Dynamical SUSY Breaking, pre-lecture 3 problems

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Consider $SU(N_c)$ SQCD, with N_f flavors $Q_f \in \mathbf{N_c}$, $f = 1 \ldots N_f$ of fundamentals, and anti-fundamentals $\widetilde{Q}_{\tilde{f}} \in \overline{\mathbf{N_c}}$. **Seiberg duality** relates this "electric" theory to a "magnetic dual," with gauge group $SU(N_f - N_c)$, matter fields $q^f \in \mathbf{N_f} - \mathbf{N_c}$ and $\tilde{q}^{\tilde{f}} \in \overline{\mathbf{N_f} - \mathbf{N_c}}$. In addition, there are the gauge singlets $\Phi_{f\tilde{g}}$; these elementary fields of the magnetic theory correspond to the composite objects $\Phi_{f\tilde{g}} = M_{f\tilde{g}}/\widehat{\Lambda} = Q_f \widetilde{Q}_{\tilde{g}}/\Lambda$ of the electric theory. $\widehat{\Lambda}$ is a dimensionful scale. These fields have the superpotential

$$W_{dual} = \Phi_{f\tilde{q}} q^f \tilde{q}^{\tilde{g}}. \tag{1}$$

1. Show that the magnetic dual theory has the same conserved $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ symmetry as the electric theory. In particular, verify that the magnetic theory has a conserved $U(1)_R$ symmetry, and that $R(\Phi)$ is compatible with $\Phi \sim Q\tilde{Q}$ and the conserved $U(1)_R$ charges of the electric theory.

2. Verify the $\mathrm{Tr}U(1)_R$ and $\mathrm{Tr}SU(N_f)_L^2 U(1)_R$ and $\mathrm{Tr}SU(N_f)^3$ 't Hooft anomalies. (If you like, feel free to verify all the others too – $\mathrm{Tr}U(1)_R^3$ is quite impressive!).

3. Adding mass terms $W_{tree} = \mathrm{Tr}mQ\tilde{Q}$ on the electric side corresponds to adding $W_{tree} = \mathrm{Tr}m\widehat{\Lambda}\Phi$ to the magnetic superpotential (1). Take the mass matrix m to have only one non-vanishing eigenvalue, m_{N_f}. Verify that the F-term equations of motion for Φ_{fN_f} and $\Phi_{N_f\tilde{g}}$ and q^{N_f} and $\tilde{q}^{\tilde{N}_f}$ lead to a similar theory, with the Higgsing of the magnetic theory $SU(N_f - N_c) \to SU(N_f - 1 - N_c)$ (as expected, since the mass decouples an electric flavor, so $N_f \to N_f - 1$ on the electric side).