Tutorial Integrability & AdS/CFT
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Complete N=4 SYM ~~& sectors~~

Adjoint fields transform as follows

$$X \to U X U^{-1}$$

where $U \in U(N)$ and X hermitean $N \times N$ matrix

Also covariant derivatives & field strengths

$$D \to U D U^{-1} \quad , \quad F \to U F U^{-1}$$

All possible "letters" W of this sort

$$W \in \{ D^n \Phi , D^n \Psi , D^n F \}$$

Words transform alike

$$W_1 \dots W_L \to U W_1 \dots W_L U^{-1}$$

and traces are invariant due to cyclicity.

$$\operatorname{tr} W_1 \dots W_L \to \operatorname{tr} W_1 \dots W_L U^{-1} U .$$

Sectors of $N=4$ SYM

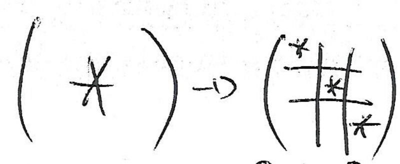
All fields are quite complicated.
Can restrict to subset to simplify? Sectors!
Hamiltonian preserves SO(6), Lorentz, cl. dimension.

W	SO(6)	Lorentz	cl. dim
ϕ	1	0	1
ψ	$1/2$	$1/2$	$3/2$
D	0	1	1

One-loop Hamitonian is NN. $2 \to 2$

eg. $\phi\phi \to ?$ $\phi\phi$ scalar sector
$\phi\psi \to$ $\phi\psi,\ \psi\phi$
$\psi\psi \to$ $\psi\psi,\ \phi D\phi,\ D\phi\,\phi$

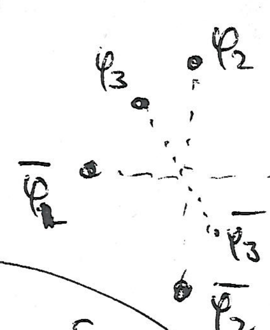
Scalar (so(6)) sector:
three Cartan generators & charges:
definite charges for $\varphi_1 = \phi_1 + i\phi_2$
$\bar\varphi_1 = \phi_1 - i\phi_2$ etc.



SO(2)^3

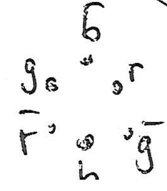
	P	J_1	J_2	J_3
φ_1	1	+1		
$\bar\varphi_1$	1	−1		
φ_2	1		+1	
$\bar\varphi_2$	1		−1	
φ_3	1			+1
$\bar\varphi_3$	1			−1

or plot



middle of faces of a cube

like QCD quarks



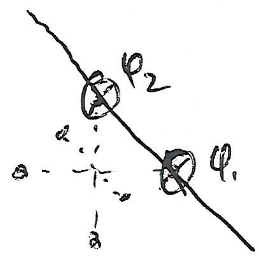
Relation among conserved charges

$D = J_1$

only φ_1 is allowed.

$\frac{1}{2}$ BPS sector

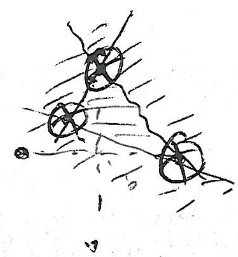
$D = J_1 + J_2$



φ_1 and φ_2 are allowed

SU(2) sector

$D = J_1 + J_2 + J_3$



φ_1 φ_2 φ_3 are allowed

SU(3) sector

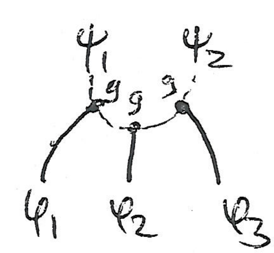
however also ψ_1 and ψ_2 satisfy

$$\tfrac{3}{2} = D = J_1 + J_2 + J_3 = \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{2}$$

larger sector at higher loops

φ_1 φ_2 φ_3 ψ_1 ψ_2 : <u>SU(2|3) sector</u>

mixing at $O(g^3)$



and many more