Homework problems for Kachru lectures, IAS, Summer 2006

1. Estimate the scaling of the flux potential for moduli with the overall radius of the Calabi-Yau space R, in the IIB theory with three-form fluxes turned on. To do this, you simply need to consider the reduction of $G_3 \cdot \overline{G_3}$ to 4d Einstein frame. What is the scaling of the resulting moduli masses with $1/R$? (Remember that the moduli arise as components of the 10d metric). What are the implications of this result for the validity of using a 4d supersymmetric effective field theory for the moduli below the KK scale, without worrying about KK modes and string modes? If you have trouble, see e.g. (3.3)-(3.9) of hep-th/0601111, or hep-th/0405068.

2. In lecture, we found that in the leading approximation, the volume modulus remains a flat direction in IIB Calabi-Yau flux vacua. This problem will explore one possible way (following the paper hep-th/0301240) that the degeneracy can be lifted by corrections to the leading approximation.

Assume that there is a non-perturbative correction to W that depends on ρ. To leading order, W was given by W_{flux} which takes some definite value W_0 in the vacuum for the complex moduli z_α and the dilaton τ. The Kähler mode ρ remains massless, while z_α and τ have masses of scale m_* that you estimated in problem 1. The appropriate low-energy effective theory at energies below the scale m_* just includes ρ, with the other moduli integrated out. The full superpotential takes the form

$$W = W_0 + Ae^{ia\rho}.$$

For the purposes of this problem, you can assume W_0 is a control parameter that can be dialed to have $|W_0| << 1$, and A, a are numbers of $\mathcal{O}(1)$ (e.g. $a \sim 1/N$ for gaugino condensation in an $SU(N)$ pure $\mathcal{N} = 1$ gauge theory).

a) Using the superpotential above, and the *leading order* Kähler potential, show that this system has a vacuum with ρ stabilized.

b) Further non-perturbative effects could yield further corrections of the form $Be^{i2a\rho}$ to W. In the small W_0 regime, are these corrections important or sub-dominant around the leading order vacuum you exhibited in a)? You should estimate this by keeping track of your solution parametrically in W_0, and checking whether the corrections are formally larger or smaller in the expansion in small W_0.

c) One expects perturbative corrections to the Kähler potential as well. Suppose K is corrected to the form

$$K(\rho, \bar{\rho}) = -3\log[-i(\rho - \bar{\rho}) + c] + \frac{d}{(\rho - \bar{\rho})} + \cdots$$

where c, d are again constants which are $\leq \mathcal{O}(1)$. By Taylor expanding the resulting corrections to V around the vacuum a), demonstrate that these corrections are expected to be small in the small W_0 limit.

Note that in all cases, the corrections may lead to global features of V which appear quite distinct from the leading approximation; the important question you are examining here is whether these corrections are important near the approximate vacuum of a) or not.

3. a) In the problem above, small values of the R-symmetry breaking order parameter W play an important role. In Nature, if supersymmetry is relevant at the electroweak scale, it is broken by an F-term which is either $F \sim (10^{11}\ \mathrm{GeV})^2$ (gravity mediation) or $F << (10^{11}\ \mathrm{GeV})^2$ (gauge mediation), with values in the latter case as low as $F \sim (50\ \mathrm{TeV})^2$ being plausible. Take these as the highest and lowest plausible SUSY-breaking F-terms in a nice, supersymmetric world. Using the fact that in supergravity

$$V \sim \left(|DW|^2 - 3\frac{|W|^2}{M_P^2}\right)$$

and that the observed vacuum energy is quite small, estimate the rough magnitude of the dimensionless quantity W/M_P^3 in vacuum, for vanilla gravity and gauge mediation scenarios. (You do not need to assume that W cancels the F-term energy to some ridiculous number of decimal places; an estimate using the leading decimal place will suffice). This provides a "bottom up" motivation to think about vacua with small values of $|W|$.

b) Find a class of IIB flux vacua (in compact models, i.e. where the dilaton is dynamical) where the superpotential is generic enough to stabilize all complex moduli, and where the value of W in vacuum is exponentially small but nonzero. This is publishable if you succeed! Existing constructions of very small W vacua either use tuning in flux-space, or only apply to non-compact Calabi-Yau spaces. But because W is the order parameter for R-symmetry breaking, it would be "natural" for such models to exist in the compact case as well.

4. In compactifications of the heterotic string theory on a Calabi-Yau manifold, one has a single real three-form flux H_3 (arising in the NS sector), which can be turned on. Except

in special circumstances, it obeys an integral flux quantization condition analogous to its IIB counterparts, and gives rise to a flux superpotential

$$W_{flux} = \int_M H_3 \wedge \Omega \ .$$

In the IIB theory, an important role is played by flux vacua where W_{flux} evaluated in the vacuum for complex structure moduli, is small. Do you expect such configurations to be attainable for the heterotic flux superpotential described above? Why or why not? Generalizations of this structure, involving "geometric fluxes" which describe heterotic strings on non-Kähler manifolds, probably make the story more analogous to the IIB one.

5. In the lectures, we mostly focused on IIB models. In this problem, you will work out part of the story for IIA models based on Calabi-Yau compactification. It differs from the IIB story in interesting ways (remember, the IIB and IIA Calabi-Yau flux models are not mirror to one another). You will basically be deriving the results of hep-th/0505160 by DeWolfe et al, without all of the excess mathematical baggage.

a) In IIA compactification on a Calabi-Yau space M, you can turn on RR fluxes of all even dimensions ($F_{0,2,4,6}$), and NS three-form flux H_3. Another important ingredient in the potential energy function for the compactification arises from the presence of O6 planes wrapping 3-cycles in M. Let us imagine the only relevant moduli are the Calabi-Yau radius R and the dilaton $g_s = e^{\phi}$. Derive the scaling of the 4d Einstein frame energy densities due to $F_{0,2,4,6}$, H_3, and O6 planes as a function of e^{ϕ} and R.

b) In models with some given O6 charge, one must either cancel the tadpole by including D6 branes, or by using the fact that $F_0 \wedge H_3$ contributes to the same tadpole. Consider a model with $\mathcal{O}(1)$ units of O6 charge, where the tadpole is cancelled with $\mathcal{O}(1)$ units of F_0 and H_3 flux. In addition, turn on N units of F_4 flux (this is not constrained by any tadpole condition). The resulting potential takes the schematic form

$$V = \frac{e^{2\phi}}{R^{12}} + N^2 \frac{e^{4\phi}}{R^{18}} + \frac{e^{4\phi}}{R^6} - \frac{e^{3\phi}}{R^9} \ .$$

Show that for $N >> 1$, this potential has vacua with g_s and R stabilized. Demonstrate by finding the N-scaling of the stabilized values, that one can make vacua with arbitrarily weak coupling and large volume.

c) In Freund-Rubin models $AdS_d \times S^p$, one cannot strictly speaking use d-dimensional effective field theory to analyze the low-energy physics. This is because the AdS curvature radius and the KK scale are parametrically the same. Is 4d effective field theory applicable to the models discussed above?

d) Full string constructions along these lines exist, with the relation between the AdS curvature and KK scale that you found in c). Find the 3d conformal field theories dual to these AdS_4 solutions. (This is very publishable if you succeed in any example!)