Problems for Lecture 1

(1) Why approximate gauge invariance for massive vectors?

We all know that gauge "symmetry" is nothing but a useful redundancy of description that helps us describe the two transverse helicities of spin 1 particles in a manifestly Lorentz invariant and local way. Nonetheless, the (non-gauge invariant) Lagrangian for a massive spin 1 particle is written as $-\frac{1}{4}F_{\mu\nu}^2 + m^2 A_\mu A^\mu$. Why must the first term be gauge invariant? What would go wrong if we also added a term proportional to $(\partial A)^2$? Similarly in the non-Abelian theory, why must the $AA\partial A$ and A^4 terms have the form dictated by gauge invariance? The goldstone formalism we discussed gives simple answers to all these questions; furthermore, for the non-Abelian theory, it dictates that all these gauge-violating effects are indeed present but with suitably small coefficients. Estimate the size of these effects. Identify how they are generated in the Higgs model with a heavy Higgs.

(2) Limits on Simple Models of New Physics

Estimate the constraints placed on the parameter space of the following simple models from existing experiments. Identify the leading constraint.

(A) An electroweak triplet field ϕ with mass m_ϕ and a cubic coupling to the Higgs $\mu h^\dagger \phi h$.

(B) Vector-like "partners" of the top and bottom quarks–Weyl fermions (Q, U^c, D^c) and (Q^c, U, D), with mass terms $M_Q Q Q^c + M_U U U^c + M_D D D^c$, and Yukawa couplings to the Higgs, $\lambda_U Q h^* U^c + \lambda'_U Q^c h U + \lambda_D Q h D^c + \lambda'_D Q^c h^* D$.

(C) A $U(1)_{B-L}$ gauge boson of mass M and coupling g.

(3) Varying the weak scale

This is a bit of an open-ended question, but there is lots of great physics in it, so have fun! Consider the Standard Model, but imagine that we vary the Higgs mass parameter m_H^2 holding all other couplings fixed. What does the long-distance world look like as m_h^2 is made more negative? What happens to the spectrum of the familiar elementary particles? How about of Nuclei and Atoms? Now, imagine that m_h^2 is positive. Is there a change of phase? Again, what does the spectrum look like?