Ken Intriligator, PITP tutorial lecture plan and homework, July 2006

The topic of dynamical susy breaking will be introduced in lecture 1. The tutorial will cover some more basic points in susy theories, listed below. The especially introductory points (1-5) will just be briefly stated in the tutorial. More detail is given here, for reference.

1. $\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2P_{\alpha\dot\alpha} \to \langle\psi|\mathcal{H}|\psi\rangle \propto \sum_\alpha \left|Q_\alpha|\psi\rangle\right|^2 + \sum_{\dot\alpha}\left|\overline{Q}_{\dot\alpha}|\psi\rangle\right|^2 \to$ *supersymmetry is spontaneously broken iff the vacuum has non-zero energy,* $V_{vac} = M_s^4$. (Global susy only in these lectures, $M_{pl} \to \infty$. But remember that in SUGRA we can add an arbitrary negative constant to the vacuum energy, via $\Delta W = const$, so the cosmological constant can still be tuned to the observed value.)

2. Chiral superfields, $\Phi = \phi + \sqrt{2}\theta_\alpha\psi^\alpha + \theta^2 F +$ (derivative terms). Susy vacua can have $\langle\phi\rangle \neq 0$. If $\langle F\rangle \neq 0$, susy is broken.

3. Consider $\mathcal{L} = \int d^4\theta K(\Phi^i, \Phi^{\dagger\bar{i}}) + \int d^2\theta W(\Phi^i) + h.c.$. E.g. $K = K_{can} = \Phi^i\overline{\Phi}^{\bar{i}}\delta_{i\bar{i}}$. EOM: $\overline{D}^2\frac{\partial K}{\partial\Phi^i} + \frac{\partial W}{\partial\Phi_i} = 0$. Implies $\langle\overline{F}^{\bar{i}}\rangle = -\langle(K^{-1})^{i\bar{i}}W_i\rangle$. In components, $\mathcal{L} \supset \frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c. - V_F$, with $V_F = (K^{-1})^{i\bar{j}}W_i\overline{W}_{\bar{j}}$. Susy vacua must have $(K^{-1})^{i\bar{i}}W_i = 0$, for all $\bar{i}$. If inverse Kahler metric $(K^{-1})^{i\bar{i}}$ is non-degenerate (i.e. using the correct effective field theory), then this is equivalent to $\frac{\partial W}{\partial\phi^i} = 0$ for all i in susy vacua. Otherwise, susy is broken.

4. Vector superfields, $V = \ldots + \theta_\alpha\overline{\theta}_{\dot\alpha}A^{\alpha\dot\alpha} - i\overline{\theta}^2\theta_\alpha\lambda^\alpha + i\theta^2\overline{\theta}_{\dot\alpha}\overline{\lambda}^{\dot\alpha} + \frac{1}{2}\theta^2\overline{\theta}^2 D$ ($\ldots$ includes gauge d.f. and derivative terms). $W_\alpha = -\frac{1}{4}\overline{D}^2 e^{-V}D_\alpha e^V$. $W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}\theta^\beta F_{\alpha\beta} + \ldots$. Glueball chiral superfield: $S = -\frac{1}{32\pi^2}\mathrm{Tr}W_\alpha W^\alpha$.

5. Classical $\mathcal{N} = 1$ susy gauge theories. $V = V_F + V_D$. $V_F = (K^{-1})^{i\bar{j}}W_i\overline{W}_{\bar{j}}$. $V_D = \frac{1}{2}\sum_a(D^a)^2$. $D_a = -g\phi^* T^a\phi$. Susy vacua must have $V_F = V_D = 0$. In addition to the F-term conditions, susy vacua have $D_a = 0$ for all $a = 1\ldots|G|$.

6. Classical $\mathcal{N} = 1$ susy gauge theories, with $W_{tree} = 0$: classical moduli spaces of vacua. $\mathcal{M}_{cl} = \{\langle\Phi\rangle|D^a = 0\}/(\text{gauge equivalence}) = \{\langle\text{gauge invt. monomials of chiral sup flds}\rangle\}/(\text{classical relations})$. The massless moduli are the chiral superfields that are left uneaten by the Higgs mechanism: $\dim_{\mathbf{C}}\mathcal{M}_{cl} = \#(\text{chiral fields}) - \#(\text{eaten})$.

7. Example: $SU(N_c)$ with $N_f = 1$ flavor, Q, $\widetilde{Q}$.

$$\mathcal{M}_{cl}: \qquad Q = \widetilde{Q}^T = (\,a \quad 0 \quad 0\ldots0\,).$$

Meson gauge invariant chiral superfield $M = Q\widetilde{Q} = a^2$. $\mathcal{M}_{cl} = \langle M \rangle$. Higgs mechanism: $SU(N_c) \to SU(N_c-1)$, one chiral superfield left uneaten: $2N_c - |SU(N_c)/SU(N_c-1)| = 1$. The light field is $\sim M$. On the classical moduli space, $K_{cl} = 2\sqrt{M^\dagger M}$. Singular at origin, interpret as additional massless fields: the $SU(N_c)/SU(N_c-1)$ gauge fields.

8. $SU(N_c)$ with $N_f < N_c$. Up to gauge/flavor rotations, $\mathcal{M}_{cl}$ is given by

$$Q = \widetilde{Q} = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \cdot & \\ & & & a_{N_f} \end{pmatrix}.$$

Gauge invariant description: $\mathcal{M}_{cl} = \langle M_{f\tilde{g}} \rangle$. $M_{fg} = Q_g \widetilde{Q}_{\tilde{g}}$, $f, \tilde{g} = 1 \dots N_f$. Higgs $SU(N_c) \to SU(N_c - N_f)$. $K_{cl} \sim \sqrt{M^\dagger M}$.

9. $SU(N_c)$ with $N_f \geq N_c$. $\dim_C \mathcal{M}_{cl} = 2N_c N_f - (N_c^2 - 1)$. Up to gauge/flavor rotations,

$$Q = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \cdot & \\ & & & a_{N_c} \\ & & & \\ & & & \end{pmatrix}, \quad \widetilde{Q} = \begin{pmatrix} \widetilde{a}_1 & & & \\ & \widetilde{a}_2 & & \\ & & \cdot & \\ & & & \widetilde{a}_{N_c} \\ & & & \\ & & & \end{pmatrix},$$

with $|a_i|^2 - |\widetilde{a}_i|^2 =$ independent of i. Gauge invariant description: fields $M = Q\tilde{Q}$, $B = Q^{N_c}$, $\tilde{B} = \tilde{Q}^{N_c}$, subject to classical relations. E.g. $M_{f\tilde{g}} = Q_{fc}\tilde{Q}^c_{\tilde{g}}$ (with $f, \tilde{g} = 1 \dots N_f$ and $c = 1 \dots N_c$) has rank$(M) \leq N_c$. E.g. for $N_f = N_c$, have $\mathcal{M}_{cl} = \{M_{f\tilde{g}},\ B, \tilde{B} |\det M - B\tilde{B} = 0.\}$ Space $\mathcal{M}_{cl}$ is singular at the origin (topologically, not just its metric).