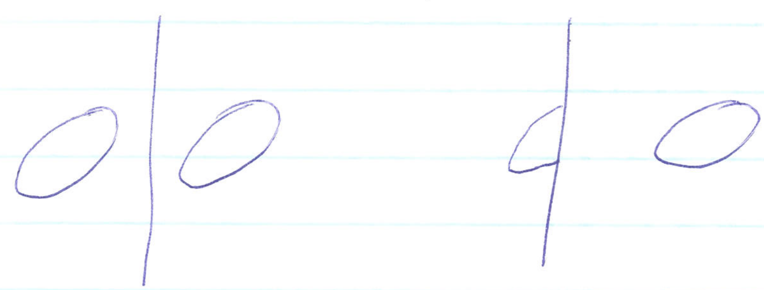
Lensing

$\delta = 8\pi G\mu$

$G\mu = 2.3 \times 10^{-7}$ (bound) $\rightarrow$ 1.2'' (arc-sec)

(string at rest $\perp$ to line of sight, much closer than lensed object)

CSL-1 was lens candidate (turned out to be binary galaxy) at ~2''. Not inconsistent: e.g. string moving transversely to line of sight as $\frac{1}{\sqrt{1-v^2}}$ enhancement.



But we are at the limit of optical lensing.

Straight string produces translated image.

~~Question~~: how straight? If string is, e.g.
Question
random walk, will produce complex multiple images.

(See later discussion)

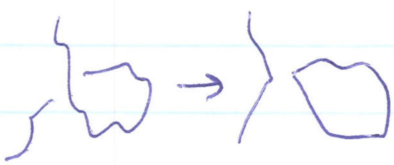
Gravity waves

Life~~time~~ cycle of string

Long string → loop → grav. radiation

O(50%) of string energy / Hubble time

lifetime $\sim \dfrac{\ell}{60 G\mu}$ (grav. radiation $\propto G\mu$)



Source of uncertainties:

- poor understanding of wiggles on long string
- fragmentation process: initially produced loop self-intersects, fragments into many pieces.

~~Lots of GW~~ Parameterized by saying typical loop produced at size αt (yesterday I gave proposed values for α).

1 Hubble time : $\frac{\rho_{\text{loops produce}}}{\rho_{\text{rad}}} \sim 200\, G\mu$

$$t_{\text{decay}} = \frac{l}{60 G\mu} \cancel{t_{\text{formed}}} + t_{\text{form}}$$

$$= \frac{\alpha\, t_{\text{formed}}}{60\, G\mu} + t_{\text{formed}}$$

$$\frac{t_{\text{decay}}}{t_{fc}} = \frac{\alpha}{60\, G\mu} + 1$$

$$\frac{\rho_{\text{loops}}}{\rho_{\text{rad}}} \begin{array}{l} \leftarrow \sim a^{-3} \quad t^{-3/2} \\ \leftarrow \sim a^{-4} \quad t^{-2} \end{array}$$

$$\frac{\Delta \rho_{\text{GW produce}}}{\rho_{\text{rad}}} \sim (200\, G\mu) \left(\frac{\alpha}{60\, G\mu} + 1\right)^{1/2}$$

$$\frac{\rho_{\text{GW}}}{\rho_{\text{rad}}} \sim (200\, G\mu) \left(\frac{\alpha}{60\, G\mu} + 1\right)^{1/2} \overbrace{\ln \frac{t}{t_{\text{inflat}}}}^{\sim 25} \times \text{dilution}$$

$\ln T_{\text{inflat}}$

$$\ln \frac{t_{\text{nuclear}}}{t_{\text{QCD}}} + \ln t_{\text{QC}}$$

$$\ln \frac{T_{\text{QCD}}^2}{\cancel{T_{\text{nuclear}}}} \sim 15 \qquad < 0.05 \text{ (BBN)}$$

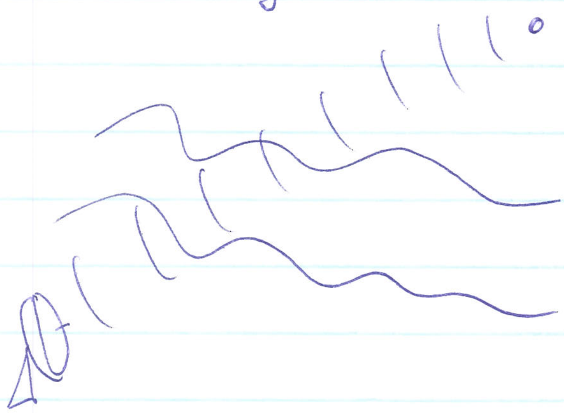
30

Assuming $\alpha \gtrsim 60\, G\mu$, find

$$G\mu \lesssim \frac{10^{-8}}{\alpha} : \text{ for } \alpha \approx 0.1$$

(largest estimate) this is stronger than CMB bound. But for $\alpha \sim 10^{-3}$ or 10^{-4} (my best estimate) it is much weaker.

Pulsar timing:



Bounds $f \frac{d\Omega_{GW}}{df} < 2\times10^{-7}$ for periods around 10 yr

$\sim 10^{-9} t_{now}$.

For modes from low harmonics of loop,

period $\sim 50\, G\mu\, t_{emission}$

(Need $G\mu > 2\times10^{-11}$) $\quad \frac{10^{-9} t_{now}}{50\, G\mu\, t_{emission}} = \frac{\text{period}_{now}}{\text{period}_{emiss}} = \frac{a_{now}}{a_{emission}}$

To get down to $G\mu = 10^{-11}$ need $\Omega < 3\times 10^{-10}$

"Square Kilometer Array" $\to 10^{-12}$

LISA, LIGO III $\to 10^{-10}$

} looks like stochastic background from inflation, but different spectral slope.

This was for low harmonics of string. High harmonics emit because of kinked structure, but even on a smooth string because of <u>cusps</u>:

Analyze motion of string in flat spacetime:

$$u = \sigma + \tau \qquad v = -\sigma + \tau$$

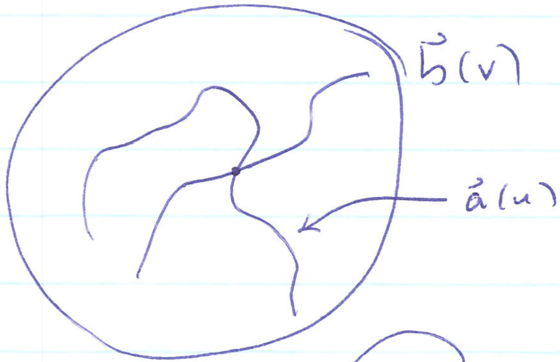
$$\partial_u X^\mu \partial_u X_\mu = \partial_v X^\mu \partial_v X_\mu = 0$$

$\vec{a} \to p$

$\vec{b} \to q$

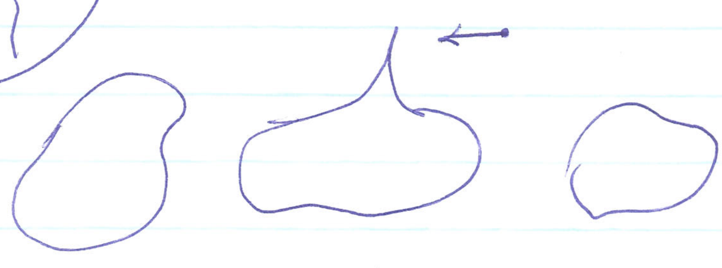
Also $\qquad X^0 = \tau$ $\qquad \vec{a}, \vec{b} \to \alpha, \beta$

$$\to \partial_u \vec{X} \equiv \vec{a} \qquad \partial_v \vec{X} \equiv \vec{b} \qquad |\vec{a}| = |\vec{b}| = 1$$



$$\dot{\vec{X}} = \frac{1}{2}(\vec{a}+\vec{b}) \to 1$$

$$\vec{X}' = \frac{1}{2}(\vec{a}-\vec{b}) \to 0$$



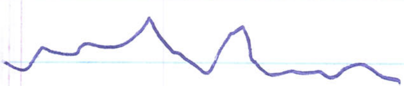
A cosmic whip!  grav waves

Near-c velocity, near Planck tension, 1000 light-years in length, emits cone of <u>high frequency</u> grav. waves.

Damour + Vilenkin: potentially observable over most of relevant $G\mu$ range at 1 event/year, LIGO (currently $\frac{1}{2}$-way through first year run at design sensitivity).

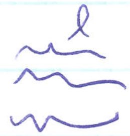
Nice paper, but Siemens, Creighton, Maor, Majumder, Cannon, Read gr-qc/0603115 improve estimate: we need LISA.

Many kinks, signals depend on network properties.

 ← small scale structure on long strings: is it fractal?

Previous analytic attempts either too crude or too complicated.

Approach like RG



short segment

Effects: intercommutation : negligible for $l < t$

grav. radiation : negligible for $l > G\mu t$

loop formation : negligible ?

expansion of univers.

Flat: $\cancel{\partial_v \vec a = \partial_u \vec b = 0}$ $\quad (\partial_\tau - \partial_\sigma)\,\vec a = 0$

$(\partial_\tau + \partial_\sigma)\,\vec b = 0$

Expanding : $\cancel{\partial_v \vec a =}$

$$\left(\partial_\tau - \frac{1}{\varepsilon(\tau,\sigma)}\partial_\sigma\right)\vec a = -\frac{\dot a}{a}\left(\vec b - (\vec a\cdot\vec b)\,\vec a\right)$$

$$\partial_\tau \varepsilon(\tau,\sigma) = \frac{\dot a}{a}\,(\vec a\cdot\vec b - 1)$$

$$a^2 = b^2 = 1$$

$\vec a(\sigma,\tau)\cdot\vec a(\sigma',\tau)$

$\vec a(\sigma,\tau)\cdot\vec b(\sigma',\tau)$: #1 drop term $O(\sigma-\sigma')$

$\approx$

$\vec a(\sigma,\tau)\cdot\vec b(\sigma',\tau) \approx$ const

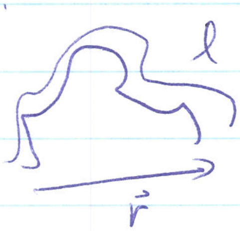
#2 drop terms which average to $O(\sigma-\sigma')$ over a Hubble time or averaged over ensemble

$$\partial_\tau \left\langle 1-\vec{a}(\sigma,\tau)\cdot\vec{a}(\sigma',\tau)\right\rangle = -\frac{\dot a}{a}\underbrace{\left(1-2\bar v^2\right)}_{\text{averages to } \left\{\begin{array}{ll}0.30 & \text{matt}\\ 0.18 & \text{rad}\end{array}\right.}\left\langle 1-\vec{a}(\sigma,\tau)\cdot\vec{a}(\sigma')\right\rangle$$

Integrate back in time to when

$$l = \varepsilon(\sigma-\sigma') = t \quad (\text{Length of segment} = \text{length of horizon})$$

+ match to simulation



Fractal dimension $\dfrac{\partial \ln l}{\partial \ln r}$ $\begin{array}{ll}0.10 & \text{rad}\\ 0.25 & \text{matt}\end{array}$

$$= 1 - A\left(\frac{l-l'}{t}\right)^{2\bar\chi}$$

from simulation

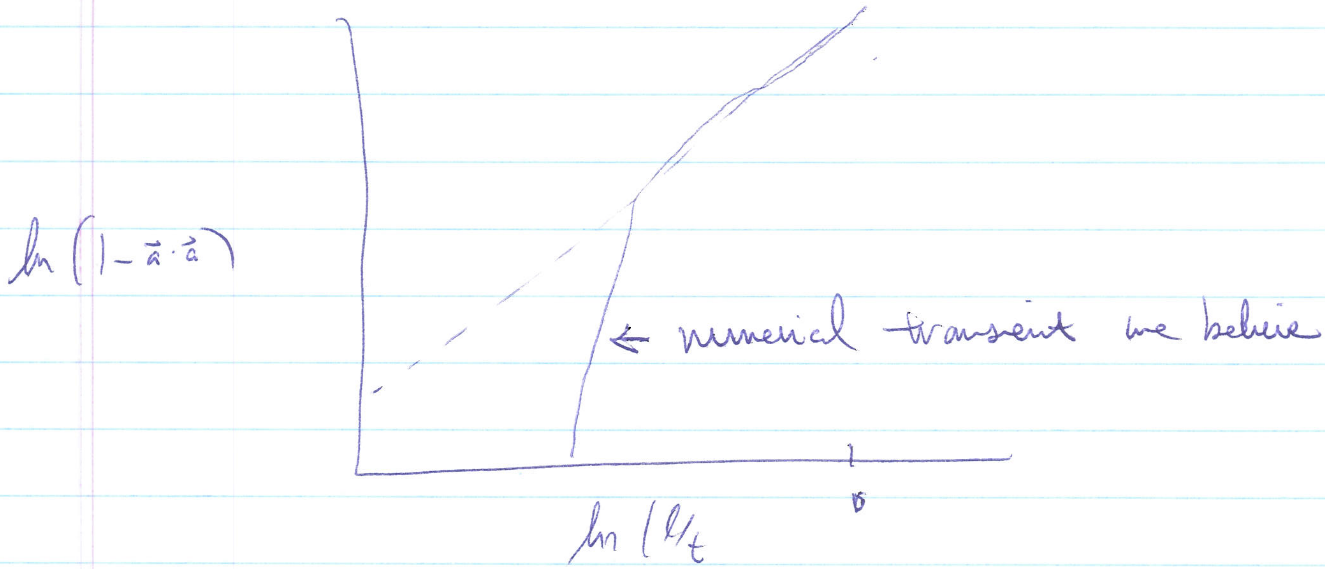
$\longrightarrow 1$ at short distance

Curves $\vec a(\sigma)$, $\vec b(\sigma)$ (derivative of curve)

have fractal dimension $\dfrac{1}{\chi} = \left\{\begin{array}{l}10\\ 4\end{array}\right.$

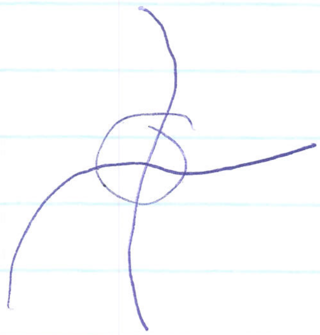
Input to loop formation: loop form if

$$0 = \int_{\sigma_1}^{\sigma_2} \vec{x}'(\sigma,\tau) = \int_{\sigma_1}^{\sigma_2}\left(\underbrace{\vec a}_{l_p} - \underbrace{\vec b}_{l_p}\right)$$



$\ln\left(1-\vec{a}\cdot\vec{a}\right)$
← numerical transient we believe
$\ln(l/t$

Calculate probability · $\delta(\ell_p - \ell_q)$ using earlier



← loops form near where curves cross (cusps)

← rate diverges at small scales.

Analytic treatment breaks down but we believe:

- loops form at 0.1t but continue to fragment
- non-self-intersecting ~~results~~ loops of finite size must form, but probably several orders of magnitude smaller
- there may be a population of very small loops.

what about the cusps?!

- grav. radiation smooths curve on scale $(G\mu)^{1+2\chi} t$

⇒ more but smaller cusps

⇒ increased signal at large $G\mu$, decreased at small $G\mu$

Will measure μ and P.

Results: bounds on string tension will reach 10^{-11} or better, but with instruments (SKA, LISA, LIGO III) that are all on the drawing board or trying to get into the funding cycle.

Shorter term: LIGO I/II / CMBPOL

if string tension is near current bound or P is small.