LHC for SUSY Theorists

3. Measurement of the SUSY Spectrum

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3. How can we use LHC observations to constrain the spectrum of SUSY particles in a way that is as model-independent as possible ?

Once we have observed excesses above the levels predicted for the Standard Model, we can use this information to estimate the masses of the new particles and to determine their properties.

It is only at this stage that we can ask the question,

Is it really SUSY, or another model with heavy particles and dark matter ?

In this lecture, I will review some methods for such analyses. I will discuss

- determination of the overall mass scale of SUSY
- specific kinematic observables that determine SUSY particle masses and mass differences
- diagnostics of the properties of SUSY particles

A warning about these methods:

Do not use them on data that has not been correctly background-subtracted.

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Analysis of top–antitop production and dilepton decay events and the top quark mass

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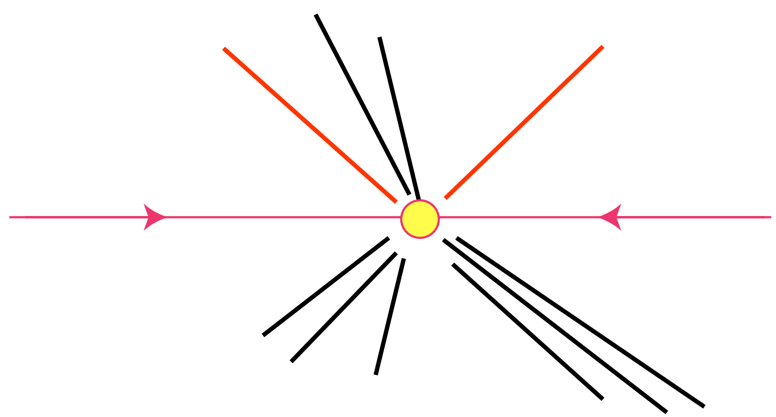
Received 22 May 1992

If this event really represents top–antitop production and decay, then the top quark mass would be 131^{+22}_{-11} GeV.

There are significant difficulties in trying to measure new particle masses at the LHC from resonances or features in kinematic distributions.

Any given process involves one quark or gluon colliding with another. We do not know the momenta of these individual particles. So we do not know the momentum of the initial state.

The final state might contain two dark matter particles. We do not observe these particles or measure their momentum. So we have incomplete information about the final state.

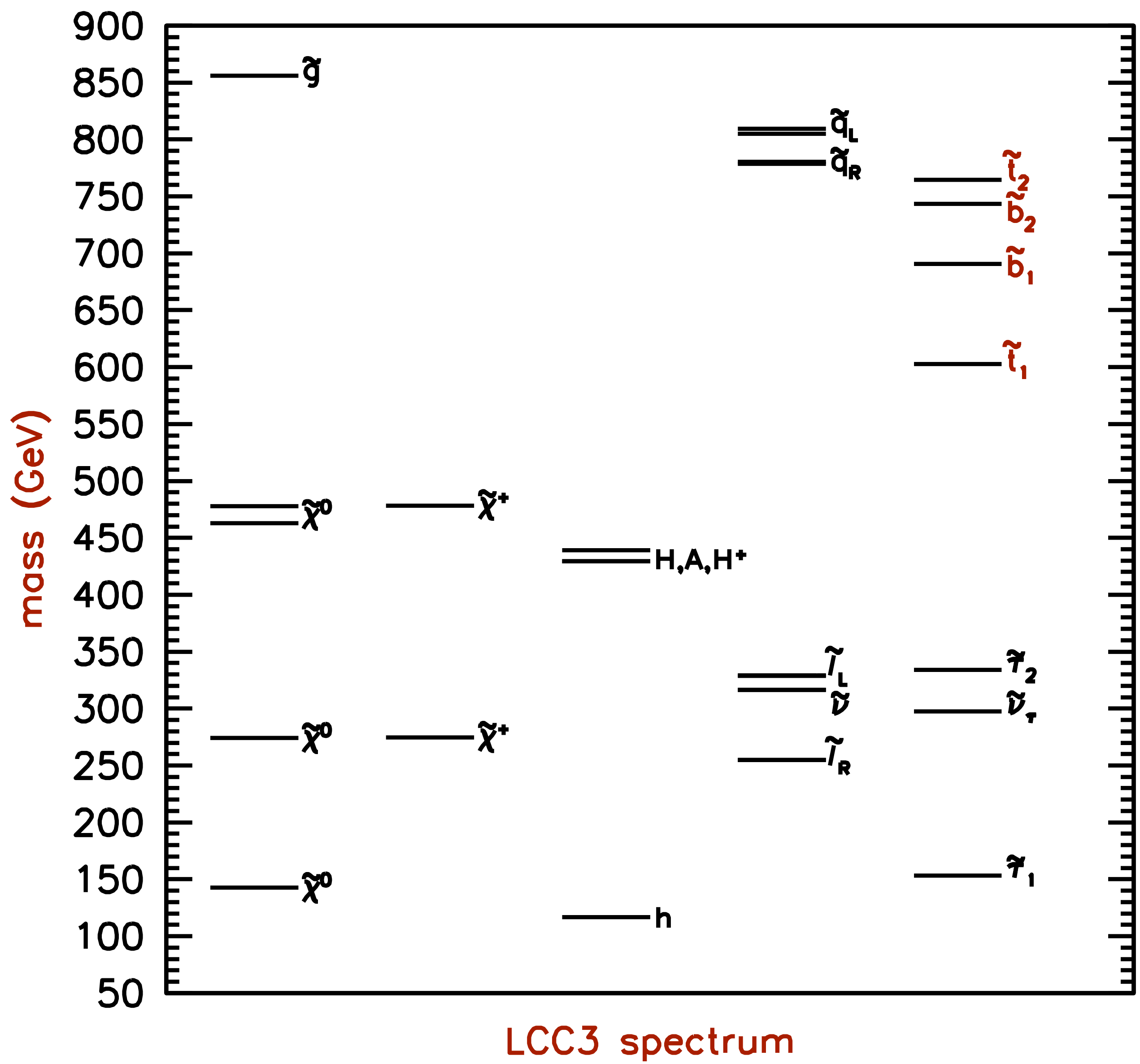


Our goal is to determine the masses of the particles and to measure their quantum numbers.

In SUSY, there are many new particles and their spectrum is complex.

The particles with large QCD production cross sections are typically the heaviest particles in the spectrum.

Our first goal should to estimate the masses of these particles.

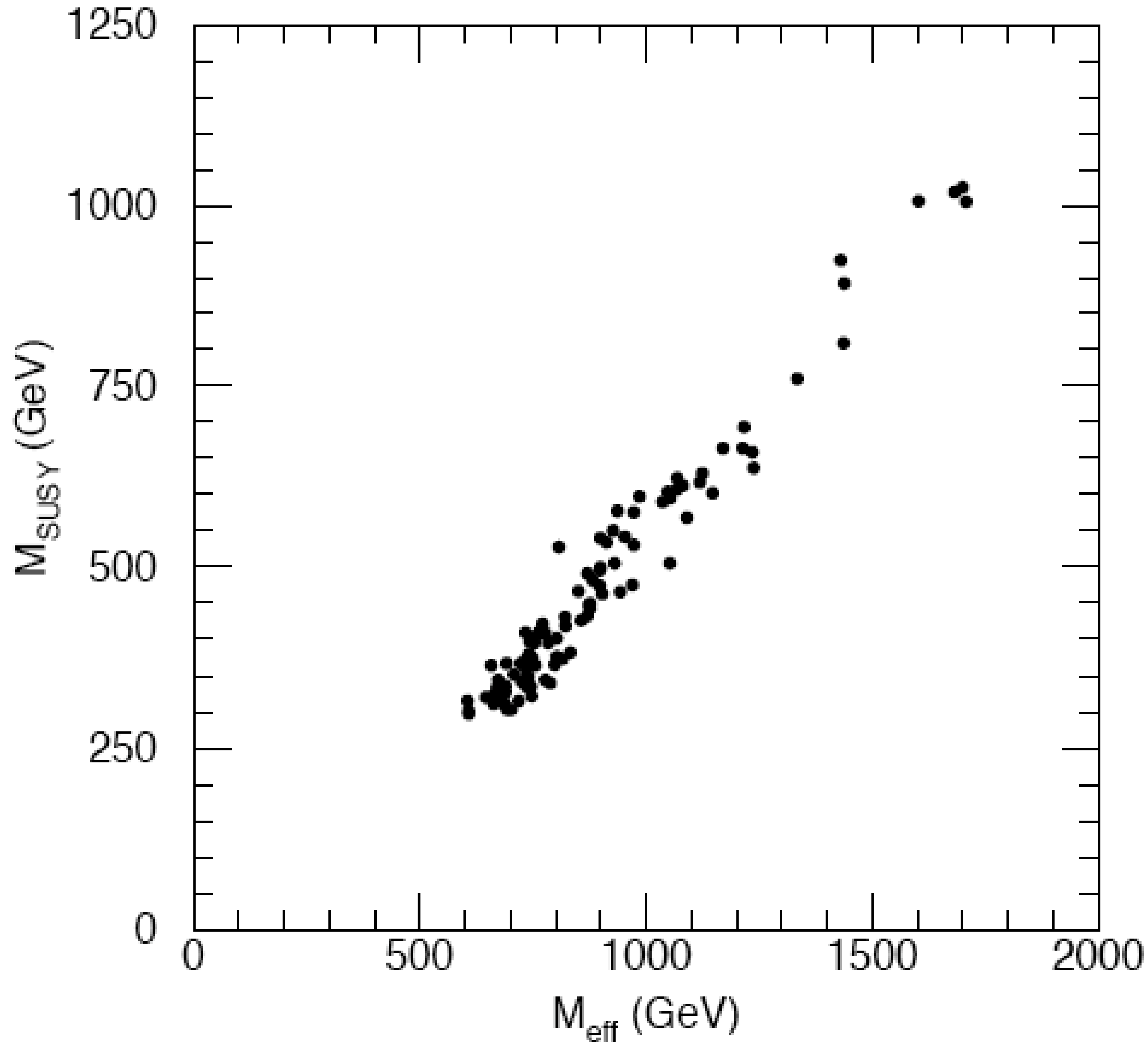


As a start, consider the overall transverse energy deposition in the detector. To remove noise from the underlying event, we might alternatively sum the ET of the hardest jets.

Hinchliffe, Paige, Shapiro, Soderqvist and Yao proposed the observable

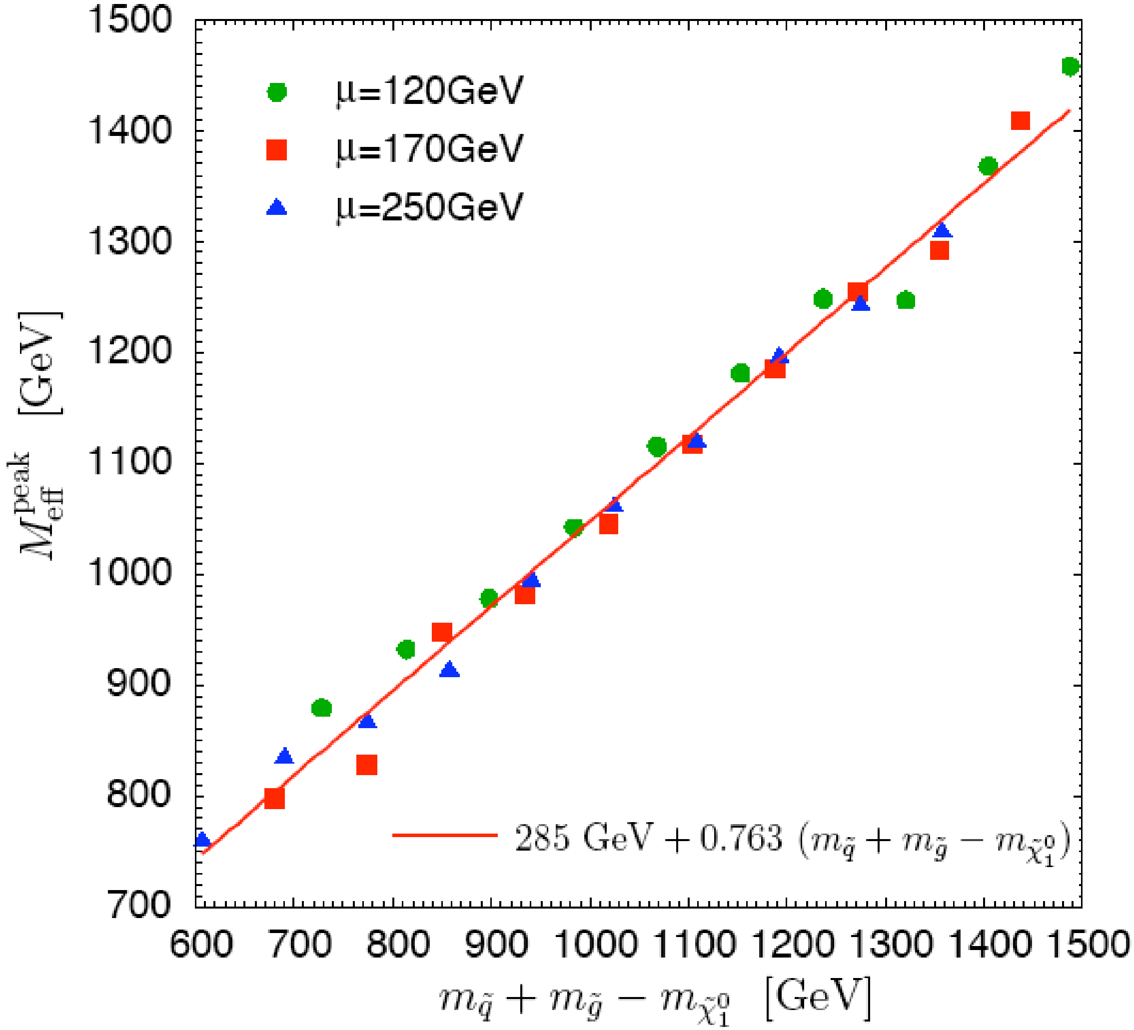
$$M_{eff} = \not{E}_T + \sum_{jets=1}^{4} E_{Ti}$$

and showed that, in a variety of 'mSUGRA' models, it correlates well with the smallest of the squark and gluino masses.



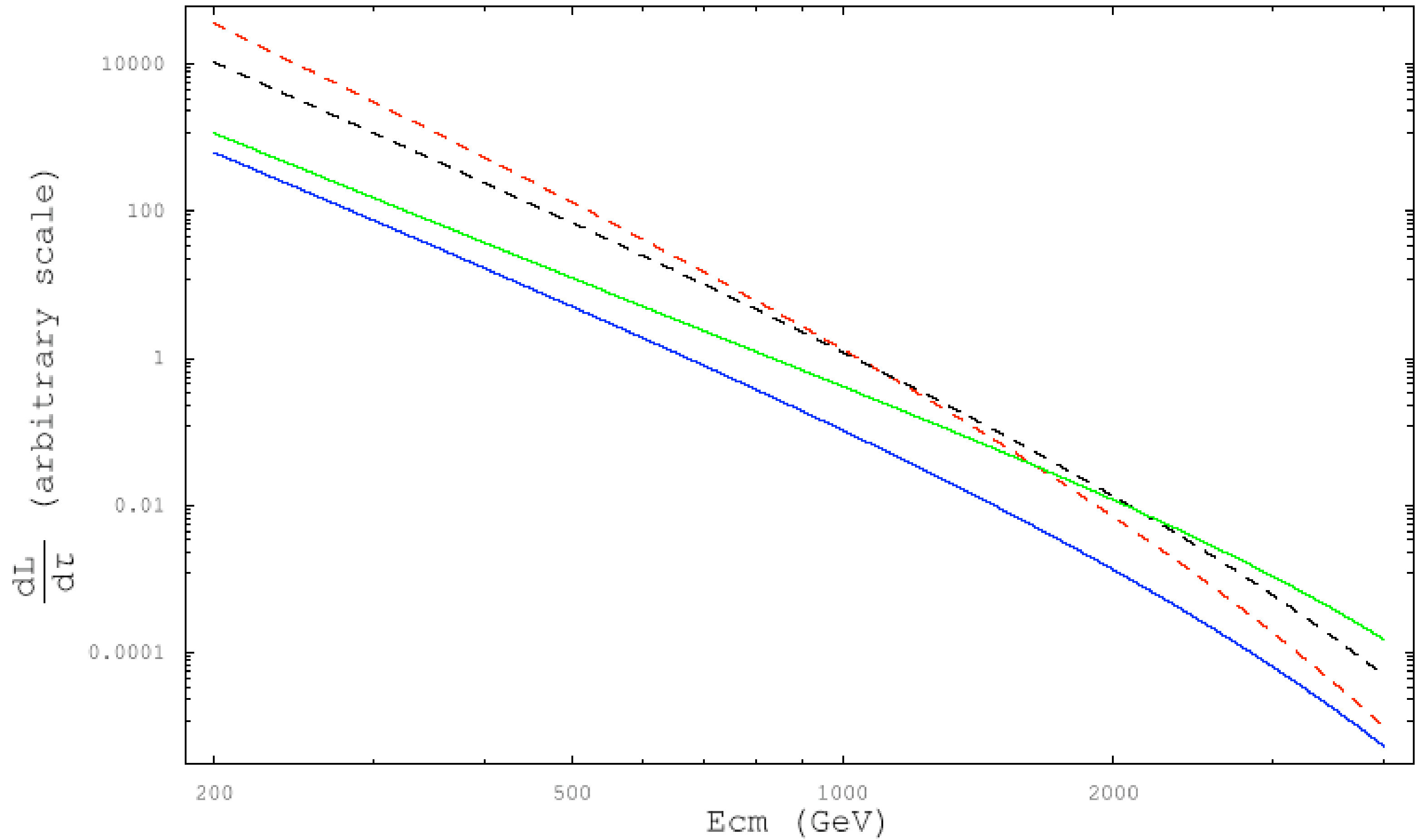
ATLAS

In models with small mass differences between the squarks and the charginos and neutralinos, much of the transverse energy in the reaction is carried off by neutalinos and is invisible. But still, the quantity M_{eff} is a reasonable indicator of the mass difference between the directly produced and the final SUSY states.



Kitano-Nomura

These relations work well because most new particle production occurs close to threshold. The production cross sections turn on at threshold and then rise only slowly when $\hat{s} \gg 4m^2$, while parton luminosities fall off very rapidly.



Recently Thaler, Schuster, Toro, et mult al. (hep-ph/0703088) suggested that we can take advantage of this to do a broader phenomenological analysis. They suggest that we

Choose an appropriate set of candidate new particles

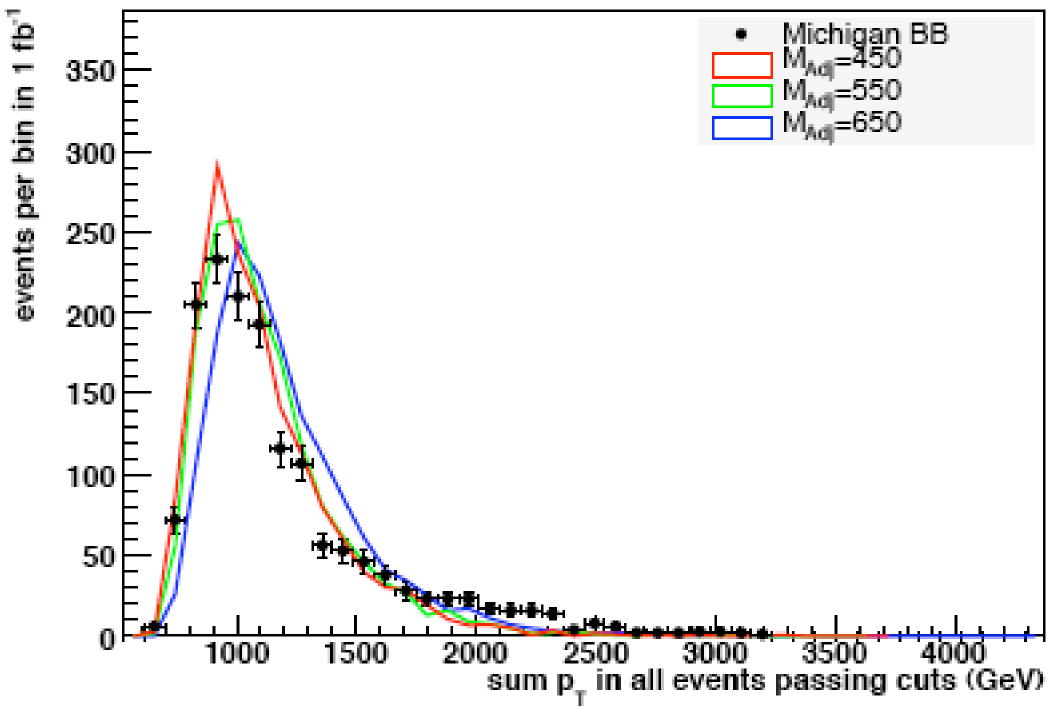
Approximate all production cross sections by constants

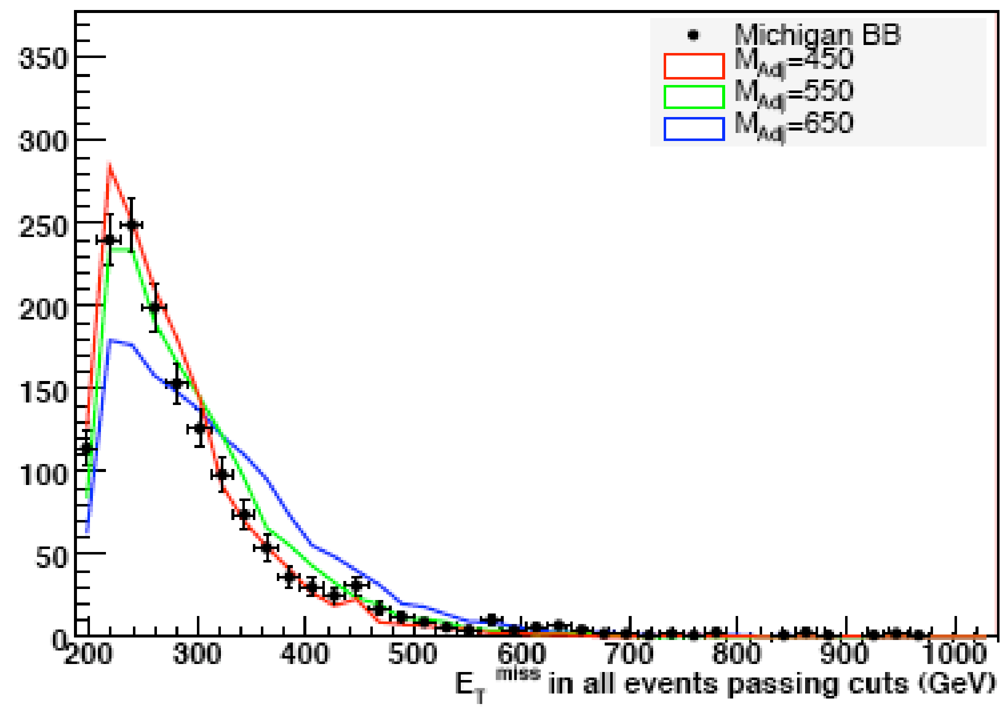
Choose appropriate decay modes for each particle. These might be 2-body decays or multi-body decays through effective operators. Approximate all decay matrix elements by constants.

Fit the data to obtain the masses, cross sections, and branching fractions.

They refer to this description as an on-shell effective theory (OSET). The program is encoded in a software package called MARMOSET.

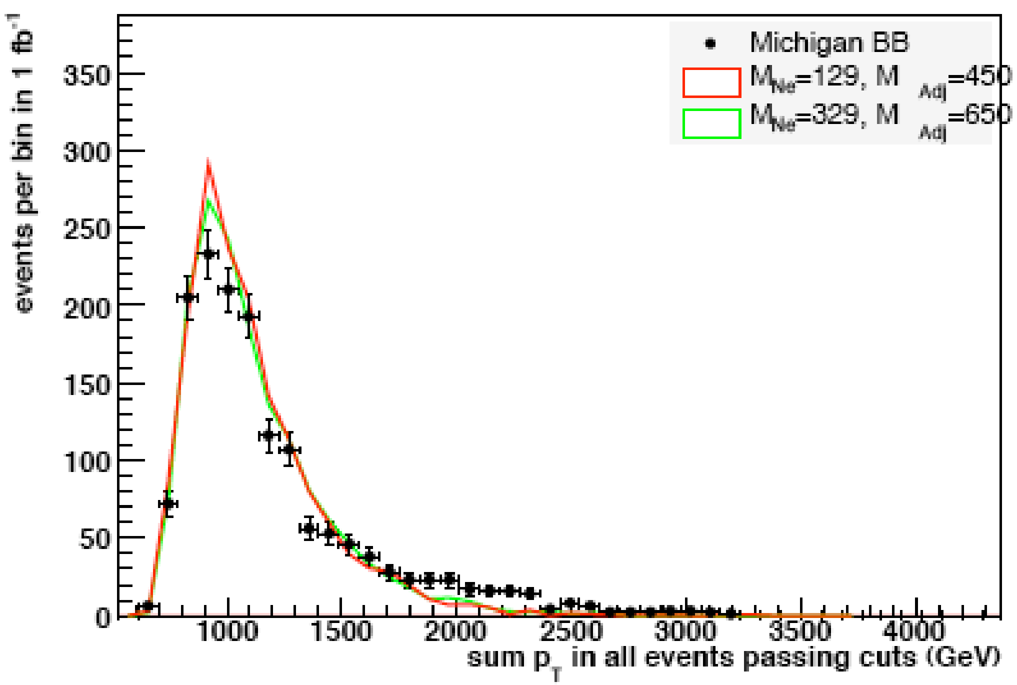
Here is a fit to the H_T and $\not{E}_T$ distributions for a specific SUSY model (red) and the corresponding curves (green, blue) when the gluino mass is shifted by 20%, 40%.

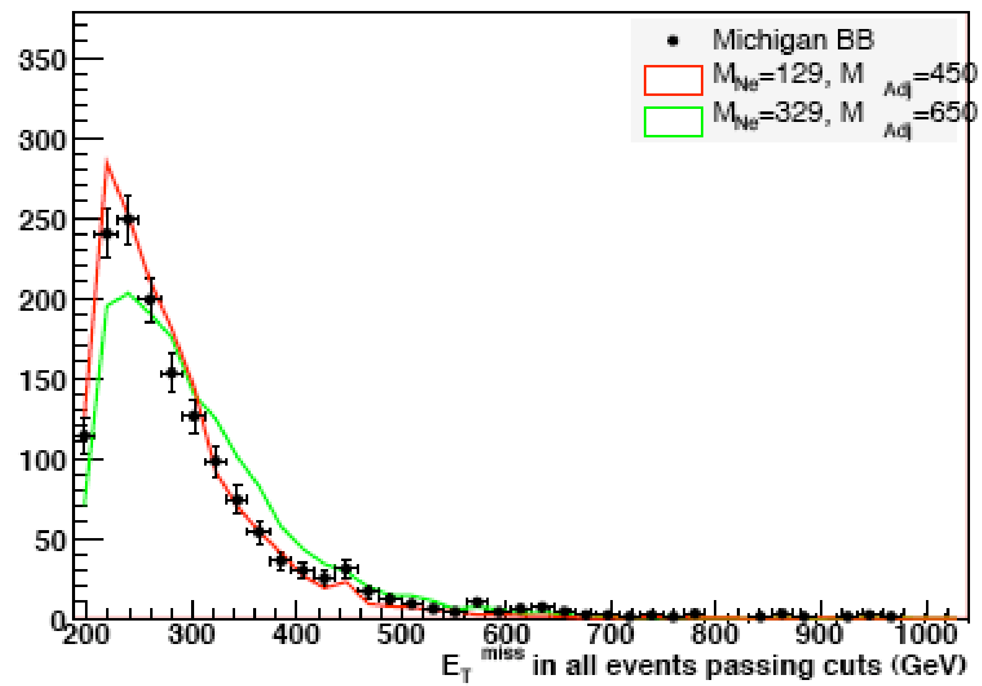




Here is the effect of shifting the gluino mass by 40%, keeping the gluino-neutralino mass difference fixed.

The H_T distribution is unaffected, but the $\not{E}_T$ distribution gives a poor fit.





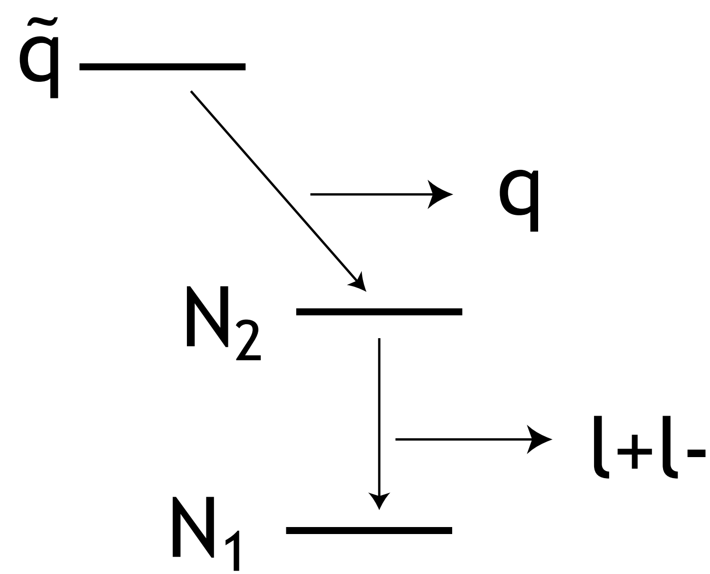
To extract more specific information, we need to perform analyses that rely on special features of the supersymmetry spectrum.

Every spectrum has special features. It is part of the art of experimentation to find and exploit them.

In the discussion to follow, I will pick out a particular feature that has been studied in a number of different analyses and use it to illustrate that level of insight that one could achieve in the hadron collider environment.

It is typical in supersymmetry models that the partners of quarks and gluons are relatively heavy states. These decay to the charginos and neutralinos, the partners of SU(2)xU(1) gauge bosons and Higgs bosons.

A feature of many supersymmetry spectra is the decay chain



The lepton momenta are measured completely, and we can construct their spectrum of invariant masses. From this point, depending on the specific model of the dilepton decay, the analysis can proceed in several different ways.

The decay of the N_2 can occur by any of the mechanisms:

$$N_2^0 \to \ell^\pm + \widetilde{\ell}^\mp \;,\; \widetilde{\ell}^\mp \to \ell^\mp + N_1^0$$
$$N_2^0 \to N_1^0 Z^0 \;,\; Z^0 \to \ell^+ \ell^-$$
$$N_2^0 \to N_1^0 Z^{*0} \;,\; Z^{*0} \to \ell^+ \ell^-$$

In a model in which $N_2 \sim \widetilde{w}^0 \;,\; N_1^0 \sim \widetilde{b}^0$, these modes are preferred in the order listed: 2-body decays dominate over 3-body decays, and the N_2 coupling to sleptons is larger than the N_2 coupling to Z^0.

The decay to an on-shell Z^0 is hard to work with, but the other two cases are interesting. To analyze them, consider the Dalitz plot associated with the 3-body system $N_1^0 \ell^+ \ell^-$

Let $$x_0 = \frac{2E(N_1)}{m(N_2)} \quad x_+ = \frac{2E(\ell^+)}{m(N_2)} \quad x_- = \frac{2E(\ell^-)}{m(N_2)}$$

in the frame of the N_2.

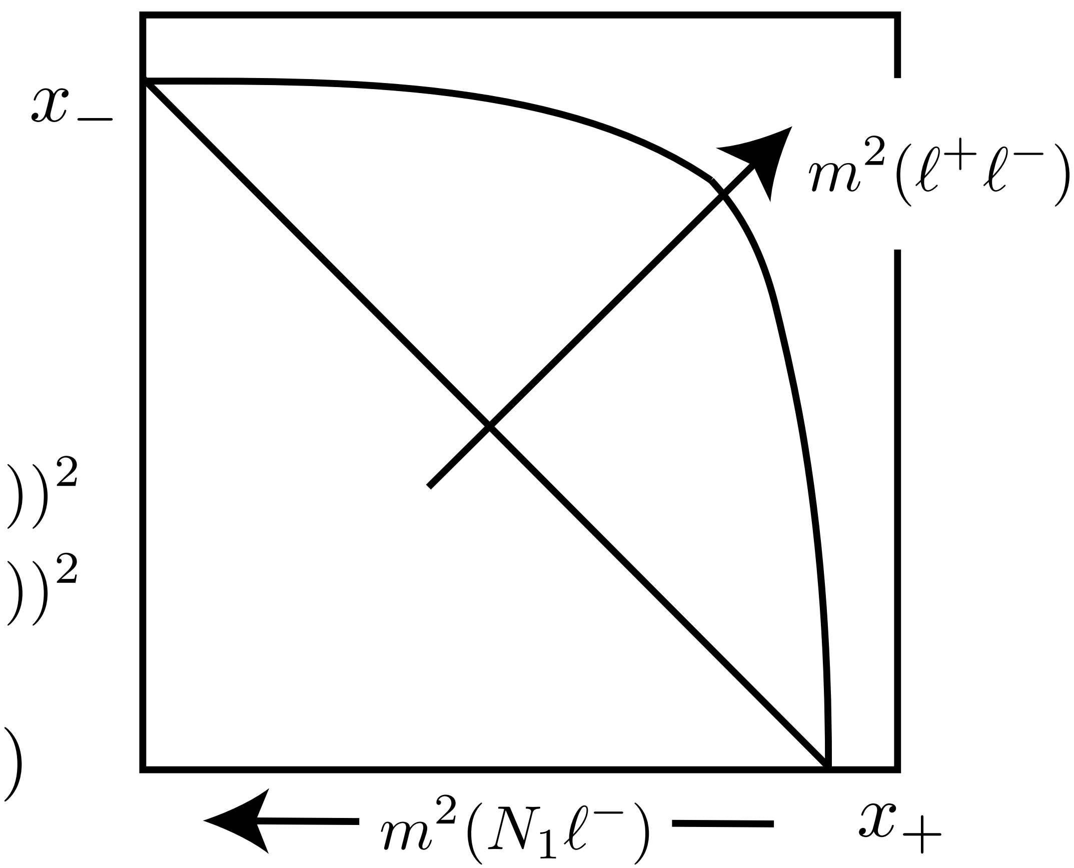
$$x_0 + x_+ + x_- = 2$$

The kinematic boundaries are located at:

$$x_+ + x_- = 1 - (m(N_1)/m(N_2))^2$$

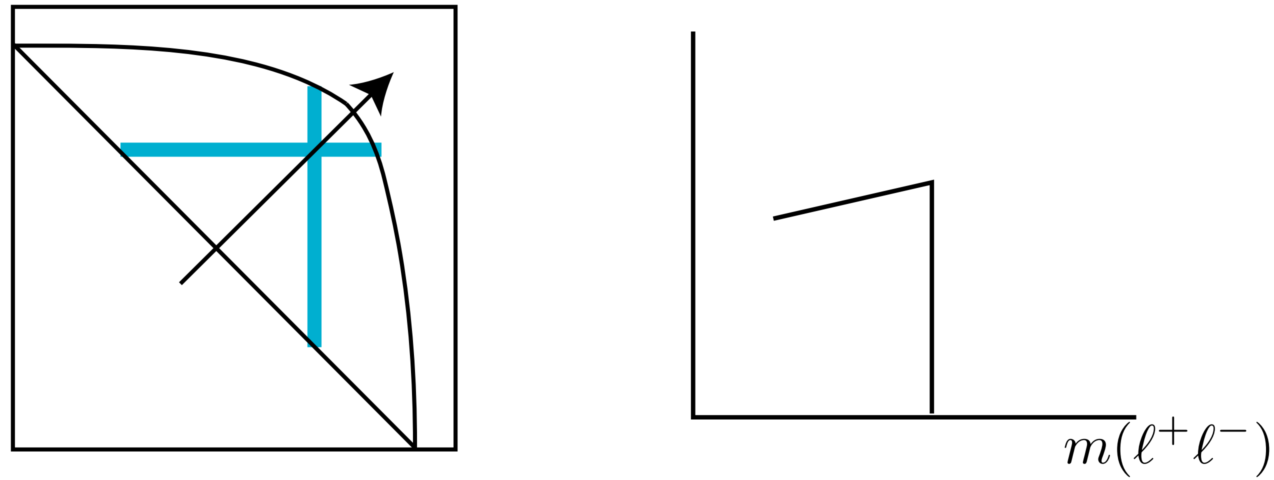
$$(1 - x_-)(1 - x_+) = (m(N_1)/m(N_2))^2$$

$$m^2(N_1\ell^-) = m^2(N_2)(1 - x_+)$$

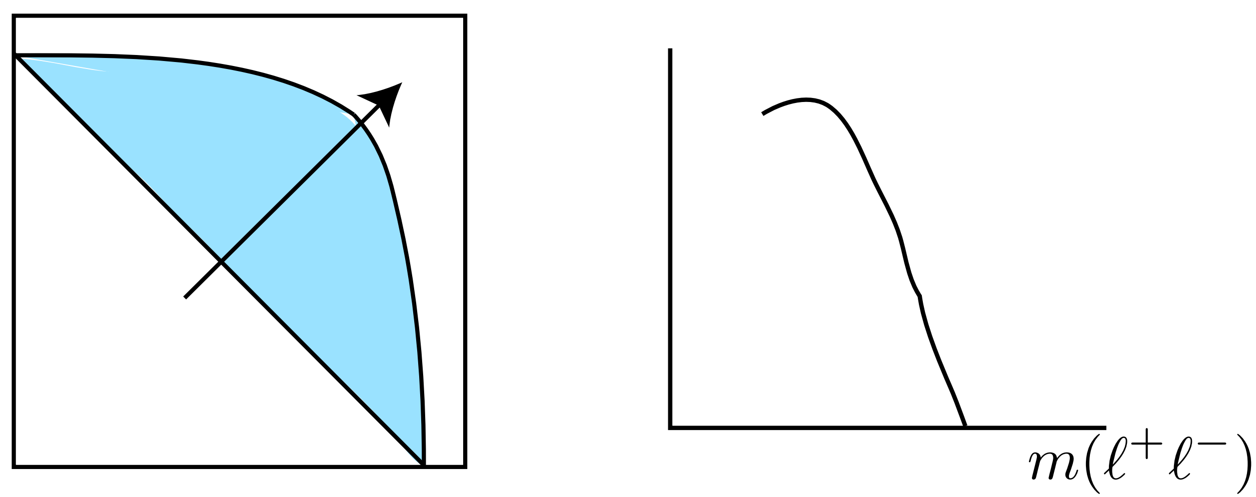


We can distinguish the cases of 2-body decay to a slepton and 3-body decay in the following way:

2-body decay populates lines on the Dalitz plot and leads to a sharp endpoint:



3-body decay populates the whole Dalitz plot and gives a slope at the endpoint:

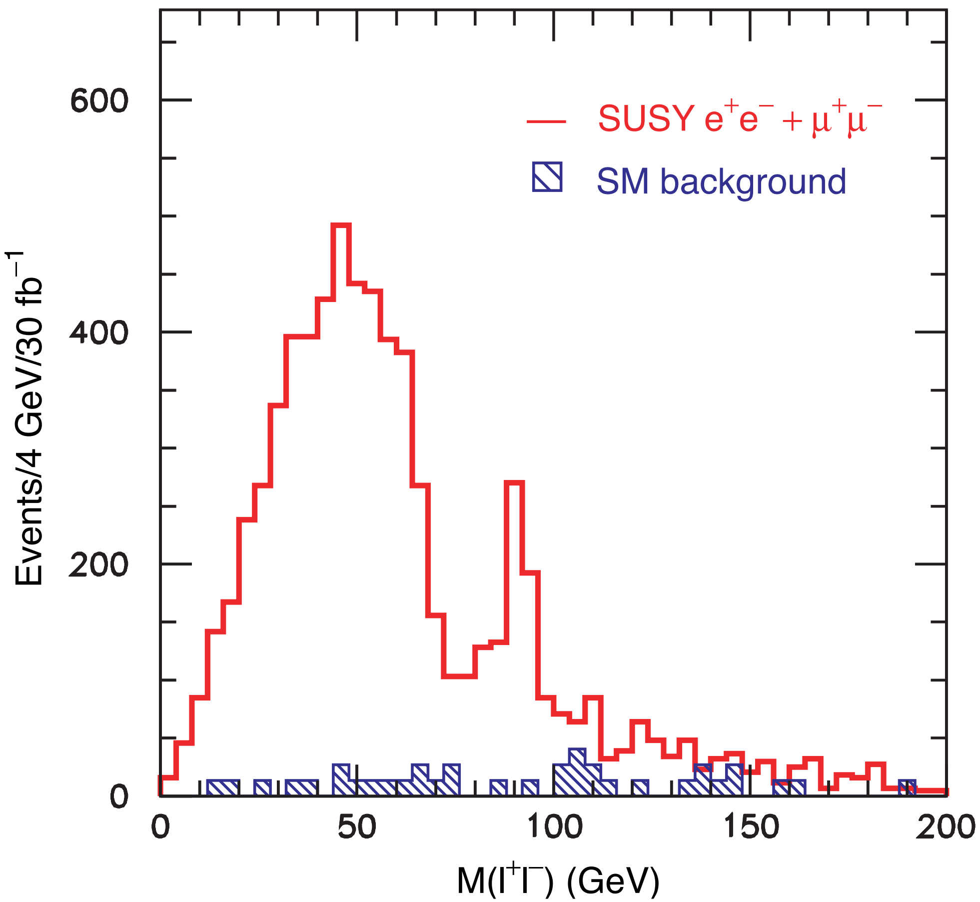


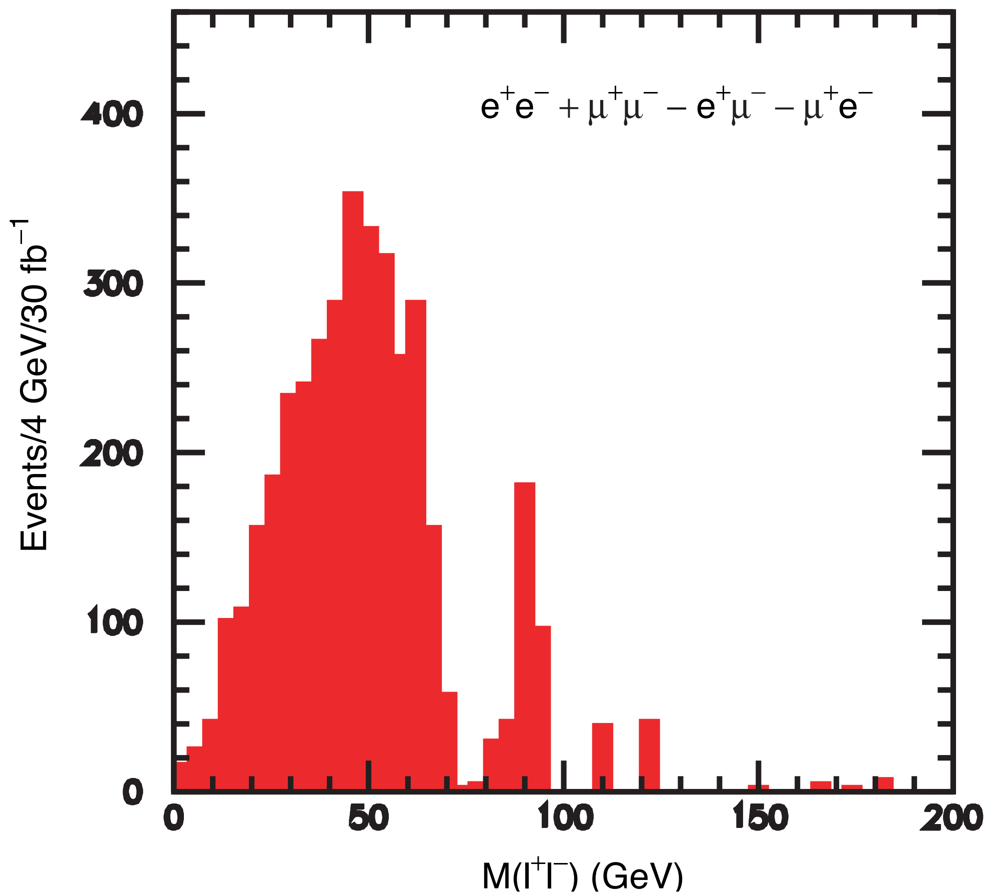
In the 3-body case, the endpoint in $m(\ell^+\ell^-)$ is exactly

$$m(N_2) - m(N_1)$$

so we obtain a precise measurement of this quantity. The shape of the spectrum has more information. For example, for heavy slepton masses, this shape is different for gaugino-like or Higgsino-like lightest neutralino.

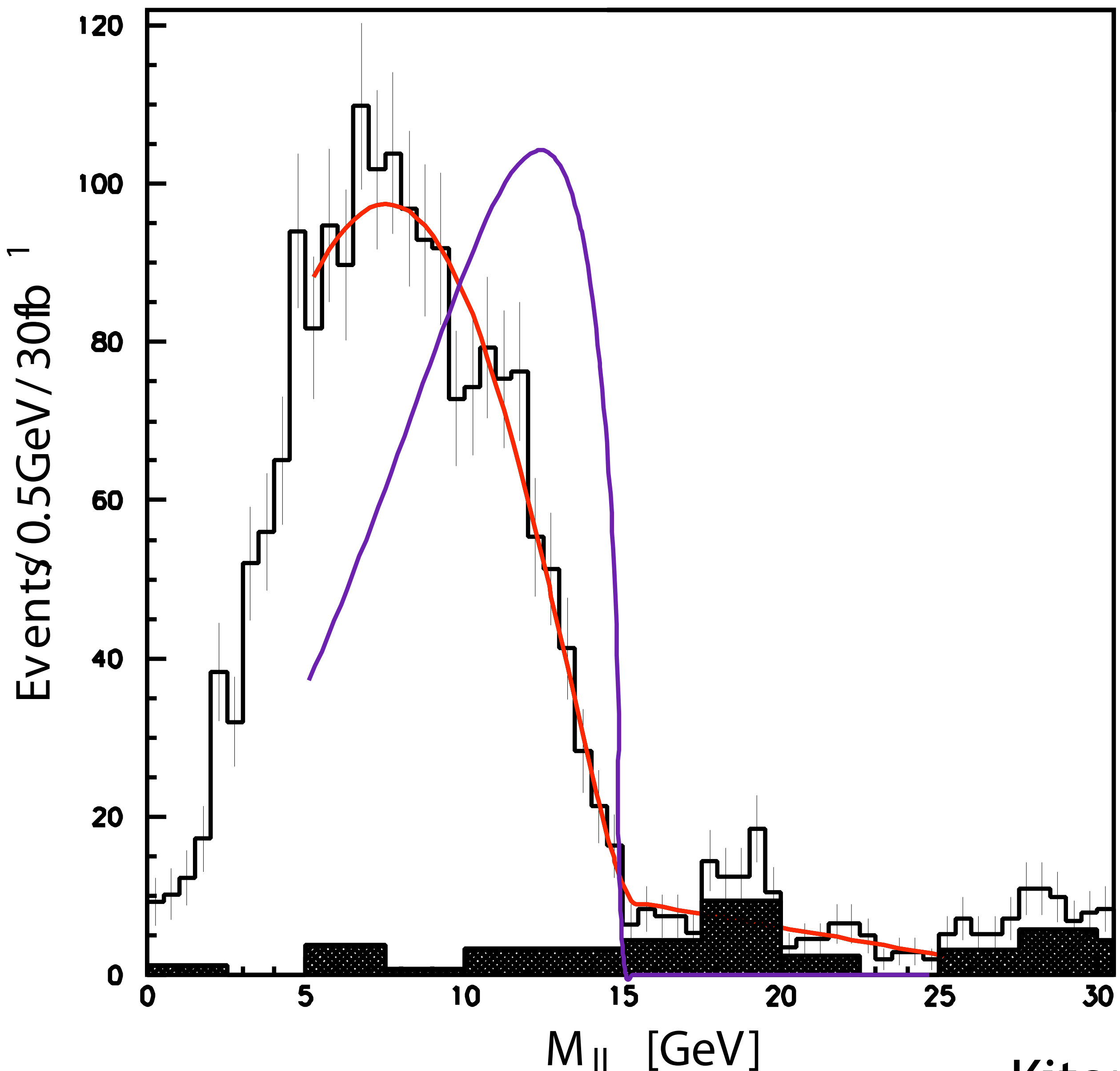
an example where the lightest neutralinos are gaugino:





Hinchliffe et al.

an example where the lightest neutralinos are Higgsino:



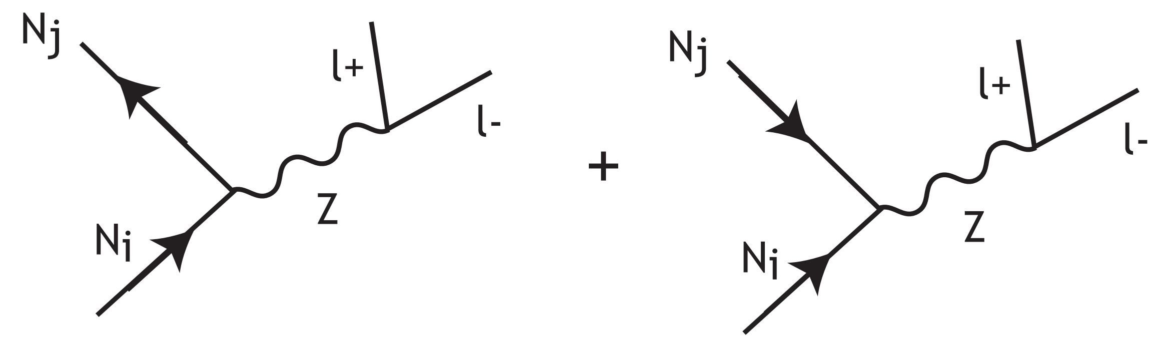
Kitano-Nomura

The origin of the shape difference is interesting. The neutralino mass matrix has the form

$$\begin{pmatrix} m_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\ 0 & m_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & -\mu & 0 \end{pmatrix}$$

so the signs of the eigenvalues m_i are (+ +) for gauginos, (-+) for Higgsinos.

The decay $N_i \to N_j \ell^+ \ell^-$ through a virtual Z involves interfering diagrams

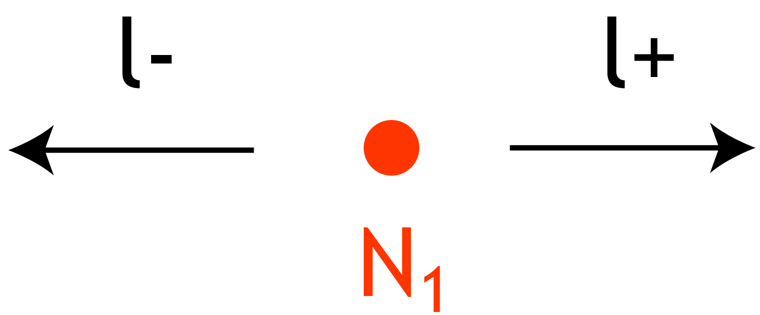


The interference term is proportional to $m_i m_j$.

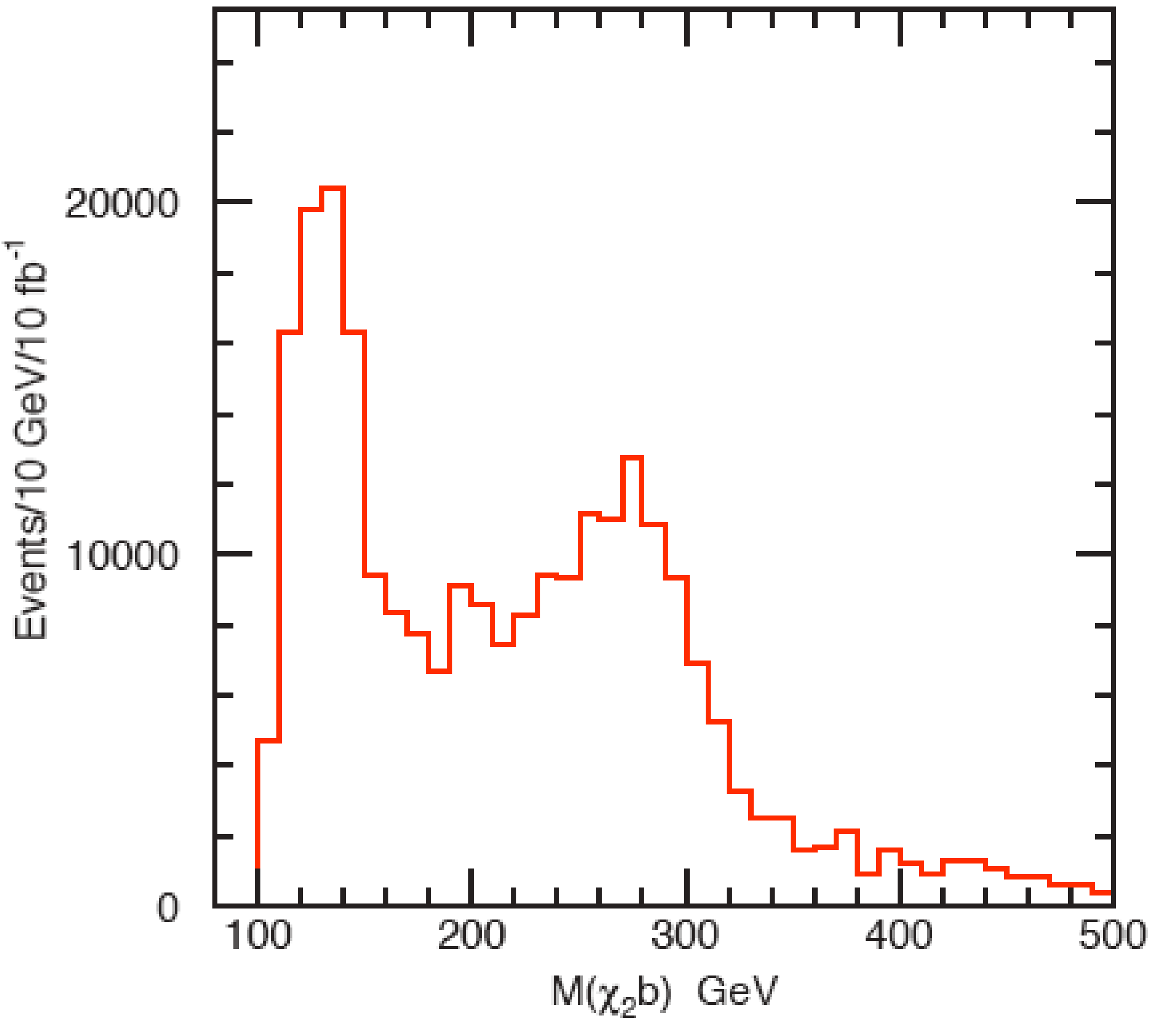
Hinchliffe et al. noticed that one could go further.

At the endpoint, the unobserved WIMP is at rest in the frame of the l+l- pair.   If we have an estimate of the mass of the WIMP, we can add back its 4-vector.

Now there is no more missing information. Add observed jets and reconstruct the parent squarks.



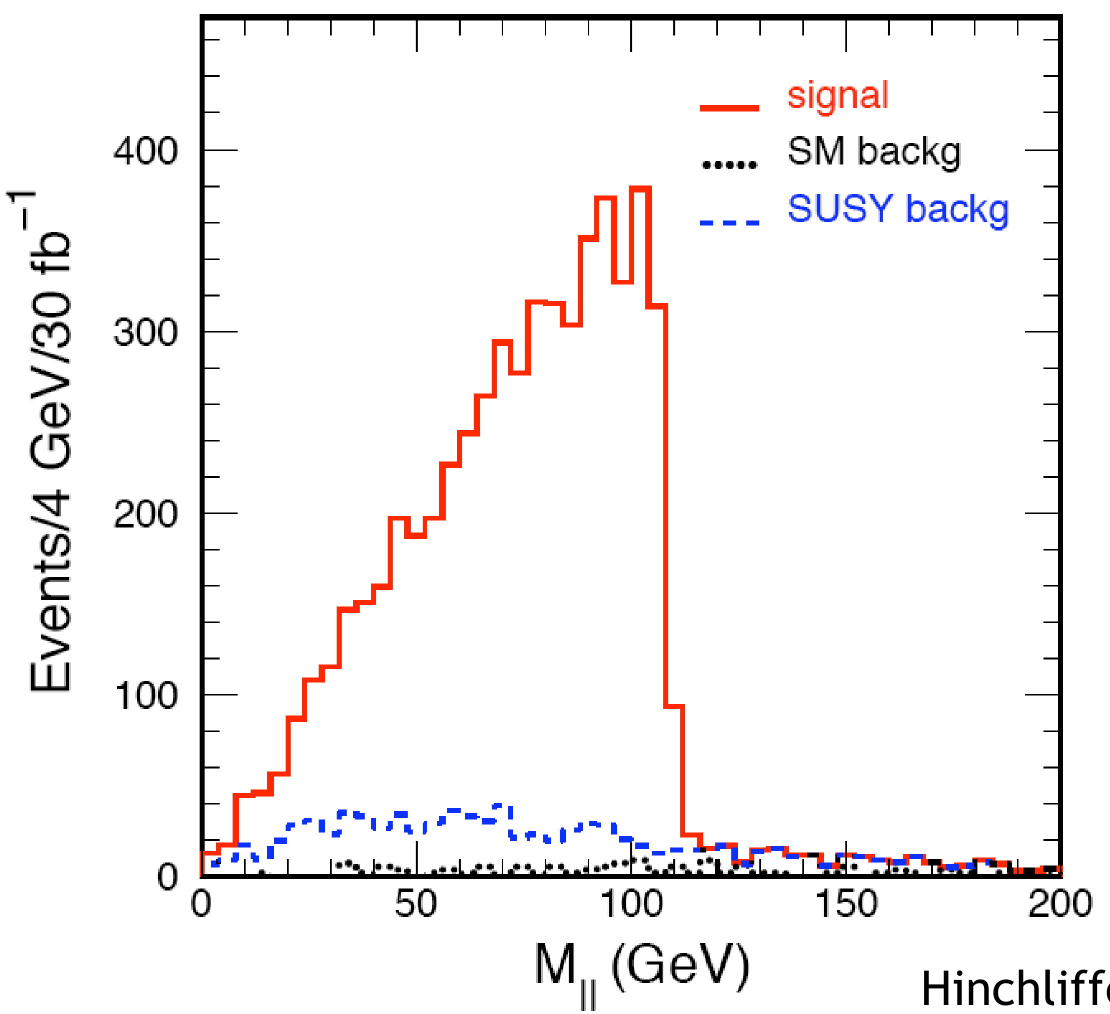
At the endpoint, the N_1 is at rest in the frame of the $\ell^+\ell^-$. If we know (or guess) the mass of the N_1, we know its 4-vector. Now we have solved the problem of missing momentum; we can add jets and try to reconstruct the parent squarks



Hinchliffe et al.

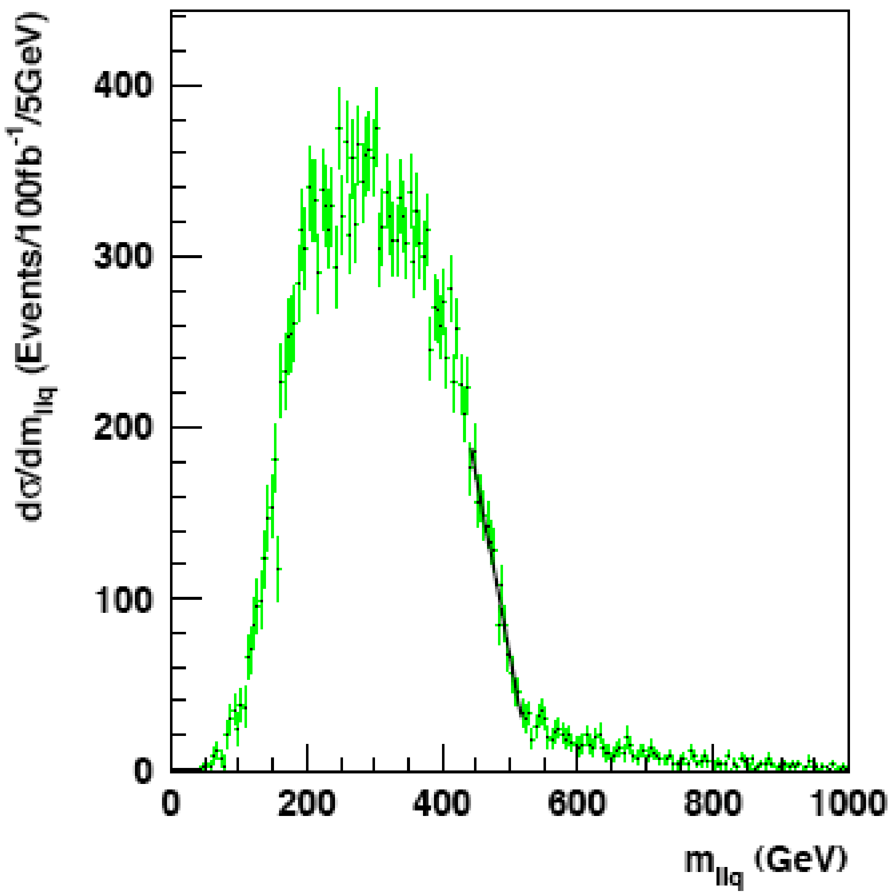
The case of a 2-body decay is even nicer. There is a sharp endpoint at

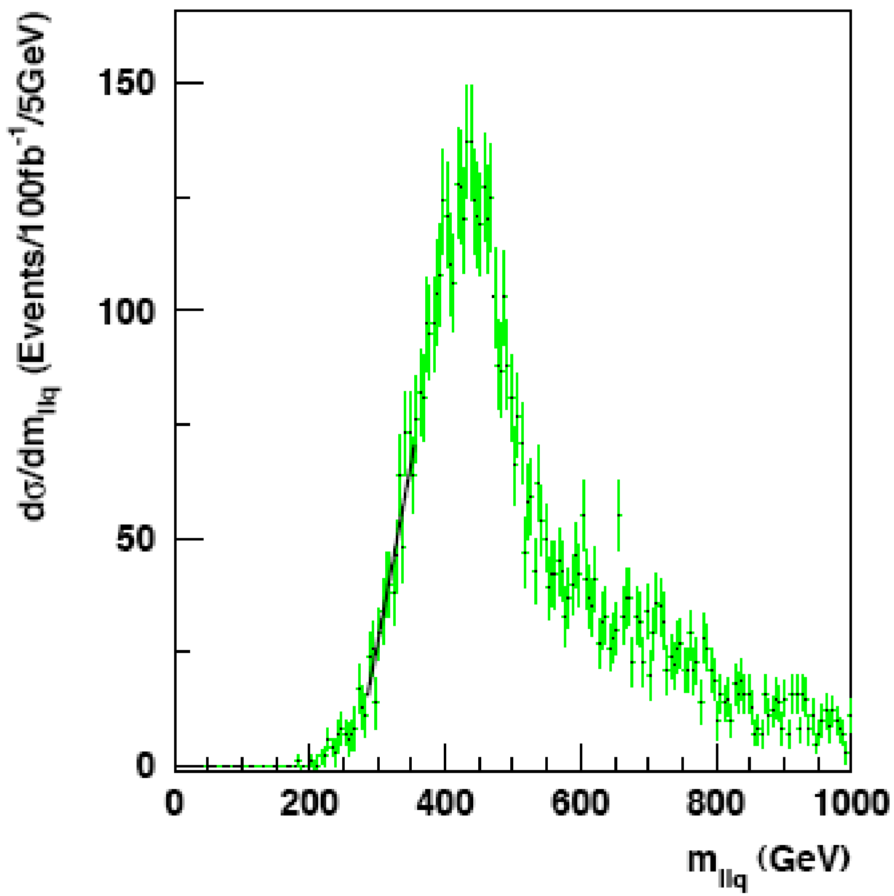
$$m(\ell^+\ell^-) = m(N_2)\sqrt{1-(\frac{m(\widetilde{\ell})}{m(N_2)})^2}\sqrt{1-(\frac{m(N_1)}{m(\widetilde{\ell})})^2}$$



Hinchliffe and Paige

The decay $\widetilde{q} \to qN_2$ is also 2-body, and so there are also upper and lower kinematic endpoints in combinations $(j\ell)$, $(j\ell\ell)$. From 4 endpoints, one can solve for the 4 unknown masses in the problem.





Hinchliffe and Paige

The case in which the decay involves an on-shell Z is the most difficult. To get definite kinematic information, we need to combine the Z with other jets in the events.

Recently, Butterworth, Ellis, and Raklev have considered the following strategy, directed at squark decay chains including

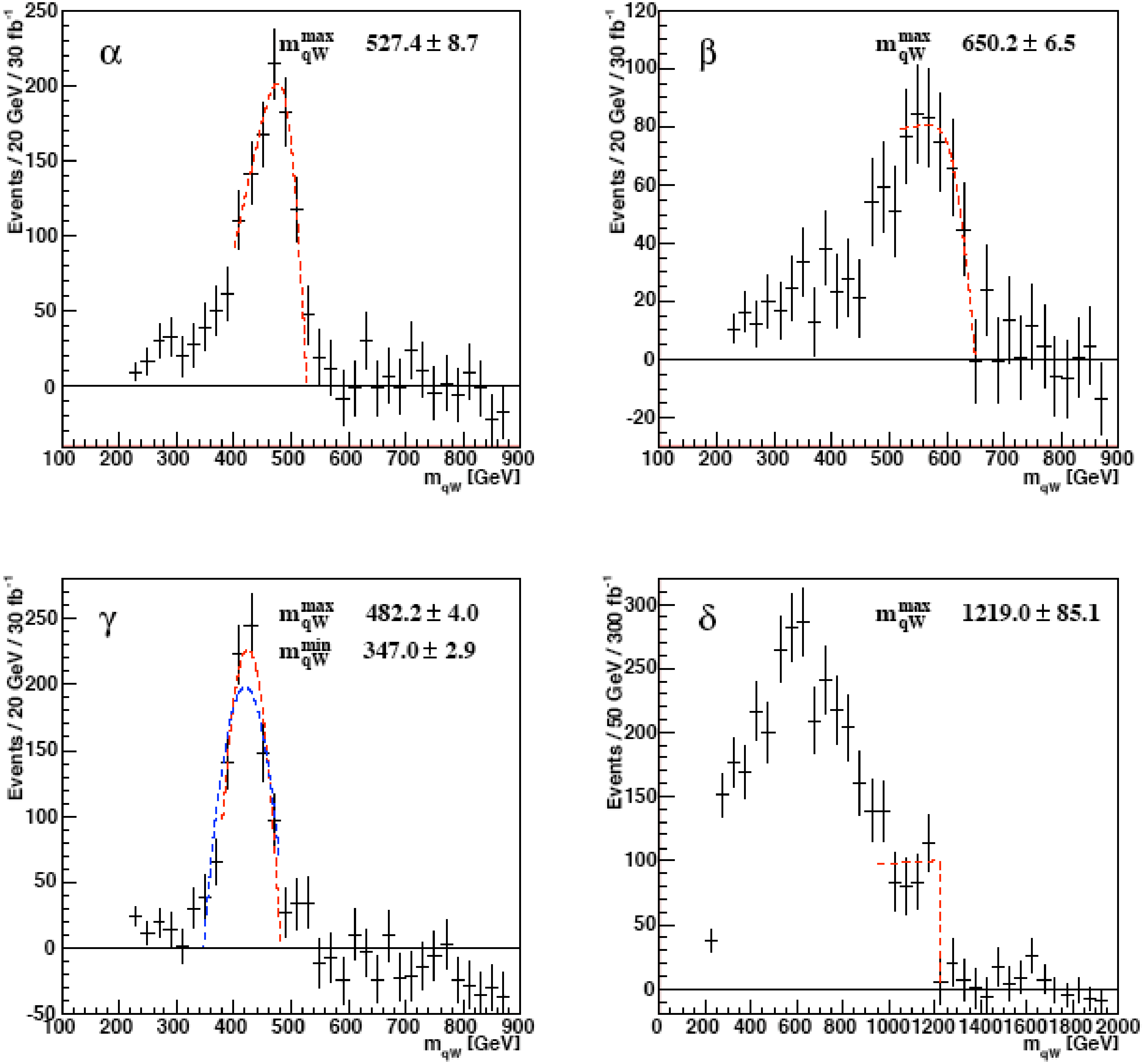
$$\widetilde{q} \to q C_1^+ \to q W^+ N_1$$

select SUSY events (jets + missing ET)

look for hadronic jets with pT > 200, 2-jet substructure and mass consistent with the W mass

combine these W candidates with the highest pT jets and look for kinematic endpoints

$m(jW)$

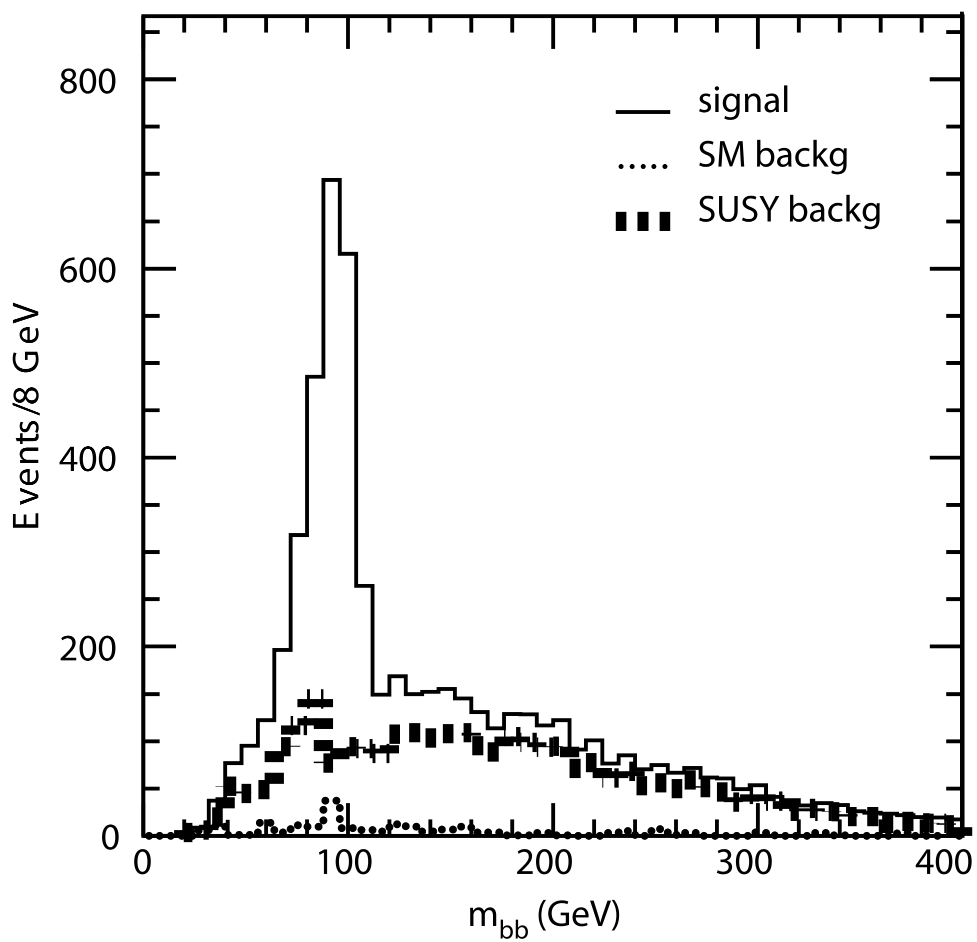


Butterworth, Ellis, and Raklev

One more case of an $N_2 \rightarrow N_1$ decay should be mentioned. If the 2-body decays to sleptons are not kinematically allowed, the dominant 2-body decay might be

$$N_2 \rightarrow N_1 + h^0$$

In this case, supersymmetry production can provide a copious source of Higgs bosons.



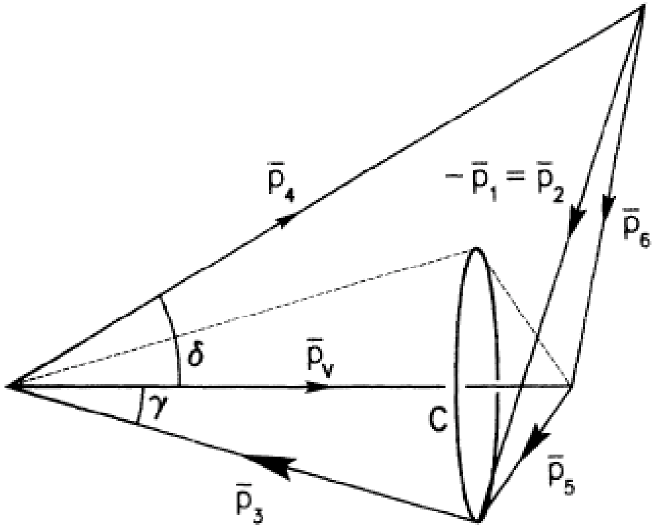
Hinchliffe et al.

If we expect that there are two missing particles in the event, we might try to partition the missing momentum into two parts.

The first analysis of this type was done for the process

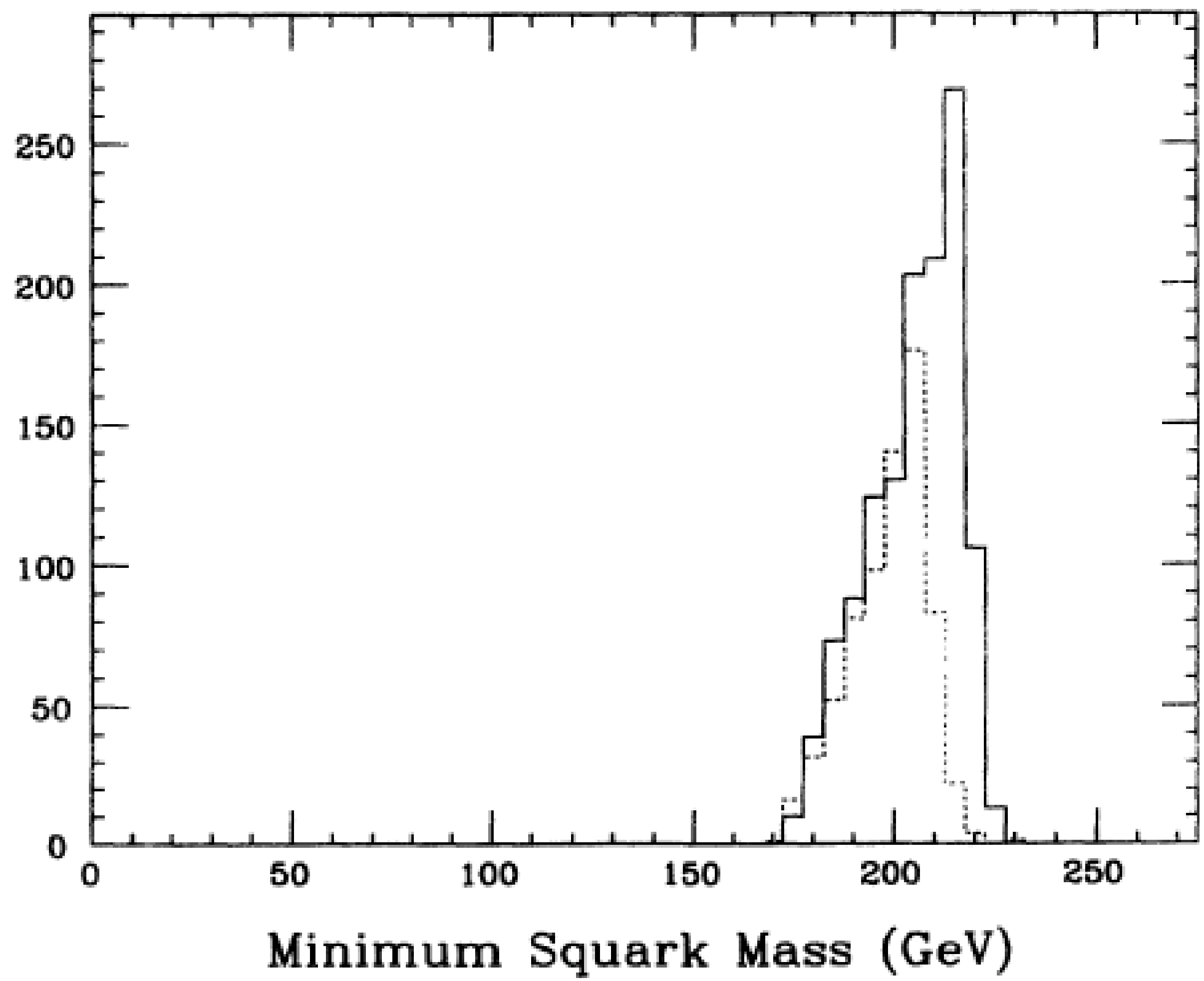
$$e^+e^- \to \widetilde{q}\widetilde{q}^* \to qN_1\overline{q}N_1$$

by Feng and Finnell. In this case, the momenta of the quark jets is measured, and the squark energies are known from the beam energy. Then the magnitudes of the neutralino energies are known. The missing momentum is measured. So it is only necessary to find the orientation of the neutralino momentum vectors. These must lie on the circle C as shown:



We can then find the point on this circle that gives the minimum value of the reconstructed squark mass. This is a lower bound on the actual squark mass.

"...momentum vectors lying on large circles C may give mass minima both close and far from the actual squark mass .. However, small circles give only accurate solutions, and thus the calculated minimum masses preferentially lie close the actual underlying squark mass." - Feng and Finnell



Lester and Summers have suggested a similar analysis in the hadron collider environment. They considered

$$pp \to \widetilde{l}^+ \widetilde{l}^- \to l^+ N_1 l^- N_1$$

Because we do not know the frame of the parton-parton collision, work with transverse momenta only.

A mass estimate from transverse momenta is the transverse mass

$$m_T^2(12) = m_1^2 + m_2^2 + 2(E_{T1} E_{T2} - \vec{p}_{T1} \cdot \vec{p}_{T2})$$

where $E_T = (p_T^2 + m^2)^{1/2}$. This is a lower bound to the actual mass

$$m^2(12) = m_1^2 + m_2^2 + 2(E_{T1} E_{T2} \cosh(\eta_1 - \eta_2) - \vec{p}_{T1} \cdot \vec{p}_{T2})$$

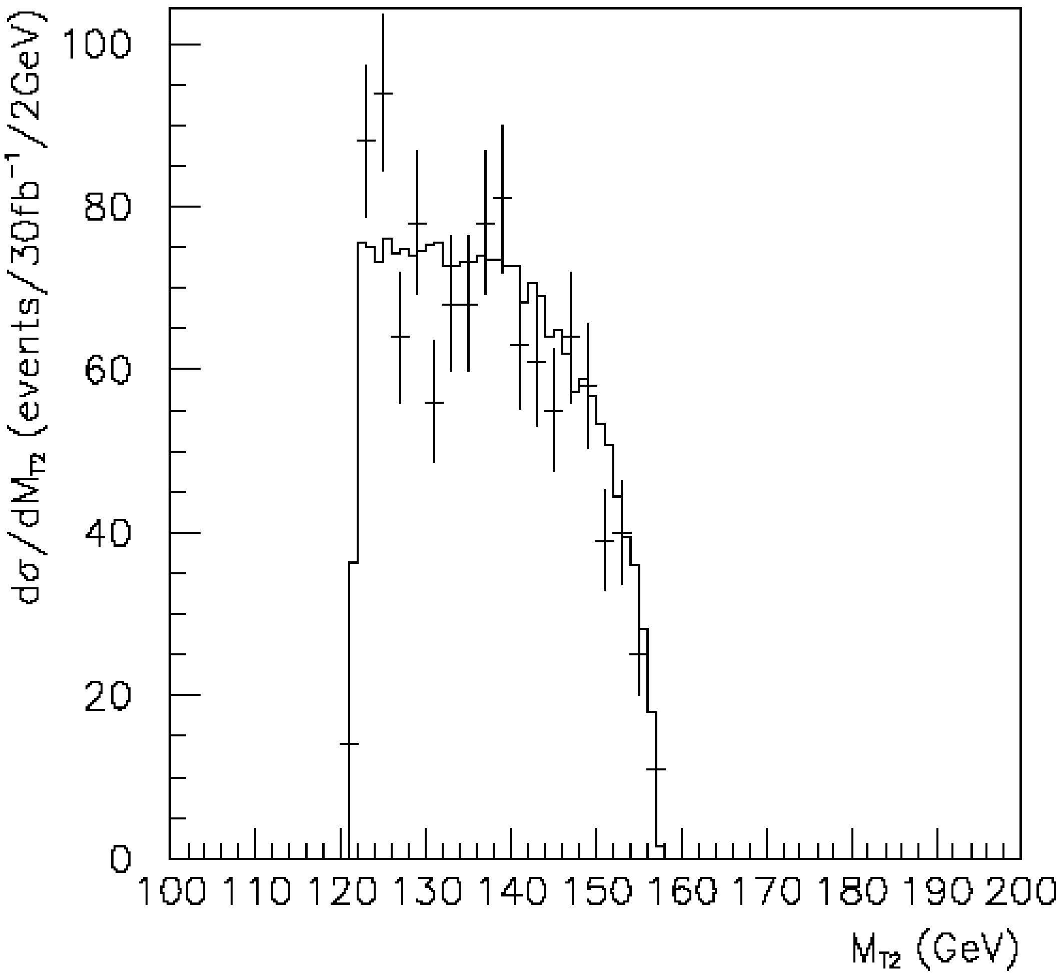
Assume that we knew the mass of the neutralino. Then if we also knew the transverse momentum of the neutralino, we could estimate

$$m(\widetilde{l})^2 \geq m_T^2(\ell N_1)$$

Since we only know the sum of the two neutralino momenta, we need to partition these two momenta in an arbitrary way. Some partition will still give a bound. Then

$$m(\widetilde{l})^2 \geq M_{T2}^2 \equiv \min_{\not{p}_T = 1+2} \left[\max(m_T^2(l,1), m_T^2(l,2)) \right]$$

The distribution of m_{T2} has a sharp endpoint at the correct value of $m(\widetilde{l})$ (assuming that we have input the correct neutralino mass):



Kawagoe, Nojiri, and Polesello and Cheng et al have discussed other methods that partition the measured missing momentum.

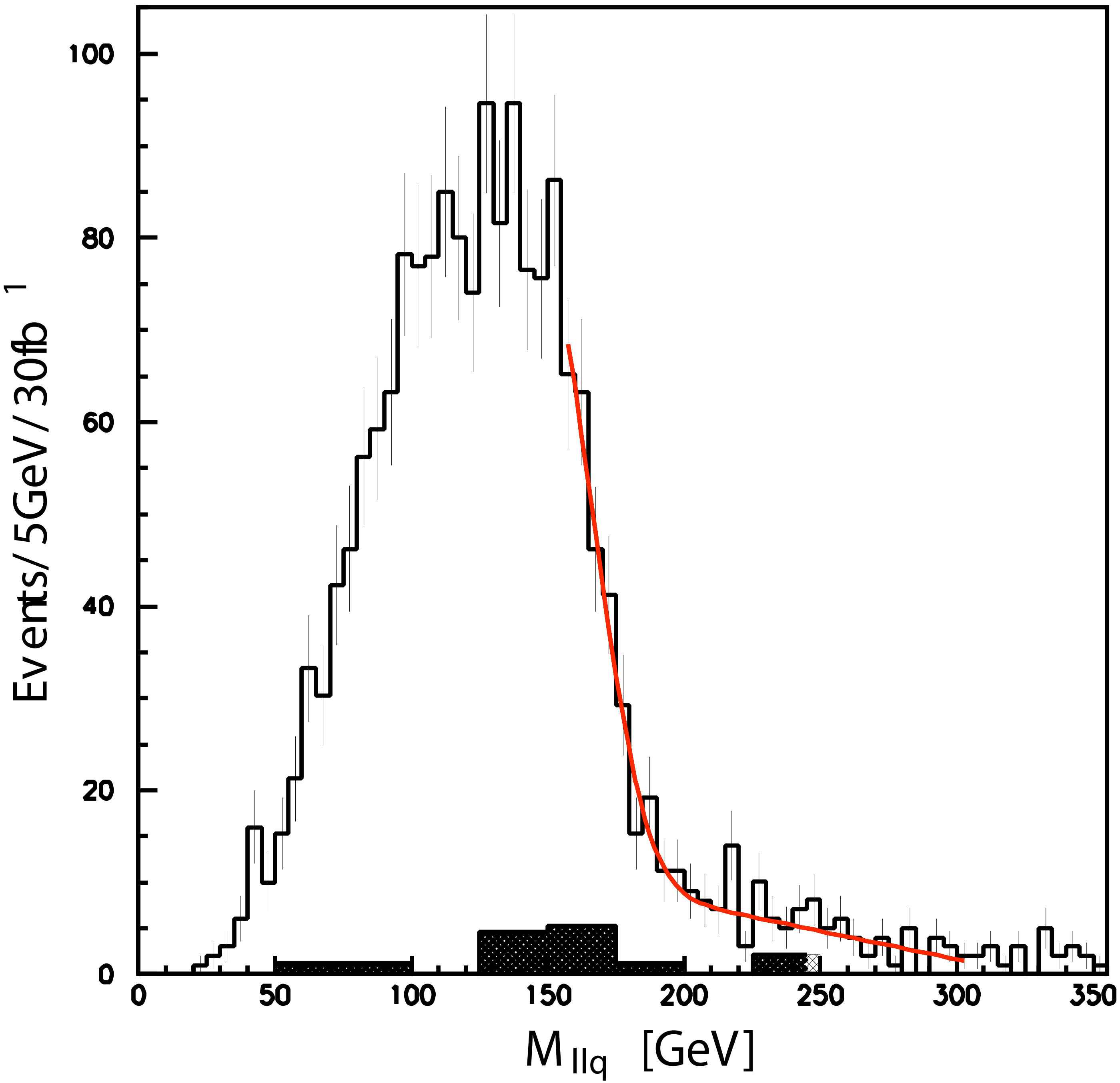
The methods I have discussed can be even more powerful in combination. Kitano and Nomura have tried a squark reconstruction by combining the variables:

$$min_{1,2}\{m(\ell\ell j)\}$$

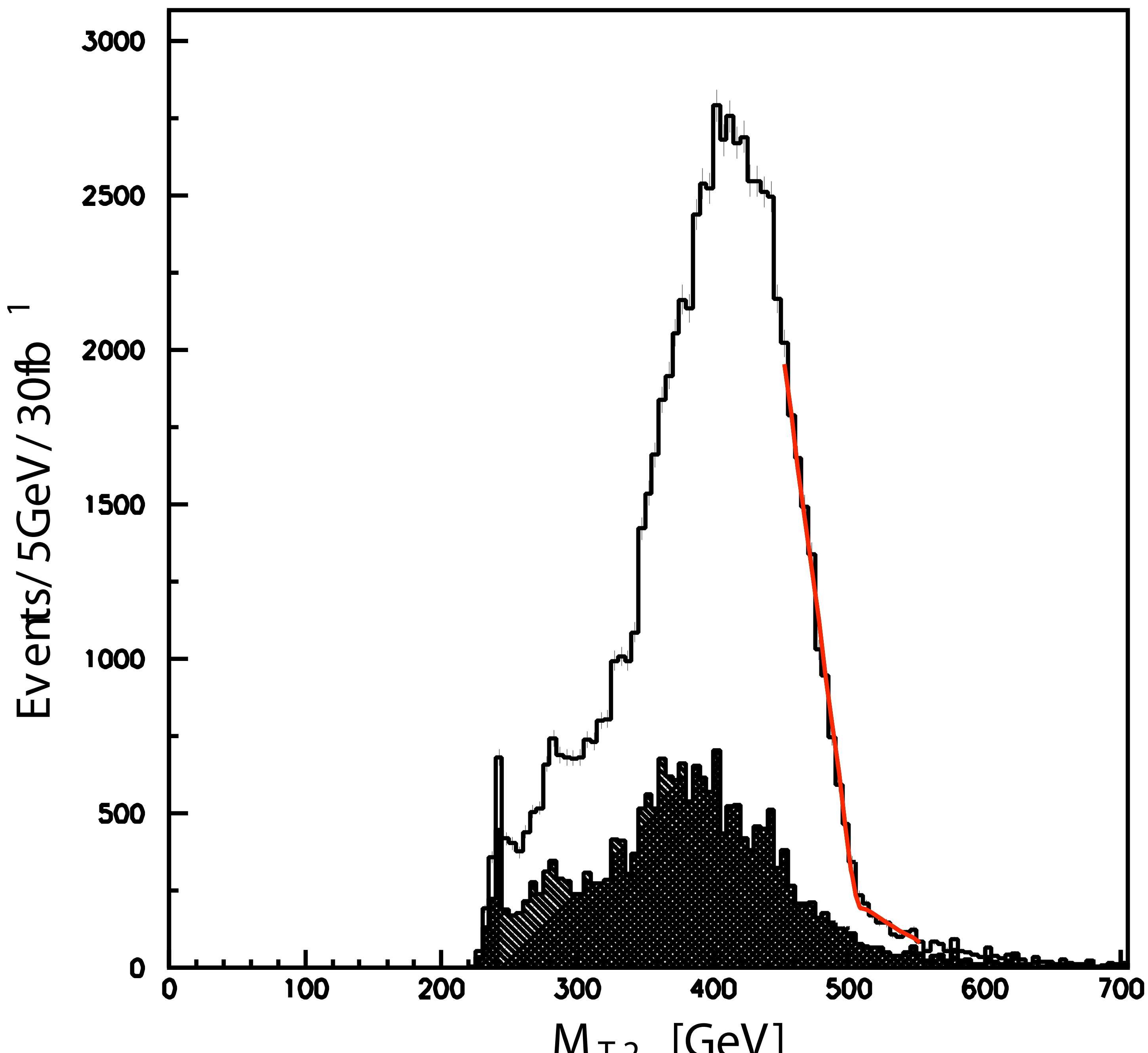
multibody endpoint

$$M_T^2 = min_{(\not{p}_{T1}+\not{p}_{T2}=\not{p}_T)} \; max\{m_T^2(p_1\ \not{p}_1), m_T^2(p_2\ \not{p}_2)\}$$

Lester and Summers

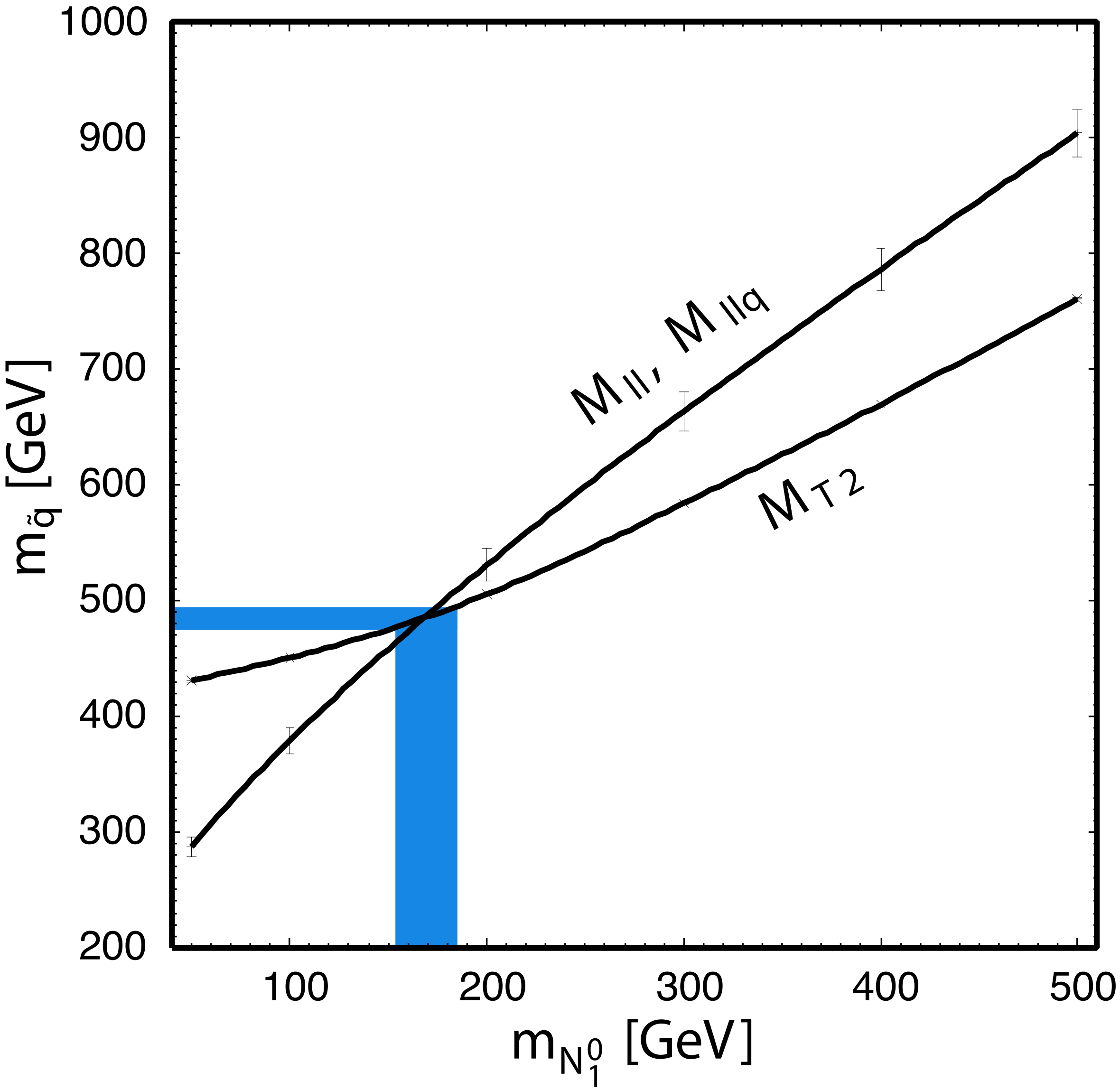


Kitano-Nomura



Kitano-Nomura

The endpoint positions have a different functional dependence on the squark and neutralino masses.   Demand consistency:



$$m_{N_1} = 169 \pm 17 \text{ GeV} \quad m_{\tilde{q}} = 486 \pm 11 \text{ GeV}$$

Kitano-Nomura

In these examples, we are obtaining rather precise SUSY particle masses, at the level of

10% or below for WIMP, squark, gluino masses

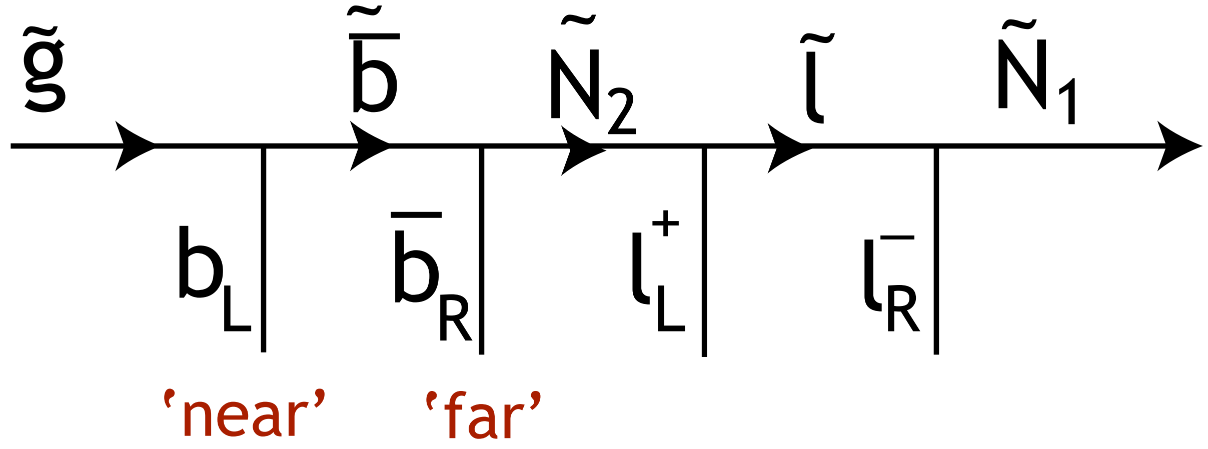
1% for mass differences in l+l- cascades

It is more subtle to obtain information on the spins and electroweak quantum numbers of new particles discovered at the LHC.

The most important information comes from the value of the pair production cross section.

In special cases, there are also asymmetries that directly test spin/chirality assignments.

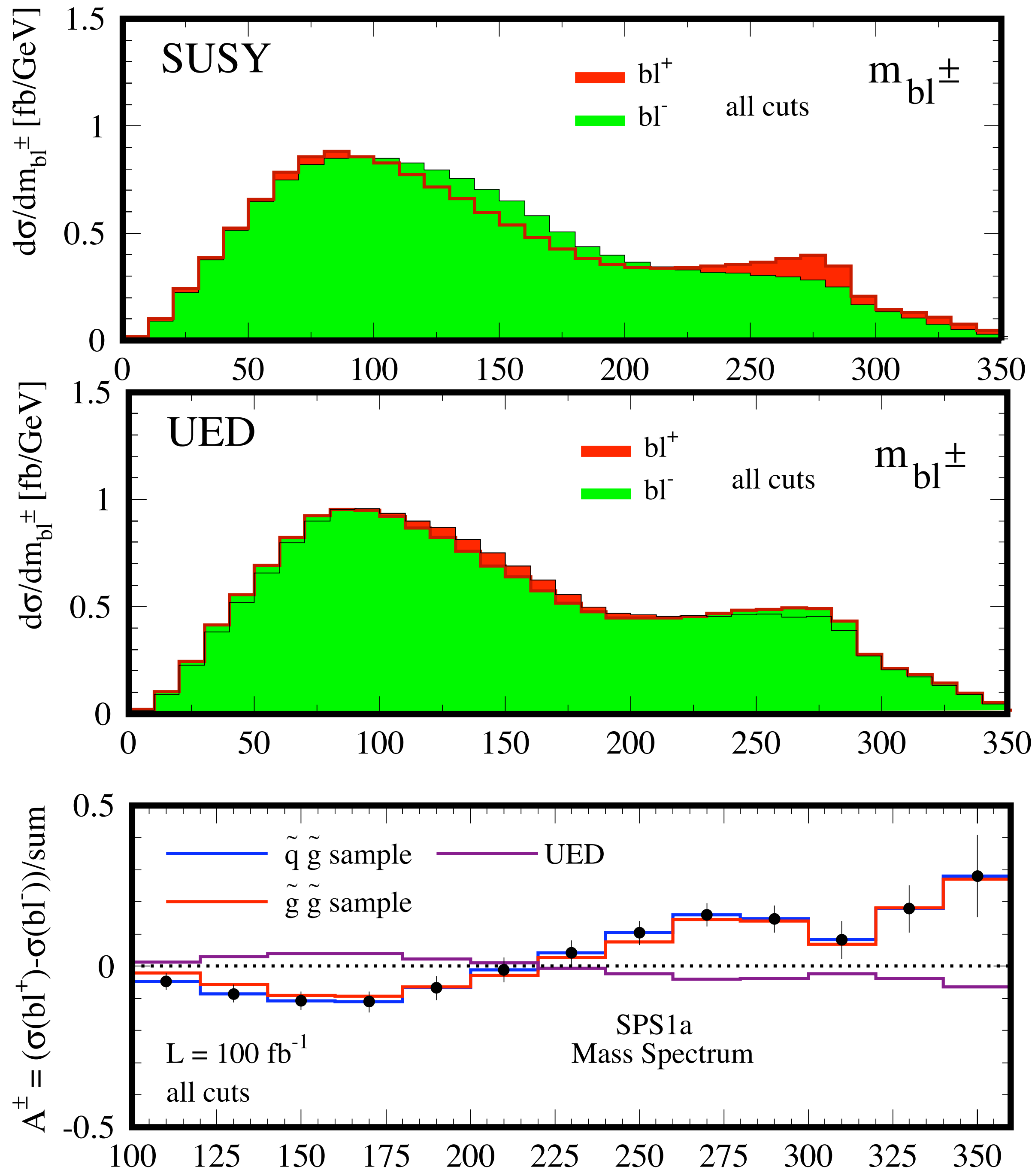
Here is an example: Consider the gluino decay scheme at a particular SUSY parameter point with the decay chain



Alves, Eboli, and Plehn have shown that the spin correlations generate distinctly different mass distributions for

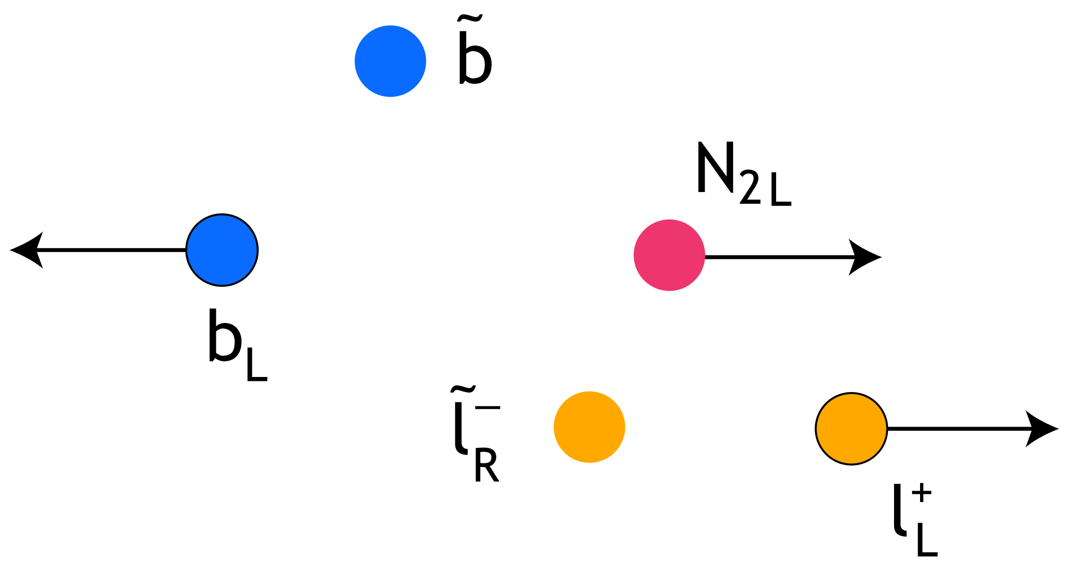
$$m(b\ell^+) \text{ vs. } m(b\ell^-)$$

The large effect comes from the far b in gluino decay paired with the near l from N decay. At the chosen spectrum point, these are the most energetic products.



Alves, Eboli, Plehn

In SUSY, the difference comes from a spin correlation in the sbottom decay noted by Barr and Kawagoe, Nojiri, Polesello:



This is very tricky: The effect comes from the fact that the $\tilde{b}$ and $\tilde{\ell}$ partner specific helicity states of b and l.

But it is important also that $\tilde{b}$ is a scalar. This leads to 100% spin correlation in its decay.

I cannot finish a discussion of SUSY mass measurements without discussing the stage in the program after the LHC.

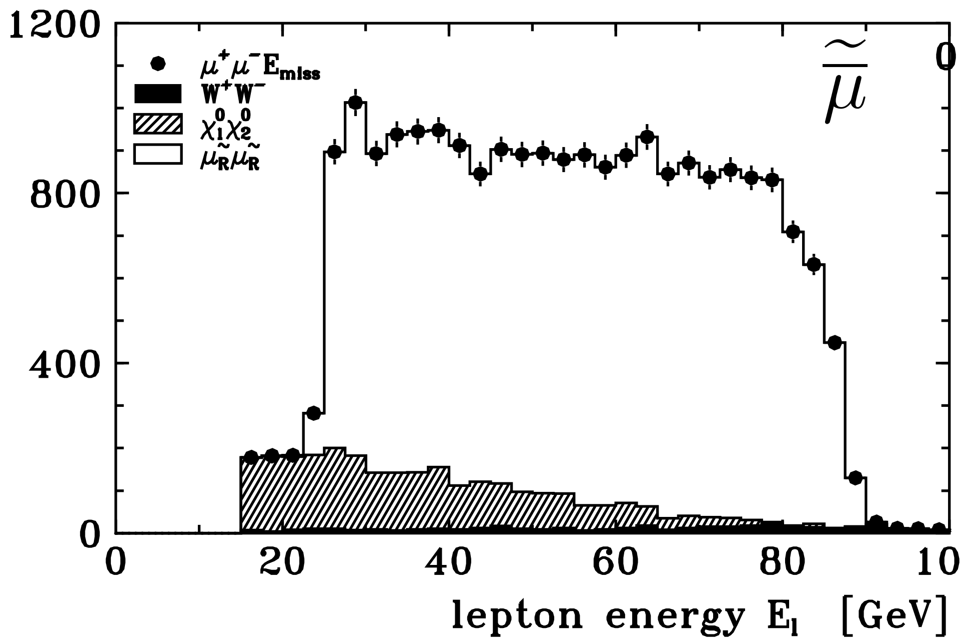
Eventually, the LHC data will be supplemented by data on SUSY particle production in electron-positron collisions from the International Linear Collder.

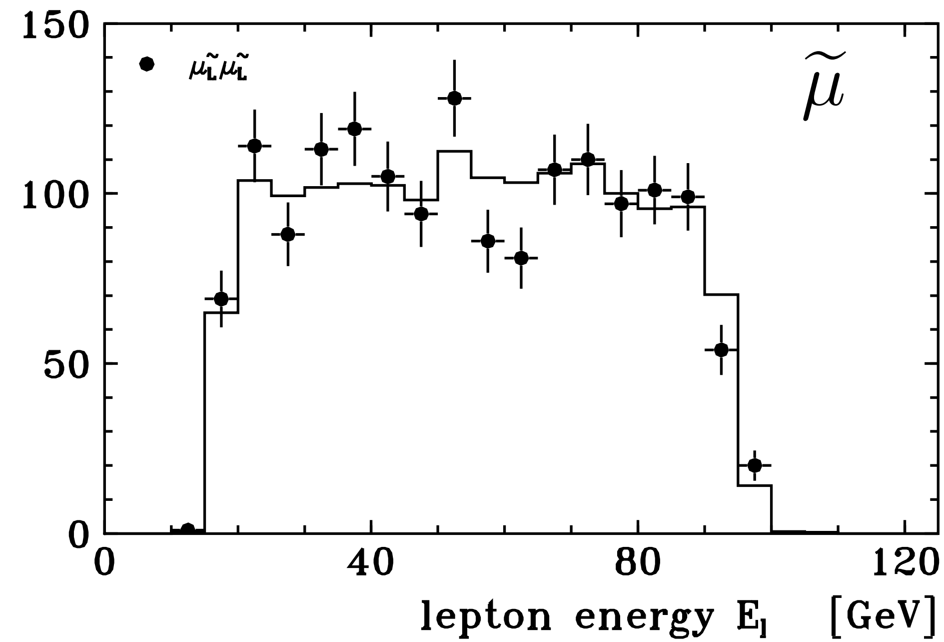
Here the center of mass energy is fixed, the events are simpler in character and easier to analyze, and initial-state polarization is available as an incisive probe of spin and of the mixing of weak-interaction eigenstates.

Here is are two examples of muon energy distributions from

$$e^+e^- \rightarrow \widetilde{\mu}^+\widetilde{\mu}^- \rightarrow \mu^+N_1\mu^-N_1$$

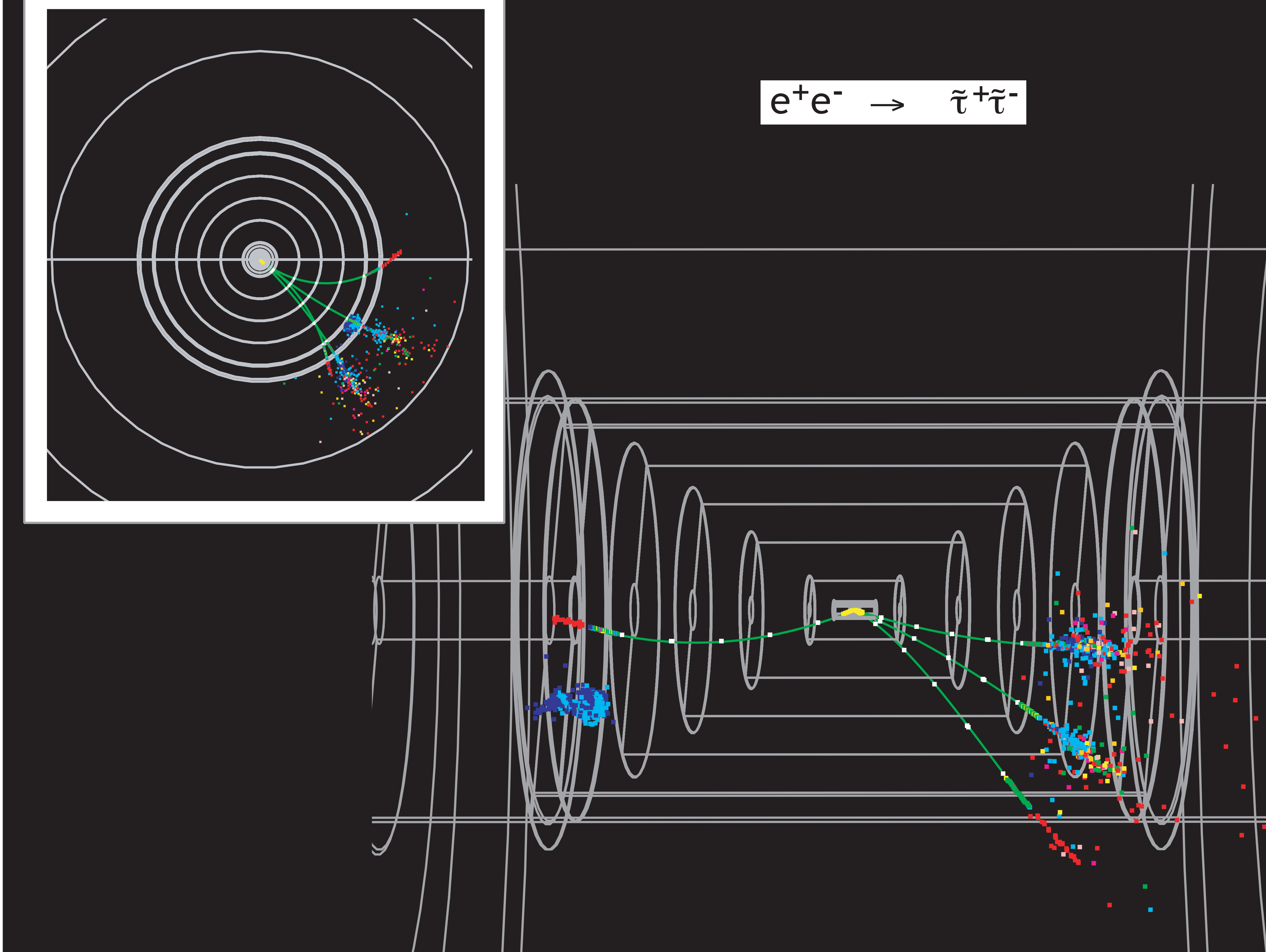
presented in the TESLA simulation studies:



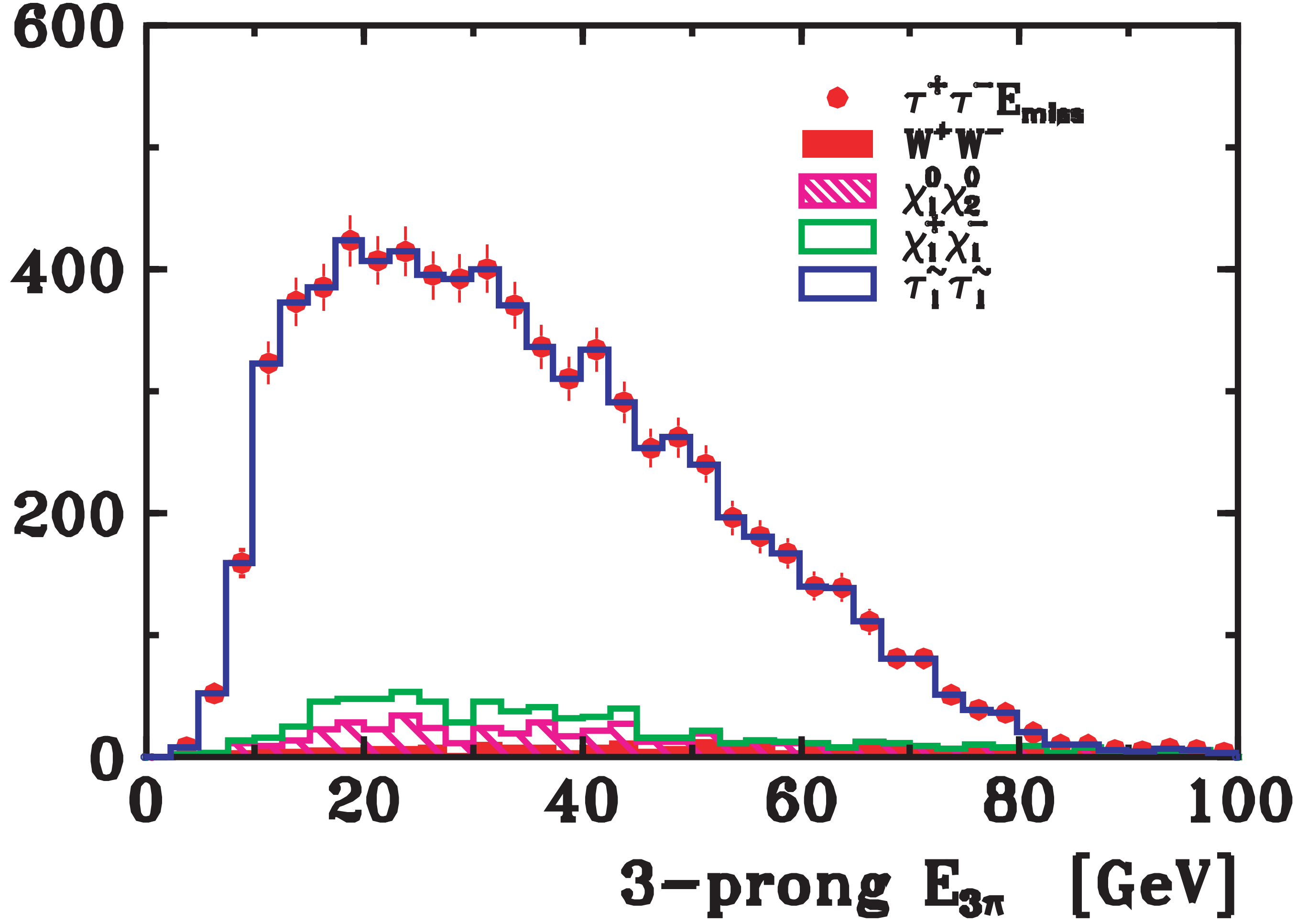


Blair and Martyn

It is expected that these masses could be measured at a next-generation e+e- collider (ILC) to a few hundred MeV (parts per mil).


$e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$

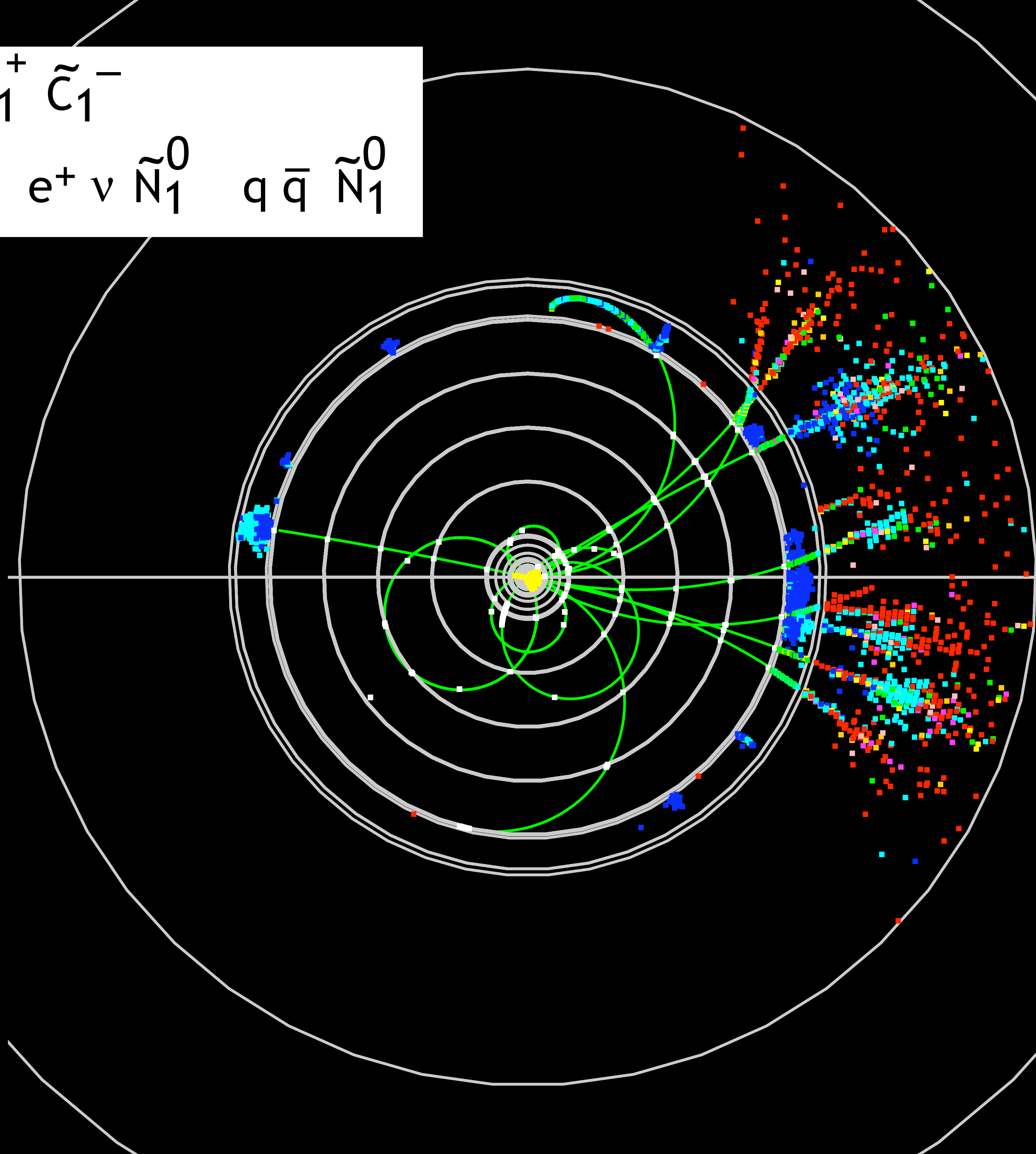
Here is the energy spectrum of visible decay products in the stau case. The kinematic endpoints are still well-defined.



Blair and Martyn

$$e^+e^- \rightarrow \tilde{C}_1^+ \, \tilde{C}_1^-$$

$$\rightarrow \; e^+ \nu \, \tilde{N}_1^0 \quad q \, \bar{q} \, \tilde{N}_1^0$$



So you see that there will also be a high-precision era of SUSY particle measurements which has the potential to bring us direct measurements of the SUSY Lagrangian parameters.

The direct experimental study of SUSY will soon begin. Don’t miss the opportunities it will present !