1) Use the method in the tutorial to prove T-duality for a two-dimensional conformal field theory in which the target space is a circle of radius R.

Hint: Start with a two-dimensional conformal field theory in which the Lagrangian is

$$L = \frac{R^2}{4\pi} \int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

where ϕ has period 2π. Add another field A which is an auxiliary $U(1)$ gauge field and modify the Lagrangian so as to get a symmetry $\phi \to \phi - u$, $A \to A + du$, for any $U(1)$-valued field u. Then follow the procedure in the tutorial, adding one more field in such a way that the path integral can be evaluated in two different ways.

2) Consider topological twistings of $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions. Show that there are essentially three ways to twist this theory so as to get a topological field theory. Show that two of the twists lead to unique TQFT's, but one leads to a one-parameter family of inequivalent TQFT's.