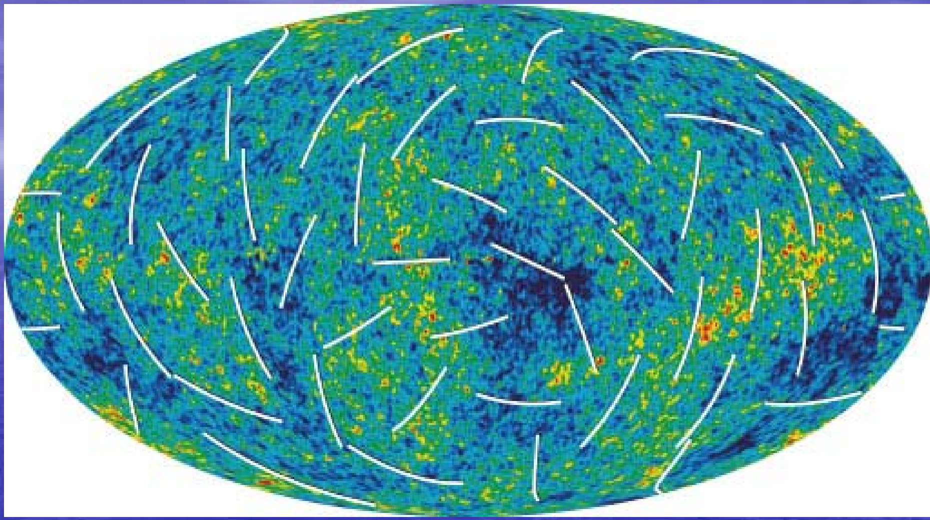
BPS Domain Walls

- A D5-brane wrapped over the 3-sphere at the bottom of the throat is the domain wall separating two adjacent vacua of the theory.
- Since it is BPS saturated, its tension cannot depend on the baryonic branch modulus. This is indeed the case. This fact provides a check on the choice of the UV boundary conditions, and on the numerical integration procedure necessary away from the KS point.
- Analytic proof?



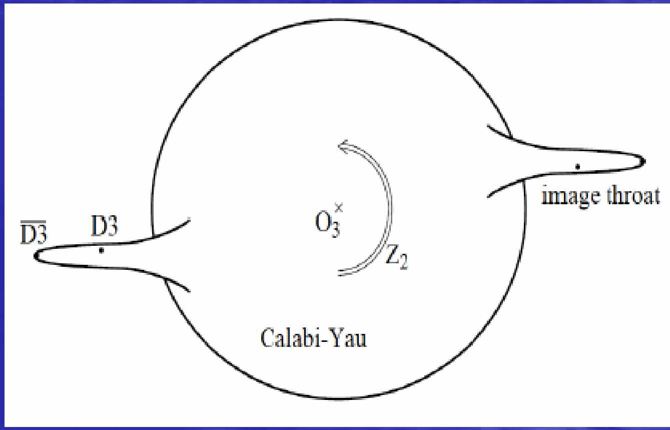
Applications to D-brane Inflation

- The Slow-Roll Inflationary Universe (Linde; Albrecht, Steinhardt) is a very promising idea for generating the CMB anisotropy spectrum observed by the WMAP.
- Finding models with very flat potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, ...



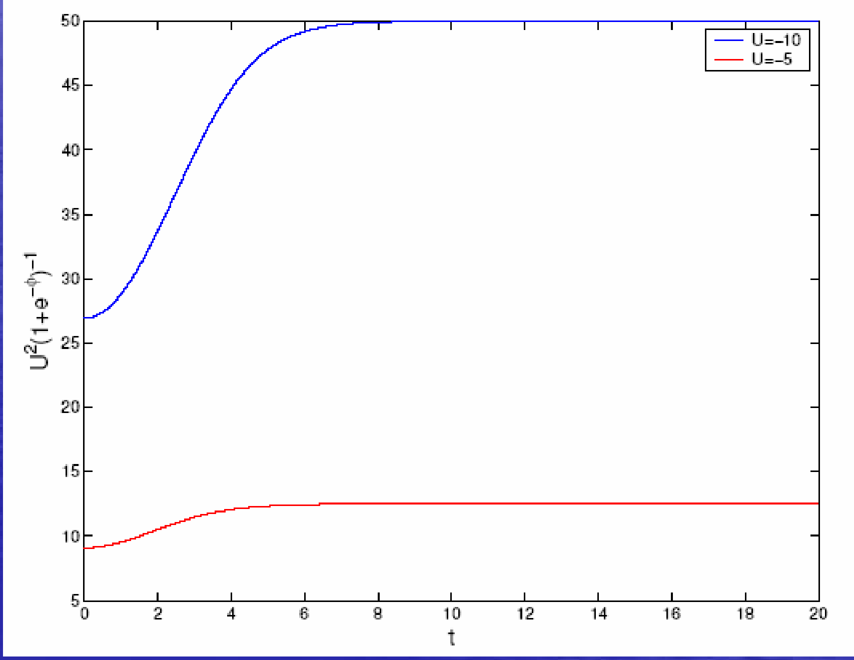
- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton.

Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi

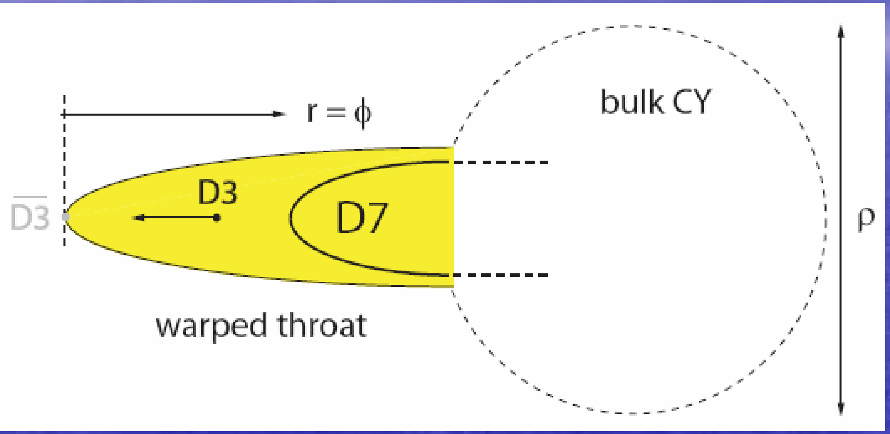


A related suggestion for D-brane inflation (A. Dymarsky, IK, N. Seiberg)

- In a flux compactification, the $U(1)_{baryon}$ is gauged. Turn on a Fayet-Iliopoulos parameter ξ .
- This makes the throat a resolved warped deformed conifold.
- The probe D3-brane potential on this space is asymptotically flat, if we ignore effects of compactification and D7-branes. The plots are for two different values of $U \sim \xi$.
- No anti-D3 needed: in presence of the D3-brane, SUSY is broken by the D-term ξ. Related to the `D-term Inflation' Binetruy, Dvali; Halyo



Slow roll D-brane inflation?



- Effects of D7-branes and of compactification generically spoil the flatness of the potential. Non-perturbative effects introduce the KKLT-type superpotential $W = W_0 + A(X)e^{-a\rho}$

  where X denotes the D3-brane position. In any warped throat D-brane inflation model, it is important to calculate $\mathrm{A(X)}$.

- The gauge theory on D7-branes wrapping a 4-cycle Σ_4 has coupling $$\frac{1}{g^2} = \frac{V^w_{\Sigma_4}}{g_7^2} = \frac{T_3 V^w_{\Sigma_4}}{8\pi^2}$$
- The non-perturbative superpotential $$\propto \exp\Big(-\frac{T_3 V^w_{\Sigma_4}}{N_{D7}}\Big)$$

  depends on the D3-brane location through the warped volume $$V^w_{\Sigma_4} \equiv \int_{\Sigma_4} \mathrm{d}^4\xi \sqrt{g^{ind}}\, h(X)$$
- In the throat approximation, the warp factor can be calculated and integrated over a 4-cycle explicitly. Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan.
- For a class of conifold embeddings $$\prod_{i=1}^{4} w_i^{p_i} = \mu^P$$ $$P \equiv \sum_{i=1}^4 p_i$$

  Arean, Crooks, Ramallo ($w_1 = z_1 + iz_2$, etc.)

  the result is $$A = A_0 \left(\frac{\mu^P - \prod_{i=1}^4 w_i^{p_i}}{\mu^P} \right)^{1/n}$$

- This formula applies both to n wrapped D7-branes, and to a wrapped Euclidean D3 ($n=1$).
- For the latter case, Ganor showed that A has a simple zero when the D3-brane approaches the 4-cycle. Our result agrees with this.
- We have also carried out such calculations for 4-cycles within the Calabi-Yau cones over $Y^{p,q}$ with analogous results: $\mathrm{A}(\mathrm{X})$ is proportional to the embedding equation raised to the power $1/n$. This appears to be a general rule for 4-cycles in the throat.

- The dependence of the non-perturbative superpotential on D3-brane position, and other compactification effects, give Hubble-scale corrections to the inflaton potential.
- Some `fine-tuning’ is generally needed to cancel different corrections to the D3-brane potential. This is currently under investigation with D. Baumann, A. Dymarsky, J. Maldacena, L. McAllister and P. Steinhardt.

Conclusions

- In the first part, we investigated non-SUSY orbifolds of AdS/CFT. At one loop, flow of double-trace couplings spoils conformal invariance even in the large N limit. There is a precise connection of this instability with presence of twisted sector closed string tachyons.
- Gauge/string dualities for confining gauge theories give a new geometrical view of such important phenomena as dimensional transmutation, chiral symmetry breaking, and quantum deformation of moduli space. We have also discussed new UV phenomena: the duality cascades.
- Embedding gauge/string dualities into string compactifications offers new possibilities for modeling inflation. In particular, D3-branes on resolved warped deformed conifolds may realize D-term inflation.
- Calculation of non-perturbative corrections to the inflaton potential is important for determining if these models can produce slow-roll inflation.