The Evolving Cosmological Constant (Problem)

Problems and Solutions

Daniel Baumann and Nissan Itzhaki

July 19, 2006

1 On the Bousso-Polchinski Mechanism for Obtaining a Small Cosmological Constant

Problem

A solution of the cosmological constant problem in the Bousso-Polchinski framework requires that

$$0 < \lambda = -|\lambda_{\rm bare}| + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2 < \Delta\lambda \sim 10^{-120}\,, \tag{1}$$

where $|\lambda_{\rm bare}|$ is the *bare* cosmological constant, J is the number of fluxes, and q_i is the size of the individual flux quanta which are labeled by incommensurate integers n_i.

1. For $\lambda_{\rm bare} \sim \mathcal{O}(M_P^4) \equiv 1$ and $J \sim \mathcal{O}(100)$ find the limit on the typical size q of the flux quanta.
2. For $\lambda_{\rm bare} \sim \mathcal{O}(M_P^4) \equiv 1$ and $q_i \equiv q \sim \mathcal{O}(0.1)$, find the minimal value of J for which there exists a set of n_i such that

$$\lambda < \Delta\lambda \sim 10^{-120}\,. \tag{2}$$

Solution

First, notice that equation (1) can be written as

$$2|\lambda_{\rm bare}| < \sum_{i=1}^{J} n_i^2 q_i^2 < 2|\lambda_{\rm bare}|\left(1 + \frac{\Delta\lambda}{2|\lambda_{\rm bare}|}\right) \tag{3}$$

This constraint can be visualized in terms of a J-dimensional grid of points, spaced by the size of the flux quanta q_i and labeled by incommensurate integers n_i (see Figure 1). Consider a sphere of radius

$$r = |2\lambda_{\text{bare}}|^{1/2} \tag{4}$$

centered at $n_i = 0$. If one of the points $(n_1, n_2, \ldots, n_J)$ is sufficently close to the sphere, the field configuration corresponding to this point will lead to an acceptable value of the cosmological constant. More precisely, one should think of a thin shell, whose width encodes the width of the observational range

$$\Delta r = \frac{1}{2}|2\lambda_{\text{bare}}|^{-1/2}\Delta\lambda\,. \tag{5}$$

Equation (5) follows from equation (3) and the binomial expansion

$$\left(1 + \frac{\Delta\lambda}{2|\lambda_{\text{bare}}|}\right)^{1/2} \approx 1 + \frac{1}{2}\frac{\Delta\lambda}{|2\lambda_{\text{bare}}|}\,. \tag{6}$$

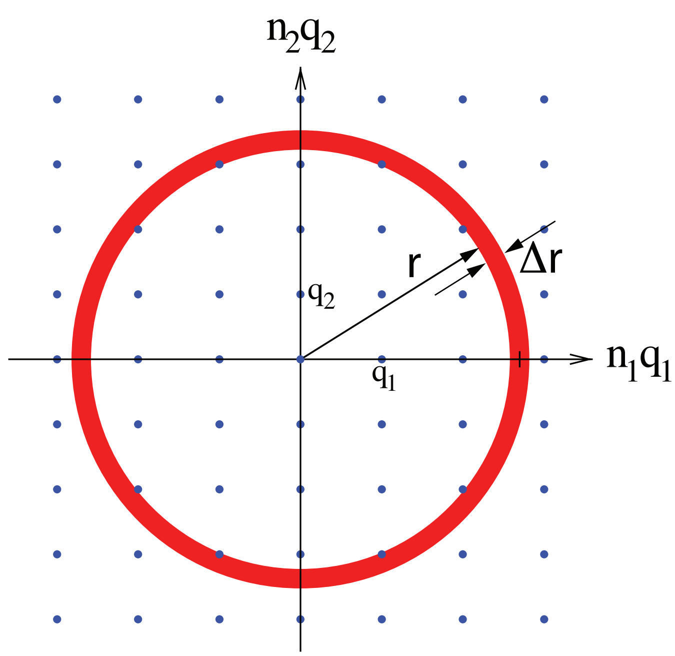

Figure 1: Visualization of the Bousso-Polchinski mechanism for solving the cosmological constant problem.

To solve the CC problem we need at least one point to lie within the shell. The volume per grid point is

$$V_{\text{grid}-\text{point}} = \prod_{i=1}^{J} q_i \sim q^J \tag{7}$$

where q is the typical size of each flux quantum. Compare this to the volume of the shell

$$V_{\text{shell}} = \omega_{J-1} r^{J-1} \Delta r = \frac{\omega_{J-1}}{2} |2\lambda_{\text{bare}}|^{\frac{J}{2}-1} \Delta\lambda \tag{8}$$

where we defined $\omega_{J-1} \equiv 2\pi^{J/2}/\Gamma(J/2)$ as the area of $(J-1)$-dimensional unit sphere. We require

$$V_{\text{grid-point}} < V_{\text{shell}} \tag{9}$$

or

$$\frac{2}{\omega_{J-1}} \left(\frac{q}{|2\lambda_{\text{bare}}|^{1/2}} \right)^J < \frac{\Delta\lambda}{|2\lambda_{\text{bare}}|} \,. \tag{10}$$

We are now in the position to answer the two questions posed in the problem:

1. Setting $|2\lambda_{\text{bare}}| \sim \mathcal{O}(1)$ and $J \sim \mathcal{O}(100)$ equation (10) implies

$$\boxed{q < \left(\frac{\omega_{99}}{2} \Delta\lambda \right)^{1/100} \approx 10^{-0.6} = 0.03} \,. \tag{11}$$

   Notice that the charges $q_i \sim q$ need not be exceedingly small if there are many fluxes. In order to achieve a small λ, it is sufficient that there be a discrepancy between the magnitude of λ_{bare} and that of the charges. For fixed charges, the task of cancellation actually becomes easier, the larger the bare cosmological constant. To understand this consider Figure 2: the larger the shell, the more points it will contain.

2. Setting $|2\lambda_{\text{bare}}| \sim \mathcal{O}(1)$ and $q \sim \mathcal{O}(0.1)$ equation (10) implies

$$\lambda(J) \equiv \frac{10^{-J}}{\pi^{J/2}/\Gamma(J/2)} < \Delta\lambda \sim 10^{-120} \,. \tag{12}$$

   Since we are only interested in an order of magnitude solution, a graphical solution of the inequality (12) seems sufficient.[1] Using `Mathematica` we plot $\log[\lambda(J)]$ vs. J (see Figure 3). We find

$$\log[\lambda(J)] < -120 \qquad \Leftrightarrow \qquad \boxed{J > 350} \,. \tag{13}$$

[1] Notice that we didn't pay careful attention to numerical factors to begin with, so being overly picky now probably wouldn't make much sense. Nevertheless, purists could try to find an approximate analytical solution by using Stirling's approximation for $\ln\Gamma(J/2)$.

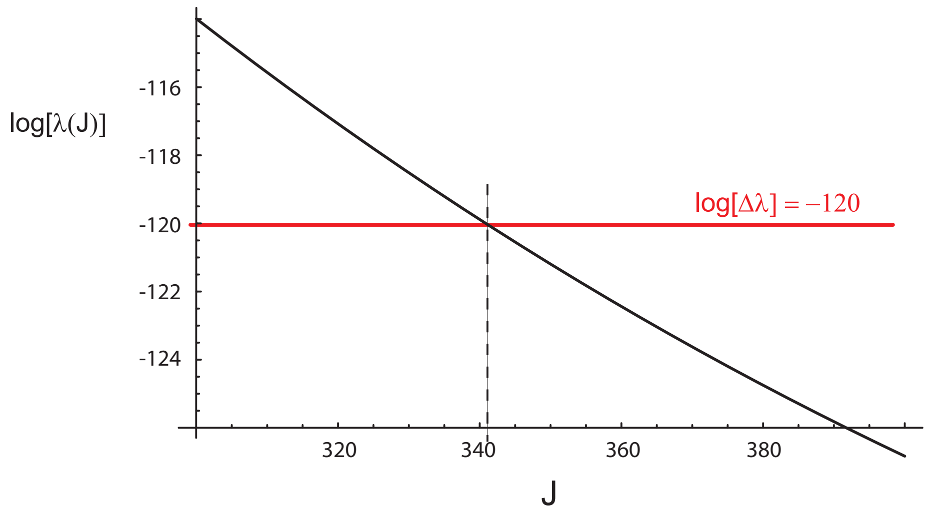

Figure 2: Graphical solution of the inequality (12) .

2 Running Coupling Constant and Dimensional Transmutation

Problem

Consider a $SU(N_c)$ gauge theory that couples to N_f fermions. Suppose that at the Planck scale the coupling constant is g_P. Use the 1-loop approximation to the β-function to find the strong coupling scale Λ associated with this gauge group. Under which conditions is it exponentially small relative to the Planck scale?

Solution

We summarize the discussion on renormalization group flow and the 1-loop β-function of Chapters 16 and 17 in Peskin and Schroeder [2].

The coupling constant α_s must be defined at some renormalization point M. The running coupling coupling

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi} \tag{14}$$

depends on the energy scale Q at which it is measured. The coupling constant g is defined to satisfy the renormalization group equation

$$\frac{d}{d\log(Q/M)} g = \beta(g, N_c, N_f) \tag{15}$$

with initial condition $\alpha_s(M) = \alpha_s$. For a $SU(N_c)$ gauge theory coupled to N_f approximately massless fermions in the fundamental representation, the β-function is given by (see Chapter 16 of [2])

$$\beta(g) = -\frac{b_0 g^3}{(4\pi)^2}, \quad \text{with} \;\; b_0 \equiv \frac{11}{3}N_c - \frac{2}{3}N_f. \tag{16}$$

Notice the all-important minus sign (indicating asymptotic freedom for small N_f) and the g^3-scaling. Then the solution of the renormalization group equation (15) is

$$\alpha_s(Q) = \frac{\alpha_s}{1 + (b_0\alpha_s/2\pi)\log(Q/M)}. \tag{17}$$

Because the fixed coupling α_s depends on the arbitrary renormalization point M, it is useful to remove it from the formula completely. To do this we define a mass scale Λ satisfying

$$1 \equiv \alpha_s \frac{b_0}{2\pi}\log(M/\Lambda). \tag{18}$$

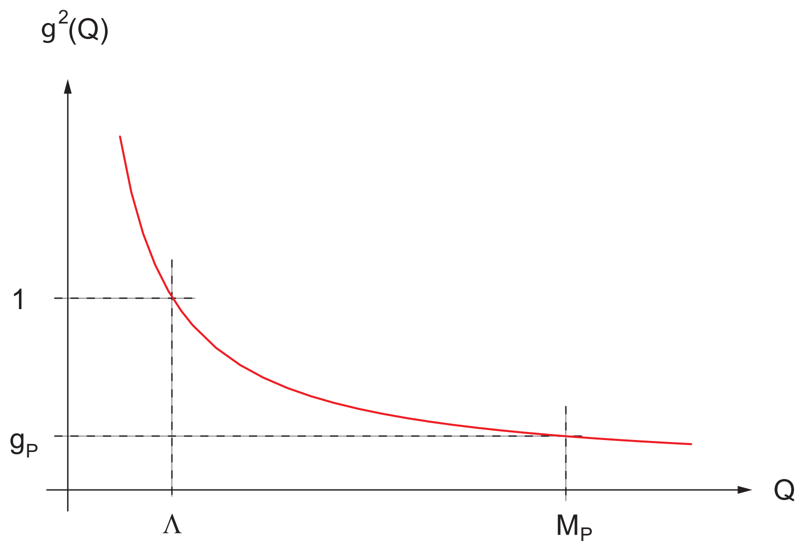

Figure 3: Running coupling constant.

Equation (17) can then be written as

$$\alpha_s(Q) = \frac{2\pi}{b_0\log(Q/\Lambda)}. \tag{19}$$

$\alpha_s(Q)$ decreases as $(\log(Q))^{-1}$ for large energies Q. Evaluating this at $Q = M_P$ and rearranging we find

$$\boxed{\Lambda = \exp\left[-\frac{8\pi^2}{g_P^2\, b_0}\right] \times M_P}, \tag{20}$$

where $g_P \equiv g_P(Q = M_P)$. This shows that the strong coupling scale Λ depends exponentially on the gauge coupling at the Planck scale g_P, *i.e.* the exponential hierachy between the strong coupling scale Λ and the Planck scale M_P requires only a moderate ratio between the running values of the coupling constants at those scales. Finally, we observe from (20) that an exponential hierarchy is also generated by a small value of b_0, *i.e.* $11N_c \sim 2N_f$.

References

[1] R. Bousso and J. Polchinski, "Quantization of four-form fluxes and dynamical neutralization of the JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].

[2] M. E. Peskin and D. V. Schroeder, "An Introduction To Quantum Field Theory."