Probing New Physics with Flavor Physics*

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Abstract

This is a **preliminary** written version of a series of lectures aimed at graduate students and postdoctoral fellows in particle theory/string theory/particle experiment familiar with the basics of the Standard Model. We begin with an overview of flavor physics and its implications for new physics. We emphasize the "new physics flavor puzzle". Then, we give four specific examples of flavor measurements and the lessons that have been (or can be) drawn from them: (i) Charm physics: lessons for supersymmetry from the upper bound on Δm_D. (ii) Bottom physics: model independent lessons on the KM mechanism and on new physics in $B^0 - \overline{B}^0$ mixing from $S_{\psi K_S}$. (iii) Top physics and beyond: testing minimal flavor violation at the LHC. (iv) Neutrino physics: interpreting the data on neutrino masses and mixing within flavor models.

I. INTRODUCTION

The Standard Model fermions appear in three generations. *Flavor physics* describes interactions that distinguish between the fermion generations.

The fermions experience two types of interactions: gauge interactions, where two fermions couple to a gauge boson, and Yukawa interactions, where two fermions couple to a scalar. Within the Standard Model, there are twelve gauge bosons, related to the gauge symmetry

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}, \tag{1}$$

and a single Higgs scalar, related to the spontaneous symmetry breaking

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{2}$$

In the *interaction basis*, gauge interactions are diagonal and universal, namely described by a single gauge coupling for each factor in $G_{\rm SM}$: g_3, g_2, and g_Y. By definition, the interaction eigenstates have no gauge couplings between fermions of different generations. The Yukawa interactions are, however, quite complicated in the interaction basis. In particular, there are Yukawa couplings that involve fermions of different generations and, consequently, the interaction eigenstates do not have well-defined masses. *Flavor physics* here refers to the part of the Standard Model that depends on the Yukawa couplings.

In the *mass basis*, Yukawa interactions are diagonal (in the Standard Model, its single-Higgs extensions and even with extended Higgs sector subject to natural flavor conservation), but not universal. The mass eigenstates have, by definition, well-defined masses. The interactions related to spontaneously broken symmetries are, however, quite complicated in the mass basis. In particular, the interactions of the charged weak force carriers $W^{\pm}$ are not diagonal, that is, they *mix* quarks of different generations. (In extensions of the Standard Model, with left-handed $SU(2)_{\rm L}$-singlet left-handed quarks, or $SU(2)_{\rm L}$-doublet right-handed quarks, also the Z-couplings involve mixing.) *Flavor physics* here refers to fermion masses and mixings.

Why is flavor physics interesting?

- Flavor physics and the physics of CP violation can discover new physics or probe it before it is directly observed in experiments. Here are some examples from the past:

  - The smallness of $\frac{\Gamma(K_L \to \mu^+ \mu^-)}{\Gamma(K^+ \to \mu^+ \nu)}$ led to predicting a fourth (the charm) quark;

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- The size of Δm_K led to a successful prediction of the charm mass;

- The size of Δm_B led to a successful prediction of the top mass;

- The measurement of ε_K led to predicting the third generation.

- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase δ_{KM} [1]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.

- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle*.

- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle*.

  The puzzle became even deeper when neutrino masses and mixings have been measured. So far, neither smallness nor hierarchy in these parameters have been established.

In these lectures, we discuss four specific measurements that relate to the four points above:

- We show how measurements of $D^0 - \overline{D}^0$ mixing allow us to explore supersymmetry and, in particular, give evidence that if there are squarks below the TeV scale, they must be quasi-degenerate.

- We explain how the measurement of the CP asymmetry in $B \to J/\psi K_S$ decays gives evidence that the KM mechanism is the dominant source of the observed CP violation, and quantitatively constrains the amount of new physics in $B^0 - \overline{B}^0$ mixing.

- We present the idea of minimal flavor violation as a solution to the new physics flavor problem, and argue that the ATLAS and CMS experiments may be able to test this solution.

- We describe the extraction of four neutrino parameters from measurements related to atmospheric and solar neutrinos, and explain their impact on models that aim to explain the Standard Model flavor puzzle.

II. FLAVOR IN THE STANDARD MODEL

A model of elementary particles and their interactions is defined by three ingredients: (i) The symmetries of the Lagrangian; (ii) The pattern of spontaneous symmetry breaking; (iii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows. (i) The gauge symmetry is $G_{\rm SM}$ of Eq. (1); (ii) Its spontaneous symmetry breaking is described by Eq. (2); (iii) There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q^I_{Li}(3,2)_{+1/6}, \quad U^I_{Ri}(3,1)_{+2/3}, \quad D^I_{Ri}(3,1)_{-1/3}, \quad L^I_{Li}(1,2)_{-1/2}, \quad E^I_{Ri}(1,1)_{-1}. \tag{3}$$

Our notations mean that, for example, left-handed quarks, Q^I_L, are triplets of $SU(3)_{\rm C}$, doublets of $SU(2)_{\rm L}$ and carry hypercharge $Y = +1/6$. The super-index I denotes interaction eigenstates. The sub-index $i = 1, 2, 3$ is the flavor (or generation) index. There is a single scalar representation,

$$\phi(1,2)_{+1/2} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{4}$$

The scalar ϕ^0 assumes a VEV,

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}, \tag{5}$$

leading to (2).

A. The interactions basis

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (1), the particle content (3,4) and the pattern of spontaneous symmetry breaking (2,5). It can be divided to three parts:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}. \tag{6}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu = \partial^\mu + ig_s G_a^\mu L_a + ig W_b^\mu T_b + ig' B^\mu Y. \tag{7}$$

Here G_a^μ are the eight gluon fields, W_b^μ the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_C$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_L$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_Y$ charges. For example, for the quark doublets Q_L^I, we have

$$\mathcal{L}_{\text{kinetic}}(Q_L) = i\overline{Q_{Li}^I}\gamma_\mu \left(\partial^\mu + \frac{i}{2}g_s G_a^\mu \lambda_a + \frac{i}{2}g W_b^\mu \tau_b + \frac{i}{6}g' B^\mu \right) Q_{Li}^I, \tag{8}$$

while for the lepton doublets L_L^I, we have

$$\mathcal{L}_{\text{kinetic}}(L_L) = i\overline{L_{Li}^I}\gamma_\mu \left(\partial^\mu + \frac{i}{2}g W_b^\mu \tau_b - ig' B^\mu \right) L_{Li}^I. \tag{9}$$

These parts of the interaction Lagrangian are flavor-universal and CP conserving.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\text{Higgs}} = \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2. \tag{10}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving.

The quark Yukawa interactions are given by

$$-\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = Y_{ij}^d \overline{Q_{Li}^I} \phi D_{Rj}^I + Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} U_{Rj}^I + \text{h.c.}, \tag{11}$$

while the lepton Yukawa interactions are given by

$$-\mathcal{L}_{\text{Yukawa}}^{\text{leptons}} = Y_{ij}^e \overline{L_{Li}^I} \phi E_{Rj}^I + \text{h.c..} \tag{12}$$

This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

B. Global symmetries and parameter counting

In the absence of the Yukawa matrices Y^d, Y^u and Y^e, the SM has a large $U(3)^5$ global symmetry:

$$G_{\text{global}}(Y^f = 0) = SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)^5, \tag{13}$$

where

$$
\begin{aligned}
SU(3)_q^3 &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\
SU(3)_\ell^2 &= SU(3)_L \times SU(3)_E, \\
U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\mathrm{PQ}} \times U(1)_E.
\end{aligned}
\tag{14}
$$

Out of the five $U(1)$ charges, three can be identified with baryon (B) and lepton (L) numbers and hypercharge (Y), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

The Yukawa interactions (11) and (12) break this symmetry (of course, the gauged $U(1)_Y$ remains a good symmetry),

$$
G_{\mathrm{global}}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau.
\tag{15}
$$

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)_q^3$ symmetry (but are neutral under $U(1)_B$),

$$
Y^u \sim (3, \bar{3}, 1)_{SU(3)_q^3}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)_q^3},
\tag{16}
$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)_\ell^2$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$
Y^e \sim (3, \bar{3})_{SU(3)_\ell^2}.
\tag{17}
$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression parameters, and the idea of minimal flavor violation.

How many independent parameters are there in $\mathcal{L}_{\mathrm{Yukawa}}^{\mathrm{quarks}}$? Each of the two Yukawa matrices Y^q $(q = u, d)$ is 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of G_{global} breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we will be able to identify the nine real parameters as six quark masses and three mixing angles, while the single phase is δ_{KM}.

How many independent parameters are there in $\mathcal{L}^{\text{leptons}}_{\text{Yukawa}}$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $U(1)^3$). We conclude that there are 3 real lepton flavor parameters. In the mass basis, we will be able to identify these parameters as the three charged lepton masses. We will have, however, to modify the model when we take into account the evidence for neutrino masses.

C. The mass basis

Upon the replacement $\mathcal{R}e(\phi^0) \to \frac{v+H^0}{\sqrt{2}}$ [see Eq. (5)], the Yukawa interactions (11) give rise to mass terms:

$$-\mathcal{L}^q_M = (M_d)_{ij}\overline{D^I_{Li}}D^I_{Rj} + (M_u)_{ij}\overline{U^I_{Li}}U^I_{Rj} + \text{h.c.}, \tag{18}$$

where

$$M_q = \frac{v}{\sqrt{2}}Y^q, \tag{19}$$

and we decomposed the $SU(2)_\text{L}$ quark doublets into their components:

$$Q^I_{Li} = \begin{pmatrix} U^I_{Li} \\ D^I_{Li} \end{pmatrix}. \tag{20}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL}M_qV^\dagger_{qR} = M^{\text{diag}}_q \quad (q=u,d), \tag{21}$$

with M^{diag}_q diagonal and real. The quark mass eigenstates are then identified as

$$q_{Li} = (V_{qL})_{ij}q^I_{Lj}, \quad q_{Ri} = (V_{qR})_{ij}q^I_{Rj} \quad (q=u,d). \tag{22}$$

The charged current interactions for quarks [that is the interactions of the charged $SU(2)_\text{L}$ gauge bosons $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$], which in the interaction basis are described by (8), have a complicated form in the mass basis:

$$-\mathcal{L}^q_{W^\pm} = \frac{g}{\sqrt{2}}\overline{U_{Li}}\gamma^\mu V_{ij}D_{Lj}W^+_\mu + \text{h.c.}. \tag{23}$$

where V is a unitary 3×3 matrix,

$$V = V_{uL} V_{dL}^\dagger, \quad (VV^\dagger = V^\dagger V = \mathbf{1}). \tag{24}$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [1, 2]. As a result of the fact that V is not diagonal, the $W^\pm$ gauge bosons couple to quark mass eigenstates of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

Exercise 1: Prove that, in the absence of neutrino masses, there is no mixing in the lepton sector.

Exercise 2: Prove that there is no mixing in the Z couplings. (In the physics jargon, there are no flavor changing neutral currents at tree level.)

The detailed structure of the CKM matrix, its parametrization, and the constraints on its elements are described in Appendix A.

III. THE NEW PHYSICS FLAVOR PUZZLE

It is clear that the Standard Model is not a complete theory of Nature:

1. It does not include gravity, and therefore it cannot be valid at energy scales above $m_{\text{Planck}} \sim 10^{19}$ GeV:

2. It does not allow for neutrino masses, and therefore it cannot be valid at energy scales above $m_{\text{seesaw}} \sim 10^{14}$ GeV;

3. The fine-tuning problem of the Higgs mass and the puzzle of the dark matter suggest that the scale where the SM is replaced with a more fundamental theory is actually much lower, $\Lambda_{\text{NP}} \lesssim 1$ TeV.

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\text{SM}}$ of Eq. (6). These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics Λ_{NP}. For example, the lowest dimension non-renormalizable terms are dimension five:

$$-\mathcal{L}_{\text{Yukawa}}^{\text{dim}-5} = \frac{Z_{ij}^\nu}{\Lambda_{\text{NP}}} L_{Li}^I L_{Lj}^I \phi\phi + \text{h.c.}. \tag{25}$$

These are the seesaw terms, leading to neutrino masses. We will return to the topic of neutrino masses in section VII.

Exercise 3: How does the global symmetry breaking pattern (15) change when (25) is taken into account?

Exercise 4: What is the number of physical lepton flavor parameters in this case? Identify these parameters in the mass basis.

As concerns quark flavor physics, consider, for example, the following dimension-six, four-fermion, flavor changing operators:

$$\mathcal{L}_{\Delta F=2} = \frac{z_{sd}}{\Lambda_{\rm NP}^2}(\overline{d_L}\gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda_{\rm NP}^2}(\overline{c_L}\gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda_{\rm NP}^2}(\overline{d_L}\gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda_{\rm NP}^2}(\overline{s_L}\gamma_\mu b_L)^2. \tag{26}$$

Each of these terms contributes to the mass splitting between the corresponding two neutral mesons. For example, the term $\mathcal{L}_{\Delta B=2} \propto (\overline{d_L}\gamma_\mu b_L)^2$ contributes to Δm_B, the mass difference between the two neutral B-mesons. We use $M_{12}^B = \frac{1}{2m_B}\langle B^0|\mathcal{L}_{\Delta F=2}|\overline{B}^0\rangle$ and

$$\langle B^0|(\overline{d_{La}}\gamma^\mu b_{La})(\overline{d_{Lb}}\gamma_\mu b_{Lb})|\overline{B}^0\rangle = -\frac{1}{3}m_B^2 f_B^2 B_B. \tag{27}$$

Analogous expressions hold for the other neutral mesons. This leads to $\Delta m_B/m_B \sim (z_{bd}/3)(f_B/\Lambda_{\rm NP})^2$. Experiments give:

$$\epsilon_K \sim 2.3 \times 10^{-3},$$
$$\Delta m_K/m_K \sim 7.0 \times 10^{-15},$$
$$\Delta m_D/m_D \lesssim 2 \times 10^{-14},$$
$$\Delta m_B/m_B \sim 6.3 \times 10^{-14},$$
$$\Delta m_{B_s}/m_{B_s} \sim 2.2 \times 10^{-13}. \tag{28}$$

These measurements give then the following constraints (the bound on $\mathcal{I}m(z_{sd})$ is stronger by a factor of $(2\sqrt{2}\epsilon_K)^{-1}$ than the bound on $|z_{sd}|$):

$$\Lambda_{\rm NP} \gtrsim \begin{cases} \sqrt{\mathcal{I}m(z_{sd})}\ 2 \times 10^4\ TeV & \epsilon_K \\ \sqrt{z_{sd}}\ 1 \times 10^3\ TeV & \Delta m_K \\ \sqrt{z_{cu}}\ 8 \times 10^2\ TeV & \Delta m_D \\ \sqrt{z_{bd}}\ 5 \times 10^2\ TeV & \Delta m_B \\ \sqrt{z_{bs}}\ 2 \times 10^2\ TeV & \Delta m_{B_s} \end{cases} \tag{29}$$

If the new physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above $10^3 - 10^4$ TeV. If indeed $\Lambda_{\rm NP} \gg TeV$, it means that we have misinterpreted the hints

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from the fine-tuning problem and the dark matter puzzle. There is, however, another way to look at these constraints:

$$\mathcal{I}m(z_{sd}) \lesssim 5 \times 10^{-9} \ (\Lambda_{\mathrm{NP}}/TeV)^2,$$
$$z_{sd} \lesssim 7 \times 10^{-7} \ (\Lambda_{\mathrm{NP}}/TeV)^2,$$
$$z_{cu} \lesssim 2 \times 10^{-6} \ (\Lambda_{\mathrm{NP}}/TeV)^2,$$
$$z_{bd} \lesssim 4 \times 10^{-6} \ (\Lambda_{\mathrm{NP}}/TeV)^2,$$
$$z_{bs} \lesssim 3 \times 10^{-5} \ (\Lambda_{\mathrm{NP}}/TeV)^2. \tag{30}$$

It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.

One can use that language of effective operators also for th SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\mathrm{SM}} \sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the z_{ij} coefficients are suppressed by α_2^2. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0 - \overline{B}^0$ mixing involves $\overline{d_L}b_L$ which transforms as $(8, 1, 1)_{SU(3)_q^3}$. The leading contribution must then be proportional to $(Y^u Y^{u\dagger})_{13} \propto y_t^2 V_{tb}V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[1]

$$\frac{2M_{12}^B}{m_B} \approx -\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2}S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{31}$$

where $x_i = m_i^2/m_W^2$ and

$$S_0(x) = \frac{x}{(1-x)^2}\left[1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2\ln x}{2(1-x)}\right]. \tag{32}$$

Similar spurion analyses, or explicit calculations, allows us to extract the weak and flavor suppression factors that apply in the SM:

$$\mathcal{I}m(z_{sd}^{\mathrm{SM}}) \sim \alpha_2^2 y_t^2 |V_{td}V_{ts}|^2 \sim 1 \times 10^{-10},$$
$$z_{sd}^{\mathrm{SM}} \sim \alpha_2^2 y_c^2 |V_{cd}V_{cs}|^2 \sim 5 \times 10^{-9},$$
$$z_{bd}^{\mathrm{SM}} \sim \alpha_2^2 y_t^2 |V_{td}V_{tb}|^2 \sim 7 \times 10^{-8},$$
$$z_{bs}^{\mathrm{SM}} \sim \alpha_2^2 y_t^2 |V_{ts}V_{tb}|^2 \sim 2 \times 10^{-6}. \tag{33}$$

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[1] A detailed derivation can be found in Appendix B of [3].

(We did not include z_{cu}^{SM} in the list because it requires a more detailed consideration. The SM contribution to Δm_D is dominated by long distance physics, with $z_{cu}^{\text{SM}} \propto \alpha_2^2 y_s^2 (\Lambda/m_D)^2 |V_{cs} V_{us}|^2 \sim 5 \times 10^{-12}$. In fact, peculiar phase space effects [4, 5] have been identified which contribute a factor of y_s^2 to the denominator, potentially enhancing $z_{cu}^{\text{SM}} \propto \alpha_2^2 (\Lambda/m_D)^2 |V_{cs} V_{us}|^2 \sim 5 \times 10^{-6}$. The short distance contribution is $\propto \alpha_2^2 (y_s^4/y_c^2)|V_{cs}V_{us}|^2 \sim 5 \times 10^{-13}$.)

It is clear than that contributions from new physics at $\Lambda_{\text{NP}} \sim 1\,TeV$ should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

IV.   LESSONS FROM $D^0 - \overline{D}^0$ MIXING

Interesting experimental results concerning $D^0 - \overline{D}^0$ mixing have been recently achieved by the BELLE and BABAR experiments. For the first time, there is evidence for width splitting (of order one percent) between the two neutral D-mesons [6, 7], while the bound on the mass splitting has become stronger [8]. We use this recent experimental information to draw important lessons on supersymmetry. This demonstrates how flavor physics – at the GeV scale – provides a significant probe of supersymmetry – at the TeV scale.

A.   Neutral meson mixing with supersymmetry

We consider the contributions from the box diagrams involving the squark doublets of the first two generations, $\tilde{Q}_{L1,2}$, to the $D^0 - \overline{D}^0$ and $K^0 - \overline{K}^0$ mixing amplitudes. The contribution that is relevant to the neutral D system is proportional to $K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}$, where K^u is the mixing matrix of the gluino couplings to a left-handed up quark and their supersymmetric squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta_{LL}^u)_{12}]^2$.) The contribution that is relevant to

the neutral K system is proportional to $K^d_{2i}K^{d*}_{1i}K^d_{2j}K^{d*}_{1j}$, where K^d is the mixing matrix of the gluino couplings to a left-handed down quark and their supersymmetric squark partners ($\propto [(\delta^d_{LL})_{12}]^2$ in the mass insertion approximation). We work in the mass basis for both quarks and squarks.

A detailed derivation [9] is given in Appendix B. It gives:

$$M^D_{12} = \frac{\alpha^2_s m_D f^2_D B_D \eta_{\rm QCD}}{108\tilde{m}^2_q}[11\tilde{f}_6(x) + 4xf_6(x)]\frac{(\Delta\tilde{m}^2_{21})^2}{\tilde{m}^4_q}(K^u_{21}K^{u*}_{11})^2, \tag{34}$$

$$M^K_{12} = \frac{\alpha^2_s m_K f^2_K B_K \eta_{\rm QCD}}{108\tilde{m}^2_q}[11\tilde{f}_6(x) + 4xf_6(x)]\frac{(\Delta\tilde{m}^2_{21})^2}{\tilde{m}^4_q}(K^d_{21}K^{d*}_{11})^2. \tag{35}$$

One can immediately identify the three generic ways in which flavor features can suppress the supersymmetric contributions to the mixing:

1. Degeneracy: $\Delta\tilde{m}^2_{21} \ll \tilde{m}^2_q$;

2. Heaviness: $\tilde{m}_q \gg 1\ TeV$;

3. Alignment: $K^{d,u}_{21} \ll 1$.

The 2×2 mass-squared matrices for the relevant squarks have the following form:

$$(\tilde{M}^2_U)_{LL} = \tilde{m}^2_{Q_L} + \left(\frac{1}{2} - \frac{2}{3}s^2_W\right)m^2_Z\cos 2\beta + M^2_u,$$

$$(\tilde{M}^2_D)_{LL} = \tilde{m}^2_{Q_L} - \left(\frac{1}{2} - \frac{1}{3}s^2_W\right)m^2_Z\cos 2\beta + M^2_d. \tag{36}$$

Here $\tilde{m}^2_{Q_L}$ is a 2×2 hermitian matrix. It does not break $SU(2)_{\rm L}$ and it is common to $\tilde{M}^2_U$ and $\tilde{M}^2_D$. Given the lower bounds on squark masses, the scale of its eigenvalues must be a factor of at least $\mathcal{O}(10)$ higher than m^2_Z. We can draw the following conclusions:

1. The scale $\tilde{m}^2_q$ is the same in Eqs. (34) and (35) up to effects of order m^2_Z, namely to an accuracy better than 0.1.

2. The mass-squared difference $\Delta\tilde{m}^2_{21}$ is the same in Eqs. (34) and (35) up to effects of order m^2_c, namely to an accuracy better than 10^{-4}.

3. Since $K_u \simeq V_{uL}\tilde{V}^\dagger_L$ and $K_d \simeq V_{dL}\tilde{V}^\dagger_L$ (the matrices V_{qL} are defined in Eq. (21), while $\tilde{V}_L$ diagonalizes $\tilde{m}^2_{Q_L}$), the mixing matrices K^u and K^d are different from each other, but they are related through the CKM matrix:

$$K^u K^{d\dagger} = V. \tag{37}$$

B. Non-degenerate squarks at the LHC?

Eqs. (34) and (35) can be translated into our generic language:

$$\Lambda_{\rm NP} = \tilde{m}_q, \tag{38}$$

$$z_{cu} = z_{12} \sin^2 \tilde{\theta}^u_{12},$$

$$z_{sd} = z_{12} \sin^2 \tilde{\theta}^d_{12},$$

$$z_{12} = \frac{11 f_6(x) + 4x \tilde{f}_6(x)}{18} \alpha_s^2 \left(\frac{\Delta \tilde{m}^2_{21}}{\tilde{m}^2_q}\right)^2, \tag{39}$$

with Eq. (37) giving

$$\sin \tilde{\theta}^u_{12} - \sin \tilde{\theta}^d_{12} \approx \sin \theta_c = 0.23. \tag{40}$$

We now ask the following question: Is it possible that the first two generation squarks, $\tilde{Q}_{L1,2}$, are accessible to the LHC (say, $\tilde{m}_q \lesssim 1\ TeV$), and are not degenerate?

To answer this question, we use Eqs. (30). For $\Lambda_{\rm NP} \lesssim 1\ TeV$, we have $z_{cu} \lesssim 2 \times 10^{-6}$ and, for a phase that is $\lll 0.1$, $z_{sd} \lesssim \times 5 \times 10^{-8}$. On the other hand, for non-degenerate squarks, $\Delta \tilde{m}^2_{21}/\tilde{m}^2_q \sim 1$ and, for example, $11 f_6(1) + 4\tilde{f}_6(1) = 1/6$, we have $z_{12} = 8 \times 10^{-5}$. Then we need, simultaneously, $\sin \tilde{\theta}^u_{12} \lesssim 0.15$ and $\sin \tilde{\theta}^d_{12} \lesssim 10^{-3}$, but this is inconsistent with Eq. (40).

There are three ways out of this situation:

1. The first two generation squarks are quasi-degenerate. The minimal level of degeneracy is $(\tilde{m}_2 - \tilde{m}_1)/(\tilde{m}_2 + \tilde{m}_1) \lesssim 0.2$. It could be the result of RGE [10].

2. The first two generation squarks are heavy. Putting $\sin \tilde{\theta}^u_{12} = 0.23$ and $\sin \tilde{\theta}^d_{12} \approx 0$, as in models of alignment [11, 12], Eq. (29) leads to

$$\tilde{m}_q \gtrsim 2\ TeV. \tag{41}$$

3. The ratio $x = \tilde{m}^2_g/\tilde{m}^2_q$ is in a fine-tuned region of parameter space where there are accidental cancellations in $11 \tilde{f}_6(x) + 4x f_6(x)$. For example, for $x = 2.33$, this combination is ~ 0.003 and the bound (41) is relaxed by a factor of 7.

Barring such accidental cancellations, the conclusion is that if squarks are within the reach of the LHC, they must be quasi-degenerate [13, 14].

Exercise 5: Does $K^d_{31} \sim |V_{ub}|$ suffice to satisfy the Δm_B constraint (with neither degeneracy nor heaviness)?

V. LESSONS FROM $S_{\psi K_S}$

Measurements of rates, mixing, and CP asymmetries in B decays in the two B factories, BaBar abd Belle, and in the two Tevatron detectors, CDF and D0, signified a new era in our understanding of CP violation. The progress is both qualitative and quantitative. Various basic questions concerning CP and flavor violation have received, for the first time, answers based on experimental information. These questions include, for example,

- Is the Kobayashi-Maskawa mechanism at work?

- Does the KM phase dominate the observed CP violation?

- How large can new physics contributions to $B^0 - \overline{B}^0$ and $B_s - \overline{B}_s$ be?

A. Is the KM mechanism at work?

The three generation standard model has room for CP violation, through the KM phase in the quark mixing matrix. Yet, one would like to make sure that indeed CP is violated by the SM interactions, namely to establish that $\sin \delta_{\rm KM} \neq 0$. Indeed, we can now say with confidence that this is the case.

In proving that the KM mechanism is at work, there is one basic assumption that we must make: Charged-current tree-level processes are dominated by the W-mediated SM diagrams. This is a very plausible assumption. I am not aware of any viable well-motivated model where this assumption is not valid.

What we would like to establish is that the CP violating Wolfenstein parameter $\eta \neq 0$ [Eq. (A4)]. The following tree level processes are useful for this purpose:

1. Charmless semileptonic B-decays, $b \to u\ell\nu$, measure R_u [see Eq. (A8)].

2. $B \to DK$ decays, which go through the quark transitions $b \to c\bar{u}s$ and $b \to u\bar{c}s$, measure the angle γ [see Eq. (A9)].

3. $B \to \rho\rho$ decays (and, similarly, $B \to \pi\pi$ and $B \to \rho\pi$ decays) go through the quark transition $b \to u\bar{u}d$. With an isospin analysis, one can determine the relative phase between the tree decay amplitude and the mixing amplitude. By incorporating the
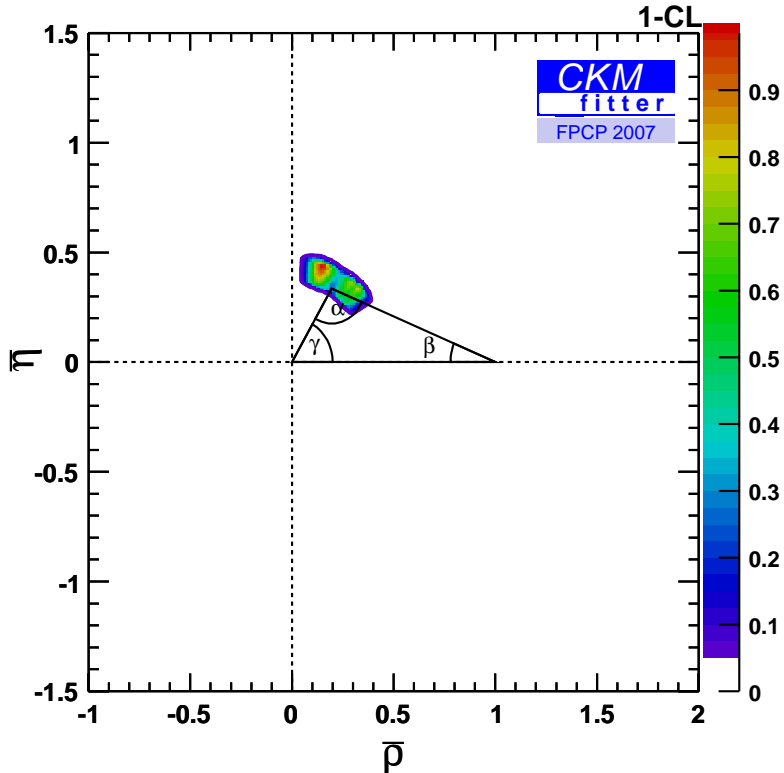
FIG. 1: The allowed region in the $\rho - \eta$ plane, assuming that tree diagrams are dominated by the Standard Model [16].

measurement of $S_{\psi K_S}$, one can subtract the phase from the mixing amplitude, finally providing a measurement of the angle α [see Eq. (A9)].

In addition, one can use measurements related to $B^0 - \overline{B}^0$ mixing. There are three relevant observables [see Eq. (56)] while the theoretical expressions, allowing for arbitrary new physics in the mixing, involve only two new parameters [see Eq. (55)]. Similarly, one can use measurements related to $B_s - \overline{B}_s$ mixing. One gains three new observables at the cost of two new parameters (see, for example, [15]).

The results of such fit, projected on the $\rho - \eta$ plane, can be seen in Fig. 1. It gives [16]

$$\eta = 0.44^{+0.05}_{-0.23} \ \ (3\sigma). \tag{42}$$

It is clear that $\eta \neq 0$ is well established:

The Kobayashi-Maskawa mechanism of CP violation is at work.

B. Is the KM mechanism dominant?

The measurement of the CP asymmetry in the $B \to J/\psi K_S$ decay and in other modes that proceed via the $b \to c\bar{c}s$ quark transition signified a new era in our understanding of CP violation. In particular, it provided the first precision test of the Kobayashi-Maskawa mechanism.

The CP asymmetry in neutral meson decays into final CP eigenstates f_{CP} is defined as follows:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\text{phys}}(t) \to f_{CP}] - d\Gamma/dt[B^0_{\text{phys}}(t) \to f_{CP}]}{d\Gamma/dt[\overline{B}^0_{\text{phys}}(t) \to f_{CP}] + d\Gamma/dt[B^0_{\text{phys}}(t) \to f_{CP}]} . \tag{43}$$

A detailed evaluation of this asymmetry is given in Appendix C. It leads to the following form:

$$\mathcal{A}_{f_{CP}}(t) = S_{f_{CP}} \sin(\Delta m t) - C_{f_{CP}} \cos(\Delta m t),$$
$$S_{f_{CP}} \equiv \frac{2\,\mathcal{I}m(\lambda_{f_{CP}})}{1 + |\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2} , \tag{44}$$
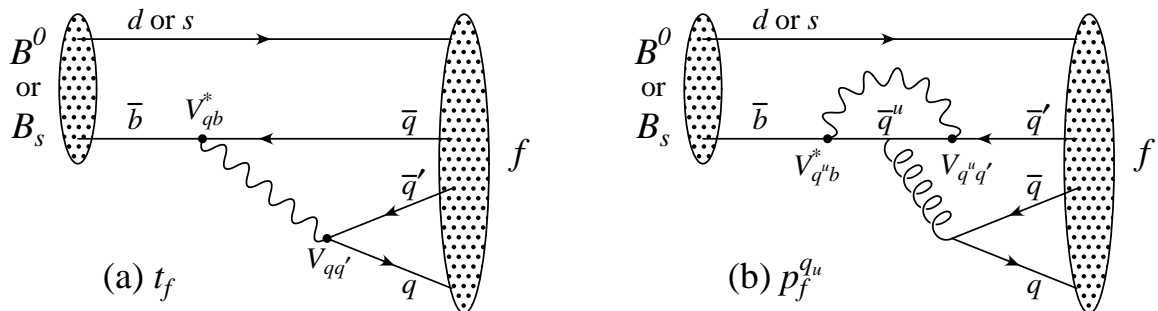
where

$$\lambda_{f_{CP}} = e^{-i\phi_B}(\overline{A}_{f_{CP}}/A_{f_{CP}}) . \tag{45}$$

Here ϕ_B refers to the phase of M_{12} [see Eq. (C23)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) . \tag{46}$$

The decay amplitudes A_f and $\overline{A}_f$ are defined in Eq. (C1).

FIG. 2: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.

16

Some of the most interesting decays involve final states that are common to B^0 and $\overline{B}^0$ [17, 18], such as $B \to J/\psi K_S$. Here Eq. (C21) applies. The processes of interest proceed via quark transitions of the form $\bar{b} \to \bar{c}c\bar{s}$. There are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 2) which carry different weak phases:

$$A_f = (V_{cb}^* V_{cs}) \, t_f + \sum_{q_u = u,c,t} \left(V_{q_u b}^* V_{q_u s} \right) p_f^{q_u} . \tag{47}$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [19].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V_{cb}^* V_{cs}) \, T_{\psi K} + (V_{ub}^* V_{us}) \, P_{\psi K}^u, \tag{48}$$

where $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$ and $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, is reached only via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing, $e^{-i\phi_K} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V_{cs}^*) \, T_{\psi K} + (V_{ub} V_{us}^*) \, P_{\psi K}^u}{(V_{cb}^* V_{cs}) \, T_{\psi K} + (V_{ub}^* V_{us}) \, P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}. \tag{49}$$

For $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c}c\bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0 , \tag{50}$$

where β is defined in Eq. (A9). (Below the percent level, several effects modify this equation [20, 21].)

Exercise 6: Show that, if the $B \to \pi\pi$ decays were dominated by tree diagrams, then $S_{\pi\pi} = \sin 2\alpha$.

Exercise 7: Estimate the accuracy of the predictions $S_{\phi K_S} = \sin 2\beta$ and $C_{\phi K_S} = 0$.

The SM prediction for $\sin 2\beta$, based on all other constraints, is [22]

$$\sin 2\beta^{S_{\psi K}} = 0.76 \pm 0.04. \tag{51}$$

A fit assuming only that tree processes are dominated by the SM diagrams (see previous section) obtains [16]

$$\sin 2\beta^{\text{tree}} = 0.82^{+0.02}_{-0.13}.\tag{52}$$

The experimental measurements give the following ranges [23]:

$$S_{\psi K_S} = 0.68 \pm 0.03, \quad C_{\psi K_S} = 0.01 \pm 0.02 \ .\tag{53}$$

The consistency of the experimental results (53) with the SM predictions (50,51,52) means that the KM mechanism of CP violation has successfully passed its first precision test. For the first time, we can make the following statement based on experimental evidence:

The Kobayashi-Maskawa mechanism is the dominant source of the CP violation observed in flavor changing processes.

There are two qualifications implicit in this statement [24]:

- *'Dominant'*: While $S_{\psi K}$ is measured with an accuracy of order 0.04, the accuracy of the SM prediction for $\sin 2\beta$ is only at the level of 0.1. Thus, it is quite possible that there is a new physics contribution at the level of $|M_{12}^{\text{NP}}/M_{12}^{\text{SM}}| \lesssim \mathcal{O}(0.1)$.

- *'Flavor changing'*: It may well happen that the KM phase, which is closely related to flavor violation through the CKM matrix, dominates meson decays while new, flavor diagonal phases (such as the two unavoidable phases in the universal version of the MSSM) dominate observables such as electric dipole moments by many orders of magnitude.

The measurement of $S_{\psi K}$ provides a significant constraint on the unitarity triangle. In the $\rho - \eta$ plane, it reads:

$$\sin 2\beta = \frac{2\eta(1 - \rho)}{\eta^2 + (1 - \rho)^2} = 0.68 \pm 0.03.\tag{54}$$

One can get an impression of the impact of this constraint by looking at Fig. 7, where the blue region represents $\sin 2\beta = 0.68 \pm 0.03$. An impression of the KM test can be achieved by observing that the blue region has an excellent overlap with the region allowed by all other measurements. A comparison between the constraints in the $\rho - \eta$ plane from CP conserving and CP violating processes is provided in Fig. 3. The impressive consistency between the two allowed regions is the basis for our statement that the KM mechanism has passed its

first precision tests. The fact that the allowed region from the CP violating processes is more strongly constrained is related to the fact that CP is a good symmetry of the strong interactions and that, therefore, various CP violating observables – in particular $S_{\psi K}$ – can be cleanly interpreted.

FIG. 3: Constraints in the $\rho - \eta$ plane from (a) CP conserving or (b) CP violating processes.
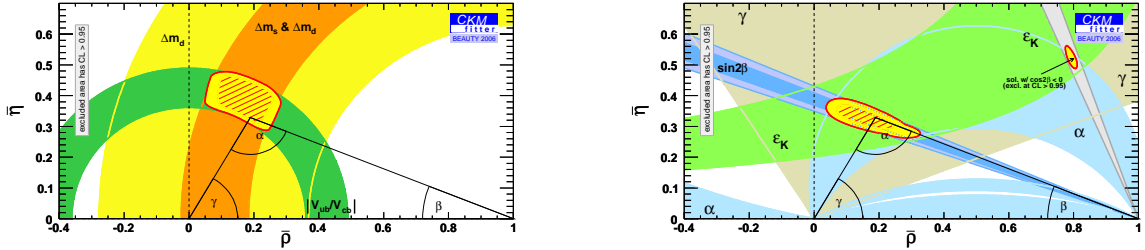
C.   How much can new physics contribute to $B^0 - \overline{B}^0$ mixing?

The measurement of $S_{\psi K_S}$ cleanly determines the relative phase between the $B^0 - \overline{B}^0$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the SM). The $b \to c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification as follows:

$$r_d^2 \, e^{2i\theta_d} = \frac{M_{12}}{M_{12}^{\rm SM}}. \tag{55}$$

Then the following observables provide constraints on r_d^2 and $2\theta_d$:

$$
\begin{aligned}
S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\
\Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\
\mathcal{A}_{\rm SL} &= -\mathcal{R}e \left(\frac{\Gamma_{12}}{M_{12}} \right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m \left(\frac{\Gamma_{12}}{M_{12}} \right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}.
\end{aligned} \tag{56}
$$

Examining whether $S_{\psi K_S}$, Δm_B and $\mathcal{A}_{\rm SL}$ fit the SM prediction, that is, whether $\theta_d \neq 0$ and/or $r_d^2 \neq 1$, we can answer the following question (see *e.g.* [25]):

Is there new physics in $B^0 - \overline{B}^0$ mixing?

Thanks to the fact that quite a few observables that are related to SM tree level processes have already been measured, we are able to refer to this question in a quantitative way. The

tree level processes are insensitive to new physics and can be used to constrain ρ and η even in the presence of new physics contributions to loop processes, such as Δm_B. Among these observables we have $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays, the phase γ from $B \to DK$ decays, and the phase α from $B \to \rho\rho$ decays (in combination with $S_{\psi K}$). One can fit these observables, and the ones in Eq. (56) to the four parameters ρ, η, r_d^2 and $2\theta_d$. The resulting constraints are shown in Fig. 4.

FIG. 4: Constraints in the (a) $\rho - \eta$ plane (b) $r_d^2 - 2\theta_d$ plane, assuming that NP contributions to tree level processes are negligible [16].
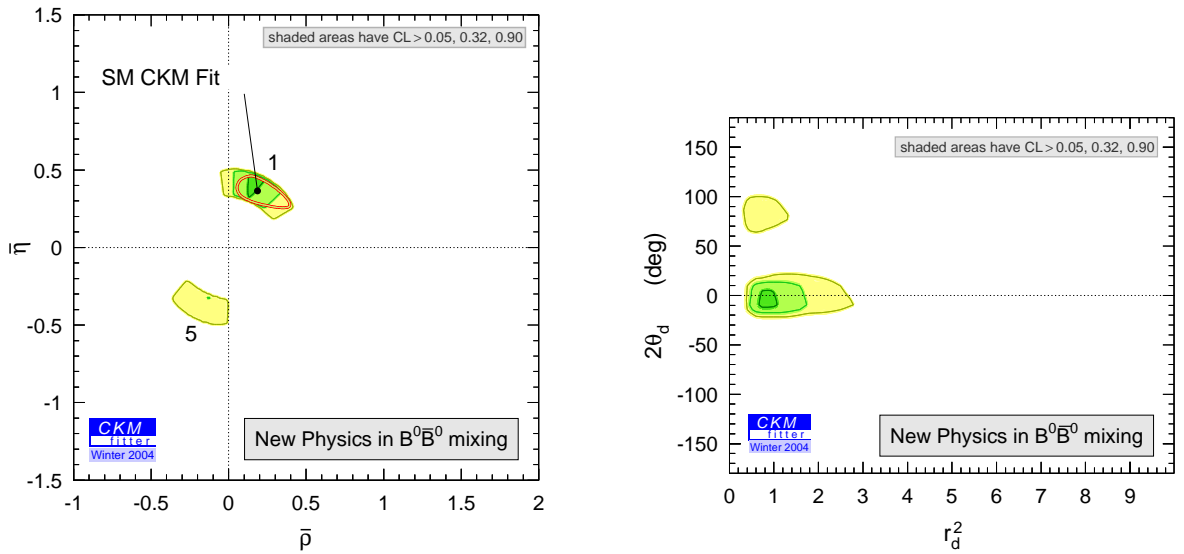
An alternative way to present the data is to use the h_d, σ_d parametrization,

$$r_d^2 e^{2i\theta_d} = 1 + h_d e^{i\sigma_d}. \tag{57}$$

While the r_d, θ_d parameters give the relation between the full mixing amplide and the SM one, and are convenient to apply to the measurements, the h_d, σ_d parameters give the relation between the new physics and SM contributions, and are more convenient in testing theoretical models:

$$h_d e^{i\sigma_d} = \frac{M_{12}^{\mathrm{NP}}}{M_{12}^{\mathrm{SM}}}. \tag{58}$$

The constraints in the $h_d - \sigma_d$ plane are shown in Fig. 5.

We conclude from Fig. 4 that **a new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude at a level higher than about 30% is now disfavored**. Consequently, a
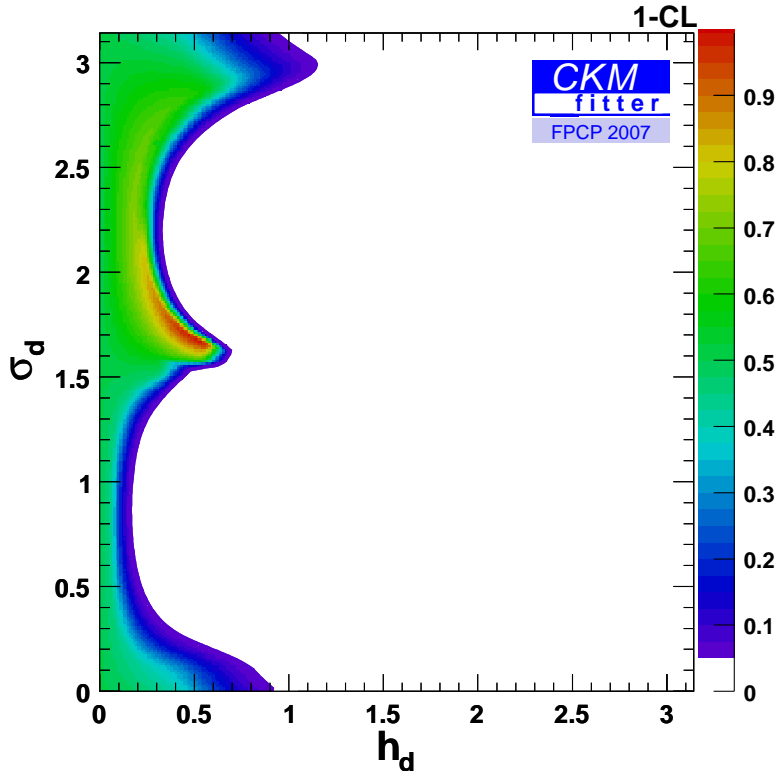
20

FIG. 5: The allowed region in the $h_d - \sigma_d$ plane, assuming that NP contributions to tree level processes are negligible [16].

long list of models that require a significant modification of the $B^0 - \overline{B}^0$ mixing amplitude are excluded.

VI. FLAVOR AT THE LHC

A. Top Physics

The LHC will study the physics of electroweak symmetry breaking. There are high hopes that it will discover not only the Higgs, but also shed light on the fine-tuning problem that is related to the Higgs mass.

The top quark plays a role in electroweak symmetry breaking. It poses the most severe fine tuning problem to the Higgs, as it gives the largest contribution to m_H^2:

$$-\frac{3}{8\pi^2} y_t^2 \Lambda_{\mathrm{NP}}^2 = -(2\ TeV)^2 \left(\frac{\Lambda_{\mathrm{NP}}}{10\ TeV}\right)^2. \tag{59}$$

It is, therefore, very likely that there exists a "top-partner" that couples to the Higgs and

cancels these quadratically divergent contributions to the Higgs mass-squared. The LHC can, in principle, measure the mass and the production cross section of the top partner and even shed light on the question of its spin [26]. Moreover, in some models – such as minimal SUGRA – it is the top quark that induces the electroweak symmetry breaking.

Given that the top quark is the only one that has an order one coupling to the Higgs, it plays a special role in various extensions of the SM. For example, in RS1 models, there is a strong enhancement of $t \to cZ$ decays (see *e.g.* [27]), and it is the only one that has a significant coupling to the Kaluza-Klein gluons (see *e.g.* [28]).

Thus, the LHC is likely to yield exciting top physics. Here, we will not focus on this issue but rather on how the LHC can shed light on the new physics flavor puzzle.

B. Minimal flavor violation (MFV)

A simple and rather generic principle that can guarantee that low energy flavor changing processes would deviate only very little from the SM predictions is that of *minimal flavor violation* (MFV) [29]. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, Y_U, Y_D and Y_E. If this remains true in the presence of the new physics, namely Y_U, Y_D and Y_E are the only flavor non-universal parameters, then the model belongs to the MFV class.

Let us now formulate this principle in a more formal way, using the language of spurions that we presented in section II B. The Standard Model with vanishing Yukawa couplings has a large global symmetry (13,14). In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks is $SU(3)_q^3$ of Eq. (14) with the three generations of quark fields transforming as follows:

$$Q_L(3,1,1), \quad U_R(1,3,1), \quad D_R(1,1,3). \tag{60}$$

The Yukawa interactions,

$$\mathcal{L}_Y = \overline{Q_L} Y_D D_R H + \overline{Q_L} Y_U U_R H_c, \tag{61}$$

($H_c = i\tau_2 H^*$) break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $SU(3)_q^3$ [see Eq. (16)]:

$$Y_U \sim (3, \bar{3}, 1), \qquad Y_D \sim (3, 1, \bar{3}). \tag{62}$$

22

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, Y_D and Y_U, must be (formally) invariant under the flavor group $SU(3)_q^3$. Of course, in reality, $\mathcal{L}_Y$ breaks $SU(3)_q^3$ precisely because $Y_{D,U}$ are *not* fields and do not transform under the symmetry.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM and Y fields, are formally invariant under G_{global}.

2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM, Y and the new fields, are formally invariant under G_{global}.

Exercise 8: Use the spurion formalism to argue that, in MFV models, the $K_L \to \pi^0 \nu \bar{\nu}$ decay amplitude is proportional to $y_t^2 V_{td} V_{ts}^*$.

Examples of MFV models include models of supersymmetry with gauge-mediation or with anomaly-mediation of its breaking. If the LHC discovers new particles that couple to the SM fermions, then it will be able to test solutions to the new physics flavor puzzle such as MFV [30]. Much of its power to test such frameworks is based on identifying top and bottom quarks.

To understand this statement, we notice that the spurions Y_U and Y_D can always be written in terms of the two diagonal Yukawa matrices λ_U and λ_D:

$$\lambda_U = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}, \quad \lambda_D = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}, \tag{63}$$

and the CKM matrix V. Thus, the only source of quark flavor changing transitions in MFV models is the CKM matrix. Next, note that to an accuracy that is better than a percent, we can write the CKM matrix as follows:

$$V = \begin{pmatrix} 1 & 0.23 & 0 \\ -0.23 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{64}$$

Exercise 9: The approximation (64) should be intuitively obvious to top-physicists, but definitely counter-intuitive to bottom-physicists. (Some of them have dedicated a large part

of their careers to experimental or theoretical efforts to determine V_{cb} and V_{ub}.) What does the approximation imply for the bottom quark? When we take into account that it is only good to $\mathcal{O}(0.05)$, what would the implications be?

We learn that the third generation of quarks is decoupled, to a good approximation, from the first two. This, in turn, means that any new particle that couples to the SM quarks (think, for example, of heavy quarks in vector-like representations of G_{SM}), decay into either third generation quark, or to non-third generation quark, but not to both. For example, in Ref. [30], MFV models with additional charge $-1/3$, $SU(2)_{\mathrm{L}}$-singlets quarks – B' – were considered. A concrete test of MFV was proposed, based on the fact that the largest mixing effect involving the third generation is of order $|V_{cb}|^2 \sim 0.002$: Is the followng prediction, concerning events of B' pair production, fulfilled:

$$\frac{\Gamma(B'\overline{B'} \to X q_{1,2} q_3)}{\Gamma(B'\overline{B'} \to X q_{1,2} q_{1,2}) + \Gamma(B'\overline{B'} \to X q_3 q_3)} \lesssim 10^{-3}. \tag{65}$$

If not, then MFV is excluded.

One can think of analogous tests in the supersymmetric framework [31]. Here, there is also a generic prediction that, in each sector (Q_L, U_R, D_R), squarks of the first two generations are quasi-degenerate, and do not decay into third generation quarks. Squarks of the third generation can be separated in mass (though, for small $\tan\beta$, the degeneracy in the $\tilde{D}_R$ sector is threefold), and decay only to third generation quarks.

We conclude that flavor physics have taught us much about the Standard Model and its extensions. Improvements in precision flavor measurements, as well as measurements at the LHC related to new particles that couple to the SM ones, are likely to teach us much more.

VII. NEUTRINO ANARCHY VERSUS QUARK HIERARCHY

A detailed presentation of the physics and the formalism of neutrino flavor transitions is given in Appendix D for both vacuum oscillations (D 1) and the matter transitions (D 2). It follows Ref. [32].

Exercise 10: For atmospheric ν_μ's with $E \sim 1 \; GeV$, the flux coming from above has $P_{\mu\mu}(L \sim 10^3 \; m) \approx 1$, while the flux from below has $P_{\mu\mu}(L \sim 10^6 \; m) \approx 0.5$. Assuming that for the flux coming from below the oscillations are averaged out, estimate Δm^2 and $\sin^2 2\theta$.

Exercise 11: For solar ν_e's, the transition between matter ($\beta_{\mathrm{MSW}} > 1$) and vacuum ($\beta_{\mathrm{MSW}} < \cos 2\theta$) flavor transitions occurs around $E \sim 2\ MeV$. The transition probability is measured to be roughly $P_{ee} \sim 0.30$ for $\beta_{\mathrm{MSW}} > 1$ and $P_{ee} \sim 0.55$ for $\beta_{\mathrm{MSW}} \ll 1$. Estimate Δm^2 and θ.

The derived ranges for the three mixing angles and two mass-squared differences at 1σ are [33]:

$$
\begin{aligned}
\Delta m_{21}^2 &= (7.9 \pm 0.3) \times 10^{-5}\ eV^2, \\
|\Delta m_{31}^2| &= (2.6 \pm 0.2) \times 10^{-3}\ eV^2, \\
\theta_{12} &= 33.7 \pm 1.3^o, \\
\theta_{23} &= 44 \pm 4^o, \\
\theta_{13} &= 0^{+5.2^o}_{-0.0}.
\end{aligned}
\tag{66}
$$

The 3σ range for the matrix elements of U are the following [33]:

$$
|U| = \begin{pmatrix} 0.79 \to 0.86 & 0.50 \to 0.61 & 0.00 \to 0.20 \\ 0.25 \to 0.53 & 0.47 \to 0.73 & 0.56 \to 0.79 \\ 0.21 \to 0.51 & 0.42 \to 0.69 & 0.61 \to 0.83 \end{pmatrix}.
\tag{67}
$$

A. New physics

The simplest and most straightforward lesson of the evidence for neutrino masses is also the most striking one: there is new physics beyond the Standard Model. This is the first experimental result that is inconsistent with the SM.

Most likely, the new physics is related to the existence of G_{SM}-singlet fermions at some high energy scale that induce, at low energies, the effective terms Eq. (25) through the seesaw mechanism. The existence of heavy singlet fermions is predicted by many extensions of the SM, especially by GUT's [beyond $SU(5)$] and left-right-symmetric theories.

There are of course other possibilities. Neutrino masses can be generated without introducing any new fermions beyond those of the SM. Instead, the existence of a scalar $\Delta_L(1,3)_{+1}$, that is, an $SU(2)_{\mathrm{L}}$-triplet, is required. The smallness of the neutrino masses is related here to the smallness of the vacuum expectation value $\langle \Delta_L^0 \rangle$ (required also by the success of the $\rho = 1$ relation) and does not have a generic natural explanation.

In left-right-symmetric models, however, where the breaking of $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ is induced by the VEV of an $SU(2)_R$-triplet, Δ_R, there must exist also an $SU(2)_L$-triplet scalar. Furthermore, the Higgs potential leads to an order of magnitude relation between the various VEVs, $\langle \Delta_L^0 \rangle \langle \Delta_R^0 \rangle \sim v^2$, and the smallness of $\langle \Delta_L^0 \rangle$ is correlated with the high scale of $SU(2)_R$ breaking. This situation can be though of as a seesaw of VEVs. In this model there are, however, also SM-singlet fermions. The light neutrino masses arise from both the seesaw mechanism ("type I") and the triplet VEV ("type II").

Neutrino masses could also be of the Dirac type. Here, again, singlet fermions are introduced, but lepton number is imposed by hand. This possibility is disfavored by theorists since it is likely that global symmetries are violated by gravitational effects. Furthermore, the lightness of the neutrinos (compared to charged fermions) is unexplained.

Another possibility is that neutrino masses are generated by mixing with singlet fermions but the mass scale of these fermions is not high. Here again the lightness of neutrino masses remains a puzzle. The best known example of such a scenario is the framework of supersymmetry without R parity.

Let us emphasize that the seesaw mechanism or, more generally, the extension of the SM with non-renormalizable terms, is the simplest explanation of neutrino masses. Models in which neutrino masses are generated by new physics at low energy imply a much more dramatic departure from the SM. Furthermore, the existence of seesaw masses is an unavoidable prediction of various extensions of the SM. In contrast, many (but not all) of the low energy mechanisms are introduced for the specific purpose of generating neutrino masses.

B. The scale of new physics

Eq. (25) gives a light neutrino mass matrix:

$$(M_\nu)_{ij} = \frac{Z_{ij}^\nu}{2} \frac{v^2}{\Lambda_{\mathrm{NP}}}.$$ (68)

It is straightforward to use the measured neutrino masses of Eq. (66) in combination with Eq. (68) to estimate the scale of new physics that is relevant to their generation. In particular, if there is no quasi-degeneracy in the neutrino masses, the heaviest of the active neutrino masses can be estimated:

$$m_h = m_3 \sim \sqrt{\Delta m_{31}^2} \approx 0.05 \ eV.$$ (69)

(In the case of inverted hierarchy, the implied scale is $m_h = m_2 \sim \sqrt{\Delta m_{31}^2} \approx 0.05 \ eV$.) It follows that the scale in the nonrenormalizable terms (25) is given by

$$\Lambda_{\rm NP} \sim v^2/m_h \approx 10^{15} \ GeV. \tag{70}$$

We should clarify two points regarding Eq. (70):

1. There could be some level of degeneracy between the neutrino masses. In such a case, Eq. (69) is modified into a lower bound and, consequently, Eq. (70) becomes an upper bound on the scale of new physics.

2. It could be that the Z_{ij} of Eq. (25) are much smaller than 1. In such a case, again, Eq. (70) becomes an upper bound on the scale of new physics.

On the other hand, in models of approximate flavor symmetries, there are relations between the structures of the charged lepton and neutrino mass matrices that give, quite generically, $Z_{33} \gtrsim m_\tau^2/v^2 \sim 10^{-4}$. We conclude that the likely range for $\Lambda_{\rm NP}$ is given by

$$10^{11} \ GeV \lesssim \Lambda_{\rm NP} \lesssim 10^{15} \ GeV. \tag{71}$$

The estimates (70) and (71) are very exciting. First, the upper bound on the scale of new physics is well below the Planck scale. This means that there is new physics in Nature which is intermediate between the two known scales, the Planck scale, $m_{\rm Pl} \sim 10^{19} \ GeV$, and the electroweak breaking scale, $v \sim 10^2 \ GeV$.

Second, the scale $\Lambda_{\rm NP} \sim 10^{15} \ GeV$ is intriguingly close to the scale of gauge coupling unification.

Third, the range (71) for the scale of lepton number breaking is optimal for leptogenesis.

C. The flavor puzzle

In the absence of neutrino masses, there are 13 flavor parameters in the SM:

$$y_t \sim 1, \quad y_c \sim 10^{-2}, \quad y_u \sim 10^{-5},$$
$$y_b \sim 10^{-2}, \quad y_s \sim 10^{-3}, \quad y_d \sim 10^{-4},$$
$$y_\tau \sim 10^{-2}, \quad y_\mu \sim 10^{-3}, \quad y_e \sim 10^{-6},$$
$$|V_{us}| \sim 0.2, \quad |V_{cb}| \sim 0.04, \quad |V_{ub}| \sim 0.004, \quad \sin\delta_{\rm KM} \sim 1. \tag{72}$$

These flavor parameters are hierarchical (their magnitudes span six orders of magnitude), and all but two or three (the top Yukawa, the CP violating phase, and perhaps the Cabibbo angle) are small. The unexplained smallness and hierarchy pose the SM *flavor puzzle*. Its solution may direct us to physics beyond the Standard Model.

Several mechanisms have been proposed in response to this puzzle. For example, approximate horizontal symmetries, broken by a small parameter, can lead to selection rules that explain the hierarchy of the Yukawa couplings.

In the extension of the SM with three active neutrinos that have Majorana masses, there are nine new flavor parameters in addition to those of Eq. (72). These are three neutrino masses, three lepton mixing angles, and three phases in the mixing matrix. Of the nine new parameters, four have been measured: two mass-squared differences and two mixing angles [see Eq. (66)]. This adds significantly to the input data on flavor physics and provides an opportunity to test and refine flavor models.

If neutrino masses arise from effective terms of the form of Eq. (25), then the overall scale of neutrino masses is related to the scale $\Lambda_{\rm NP}$ and, in most cases, does not tell us anything about flavor physics. More significant information for flavors models can be written in terms of three dimensionless parameters whose values can be read from Eq. (66):

$$
\begin{aligned}
|U_{\mu 3}U_{\tau 3}^*| &\sim 0.4 - 0.6, \\
|U_{e1}U_{e2}^*| &\sim 0.4 - 0.5, \\
\Delta m_{21}^2/|\Delta m_{31}^2| &\sim 0.027 - 0.034.
\end{aligned}
\tag{73}
$$

In addition, the upper bound on θ_{13} often plays a significant role in flavor model building:

$$
|U_{e3}| \lesssim 0.20 \ (3\sigma).
\tag{74}
$$

There are several features in the numerical estimates (73,74) that have drawn much attention and have driven numerous investigations:

(i) Large mixing and strong hierarchy: The mixing angle that is relevant to the $2-3$ sector is large, $|U_{\mu 3}| \sim 0.7$. On the other hand, if there is no quasi-degeneracy in the neutrino masses, the corresponding mass ratio is small, $m_2/m_3 \sim 0.17$. It is difficult to explain in a natural way a situation where there is an $\mathcal{O}(1)$ mixing but the corresponding masses are hierarchical.

(ii) Two large and one small mixing angles: The mixing angles relevant to the $2-3$ sector ($|U_{\mu 3}| \sim 0.7$) and $1-2$ sector ($|U_{e2}| \sim 0.55$) are large, yet the $1-3$ mixing angle is small ($|U_{e3}| \lesssim 0.20$). Such a situation is, again, difficult – though not impossible – to explain from approximate symmetries. An example of a symmetry that does predict such a pattern is that of $L_e - L_\mu - L_\tau$. This symmetry predicts, however, $\theta_{12} \simeq \pi/4$, which is experimentally excluded.

(iii) Maximal mixing: The value of θ_{23} is intriguingly close to maximal mixing ($\sin^2 2\theta_{23} = 1$). It is interesting to understand whether a symmetry could explain this special value.

(iv) Tribimaximal mixing: The mixing matrix (67) has a structure that is consistent with the following unitary matrix [34]:

$$
U = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}
\end{pmatrix}.
\tag{75}
$$

It is interesting to understand whether a symmetry could explain this special structure.

All four features enumerated above are difficult to explain in a large class of flavor models that do very well in explaining the flavor features of the quark sector. In particular, models with Abelian horizontal symmetries (Froggatt-Nielsen type [35]) predict that, in general, $|V_{ub}| \sim |V_{us} V_{cb}|$, $|V_{ij}| \gtrsim m_i/m_j$ ($i < j$) and $V \sim \mathbf{1}$ [12, 36]. All of these are successful predictions. At the same time, however, these models predict [37] that for the neutrinos, in general, $|U_{ij}|^2 \sim m_i/m_j$ and $|U_{e3}| \sim |U_{e2} U_{\mu 3}|$, in contradiction to, respectively, points (i) and (ii) above (and there is no way to make θ_{23} parametrically close to $\pi/4$). On the other hand, there exist very specific models where these features are related to a symmetry.

It is possible, however, that the above interpretation of the results is wrong. Indeed, the data can be interpreted in a very different way:

(iv) No small parameters. The two measured mixing angles are larger than any of the quark mixing angles. Indeed, they are both of order one. The measured mass ratio, $m_2/m_3 \gtrsim 0.16$ is larger than any of the quark and charged lepton mass ratios, and could be interpreted as an $\mathcal{O}(1)$ parameter (namely, it is accidentally small, without any parametric suppression). If this is the correct way of reading the data, the measured neutrino parameters may actually reflect the absence of any hierarchical structure in the neutrino mass matrices [38]. The possibility that there is no structure – neither hierarchy, nor degeneracy – in the neutrino sector has been called "neutrino mass anarchy". An important test of this idea will be

provided by the measurement of $|U_{e3}|$. If indeed the entries in M_ν have random values of the same order, all three mixing angles are expected to be of order one. If experiments measure $|U_{e3}| \sim 0.1$, that is, close to the present bound, it can be argued that its smallness is accidental. The stronger the upper bound on this angle becomes, the more difficult it will be to maintain this view.

Neutrino mass anarchy can be accommodated within models of Abelian flavor symmetries, if the three lepton doublets carry the same charge. Indeed, consider a supersymmetric model with a $U(1)_H$ symmetry that is broken by a single small spurion ϵ_H of charge -1. Let us assume that the three fermion generations contained in the 10-representation of $SU(5)$ carry charges $(2, 1, 0)$, while the three $\bar{5}$-representations carry charges $(0, 0, 0)$. (The Higgs fields carry no H charges.) Such a model predicts ϵ_H^2 hierarchy in the up sector, ϵ_H hierarchy in the down and charged lepton sectors, and anarchy in the neutrino sector.

Exercise 12: The selection rule for this model is that a term in the superpotential that carries H charge $n \geq 0$ is suppressed by ϵ_H^n. Find the parametric suppression of the various entries in M_u, M_d, M_ℓ and M_ν. Find the parametric suppression of the mixing angles.

It would be nice if the features of quark mass hierarchy and neutrino mass anarchy can be traced back to some fundamental principle or to a stringy origin (see, for example, [39]).

VIII. CONCLUSIONS

We have described four topics in flavor physics, each demonstrating a different point of interest:

(i) The upper bound on Δm_D shows that alignment cannot be the only flavor mechanism that suppresses the supersymmetric flavor changing contributions. It demonstrates how flavor physics at the GeV scale probes new physics at the TeV scale.

(ii) The measurement of $S_{\psi K}$ provides a precision test of the Kobayashi-Maskawa mechanism of CP violation. It strengthens the evidence that this is the dominant source of CP violation in flavor changing processes.

(iii) The LHC may discover new particles that couple to the standard model fermions. If that happens, we will be able to use the new physics for better understanding of the flavor puzzle, and the flavor physics for better understanding of the new physics.

(iv) The measurements of neutrino flavor parameters – mass-squared differences and

mixing angles – have tested models that aim to explain the hierarchy in the quark sector, and have added novel aspects to the question of whether the flavor structure has a symmetry-related explanation.

The huge progress in flavor physics in recent years has provided answers to many questions. At the same time, new questions arise. We look forward to the LHC era for more answers and more questions.

APPENDIX A: THE CKM MATRIX

The CKM matrix V is a 3×3 unitary matrix. Its form, however, is not unique:

(i) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, $i.e.$ $(u_1, u_2, u_3) \rightarrow (u, c, t)$ and $(d_1, d_2, d_3) \rightarrow (d, s, b)$. The elements of V are written as follows:

$$
V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{A1}
$$

(ii) There is further freedom in the phase structure of V. This means that the number of physical parameters in V is smaller than the number of parameters in a general unitary 3×3 matrix which is nine (three real angles and six phases). Let us define P_q ($q = u, d$) to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (22) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since M_q^{diag} remains unchanged by such transformations. However, V does change:

$$
V \rightarrow P_u V P_d^*. \tag{A2}
$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase δ_{KM} which is the single source of CP violation in the quark sector of the Standard Model [1].

The fact that V is unitary and depends on only four independent physical parameters

can be made manifest by choosing a specific parametrization. The standard choice is [40]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{A3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The θ_{ij}'s are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$. It is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter and η representing the CP violating phase [41, 42]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{A4}$$

A very useful concept is that of the *unitarity triangles*. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{A5}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{A6}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{A7}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (A7) only. The unitarity triangle related to Eq. (A7) is depicted in Fig. 6.

The rescaled unitarity triangle is derived from (A7) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left| \frac{V_{ud}V_{ub}}{V_{cd}V_{cb}} \right| = \sqrt{\rho^2 + \eta^2}, \quad R_t \equiv \left| \frac{V_{td}V_{tb}}{V_{cd}V_{cb}} \right| = \sqrt{(1-\rho)^2 + \eta^2}. \tag{A8}$$
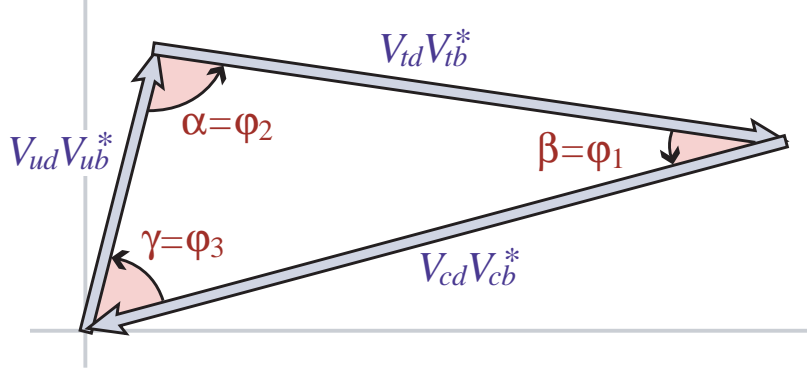
FIG. 6: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

The three angles of the unitarity triangle are defined as follows [43, 44]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{A9}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (A6,A5):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{A10}$$

The λ and A parameters are very well determined at present:

$$\lambda = 0.226 \pm 0.001, \quad A = 0.818 \pm 0.015. \tag{A11}$$

The main effort in CKM measurements is thus aimed at improving our knowledge of ρ and η:

$$\rho = 0.14^{+0.04}_{-0.02}, \quad \eta = 0.35 \pm 0.02. \tag{A12}$$

The present status of our knowledge is best seen in a plot of the various constraints and the final allowed region in the $\rho - \eta$ plane. This is shown in Fig. 7.

APPENDIX B: SUPERSYMMETRIC CONTRIBUTIONS TO NEUTRAL ME-SON MIXING

We consider the squark-gluino box diagram contribution to $D^0 - \overline{D}^0$ mixing amplitude that is proportional to $K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}$, where K^u is the mixing matrix of the gluino couplings
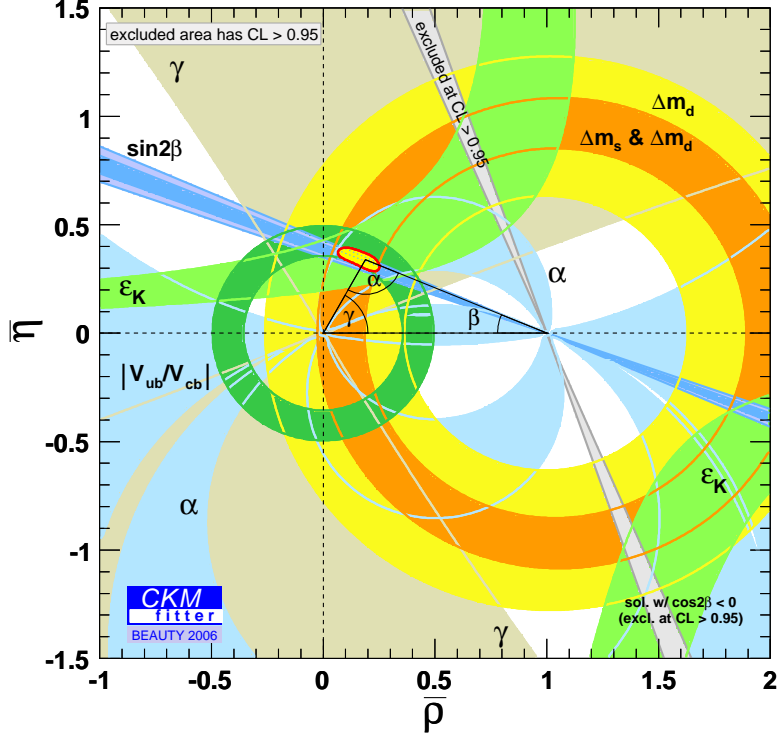
FIG. 7: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}/V_{cb}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [16].

to a left-handed up quark and their supersymmetric squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta^u_{LL})_{12}]^2$.) We work in the mass basis for both quarks and squarks.

The contribution is given by

$$M_{12}^D = -i\frac{4\pi^2}{27}\alpha_s^2 m_D f_D^2 B_D \eta_{\text{QCD}} \sum_{i,j}(K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*})(11\tilde{I}_{4ij} + 4\tilde{m}_g^2 I_{4ij}). \tag{B1}$$

where

$$
\begin{aligned}
\tilde{I}_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2 - \tilde{m}_g^2)^2(p^2 - \tilde{m}_i^2)(p^2 - \tilde{m}_j^2)} \\
&= \frac{i}{(4\pi)^2}\left[\frac{\tilde{m}_g^2}{(\tilde{m}_i^2 - \tilde{m}_g^2)(\tilde{m}_j^2 - \tilde{m}_g^2)} \right. \\
&\left. + \frac{\tilde{m}_i^4}{(\tilde{m}_i^2 - \tilde{m}_j^2)(\tilde{m}_i^2 - \tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^4}{(\tilde{m}_j^2 - \tilde{m}_i^2)(\tilde{m}_j^2 - \tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right],
\end{aligned}
\tag{B2}
$$

34

$$I_{4ij} \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - \tilde{m}_g^2)^2(p^2 - \tilde{m}_i^2)(p^2 - \tilde{m}_j^2)}$$

$$= \frac{i}{(4\pi)^2} \left[\frac{1}{(\tilde{m}_i^2 - \tilde{m}_g^2)(\tilde{m}_j^2 - \tilde{m}_g^2)} \right.$$

$$\left. + \frac{\tilde{m}_i^2}{(\tilde{m}_i^2 - \tilde{m}_j^2)(\tilde{m}_i^2 - \tilde{m}_g^2)^2} \ln \frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^2}{(\tilde{m}_j^2 - \tilde{m}_i^2)(\tilde{m}_j^2 - \tilde{m}_g^2)^2} \ln \frac{\tilde{m}_j^2}{\tilde{m}_g^2} \right]. \quad (B3)$$

We now follow the discussion in refs. [9, 10]. To see the consequences of the super-GIM mechanism, let us expand the expression for the box integral around some value $\tilde{m}_q^2$ for the squark masses-squared:

$$I_4(\tilde{m}_g^2, \tilde{m}_i^2, \tilde{m}_j^2) = I_4(\tilde{m}_g^2, \tilde{m}_q^2 + \delta\tilde{m}_i^2, \tilde{m}_q^2 + \delta\tilde{m}_j^2)$$

$$= I_4(\tilde{m}_g^2, \tilde{m}_q^2, \tilde{m}_q^2) + (\delta\tilde{m}_i^2 + \delta\tilde{m}_j^2) I_5(\tilde{m}_g^2, \tilde{m}_q^2, \tilde{m}_q^2, \tilde{m}_q^2)$$

$$\frac{1}{2} \left[(\delta\tilde{m}_i^2)^2 + (\delta\tilde{m}_j^2)^2 + 2(\delta\tilde{m}_i^2)(\delta\tilde{m}_j^2) \right] I_6(\tilde{m}_g^2, \tilde{m}_q^2, \tilde{m}_q^2, \tilde{m}_q^2, \tilde{m}_q^2) + \cdots \text{(B4)}$$

where

$$I_n(\tilde{m}_g^2, \tilde{m}_q^2, \ldots, \tilde{m}_q^2) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - \tilde{m}_g^2)^2(p^2 - \tilde{m}_q^2)^{n-2}}, \quad (B5)$$

and similarly for $\tilde{I}_{4ij}$. Note that $I_n \propto (\tilde{m}_q^2)^{n-2}$ and $\tilde{I}_n \propto (\tilde{m}_q^2)^{n-3}$. Thus, using $x \equiv \tilde{m}_g^2/\tilde{m}_q^2$, it is customary to define

$$I_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-2}} f_n(x), \quad \tilde{I}_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-3}} \tilde{f}_n(x). \quad (B6)$$

The unitarity of the mixing matrix implies that

$$\sum_i (K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}) = \sum_j (K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}) = 0. \quad (B7)$$

We learn that the terms that are proportional f_4, $\tilde{f}_4$, f_5 and $\tilde{f}_5$ vanish in their contribution to M_{12}. When $\delta\tilde{m}_i^2 \ll \tilde{m}_q^2$ for all i, the leading contributions to M_{12} come from f_6 and $\tilde{f}_6$. We learn that for quasi-degenerate squarks, the leading contribution is quadratic in the small mass-squared difference. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are given by

$$f_6(x) = \frac{6(1 + 3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(1 - x)^5},$$

$$\tilde{f}_6(x) = \frac{6x(1 + x)\ln x - x^3 - 9x^2 + 9x + 1}{3(1 - x)^5}. \quad (B8)$$

For example, with $x = 1$, $f_6(1) = -1/20$ and $\tilde{f}_6 = +1/30$; with $x = 2.33$, $f_6(2.33) = -0.015$ and $\tilde{f}_6 = +0.013$.

To further simplify things, let us consider a two generation case. Then

$$
\begin{aligned}
M_{12}^D &\propto 2(K_{21}^u K_{11}^{u*})^2 (\delta \tilde{m}_1^2)^2 + 2(K_{22}^u K_{12}^{u*})^2 (\delta \tilde{m}_2^2)^2 + (K_{21}^u K_{11}^{u*} K_{22}^u K_{12}^{u*})(\delta \tilde{m}_1^2 + \delta \tilde{m}_2^2)^2 \\
&= (K_{21}^u K_{11}^{u*})^2 (\tilde{m}_2^2 - \tilde{m}_1^2)^2 .
\end{aligned}
\tag{B9}
$$

We thus rewrite Eq. (B1) for the case of quasi-degenerate squarks:

$$
M_{12}^D = \frac{\alpha_s^2 m_D f_D^2 B_D \eta_{\text{QCD}}}{108 \tilde{m}_q^2} [11 \tilde{f}_6(x) + 4x f_6(x)] \frac{(\Delta \tilde{m}_{21}^2)^2}{\tilde{m}_q^4} (K_{21}^u K_{11}^{u*})^2 .
\tag{B10}
$$

For example, for $x = 1$, $11\tilde{f}_6(x) + 4x f_6(x) = +0.17$. For $x = 2.33$, $11\tilde{f}_6(x) + 4x f_6(x) = +0.003$.

APPENDIX C: CP VIOLATION IN NEUTRAL B DECAYS TO FINAL CP EIGENSTATES

We define decay amplitudes of B (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$
A_f = \langle f | \mathcal{H} | B \rangle \quad , \quad \overline{A}_f = \langle f | \mathcal{H} | \overline{B} \rangle \quad , \quad A_{\overline{f}} = \langle \overline{f} | \mathcal{H} | B \rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f} | \mathcal{H} | \overline{B} \rangle ,
\tag{C1}
$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_B and ξ_f according to

$$
\begin{aligned}
CP\,|B\rangle &= e^{+i\xi_B} |\overline{B}\rangle \quad , \quad CP\,|f\rangle = e^{+i\xi_f} |\overline{f}\rangle , \\
CP\,|\overline{B}\rangle &= e^{-i\xi_B} |B\rangle \quad , \quad CP\,|\overline{f}\rangle = e^{-i\xi_f} |f\rangle ,
\end{aligned}
\tag{C2}
$$

so that $(CP\,)^2 = 1$. The phases ξ_B and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\,,\mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$
\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)} A_f .
\tag{C3}
$$

A state that is initially a superposition of B^0 and $\overline{B}^0$, say

$$
|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle ,
\tag{C4}
$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$
|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots .
\tag{C5}
$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [45]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\Gamma . \tag{C6}$$

M and Γ are associated with $(B^0, \overline{B}^0) \leftrightarrow (B^0, \overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters $p_{L,H}$ and $q_{L,H}$ to specify the components of the strong interaction eigenstates, B^0 and $\overline{B}^0$, in the light (B_L) and heavy (B_H) mass eigenstates:

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\overline{B}^0\rangle \tag{C7}$$

with the normalization $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$. If either CP or CPT is a symmetry of $\mathcal{H}$ (independently of whether T is conserved or violated) then $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, and solving the eigenvalue problem for $\mathcal{H}$ yields $p_L = p_H \equiv p$ and $q_L = q_H \equiv q$ with

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}} . \tag{C8}$$

From now on we assume that CPT is conserved. If either CP or T is a symmetry of $\mathcal{H}$ (independently of whether CPT is conserved or violated), then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 , \tag{C9}$$

where ξ_B is the arbitrary unphysical phase introduced in Eq. (C2).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L. \tag{C10}$$

Note that here Δm_B is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H + M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{C11}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{C12}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \quad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{C13}$$

All CP-violating observables in B and $\overline{B}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

For neutral D, B, and B_s mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time t as $|B^0{}_{\text{phys}}(t)\rangle$ or $|\overline{B}^0_{\text{phys}}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$
\begin{aligned}
|B^0{}_{\text{phys}}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\,g_-(t)|\overline{B}^0\rangle, \\
|\overline{B}^0_{\text{phys}}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\,g_-(t)|B^0\rangle ,
\end{aligned}
\tag{C14}
$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{C15}$$

One obtains the following time-dependent decay rates:

$$
\begin{aligned}
\frac{d\Gamma[B^0{}_{\text{phys}}(t) \to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) \\
&\quad + 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t) , \tag{C16} \\
\frac{d\Gamma[\overline{B}^0_{\text{phys}}(t) \to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) \\
&\quad + 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t) , \tag{C17}
\end{aligned}
$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (C16,C17). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B \leftrightarrow \overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (C16,C17) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

One possible manifestation of CP-violating effects in meson decays [46] is in the interference between a decay without mixing, $B^0 \to f$, and a decay with mixing, $B^0 \to \overline{B}^0 \to f$ (such an effect occurs only in decays to final states that are common to B^0 and $\overline{B}^0$, including all CP eigenstates). It is defined by

$$\mathcal{I}m(\lambda_f) \neq 0 \ , \tag{C18}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \ . \tag{C19}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\mathrm{phys}}(t) \to f_{CP}] - d\Gamma/dt[B^0_{\mathrm{phys}}(t) \to f_{CP}]}{d\Gamma/dt[\overline{B}^0_{\mathrm{phys}}(t) \to f_{CP}] + d\Gamma/dt[B^0_{\mathrm{phys}}(t) \to f_{CP}]} \ . \tag{C20}$$

For $\Delta\Gamma = 0$ and $|q/p| = 1$ (which is a good approximation for B mesons), $\mathcal{A}_{f_{CP}}$ has a particularly simple form [47–49]:

$$\begin{aligned} \mathcal{A}_f(t) &= S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \\ S_f &\equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \ , \end{aligned} \tag{C21}$$

Consider the $B \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{B} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^{\pm}$ bosons and hence are often called "weak phases". The weak

phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in A_f is convention independent. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (C3). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$
\begin{aligned}
A_f &= |a_1| e^{i(\delta_1 + \phi_1)} + |a_2| e^{i(\delta_2 + \phi_2)}, \\
\overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1 - \phi_1)} + |a_2| e^{i(\delta_2 - \phi_2)}.
\end{aligned}
\tag{C22}
$$

Similarly, for neutral meson decays, it is useful to write

$$
M_{12} = |M_{12}| e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}| e^{i\phi_\Gamma} .
\tag{C23}
$$

Each of the phases appearing in Eqs. (C22,C23) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f + \phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays to a final CP eigenstate f [Eq. (C20)] with eigenvalue $\eta_f = \pm 1$ are given by

$$
\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f) \sin(\Delta m t) \quad \text{with} \quad \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f).
\tag{C24}
$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

APPENDIX D: NEUTRINO FLAVOR TRANSITIONS

1.   Neutrinos in vacuum

Neutrino oscillations in vacuum [50] arise since neutrinos are massive and mix. In other words, the neutrino state that is produced by electroweak interactions is not a mass eigenstate. The weak eigenstates ν_α ($\alpha = e, \mu, \tau$ denotes the charged lepton mass eigenstates and their neutrino doublet-partners) are linear combinations of the mass eigenstates ν_i ($i = 1, 2, 3$):

$$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle. \tag{D1}$$

After traveling a distance L (or, equivalently for relativistic neutrinos, time t), a neutrino originally produced with a flavor α evolves as follows:

$$|\nu_\alpha(t)\rangle = U^*_{\alpha i}|\nu_i(t)\rangle. \tag{D2}$$

It can be detected in the charged-current interaction $\nu_\alpha(t)N' \to \ell_\beta N$ with a probability

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu_\alpha(t)\rangle|^2 = \left| \sum_{i=1}^{3}\sum_{j=1}^{3} U^*_{\alpha i} U_{\beta j}\langle \nu_j(0)|\nu_i(t)\rangle \right|^2. \tag{D3}$$

We follow the analysis of ref. [32]. We use the standard approximation that $|\nu\rangle$ is a plane wave (for a pedagogical discussion of the possible quantum mechanical problems in this naive description of neutrino oscillations we refer the reader to [51, 52]), $|\nu_i(t)\rangle = e^{-iE_it}|\nu_i(0)\rangle$. In all cases of interest to us, the neutrinos are relativistic:

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}, \tag{D4}$$

where E_i and m_i are, respectively, the energy and the mass of the neutrino mass eigenstate. Furthermore, we can assume that $p_i \simeq p_j \equiv p \simeq E$. Then, we obtain the following transition probability:

$$P_{\alpha\beta} = \delta_{\alpha\beta} = 4\sum_{i=1}^{2}\sum_{j=i+1}^{3} \mathcal{R}e\left(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right)\sin^2 x_{ij}, \tag{D5}$$

where $x_{ij} \equiv \Delta m_{ij}^2 L/(4E)$, $\Delta m_{ij}^2 = m_i^2 - m_j^2$ and $L = t$ is the distance between the source (that is, the production point of ν_α) and the detector (that is, the detection point of ν_β). In deriving Eq. (D5) we used the orthogonality relation $\langle \nu_j(0)|\nu_i(0)\rangle = \delta_{ij}$. It is convenient to use the following units:

$$x_{ij} = 1.27\,\frac{\Delta m_{ij}^2}{eV^2}\,\frac{L/E}{m/MeV}. \tag{D6}$$

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TABLE I: Characteristic values of L and E for various neutrino sources and experiments.

Experiment	L (m)	E (MeV)	Δm^2 (eV^2)
Solar	10^{10}	1	10^{-10}
Atmospheric	$10^4 - 10^7$	$10^2 - 10^5$	$10^{-1} - 10^{-4}$
Reactor	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
Kamland	10^5	1	10^{-5}
Accelerator	10^2	$10^3 - 10^4$	$\gtrsim 10^{-1}$
Long-baseline Accelerator	$10^5 - 10^6$	10^4	$10^{-2} - 10^{-3}$

The transition probability [Eq. (D5)] has an oscillatory behavior, with oscillation lengths

$$L_{0,ij}^{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2} \tag{D7}$$

and amplitude that is proportional to elements of the mixing matrix. Thus, in order to have oscillations, neutrinos must have different masses ($\Delta m_{ij}^2 \neq 0$) and they must mix ($U_{\alpha i} U_{\beta i} \neq 0$).

An experiment is characterized by the typical neutrino energy E and by the source-detector distance L. In order to be sensitive to a given value of Δm_{ij}^2, the experiment has to be set up with $E/L \approx \Delta m_{ij}^2$ ($L \sim L_{0,ij}^{\text{osc}}$). The typical values of L/E for different types of neutrino sources and experiments are summarized in Table I.

If $(E/L) \gg \Delta m_{ij}^2$ ($L \ll L_{0,ij}^{\text{osc}}$), the oscillation does not have time to give an appreciable effect because $\sin^2 x_{ij} \ll 1$. The case of $(E/L) \ll \Delta m_{ij}^2$ ($L \gg L_{0,ij}^{\text{osc}}$) requires more careful consideration. One must take into account that, in general, neutrino beams are not monochromatic. Thus, rather than measuring $P_{\alpha\beta}$, the experiments are sensitive to the average probability

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} - 4 \sum_{i=1}^{2} \sum_{j=i+1}^{3} \mathcal{R}e \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \langle \sin^2 x_{ij} \rangle. \tag{D8}$$

For $L \gg L_{0,ij}^{\text{osc}}$, the oscillation phase goes through many cycles before the detection and is averaged to $\langle \sin^2 x_{ij} \rangle = 1/2$.

For a two neutrino case,

$$P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 x. \tag{D9}$$

2. Neutrinos in matter

a. The effective potential

When neutrinos propagate in dense matter, the interactions with the medium affect their properties. These effects are either coherent or incoherent. For purely incoherent $\nu - p$ scattering, the characteristic cross section is very small,

$$\sigma \sim \frac{G_F^2 s}{\pi} \sim 10^{-43} \ \text{cm}^2 \left(\frac{E}{1 \ MeV}\right)^2 . \tag{D10}$$

The smallness of this cross section is demonstrated by the fact that if a beam of 10^{10} neutrinos with $E \sim 1 \ MeV$ was aimed at Earth, only one would be deflected by the Earth's matter. It may seem then that for neutrinos matter is irrelevant. However, one must take into account that Eq. (D10) does not contain the contribution from forward elastic coherent interactions. In coherent interactions, the medium remains unchanged and it is possible to have interference of scattered and unscattered neutrino waves which enhances the effect. Coherence further allows one to decouple the evolution equation of neutrinos from the equations of the medium. In this approximation, the effect of the medium is described by an effective potential which depends on the density and composition of the matter [53].

Consider, for example, the evolution of ν_e in a medium with electrons. The effective low-energy Hamiltonian describing the relevant neutrino interactions is given by

$$H_W = \frac{G_F}{\sqrt{2}} \left[\overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x)\right] \times \left[\overline{e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x)\right] . \tag{D11}$$

The effective charged-current Hamiltonian due to the electrons in the medium is

$$
\begin{aligned}
H_C^{(e)} &= \frac{G_F}{\sqrt{2}} \int d^3 p_e f(E_e, T) \langle\langle e(s, p_e)|\overline{e}(x)\gamma^\alpha(1-\gamma_5)\nu_e(x)\overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x)|e(s, p_e)\rangle\rangle \\
&= \frac{G_F}{\sqrt{2}} \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x) \int d^3 p_e f(E_e, T) \langle\langle e(s, p_e)|\overline{e}(x)\gamma^\alpha(1-\gamma_5)e(x)|e(s, p_e)\rangle\rangle,
\end{aligned}
$$

where s is the electron spin and p_e its momentum. Coherence implies that s, p_e are the same for the initial and final electrons.

Expanding the electron fields $e(x)$ in plane waves and using $a_s^\dagger(p_e)a_s(p_e) = N_e^{(s)}(p_e)$ (the number operator), we obtain

$$
\begin{aligned}
\langle\langle e(s, p_e) \ | \ \overline{e}(x)\gamma^\alpha(1-\gamma_5)e(x)|e(s, p_e)\rangle\rangle &= N_e(p_e)\frac{1}{2}\sum_s \overline{u_{(s)}}(p_e)\gamma_\alpha(1-\gamma_5)u_{(s)}(p_e) \\
&= \frac{N_e(p_e)}{2}\text{Tr}\left[\frac{m_e + \not{p}}{2E_e}\gamma_\alpha(1-\gamma_5)\right] = N_e(p_e)\frac{p_e^\alpha}{E_e}. \tag{D12}
\end{aligned}
$$

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Isotropy implies that $\int d^3 p_e \vec{p}_e f(E_e, T) = 0$. Thus only the p^0 term contributes upon integration, with $\int d^3 p_e f(E_e, T) N_e(p_e) = N_e$ (the electron number density). We obtain:

$$H_C^{(e)} = \frac{G_F N_e}{\sqrt{2}} \overline{\nu_e}(x) \gamma_0 (1 - \gamma_5) \nu_e(x). \tag{D13}$$

The effective potential for ν_e induced by its charged-current interactions with electrons in matter is then given by

$$V_C = \langle \nu_e | \int d^3 x H_C^{(e)} | \nu_e \rangle = \frac{G_F N_e}{\sqrt{2}} = \sqrt{2} G_F N_e. \tag{D14}$$

For $\overline{\nu_e}$ the sign of V is reversed. The potential can also be expressed in terms of the matter density ρ:

$$V_C = 7.6 \frac{N_e}{N_p + N_n} \frac{\rho}{10^{14} \text{ g/cm}^3} \text{ eV}. \tag{D15}$$

Two examples that are relevant to observations are the following:

- At the Earth's core $\rho \sim 10$ g/cm^3 and $V \sim 10^{-13}$ eV;

- At the solar core $\rho \sim 100$ g/cm^3 and $V \sim 10^{-12}$ eV.

b.   Evolution equation

Consider a state that is an admixture of two neutrino species, $|\nu_e\rangle$ and $|\nu_a\rangle$ or, equivalently, $|\nu_1\rangle$ and $|\nu_2\rangle$:

$$
\begin{aligned}
|\Phi(x) &= \Phi_e(x)|\nu_e\rangle + \Phi_a(x)|\nu_a\rangle \\
&= \Phi_1(x)|\nu_1\rangle + \Phi_2(x)|\nu_2\rangle.
\end{aligned}
\tag{D16}
$$

The evolution of Φ in a medium is described by a system of coupled Dirac equations:

$$
\begin{aligned}
E\Phi_1 &= \left(\frac{\hbar}{i} \gamma_0 \gamma_1 \frac{\partial}{\partial x} + \gamma_0 m_1 + V_{11} \right) \Phi_1 + V_{12}\Phi_2, \\
E\Phi_2 &= \left(\frac{\hbar}{i} \gamma_0 \gamma_1 \frac{\partial}{\partial x} + \gamma_0 m_2 + V_{22} \right) \Phi_2 + V_{12}\Phi_1.
\end{aligned}
\tag{D17}
$$

The V_{ij} terms give the effective potential for neutrino mass eigenstates. They can be simply derived from the effective potential for interaction eigenstates [such as V_{ee} of Eq. (D14)]:

$$V_{ij} = \langle \nu_i | \int d^3 x H_{\text{int}}^{\text{medium}} | \nu_j \rangle = U_{i\alpha} V_{\alpha\alpha} U_{j\alpha}^*. \tag{D18}$$

We decompose the neutrino state: $\Phi_i(x) = C_i(x)\phi_i(x)$, where $\phi_i(x)$ is the Dirac spinor part satisfying

$$(\gamma_0\gamma_1\{[E - V_{ii}(x)]^2 - m_i^2\}^{1/2} + \gamma_0 m_i + V_{ii}) = E\phi_i(x). \tag{D19}$$

We make the following approximations:

1. The scale over which V changes is much larger than the microscopic wavelength of the neutrino, $(\partial V/\partial x)V \ll \hbar m/E^2$.

2. Expanding to first order in V implies that $V_{12}\gamma_0\gamma_1\phi_2 \simeq \phi_1$, $V_{12}\gamma_0\gamma_1\phi_1 \simeq \phi_2$, and $\{[E - V_{ii}(x)]^2 - m_i^2\}^{1/2} \simeq E - V_{ii}(x) - m_i^2/2E$.

From 1 we find that the Dirac equations take the form

$$EC_1\phi_1 = \frac{\hbar}{i}\gamma_0\gamma_1\frac{\partial C_1}{\partial x}\phi_1 + (\gamma_0 m_1 + V_{11})C_1\phi_1 + V_{12}C_2\phi_2,$$
$$EC_2\phi_2 = \frac{\hbar}{i}\gamma_0\gamma_1\frac{\partial C_2}{\partial x}\phi_2 + (\gamma_0 m_2 + V_{22})C_2\phi_2 + V_{12}C_1\phi_1. \tag{D20}$$

Then multiplying by $\gamma_0\gamma_1$ and using the equation of motion of ϕ and 2, we can drop the dependence on the spinor ϕ and obtain

$$\frac{\hbar}{i}\frac{\partial C_1}{\partial x} = \left(E - V_{11}(x) - \frac{m_1^2}{2E}\right)C_1 - V_{12}C_2,$$
$$\frac{\hbar}{i}\frac{\partial C_2}{\partial x} = \left(E - V_{22}(x) - \frac{m_2^2}{2E}\right)C_2 - V_{12}C_1. \tag{D21}$$

Changing notations $C_{i,\alpha}(x) \to \nu_{i,\alpha}(x)$ (and $\hbar = 1$), removing the diagonal piece that is proportional to E, and rotating to the flavor basis, we can rewrite Eq. (D21) in matrix form [53]:

$$-i\frac{\partial}{\partial x}\begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = -\frac{1}{2E}M_w^2\begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}, \tag{D22}$$

where we have defined an effective mass matrix in matter,

$$M_w^2 = \frac{1}{2}\begin{pmatrix} m_1^2 + m_2^2 + 4EV_e - \Delta m^2\cos 2\theta & \Delta m^2\sin 2\theta \\ \Delta m^2\sin 2\theta & m_1^2 + m_2^2 + 4EV_a + \Delta m^2\cos 2\theta \end{pmatrix}, \tag{D23}$$

with $\Delta m^2 = m_2^2 - m_1^2$.

We define the instantaneous mass eigenstates in matter, ν_i^m, as the eigenstates of M_w for a fixed value of x. They are related to the interaction eigenstates by a unitary transformation,

$$\begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = U(\theta_m)\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}. \tag{D24}$$

The eigenvalues of M_w, that is, the effective masses in matter, are given by [53, 54]

$$\mu_{1,2}^2 = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_a) \mp \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}, \tag{D25}$$

while the mixing angle in matter is given by

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}, \tag{D26}$$

where

$$A \equiv 2E(V_e - V_a). \tag{D27}$$

The instantaneous mass eigenstates ν_i^m are, in general, not energy eigenstates: they mix in the evolution. The importance of this effect is controlled by the relative size of $4E\dot{\theta}_m(t)$ with respect to $\mu_2^2(t) - \mu_1^2(t)$. When the latter is much larger than the first, ν_i^m behave approximately as energy eigenstates and do not mix during the evolution. This is the adiabatic transition approximation. The adiabaticity condition reads

$$\mu_2^2(t) - \mu_1^2(t) \gg 2EA\Delta m^2 \sin 2\theta \left| \dot{A}/A \right|. \tag{D28}$$

The transition probability for the adiabatic case is given by

$$P_{ee}(t) = \left| \sum_i U_{ei}(\theta) U_{ei}^*(\theta_p) \exp\left(-\frac{i}{2E} \int_{t_0}^t \mu_i^2(t') dt' \right) \right|^2, \tag{D29}$$

where θ_p is the mixing angle at the production point. For the case of two-neutrino mixing, Eq. (D29) takes the form

$$P_{ee}(t) = \cos^2 \theta_p \cos^2 \theta + \sin^2 \theta_p \sin^2 \theta + \frac{1}{2} \sin 2\theta_p \sin 2\theta \cos\left(\frac{\delta(t)}{2E} \right), \tag{D30}$$

where

$$\delta(t) = \int_{t_p}^t [\mu_2^2(t') - \mu_1^2(t')] dt'. \tag{D31}$$

For $\mu_2^2(t) - \mu_1^2(t) \gg E$, the last term in Eq. (D30) is averaged out and the survival probability takes the form

$$P_{ee} = \frac{1}{2}[1 + \cos 2\theta_p \cos 2\theta]. \tag{D32}$$

The relative importance of the MSW matter term [A of Eq. (D27)] and the kinematic vacuum oscillation term in the Hamiltonian [the off-diagonal term in Eq. (D23)] can be

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parametrized by the quantity β_{MSW}, which represents the ratio of matter to vacuum effects (see, for example [55]). From Eq. (D23) we see that the appropriate ratio is

$$\beta_{\text{MSW}} = \frac{2\sqrt{2}G_F n_e E_\nu}{\Delta m^2}. \tag{D33}$$

The quantity β_{MSW} is the ratio between the oscillation length in matter and the oscillation length in vacuum. In convenient units, β_{MSW} can be written as

$$\beta_{\text{MSW}} = 0.19 \left(\frac{E_\nu}{1\ MeV}\right) \left(\frac{\mu_e \rho}{100\ \text{g cm}^{-3}}\right) \left(\frac{8 \times 10^{-5}\ eV^2}{\Delta m^2}\right). \tag{D34}$$

Here μ_e is the electron mean molecular weight ($\mu_e \approx 0.5(1+X)$, where X is the mass fraction of hydrogen) and ρ is the total density. If $\beta_{\text{MSW}} \lesssim \cos 2\theta$, the survival probability corresponds to vacuum averaged oscillations [see Eq. (D9)],

$$P_{ee} = \left(1 - \frac{1}{2}\sin^2 2\theta\right) \quad (\beta_{\text{MSW}} < \cos 2\theta, \text{ vacuum}). \tag{D35}$$

If $\beta_{\text{MSW}} > 1$, the survival probability corresponds to matter dominated oscillations [see Eq. (D32)],

$$P_{ee} = \sin^2 \theta \quad (\beta_{\text{MSW}} > 1, \text{ MSW}). \tag{D36}$$

The survival probability is approximately constant in either of the two limiting regimes, $\beta_{\text{MSW}} < \cos 2\theta$ and $\beta_{\text{MSW}} > 1$. There is a strong energy dependence only in the transition region between the limiting regimes.

For the Sun, $N_e(R) = N_e(0)\exp(-R/r_0)$, with $r_0 \equiv R_\odot/10.54 = 6.6 \times 10^7$ m $= 3.3 \times 10^{14}\ eV^{-1}$. Then, the adiabaticity condition for the Sun reads

$$\frac{(\Delta m^2/eV^2)\sin^2 2\theta}{(E/MeV)\cos 2\theta} \gg 3 \times 10^{-9}. \tag{D37}$$

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[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[3] G. C. Branco, L. Lavoura and J. P. Silva, *CP violation,* Clarendon Press, Oxford (1999).

[4] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65**, 054034 (2002) [arXiv:hep-ph/0110317].

[5] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Rev. D **69**, 114021 (2004) [arXiv:hep-ph/0402204].

[6] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0703020.

[7] K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0703036.

[8] K. Abe *et al.* [BELLE Collaboration], arXiv:0704.1000 [hep-ex].

[9] G. Raz, Phys. Rev. D **66**, 037701 (2002) [arXiv:hep-ph/0205310].

[10] Y. Nir and G. Raz, Phys. Rev. D **66**, 035007 (2002) [arXiv:hep-ph/0206064].

[11] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993) [arXiv:hep-ph/9304307].

[12] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **420**, 468 (1994) [arXiv:hep-ph/9310320].

[13] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini,
arXiv:hep-ph/0703204.

[14] Y. Nir, JHEP, in press [arXiv:hep-ph/0703235].

[15] Y. Grossman, Y. Nir and G. Raz, Phys. Rev. Lett. **97**, 151801 (2006) [arXiv:hep-ph/0605028].

[16] CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), [hep-ph/0406184],
updated results and plots available at: http://ckmfitter.in2p3.fr

[17] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981).

[18] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. B **193**, 85 (1981).

[19] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996)
[arXiv:hep-ph/9512380].

[20] Y. Grossman, A. L. Kagan and Z. Ligeti, Phys. Lett. B **538**, 327 (2002) [arXiv:hep-ph/0204212].

[21] H. Boos, T. Mannel and J. Reuter, Phys. Rev. D **70**, 036006 (2004) [arXiv:hep-ph/0403085].

[22] M. Bona *et al.* [UTfit Collaboration], JHEP **0610**, 081 (2006) [arXiv:hep-ph/0606167].

[23] E. Barberio *et al.* [HFAG Group], arXiv:0704.3575 [hep-ex] and online update at
http://www.slac.stanford.edu/xorg/hfag

[24] Y. Nir, Nucl. Phys. Proc. Suppl. **117**, 111 (2003) [arXiv:hep-ph/0208080].

[25] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B **407**, 307 (1997) [hep-ph/9704287].

[26] P. Meade and M. Reece, Phys. Rev. D **74**, 015010 (2006) [arXiv:hep-ph/0601124].

[27] P. J. Fox, Z. Ligeti, M. Papucci, G. Perez and M. D. Schwartz, arXiv:0704.1482 [hep-ph].

[28] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez and J. Virzi, arXiv:hep-ph/0612015.

[29] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002)
[arXiv:hep-ph/0207036].

[30] Y. Grossman, Y. Nir, J. Thaler, T. Volansky and J. Zupan, arXiv:0706.1845 [hep-ph].

[31] Y. Nir and Y. Shadmi, work in progress.

[32] M. C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. **75**, 345 (2003) [arXiv:hep-ph/0202058].

[33] M. C. Gonzalez-Garcia and M. Maltoni, arXiv:0704.1800 [hep-ph].

[34] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [arXiv:hep-ph/0202074].

[35] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147**, 277 (1979).

[36] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **398**, 319 (1993) [arXiv:hep-ph/9212278].

[37] Y. Grossman and Y. Nir, Nucl. Phys. B **448**, 30 (1995) [arXiv:hep-ph/9502418].

[38] L. J. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000) [arXiv:hep-ph/9911341].

[39] Y. E. Antebi, Y. Nir and T. Volansky, Phys. Rev. D **73**, 075009 (2006) [arXiv:hep-ph/0512211].

[40] L. Chau and W. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[41] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[42] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994) [arXiv:hep-ph/9403384].

[43] C. Dib, I. Dunietz, F. J. Gilman and Y. Nir, Phys. Rev. D **41**, 1522 (1990).

[44] J. L. Rosner, A. I. Sanda and M. P. Schmidt, EFI-88-12-CHICAGO [Presented at Workshop on High Sensitivity Beauty Physics, Batavia, IL, Nov 11-14, 1987].

[45] V. Weisskopf and E. P. Wigner, Z. Phys. **63**, 54 (1930); Z. Phys. **65**, 18 (1930). [See Appendix A of P. K. Kabir, "The CP Puzzle: Strange Decays of the Neutral Kaon", Academic Press (1968).]

[46] Y. Nir, SLAC-PUB-5874 [Lectures given at 20th Annual SLAC Summer Institute on Particle Physics (Stanford, CA, 1992)].

[47] I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986).

[48] Ya. I. Azimov, N. G. Uraltsev, and V. A. Khoze, Sov. J. Nucl. Phys. **45**, 878 (1987) [Yad. Fiz. **45**, 1412 (1987)].

[49] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **281**, 41 (1987).

[50] B. Pontecorvo, Sov. Phys. JETP **6**, 429 (1957) [Zh. Eksp. Teor. Fiz. **33**, 549 (1957)].

[51] H. J. Lipkin, arXiv:hep-ph/9901399.

[52] C. W. Kim and A. Pevsner, Contemp. Concepts Phys. **8**, 1 (1993).

[53] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[54] S.P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985) [Yad. Fiz. **42**, 1441 (1985)].

[55] J. N. Bahcall and C. Pena-Garay, New J. Phys. **6**, 63 (2004) [arXiv:hep-ph/0404061].