PiTP Homework for Kachru lectures

1. Show that one can obtain the Randall-Sundrum metric by truncating the metric of N D3-branes to the near horizon limit and forgetting the "extra" five dimensions (and, perhaps, performing some Z_2 identifications).

2. Consider a scalar field propagating in the metric

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

with μ running over four Minkowski directions. Let y run between 0 and πR, as in the Randall-Sundrum models.

a) Consider a bulk scalar field with mass $m^2 = ak^2$. What is its equation of motion?

b) Using separation of variables, find the dependence of bulk solutions on the y coordinate.

c) In absence of boundary terms, there is no zero mode. Show that if one adds an appropriate boundary mass at both $y = 0$ and $y = \pi R$, one can find a bulk zero mode.

d) Where is the bulk zero mode "localized" in the 5th dimension? (This depends on the mass....).

3. Consider p anti-D3 branes in the Klebanov-Strassler geometry.

a) Using the DBI action and the form of the background fluxes and metric, derive their potential function if they are coincident and localized at the IR tip of the geometry.

b) Enumerate the critical points of the near-tip potential.

c) Which vacua are tachyonic and which are potentially (meta)stable, at the level of this analysis?

4. Let us return to the scalar ("sfermion") wavefunctions of problem 2.

a. Show that the background metric in the slice of AdS can be re-written as

$$ds^2 = A^2(z)(-dt^2 + d\mathbf{x}^2 + dz^2)$$

with

$$A(z) = \frac{1}{z^2}.$$

b. Assume $A^2(z)$ is deformed to $A^2(z) \to A^2(z) + \delta A^2$, where

$$\delta A^2(z) = -\frac{1}{z^2}(z^4/z_*^4)$$

for some z_*. Assuming that there was a zero mode for the "sfermion" before including this perturbation, localized at some $z' < z_*$, what is the (approximate) sfermion mass after including the perturbation to the metric?