

# **D-BRANE MODEL BUILDING, PART I**

**Luis E. Ibáñez**

Instituto de Física Teórica (IFT-UAM/CSIC)

Universidad Autónoma de Madrid

*PITP School, IAS Princeton, July 2008*

# I-INTRODUCTION

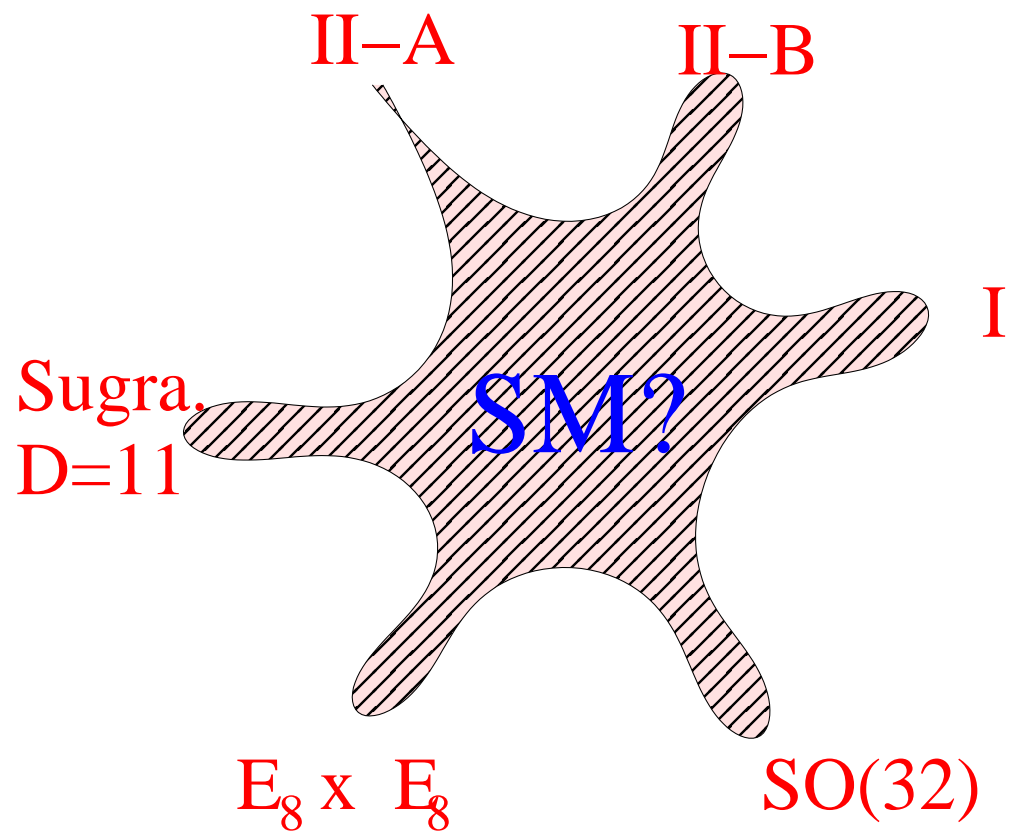
## String Phenomenology

- The **String Model-Building Program** aims at a general study of **possible compactifications/constructions** giving rise to a low-energy theory **resembling as much as possible the SM (or the MSSM)** .
- **Hope:**
  - If completely realistic models are found, this would provide a **proof that string theory may be a unified theory of all particles and interactions**
  - In our way we may **identify general patterns** (e.g. symmetries, extra particle content etc.) which could be present in large classes of realistic vacua.
  - **Get new ideas in addressing old problems** like e.g. CP violation, fermion masses and mixings, dark matter/energy,...
  - **Obtain, if possible, predictions** which could be tested experimentally (e.g. **SUSY-breaking MSSM soft terms, dark matter,..**)

- It is conceivable that only very few classes (if any) of compactifications will be able to fit all experimental data. If that were the case, it would be also likely that other new **testable predictions could be derived** from these.
- This may be a formidable task but **we believe we now have a much more accurate view of the Flavor Landscape** of semirealistic compactifications.
- In addition there is now an extra motivation: **With the advent of LHC we have to explore whether experimental information can provide tests of string vacua!**
- As physicists we have to try and check **what information on possible string compactifications may be extracted from LHC data.**

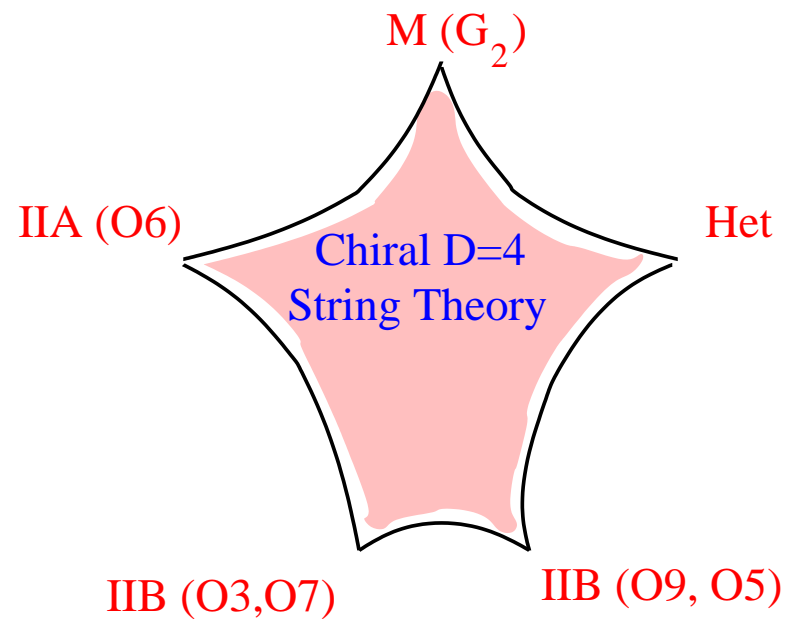
## The M-theory Landscape

- Where is the SM



## The Chiral M-theory Landscape

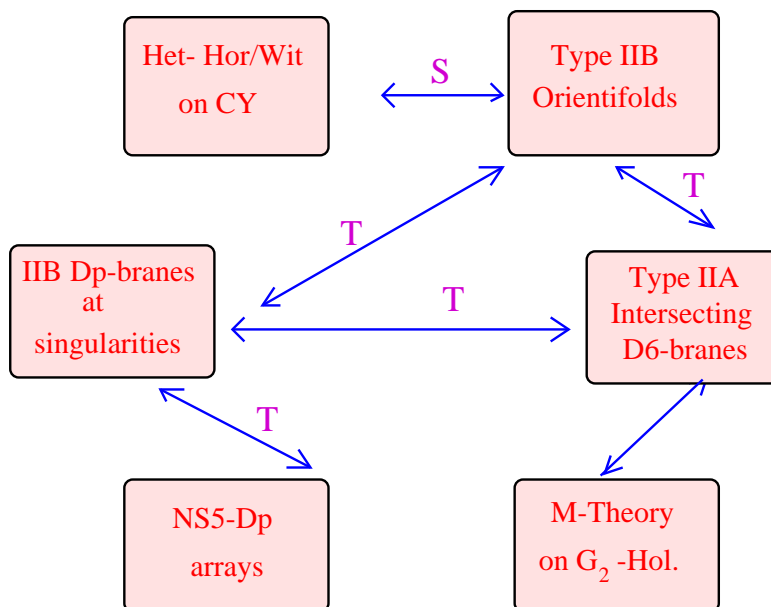
- The deepest property of the SM is its chirality
- We have to look for string vacua which are chiral in 4D.



- There is a number of different ways for obtaining chiral string vacua in string theory. They may be summarized in the five general classes in the figure.

## Connections among chiral vacua

- Different dualities connect connect the different classes of vacua. So a given vacuum may be sometimes studied in different limits.

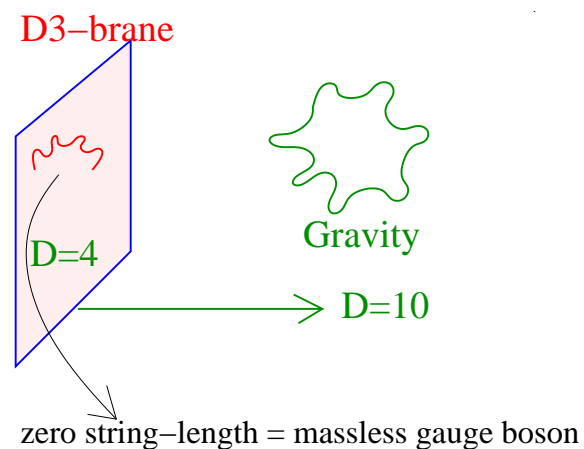


### CHIRALITY IN STRING COMPACTIFICATIONS

- In these lectures we are going to concentrate on **IIA and IIB orientifold vacua with the SM living on  $Dp$ -branes**

## Some basic $D_p$ -brane properties revisited

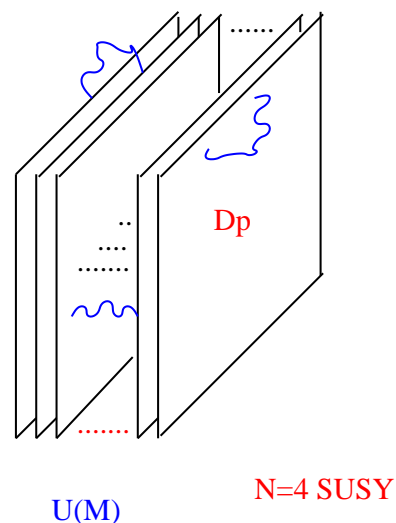
- They correspond to solitonic solutions of Type II string theory. But for us they will be mostly a subspace of  $D = 10$  where open strings may end and start:



- We want those open strings to describe the SM so that  $D_p$  branes must contain Minkowski space
  - Type IIA:  $D_4$ ,  $D_6$ ,  $D_8$
  - IIB:  $D_3$ ,  $D_5$ ,  $D_7$ ,  $D_9$

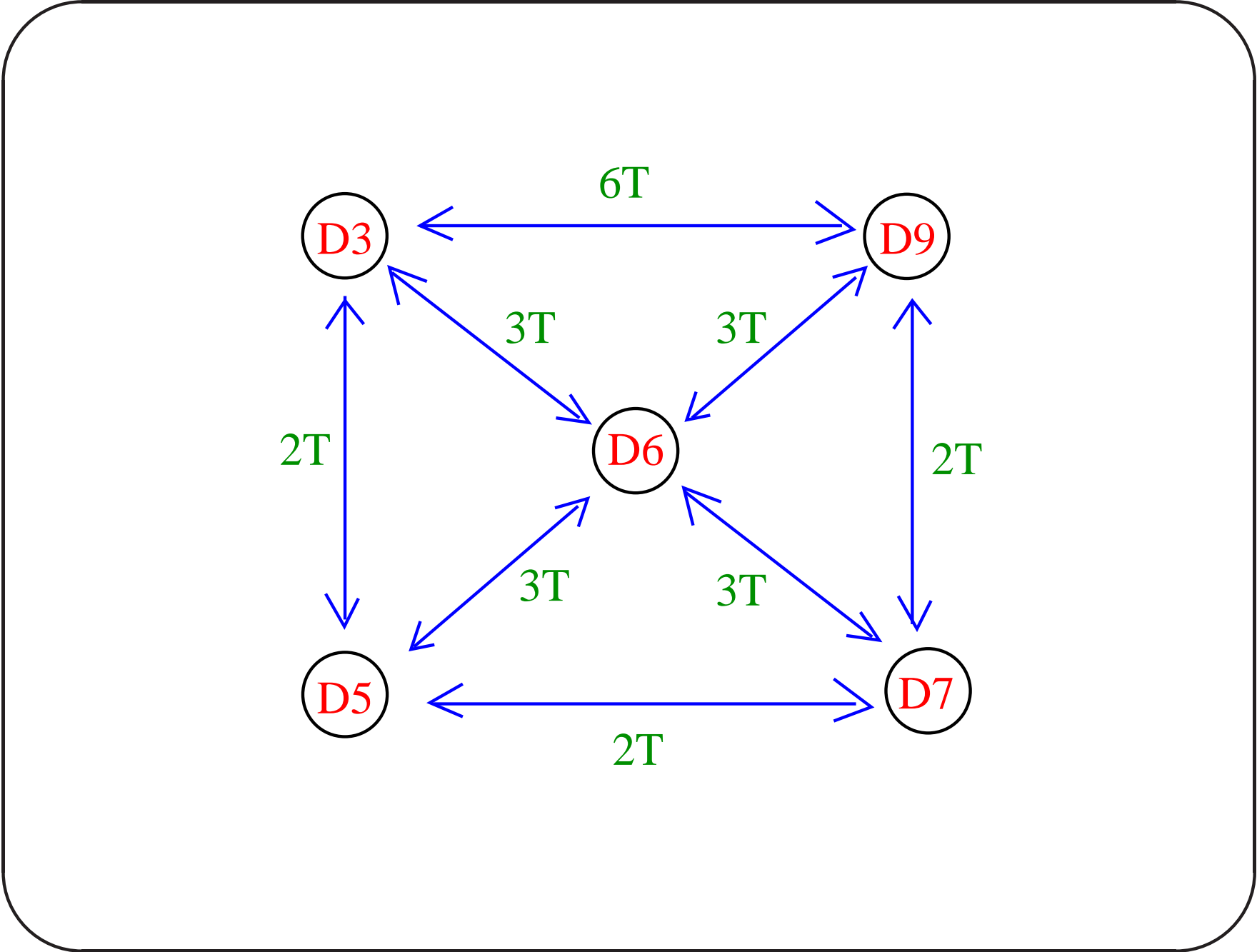


- $Dp$ -branes are **charged under antisymmetric RR tensors with  $(p + 1)$  indices** present in the massless spectrum of II string theory.
- The **overall RR charge in a given brane configuration has to cancel in a compact space**. This leads to constraints on vacua and also to **anomaly cancellation**.
- D-branes contain the gauge and matter degrees of freedom.  $M$  of them in flat space have  $U(M)$  gauge symmetry with  $N = 4$  SUSY



- This is not chiral. In order to get **chirality**:

- Brane intersections
- Magnetized branes
- Branes at singularities
- There are also non-dynamical extended objects, orientifold  $O_p$ -planes.
  - Type IIA:  $O6$
  - Type IIB:  $O3, O5, O7, O9$
- They have negative tension and RR charge. They are generically needed to obtain consistent Minkowski vacua  $\rightarrow$  IIA and IIB orientifolds.
- There is a Mirror Symmetry which exchanges IIA and IIB compactifications.
- In simple cases it corresponds to T-duality. T-dualities in toroidal/orbifold settings exchange Neumann and Dirichlet boundary conditions.
- An odd number of T-dualities exchanges IIA and IIB and the dimensionality of  $Dp$ -branes changes accordingly

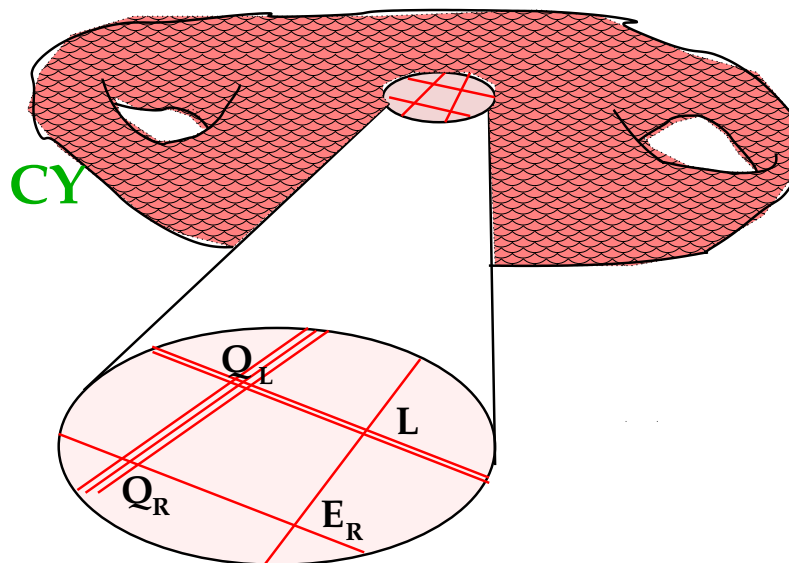


## Some basic rules for model building

- In order to obtain interesting classical string vacua:
  - One starts with **Type II theory compactified on a CY**. (In many of our examples we will consider tori or toroidal orbifolds).
  - **One considers distributions of  $Dp$ -branes containing Minkowski space**. If they have  $N = 1$  **supersymmetry they will be perturbatively stable**. The branes wrap or are located in specific regions of the CY.
  - Appropriate  **$Op$  planes will also in general be required to cancel the vacuum energy**. This will require to have rather a **CY orientifold**.
  - The brane distribution is so chosen that the **massless sector resembles as much as possible the SM or the MSSM**.

## Global versus local models

- One may consider two approaches:
  - **Global models**. One insists in having a complete compact CY compactification with e.g. RR tadpole cancellation, consistent at the global level.
  - **Local models**. One considers local sets of lower dimensional  $Dp$ -branes,  $p \leq 7$  which are localized on some area of the CY and reproduce SM physics. One does not then care about global aspects of the compactification and assume that eventually the configuration may be embedded inside a fully consistent global model.
- The latter is often called the **bottom-up approach**, since one first constructs the local (bottom) model and eventually may embed it inside a variety of global models.



- This **bottom-up approach is not available in heterotic or Type I models** since the **SM fields live in the bulk 6 dimensions of the CY**.
- In principle **a globally consistent compactification is more satisfactory**. On the other hand **local configurations of Dp-branes may be more efficient in trying to identify promising string vacua, independent of details of the global theory**.

## Topics to be discussed

- **1)** Introduction.
- **2)** Type IIA orientifolds and intersecting D6-brane models.
- **3)** Type IIB orientifolds and magnetized branes.
- **4)** The effective action
- **5)** D3-branes at singularities.
- **6)** Superpotential interactions from stringy instantons.
- **7)** Fluxes, SUSY-breaking soft terms, LHC.

## II- TYPE IIA ORIENTIFOLDS AND INTERSECTING D6-BRANES

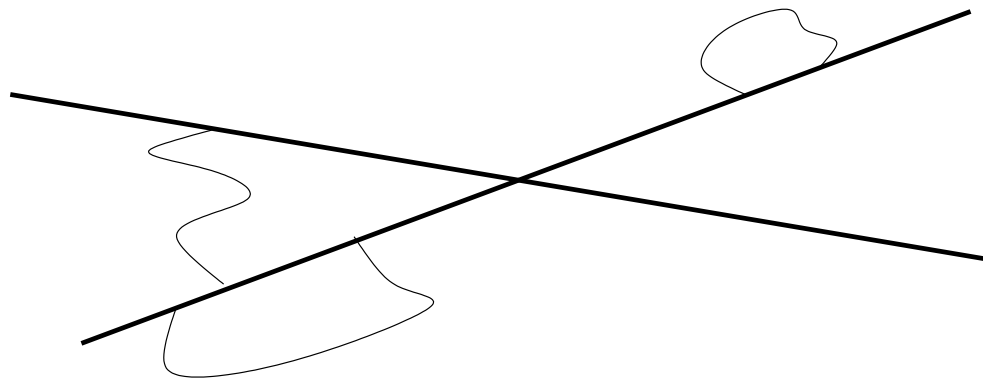
- F. Marchesano, “Progress in D-brane model building,” arXiv:hep-th/0702094
- A. M. Uranga, “The standard model in string theory from D-branes,” Nucl. Phys. Proc. Suppl. **171** (2007) 119. “Chiral four-dimensional string compactifications with intersecting D-branes,” Class. Quant. Grav. **20** (2003) S373 [arXiv:hep-th/0301032]
- R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, “Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,” Phys. Rept. **445** (2007) 1 [arXiv:hep-th/0610327]
- R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, “Toward realistic intersecting D-brane models,” Ann. Rev. Nucl. Part. Sci. **55** (2005) 71 [arXiv:hep-th/0502005]
- F. G. Marchesano, “Intersecting D-brane models,” [arXiv:hep-th/0307252]
- D. Cremades, L. E. Ibáñez and F. Marchesano, “More about the standard model at intersecting branes,” arXiv:hep-ph/0212048.



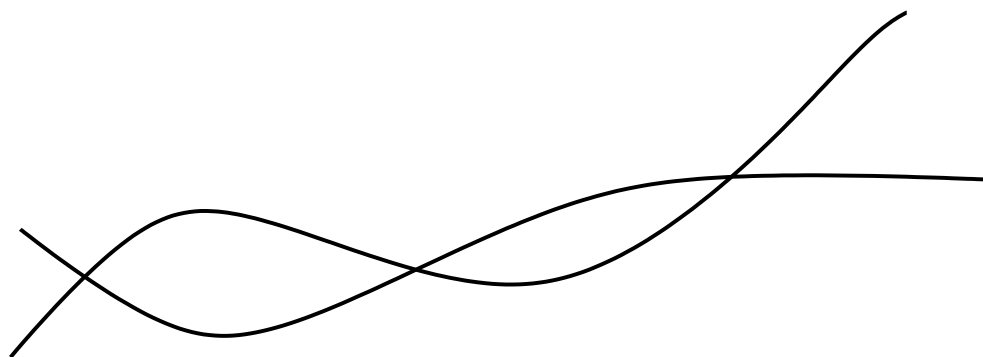
## Why intersecting branes?

They have a number of properties present in the **SM** :

- ① **Gauge group**: Each stack of  $N$  branes carries a  $U(N)$  gauge theory.
- ② **Chirality**: Two intersecting branes present chiral fermions at their intersection, transforming in bifundamental  $(N, \bar{M})$  or  $(N, M)$ .

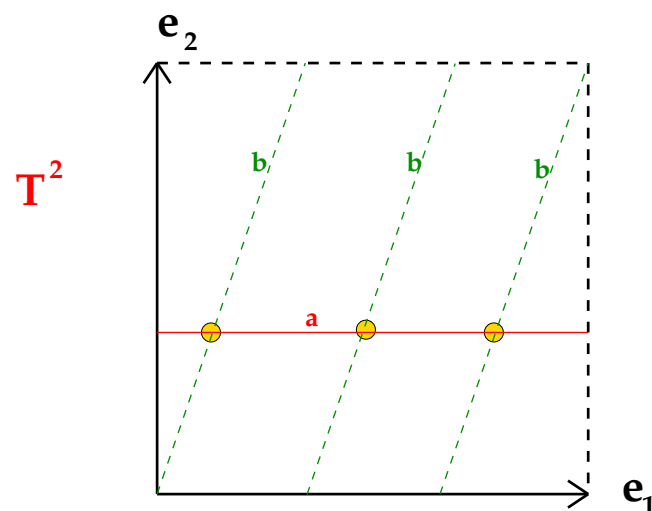


- ① **Family replication:** Branes at angles wrapping a compact manifold may intersect several times.



## Example of triplication

- Consider a pair of D-branes wrapping a cycle on a 2-torus. The torus is defined as  $T^2/\Lambda$ , with  $\Lambda$  generated by translations by the vectors  $\{e_1, e_2\}$ .



'a' and 'b' D-branes intersect at three points

- wrapping numbers  $(n_a, m_a) = (1, 0)$ ;  $(n_b, m_b) = (1, 3)$

### Why D6-branes?

- In **Type II** we have only **D4, D6, D8** which can contain  $M_4$  inside. They can wrap respectively **1-, 3- and 5-cycles in compact dimensions**. But Calabi-Yau compact spaces do not admit 1- or 5-cycles.: **in IIA only D6-branes do the trick**. (see however later..)

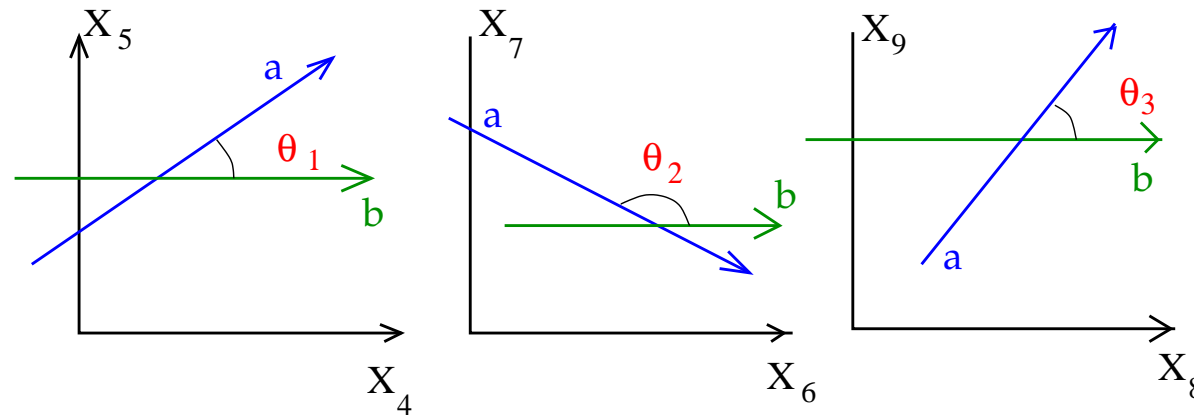
### Why orientifolds ?

- **D6-branes have positive tension**. They contribute to vacuum energy and then **it is not possible to get Minkowski vacua** . In the presence of an 'orientifold twist' **new contributions appear from 'orientifold planes' O6 with negative tension**:

$$V = T_{D6} + T_{O6} = 0 \quad (1)$$

- Furthermore **the  $Dp$ -branes are charged with respect to RR antisymmetric fields**. **Orientifolds contribute to cancel the RR charge** .

## Intersecting D6-branes in flat space



$$M_{ab}^2 = N + \frac{(r + r_\theta)^2}{2} - \frac{1}{2} + \sum_{i=1}^3 \frac{1}{2} |\theta_i| (1 - |\theta_i|) \quad (2)$$

where

$r_\theta = (\theta^1, \theta^2, \theta^3, 0)$  and  $r_i = \mathbf{Z}, \mathbf{Z} + \frac{1}{2}$  for NS, R sectors respectively. GSO projection imposes  $\sum_i r_i = \text{odd}$ .

## Chiral fermions

- In the R sector, after GSO projection **there is always one massless chiral fermion state:**

$$r + r_\theta = \left(-\frac{1}{2} + \theta_1, -\frac{1}{2} + \theta_2, -\frac{1}{2} + \theta_3, +\frac{1}{2}\right) \quad (3)$$

Indeed, **for any value of the angles:**

$$M_f^2 = \frac{1}{2} \left[ \left( \sum_{i=1}^3 \left(-\frac{1}{2} + \theta_i\right)^2 \right) + \frac{1}{4} \right] - \frac{1}{2} + \frac{1}{2} \left( \sum_{i=1}^3 (\theta_i - \theta_i^2) \right) = 0 \quad (4)$$

## Scalars

- There are three lightest (adjoint) scalars at each intersection with

$$\begin{aligned}
 &(-1 + \theta_1, \theta_2, \theta_3, 0) \\
 &(\theta_1, -1 + \theta_2, \theta_3, 0) \\
 &(\theta_1, \theta_2, -1 + \theta_3, 0)
 \end{aligned} \tag{5}$$

- They have masses **which depend on the angles** (in string units):

$$\begin{aligned}
 M_1^2 &= \frac{1}{2}(-|\vartheta^1| + |\vartheta^2| + |\vartheta^3|) \\
 M_2^2 &= \frac{1}{2}(|\vartheta^1| - |\vartheta^2| + |\vartheta^3|) \\
 M_3^2 &= \frac{1}{2}(|\vartheta^1| + |\vartheta^2| - |\vartheta^3|)
 \end{aligned} \tag{6}$$

- For wide choices of angles **scalars are non-tachyonic**. For particular choices there is a **massless scalar**, signaling the presence of  **$N = 1$  SUSY at that intersection** (see later).

## Intersecting brane worlds

- To construct an specific model one **compactifies type IIA down to four dimensions on a Ricci-flat manifold CY**. The theory has  $N = 2$  supersymmetry in  $D = 4$
- One further **constructs an orientifold** by modding the theory by  $\Omega\mathcal{R}$  with  $\Omega =$  worldsheet parity and  $\mathcal{R}$  a  $Z_2$  antiholomorphic involution in the CY with  $\mathcal{R}J = -J$  and  $\mathcal{R}\Omega_3 = \overline{\Omega_3}$ . The theory has now  $N = 1$  in  $D = 4$ .
- The submanifolds left invariant under the orientifold operation are **orientifold O6 planes**. They carry RR charge.
- Sets of **D6-branes may be added filling  $M_4$  and wrapping a 3-cycle in the CY**.
- In order for the configuration to be supersymmetric the 3-cycle the D6-branes must wrap '**Special Lagrangian Submanifolds**' (SLAG's). These SLAG's minimize the volume wrapped by the brane.

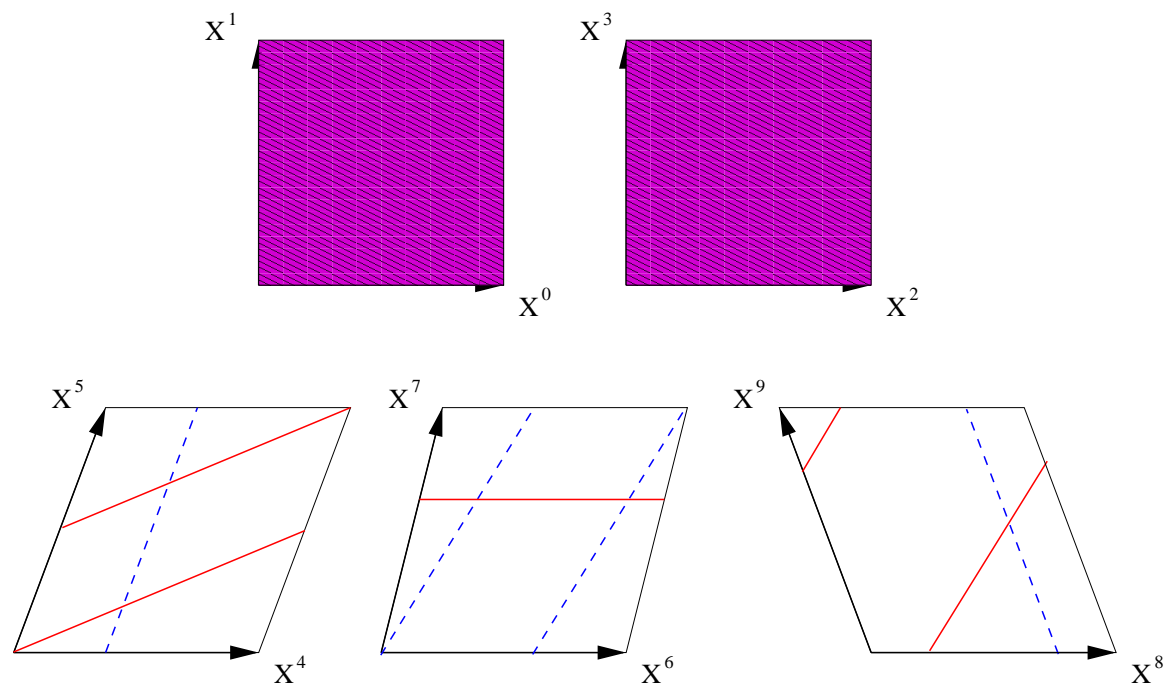


- In physical terms this implies **vanishing F- and D- terms in the effective Lagrangian**. Also  $D6$ -branes admit only flat gauge bundles in their **worldvolume** (i.e. Wilson lines, no fluxes).
- There are **massless chiral fields** at intersections of two branes  $D6_a, D6_b$  with **multiplicity given by the intersection number of the 3-cycles**  $I_{ab} = \Pi_a \cdot \Pi_b$ .
- Unfortunately **our knowledge of SLAG's in generic CY manifolds is very limited**. **In our examples we will work with either tori or toroidal orbifolds (or else with local configurations)**.
- Even in this simple class of theories **one can construct specific models with a chiral content quite close to that of the SM**.

## D6-branes wrapping at angles in $T^6$

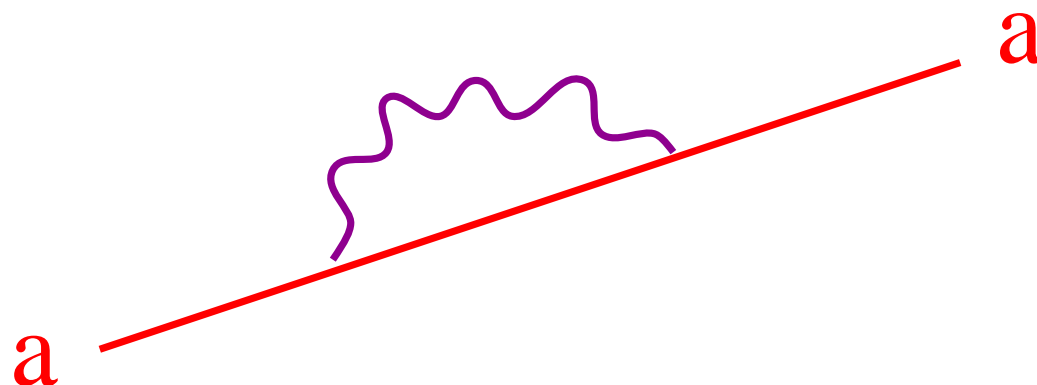
Setup: **type IIA D6-branes** filling  $M_4$  and wrapping **3-cycles** on  $T^6$ . Assume a factorized torus and 3-cycles:

$$T^6 = T^2 \times T^2 \times T^2 ; \text{ 3-cycle} = \text{1-cycle} \times \text{1-cycle} \times \text{1-cycle}$$

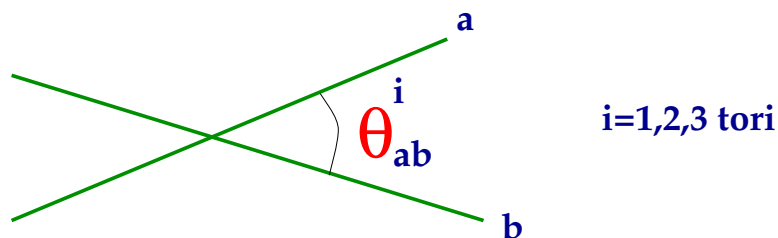


$$[\Pi_a] = (n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3) ; (n_a^i, m_a^i \text{ coprime})$$

## Light spectrum



- $D6_a$ - $D6_a$ . Massless  $U(n_a)$ .
- 3 sets of adjoint scalars and fermionic partners. Their vevs parametrize the position of the brane (real part) and a Wilson line along each 1-cycle (imaginary part).
- There are orbifold orientifolds in which the branes wrap rigid 3-cycles and these adjoints are absent. This would also be the generic case in a CY.



- $D6_a - D6_b + D6_b - D6_a$ . Chiral fermions transforming in the  $(N_a, \bar{N}_b)$  representation. Their multiplicity given by

$$\begin{aligned} \text{Intersection number: } I_{ab} &= I_{ab}^1 \times I_{ab}^2 \times I_{ab}^3 = \\ &= (n_a^1 m_b^1 - m_a^1 n_b^1) (n_a^2 m_b^2 - m_a^2 n_b^2) (n_a^3 m_b^3 - m_a^3 n_b^3) \end{aligned}$$

Note  $I_{ab} = -I_{ba}$ . Opposite signs means opposite  $D = 4$  chirality.

## Toroidal orientifolds

- Orientifold: moding the theory by a discrete operation involving the **worldsheet parity operator**  $\Omega$ ,  $\Omega(\tau, \sigma) = (\tau, -\sigma)$ .
- The simplest orientifold is **Type I theory**

$$\text{Type I} = \frac{\text{Type IIB}}{\Omega} \quad (7)$$

- It is an orientifold of Type IIB theory with **D9-branes**. The orientifold projection breaks  $U(32)$  to  $SO(32)$ .
- More generally **orientifolds involve a geometrical action**  $\mathcal{R}$ :  $\Omega\mathcal{R}$  for **lower dimensional Dp-branes**.  $\mathcal{R}$  may be obtained from Type I by T-duality. (There is also a  $(-1)^{F_L}$  factor for technical reasons which we will ignore).
- The submanifolds of the compact space left invariant by this action are called '**orientifold planes**'.

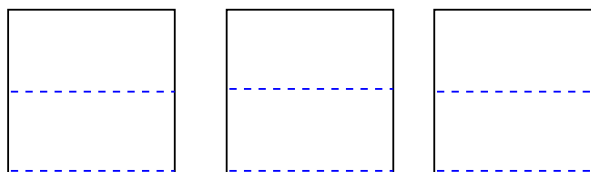
- In our case case we will be interested in the orientifold:

$$\frac{T^2 \times T^2 \times T^2}{\Omega\mathcal{R}} \quad (8)$$

with

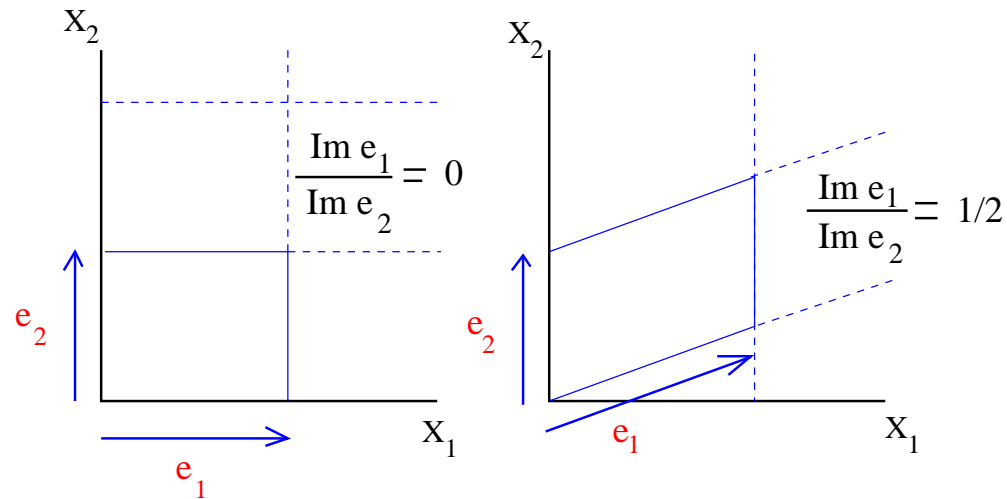
$$\mathcal{R} = \mathcal{R}_{(5)}\mathcal{R}_{(7)}\mathcal{R}_{(9)} ; \mathcal{R}_{(i)}X_i = -X_i \quad (9)$$

- We will have **invariant  $O6$ -planes**.



- This may be obtained by making 3 simultaneous **T-dualities along  $X_5, X_7, X_9$  compact directions** starting from Type I..

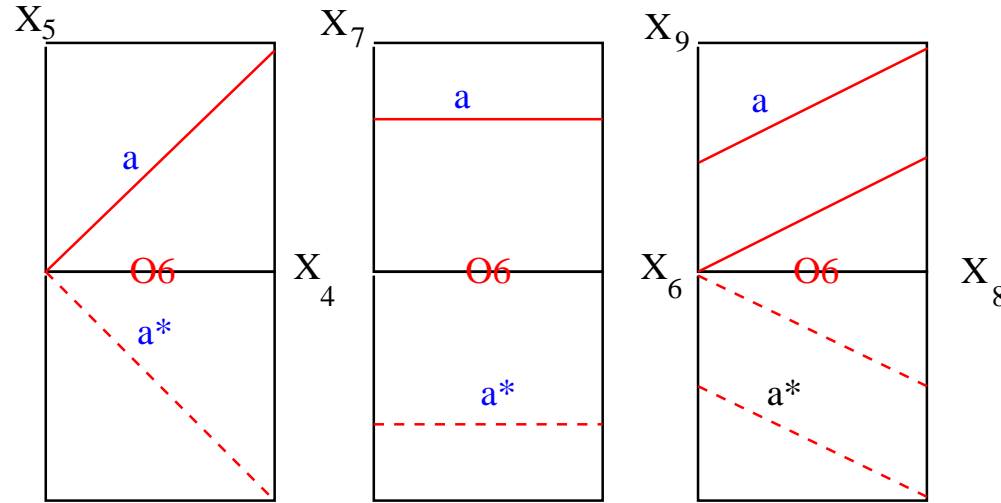
- Each torus defined as  $T^2/\Lambda$ , with  $\Lambda$  is a lattice generated by two complex numbers  $\{e_1, e_2\}$ .
- Only 2 complex structures of  $T^2$  are invariant, i.e., **square and tilted tori**:



- Still  $\text{Re } e_1 / \text{Im } e_2$  is arbitrary.
- Effective wrapping numbers for  $D6$ -branes:

$$(n^i, m^i)_{\text{eff}} \equiv (n^i, m^i) + b^{(i)}(0, n^i) ; \quad b^{(i)} = 0, 1/2 \quad (10)$$

- For the setting to be orientifold invariant, for each D6-brane  $a$ , we must add its mirror image  $a^*$  under  $\Omega\mathcal{R}$ .



- Then  $\Omega\mathcal{R}$  action reduces to

$$\begin{aligned}
 \text{D6 - brane } a &\mapsto \text{D6 - brane } a^* \\
 (n_a^i, m_a^i) &\quad (n_a^i, -m_a^i)
 \end{aligned}
 \tag{11}$$

- There are 8 reflection invariant O6-planes at



$(X_5; X_7; X_9) = (0, 1/2; 0, 1/2; 0, 1/2)$  for the square tori. I.e. there is an orientifold at the origin wrapping the 3-cycle :

$$[\Pi_{O_6}] = (1, 0) \times (1, 0) \times (1, 0)$$

- If some tori are tilted the invariant cycles are

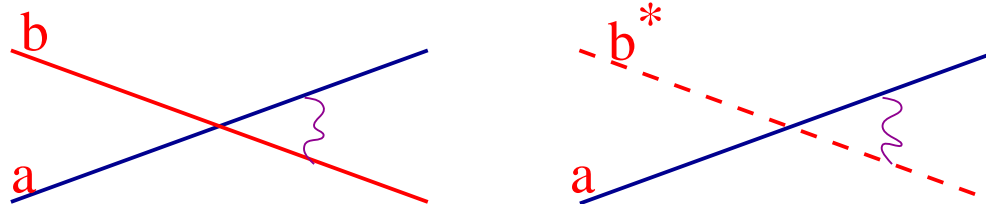
$$[\Pi_{O_6}] = (1/\beta_1, 0) \times (1/\beta_2, 0) \times (1/\beta_3, 0) ; \beta_i = 1/(1 - b_i) = 1/2$$

- For D6-branes with geometric configuration invariant under R  $U(N)$  is projected out to  $SO(N)$  or  $Sp(N)$ , depending on details of this projection.

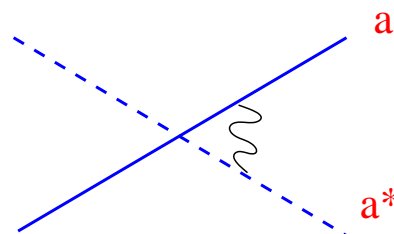
## New massless fermions in orientifolds

- $D6_a D6_b$  and  $D6_a D6_b^*$  sector: **bifundamental** fermion representations

$$I_{ab}(N_a, \overline{N}_b) + I_{ab^*}(N_a, N_b)$$



- $D6_a D6_a^*$  Brane intersection with its mirror



One get fermions in **symmetric**  $S_a$  and **antisymmetric**  $A_a$  of  $U(N)_a$ :

$$n_{A_a} = \frac{1}{2}(I_{aa^*} - I_{aO(6)}) ; n_{S_a} = \frac{1}{2}(I_{aa^*} + I_{aO(6)}) \tag{12}$$

## Tadpole cancellation conditions

- The D6-branes are charged under a RR 7-form  $C^{(7)}$ . Orientifold O6-planes have charge = -4. Overall  $C^{(7)}$  charge in a compact space should vanish:

$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_a^*] - 4[\Pi_{O6}] = 0 \quad (13)$$

- In terms of wrapping numbers one has ( $\mathcal{R}$  leaves  $2 \times 2 \times 2 = 8$  fixed orientifold planes)

$$\begin{aligned} \sum_a N_a n_a^1 n_a^2 n_a^3 &= 16 \\ \sum_a N_a n_a^1 m_a^2 m_a^3 &= 0 \\ \sum_a N_a m_a^1 n_a^2 m_a^3 &= 0 \\ \sum_a N_a m_a^1 m_a^2 n_a^3 &= 0 \end{aligned} \quad (14)$$

- These conditions automatically **guarantee anomaly cancellation**. Indeed, recalling that  $I_{ab} = [\Pi_a] \cdot [\Pi_b]$  and using tadpole cancellation (for the case with  $I_{aO6} = 0$  for simplicity):

$$[\Pi_a] \cdot \left( \sum_b N_b [\Pi_b] + \sum_b N_b [\Pi_b^*] - 4[\Pi_{O6}] \right) = \quad (15)$$

$$\sum_b N_b [I_{ab} + I_{ab^*}] = \#N_a - \#\bar{N}_a = 0 \quad (16)$$

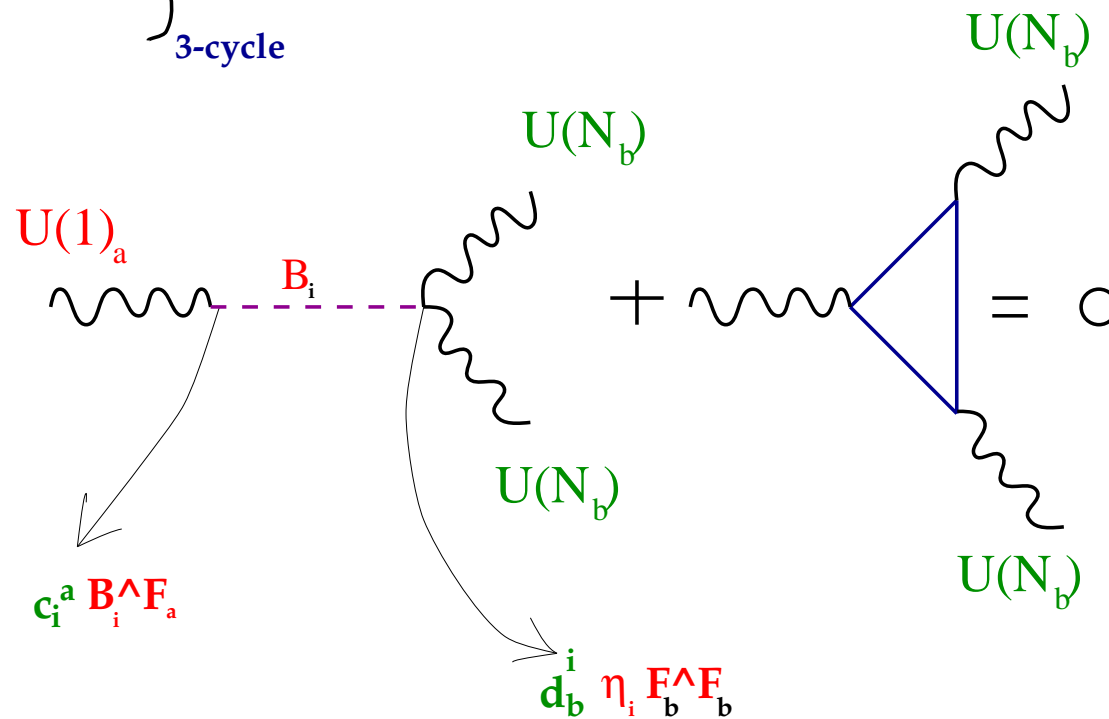
i.e. same number of fundamentals and antifundamentals, **which is just non-Abelian anomaly cancellation**.

## U(1) anomalies and couplings

In general there are **U(1) anomalies cancelled** by a version of the **GS mechanism**

$$\mathbf{B}_i^{\mu\nu} = \int_{\text{3-cycle}} \mathbf{C}^5$$

$$\mathbf{C}^5 = \text{RR-form}$$



$$\sum_i c_i^a d_b^i + A_b^a = 0 \quad \mathbf{B}_i^{\mu\nu} \xleftrightarrow{\text{duals}} \eta_i$$

- More precisely there are **RR 2-forms**  $B_2^I, I = 0, 1, 2, 3$  with **couplings to**  $U(1)$ 's

$$\sum_a N_a (m_a^1 m_a^2 m_a^3 B_2^0 + \sum_{I=1}^3 n_a^K n_a^J m_a^I B_2^I) \wedge F_a, \quad I \neq J \neq K \quad (17)$$

and **their dual pseudoscalars**  $\eta^I$  with **couplings to Abelian and non-Abelian**  $F_b \wedge F_b$

$$\sum_b (n_b^1 n_b^2 n_b^3 \eta^0 + \sum_{I=1}^3 n_b^I m_b^J m_b^K \eta^I) \wedge F_b \wedge F_b \quad (18)$$

Both couplings combine to cancel the triangle anomalies.

- **If a  $U(1)$  is anomalous it is necessary massive**, due to the  $B \wedge F_a$  couplings:

$$\epsilon_{\mu\nu\rho\sigma} B^{\mu\nu} (\partial^\rho A^\sigma) = (\partial_\sigma \eta) A^\sigma = \text{Higgs - like coupling}$$

- But **not the other way round**: **anomaly free  $U(1)$ 's may become massive!!**  
(e.g.  $U(1)_{B-L}$ )
- **Condition for a  $U(1) = \sum_a c_a Q_a$  (e.g. hypercharge !) to remain massless:**

$$\sum_a N_a m_a^1 m_a^2 m_a^3 c_a = 0 \quad (19)$$

$$\sum_a N_a n_a^J n_a^I m_a^I c_a = 0 \quad (\text{for any } I \neq J \neq K) \quad (20)$$

- In above orientifold models, there **are 4  $D = 4$  RR fields** involved, thus **at most 4  $U(1)$ 's can gain mass this way.**
- Even if the abelian gauge symmetry is lost, the  $U(1)$ 's remain as **perturbative global symmetries**. **This symmetries may however be broken by string instanton effects.**

## MODEL BUILDING WITH TOROIDAL IIA ORIENTIFOLDS

- R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, “Noncommutative compactifications of type I strings on tori with magnetic JHEP **0010** (2000) 006 [arXiv:hep-th/0007024].
- G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, “Intersecting brane worlds,” JHEP **0102** (2001) 047 [arXiv:hep-ph/0011132]; “D = 4 chiral string compactifications from intersecting branes,” J. Math. Phys. **42** (2001) 3103 [arXiv:hep-th/0011073].
- L. E. Ibanez, F. Marchesano and R. Rabadan, “Getting just the standard model at intersecting branes,” JHEP **0111**, 002 (2001) [arXiv:hep-th/0105155].
- M. Cvetič, G. Shiu and A. M. Uranga, “Chiral four-dimensional N = 1 supersymmetric type IIA orientifolds from intersecting D6-branes,” Nucl. Phys. B **615** (2001) 3 [arXiv:hep-th/0107166].
- D. Cremades, L. E. Ibanez and F. Marchesano, “Intersecting brane models of particle physics and the Higgs mechanism,” JHEP **0207**, 022 (2002) [arXiv:hep-th/0203160]; “SUSY quivers, intersecting branes and the modest hierarchy problem,” JHEP **0207**, 009 (2002) [arXiv:hep-th/0201205].
- F. Marchesano and G. Shiu, “Building MSSM flux vacua,” JHEP **0411** (2004) 041 [arXiv:hep-th/0409132].

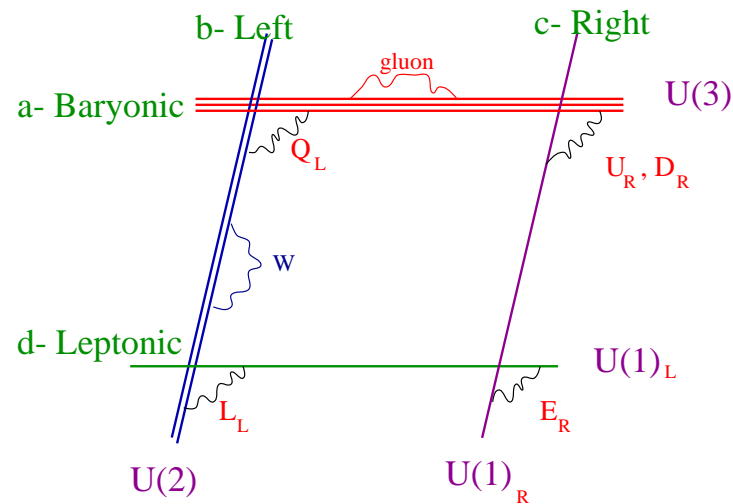


## Just the SM at intersecting branes

### Minimal Structure of SM D-brane settings

Configuration of 4 stacks of branes:

stack <b>a</b>	$N_a = 3$	$SU(3) \times U(1)_a$	Baryonic brane
stack <b>b</b>	$N_b = 2$	$SU(2) \times U(1)_b$	Left brane
stack <b>c</b>	$N_c = 1$	$U(1)_c$	Right brane
stack <b>d</b>	$N_d = 1$	$U(1)_d$	Leptonic brane



## # Generations = # Colours ?

- **Important constraint** in **any D-brane model** with fermions in  $U(N)$  bifundamentals (comes from RR-tadpole cancellation):

$$\text{Number of } N\text{-plets} = \text{Number of } \bar{N}\text{-plets of } U(N)$$

This is true **even for**  $U(2)$  or  $U(1)$ .

- **Impose** **Number of 2-plets = Number of  $\bar{2}$ -plets of  $U(2)$**

Left-handed SM fermions:

$$3 Q_L = 3 (3, 2) \longrightarrow 9 \text{ 2-plets}$$

$$3 L = 3 (1, \bar{2}) \longrightarrow 3 \bar{2}\text{-plets}$$

→ **Minimal SM has 'U(2) anomalies'**

**6 extra fermion  $SU(2)$  doublets needed to cancel anomalies.**

- This is **generic for any Dp-brane model with  $U(2)_L$**  . (Also true for models with **branes at singularities**).

- Simple way to **Cancel Anomalies** :

$$2 (3, 2) + 1 (3, \bar{2}) \longrightarrow 3 \text{ net } 2\text{-plets}$$

$$3 (1, \bar{2}) \longrightarrow 3 \text{ net } \bar{2}\text{-plets}$$

→ **U(2) anomalies cancel !!**

- this works only because **# COLORS = # GENERATIONS**
- But **in the SUSY case** this doesnot work because of the contribution of Higgsinos to the  $U(2)$  anomalies. An alternative option is having  $SU(2)_L = Sp(2)$ , an option exploited in some SUSY models, see later.

## Quantum numbers of SM in intersecting brane models

Assuming all fermions come from **bifundamentals** and imposing  $\#N$ -plets  $=\#\bar{N}$ -plets leads to the following model independent **unique** structure (up to redefinitions):

Intersection	Matter fields		$Q_a$	$Q_b$	$Q_c$	$Q_d$	$Q_Y$
(ab)	$Q_L$	$(3, 2)$	1	-1	0	0	1/6
(ab*)	$q_L$	$2(3, 2)$	1	1	0	0	1/6
(ac)	$U_R$	$3(3, 1)$	-1	0	1	0	-2/3
(ac*)	$D_R$	$3(3, 1)$	-1	0	-1	0	1/3
(bd*)	$L$	$3(1, 2)$	0	-1	0	-1	-1/2
(cd)	$N_R$	$3(1, 1)$	0	0	1	-1	0
(cd*)	$E_R$	$3(1, 1)$	0	0	-1	-1	1

$$SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$$

Where **hypercharge** is defined as:

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d \quad (21)$$

(Orthogonal linear combinations will be massive, see later)

## Wrapping numbers $(n_a^i, m_a^i)$ yielding the SM

$N_i$	$(n_i^1, m_i^1)$	$(n_i^2, m_i^2)$	$(n_i^3, m_i^3)$
$N_a = 3$	$(1/\beta^1, 0)$	$(n_a^2, \epsilon\beta^2)$	$(1/\rho, -\tilde{\epsilon}/2)$
$N_b = 2$	$(n_b^1, \epsilon\tilde{\epsilon}\beta^1)$	$(1/\beta^2, 0)$	$(1, -3\rho\tilde{\epsilon}/2)$
$N_c = 1$	$(n_c^1, 3\rho\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(0, 1)$
$N_d = 1$	$(1/\beta^1, 0)$	$(n_d^2, \epsilon\beta^2/\rho)$	$(1, 3\rho\tilde{\epsilon}/2)$

Table 1: D6-brane wrapping numbers giving rise to a SM spectrum. The general solutions are parametrized by two phases  $\epsilon, \tilde{\epsilon} = \pm 1$ , the NS background on the first two tori  $\beta^i = 1 - b^i = 1, 1/2$ , four integers  $n_a^2, n_b^1, n_c^1, n_d^2$  and a parameter  $\rho = 1, 1/3$ .

- **Large family** of models with the **chiral fermion spectrum of a 3 generation SM**.
- The  $2^{nd} - 4^{th}$  **RR-tadpole conditions** automatic. The first yield:

$$\frac{3n_a^2}{\rho\beta^1} + \frac{2n_b^1}{\beta^2} + \frac{n_d^2}{\beta^1} = 16 . \quad (22)$$

- One can also add **hidden sector D6-branes** parallel to the O6 plane

$$N_h(1/\beta_1, 0)(1/\beta_2, 0)(2, 0) \quad (23)$$

- Then one has **more flexibility**

$$\frac{3n_a^2}{\rho\beta^1} + \frac{2n_b^1}{\beta^2} + \frac{n_d^2}{\beta^1} + 2N_h\beta^1\beta^2 = 16 . \quad (24)$$

## U(1) symmetries

Intersection	Matter fields		$Q_a$	$Q_b$	$Q_c$	$Q_d$	$Q_Y$
(ab)	$Q_L$	$(3, 2)$	1	-1	0	0	1/6
(ab*)	$q_L$	$2(3, 2)$	1	1	0	0	1/6
(ac)	$U_R$	$3(3, 1)$	-1	0	1	0	-2/3
(ac*)	$D_R$	$3(3, 1)$	-1	0	-1	0	1/3
(bd*)	$L$	$3(1, 2)$	0	-1	0	-1	-1/2
(cd)	$N_R$	$3(1, 1)$	0	0	1	-1	0
(cd*)	$E_R$	$3(1, 1)$	0	0	-1	-1	1

$$SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$$

- $Q_a = 3B$  ;  $Q_d = L$  ;  $Q_c = 2I_R$

These known global symmetries of the SM are in fact gauge symmetries !!

- Two are anomaly-free :

$$\begin{aligned} \frac{Q_a}{3} - Q_d &= B - L \\ Q_c &= 2I_R \end{aligned} \tag{25}$$

with

$$Y = \frac{1}{2}(B - L) - I_R$$

- Two have triangle anomalies:

$$3Q_a + Q_d ; Q_b$$

Anomalies are cancelled by a Generalized Green-Schwarz mechanism as discussed above.

- Only one  $U(1)$  remains massless:

$$Q_0 = n_c^1(Q_a - 3Q_d) - \frac{3\tilde{\epsilon}\beta^2}{2\beta^1}(n_a^2 + 3\rho n_d^2)Q_c \quad (26)$$

It coincides with standard hypercharge if:

$$n_c^1 = \frac{\tilde{\epsilon}\beta^2}{2\beta^1}(n_a^2 + 3\rho n_d^2). \quad (27)$$

- $U(1)_{B-L}$  gets a mass ( even though it is anomaly free ! )

⇒ **Just the SM group and 3 generations**



EXAMPLE	$(n_i^1, m_i^1)$	$(n_i^2, m_i^2)$	$(n_i^3, m_i^3)$
$N_a = 3$	(1, 0)	(1, 1)	(3, -1/2)
$N_b = 2$	(1, 1)	(1, 0)	(1, -1/2)
$N_c = 1$	(-1, 1)	(1, 0)	(0, 1)
$N_d = 1$	(1, 0)	(1, 3)	(1, 1/2)

$$I_{ab} = (1) \times (-1) \times (-3/2 + 1/2) = 1 q_L; I_{ab}^* = (-1) \times (-1) \times (3/2 + 1/2) = 2 Q_L$$

$$I_{ac} = (1) \times (-1) \times (3) = -3 U_R; I_{ac}^* = (-1) \times (-1) \times (-3) = -3 D_R$$

$$I_{db} = (1) \times (-3) \times (-1) = 3 L; I_{db}^* = 0$$

$$I_{dc} = (1) \times (-3) \times (1) = -3 N_R; I_{dc}^* = (-1) \times (-3) \times (-1) = -3 E_R$$

- **RR-tadpole cancellations:**

$$\sum_{i=a,b,c,d} n_i^1 n_i^2 n_i^3 = 9 + 2 + 1 + N_{hidden} = 16 \quad (28)$$

They cancel if we add two branes along the orientifold plane at the origin with  $(1, 0)(1, 0)(2, 0)$ . This leads to no new extra matter.

- **Couplings of  $U(1)$ 's to RR fields:** Recalling the couplings

$$\sum_a N_a (m_a^1 m_a^2 m_a^3 B_2^0 + \sum_{I=1}^3 n_a^I m_a^I B_2^I) \wedge F_a, \quad I \neq J \neq K \quad (29)$$

one finds in our case

$$B_2^1 \wedge (2F_b)$$

$$B_2^2 \wedge (9F_a + 3F_d)$$

$$B_2^3 \wedge ((-3/2)F_a - F_b - F_c + (1/2)F_d)$$

(no coupling to  $B_2^0$  since  $mmm = 0$ ).

- This means that the linear combinations of  $U(1)$ 's which get a mass are

$$Q_\alpha = \sum_{i=a,b,c,d} c_\alpha^i Q_i \quad (30)$$

with  $(c_a, c_b, c_c, c_d) = (0, 1, 0, 0), (3, 0, 0, 1), (-3/2, -1, -1, 1/2)$ .

- The orthogonal combination  $(1,0,-3,-3)$  is **hypercharge, the only  $U(1)$  which remains massless:**

$$Q_Y = \frac{1}{6} Q_a - \frac{1}{2} Q_c - \frac{1}{2} Q_d \quad (31)$$

- $B_2^0$  (or its dual  $\eta^0$ ) remain massless, do not combine with any  $U(1)$  since  $m_a^1 m_a^2 m_a^3 = 0$  for all branes.
- From the general couplings

$$\sum_b (n_b^1 n_b^2 n_b^3 \eta^0 + \sum_{I=1}^3 n_b^I m_b^J m_b^K \eta^I) \wedge F_b \wedge F_b \quad (32)$$

one has couplings

$$\eta^0 (3F_a \wedge F_a + F_b \wedge F_b + F_d \wedge F_d) \quad (33)$$

- $\eta^0$  has axionic couplings to QCD which could solve the strong CP problem.

## Baryon and lepton number violation

- **Baryon number** is a **gauge** symmetry. So the **proton is automatically stable**. (Baryogenesis should take place non-perturbatively).
- This **could be good news for proton stability for models with 1 TeV string scale**.
- **Lepton number** is also a **gauge** symmetry. Only **Dirac** masses exist at the perturbative level.
- **Majorana neutrino** masses may appear at the non-perturbative level (see later).

## Gauge coupling constants

$$SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$$

- Gauge couplings **are not unified** :

$$\frac{1}{g_i^2} = \frac{M_s^3}{(2\pi)^4 \lambda} \text{Vol}(\Pi_i) ; \quad i = a, b, c, d \quad (34)$$

$\text{Vol}(\Pi_i)$  being the volume each D6-brane is wrapping.

- Thus, e.g.,  $SU(3)$  interactions are stronger than  $SU(2)$  because **'baryonic' branes wrap less volume than 'left' branes**

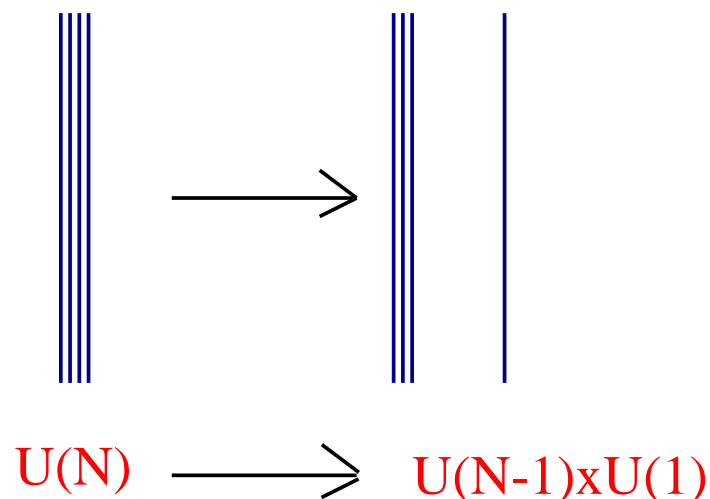
$$g_a^2 = \frac{g_{QCD}^2}{6} ; \quad g_b^2 = \frac{g_L^2}{4} ; \quad \frac{1}{g_Y^2} = \frac{1}{36g_a^2} + \frac{1}{4g_c^2} + \frac{1}{4g_d^2}. \quad (35)$$

- In this class of models there is **sufficient freedom to accommodate observed couplings** (not so easy in SUSY examples).

## Higgs mechanism and brane recombination

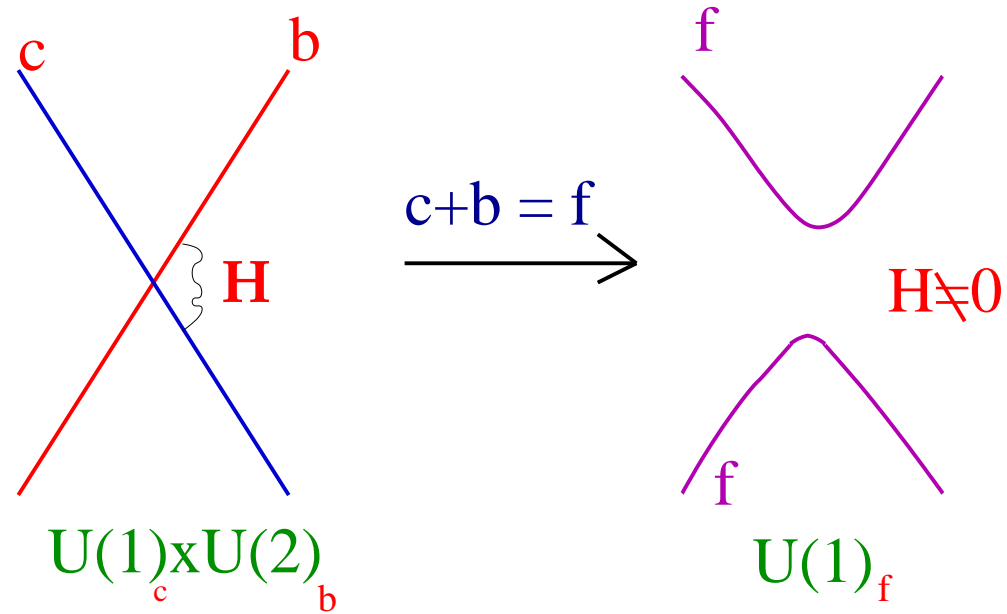
- ① Brane separation = Adjoint Higgsing Does not lower the rank.

$N$  D $p$ -branes



i.e.,  $U(4)_{PS} \rightarrow U(3) \times U(1)$

- ② Brane recombination lowers the rank. Happens when a bifundamental scalar gets a vev. This may happen varying the angles through the complex structure.



In SM the rank is lowered  $\rightarrow$  brane recombination of the branes  $b$  and  $c(c^*)$  at which intersection the Higgs scalars lie.

- In this class of models SM Higgs fields come from  $b$ - $c$  intersections once one is on top of each other in second plane:



$N_i$	$(n_i^1, m_i^1)$	$(n_i^2, m_i^2)$	$(n_i^3, m_i^3)$
$N_b = 2$	$(n_b^1, \epsilon\tilde{\beta}^1)$	$(1/\beta^2, 0)$	$(1, -3\rho\tilde{\epsilon}/2)$
$N_c = 1$	$(n_c^1, 3\rho\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(0, 1)$

There are two varieties depending on  $U(1)_b$  charge:

Higgs	$Q_b$	$Q_c$	Y
$h$	1	-1	1/2
$H$	-1	-1	1/2

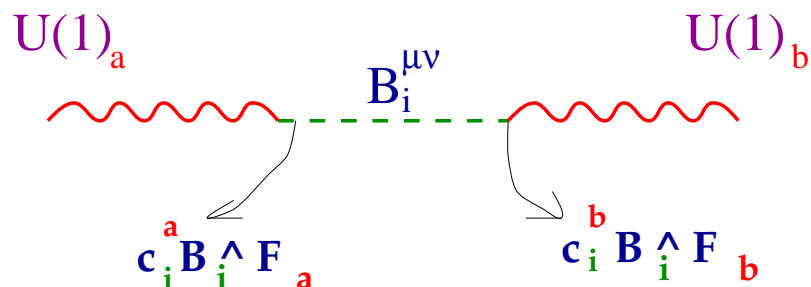
$$n_h = \beta^1 |n_c^1 + 3\rho n_b^1|; \quad n_H = \beta^1 |n_c^1 - 3\rho n_b^1| \quad (36)$$

- Previous example has  $n_h = 2, n_H = 0$ . (Many examples with only one).

## TeV-Scale $Z'$ Bosons from D-branes ?

- It could so happen that the string scale could be close to 1 TeV. Then KK and string excitations could be produced <sup>a</sup>. But also extra  $Z$ 's with masses from the GS mechanism.
- There is quite a generic structure of extra  $U(1)$ 's. They are NOT of  $E_6$  type:  
 $(B - L); I_R; (3B + L); Q_b$
- If  $M_s \propto 1 - 10$  TeV, could perhaps be tested at LHC.
- Extra  $Z$ 's get masses by combining with RR string fields  $B_i^{\mu\nu}$ .

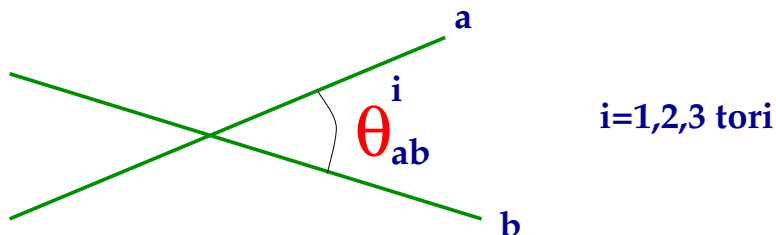
<sup>a</sup>D. Ghilencea, L.E.I., N. Irges, F. Quevedo, hep-ph/0205083.



$$M_{ab}^2 = \sum_i g_a g_b c_i^a c_i^b M_{\text{string}}^2$$

- There is a  $4 \times 4$  mass matrix for the  $U(1)$ 's
- Four Eigenvalues =  $(0, M_2, M_3, M_4)$ . One finds typically at least one of them  $M_3 < \frac{1}{3} M_{\text{string}}$
- There are then three massive  $Z$ 's mix with SM  $Z^0$ . One can put constraints on  $M_i$  from the  $\rho$ -parameter.

## Scalars at D6-brane intersections

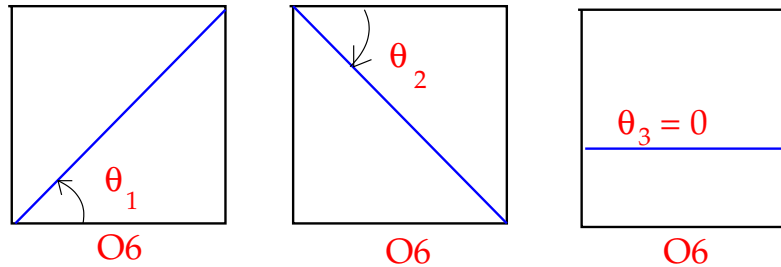


- There are three lightest scalars (“squarks/sleptons”) at each intersection with masses in string units:

$$\begin{aligned}
 M_1^2 &= \frac{1}{2}(-|\vartheta^1| + |\vartheta^2| + |\vartheta^3|) \\
 M_2^2 &= \frac{1}{2}(|\vartheta^1| - |\vartheta^2| + |\vartheta^3|) \\
 M_3^2 &= \frac{1}{2}(|\vartheta^1| + |\vartheta^2| - |\vartheta^3|)
 \end{aligned}
 \tag{37}$$

- For wide ranges of parameters **scalars are non-tachyonic**
- For particular choices of radii and wrappings  $n_a^i, m_a^i$  there is a **massless scalar**, signaling the presence of  **$N = 1$  SUSY at THAT intersection**
- But fully  **$N = 1$  SUSY** toroidal brane configuration in which ALL intersections respect the **same** supersymmetry is **not possible** (due to RR-tadpole cancellation).

## N = 1 SUSY MODELS



- For a system of D6-branes to **preserve N = 1 SUSY**, each brane should be related to the O6 plane by a SU(3) rotation. This implies for all  $D6_a$  branes

$$(\theta_1^a + \theta_2^a + \theta_3^a) = 0 ; \quad \text{tg} \theta_a^i = \frac{m_i^a}{n_i^a} \tau_i ; \quad \tau_i = (\text{Im}(e_2^{(i)}) / \text{Re}(e_1^{(1)})) \quad (38)$$

This means:

$$\sum_i \text{tg}(\theta_i^a) - \prod_i \text{tg}(\theta_i^a) = 0 \quad (39)$$

- Note that, for fixed  $n_a^i, m_a^i$ , this condition depends on the geometric complex

structure  $\mathcal{T}_i$ , it is a dynamical condition.

- When that happens **one bifundamental scalar becomes massless at each intersection** providing for the  $N = 1$  SUSY partners of the massless chiral fermion.
- A **small deviation corresponds to turning on of a FI-term.**
- It turns out that SUSY configurations of **a,b,c,d** branes necessarily have some wrapping numbers contributing **negatively to the nmm, mnm, mnm RR-tadpole conditions.**
- **We need new orientifold O6 planes contributing positively** to  $2^{nd} - 4^{th}$  RR-tadpole.
- Precisely this **happens in the simple  $\Omega\mathcal{R}$  orientifold** of the  $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$  orbifold.

## N = 1 Type IIA $Z_2 \times Z_2$ orientifold

- The  $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$  orbifold is generated by the reflections

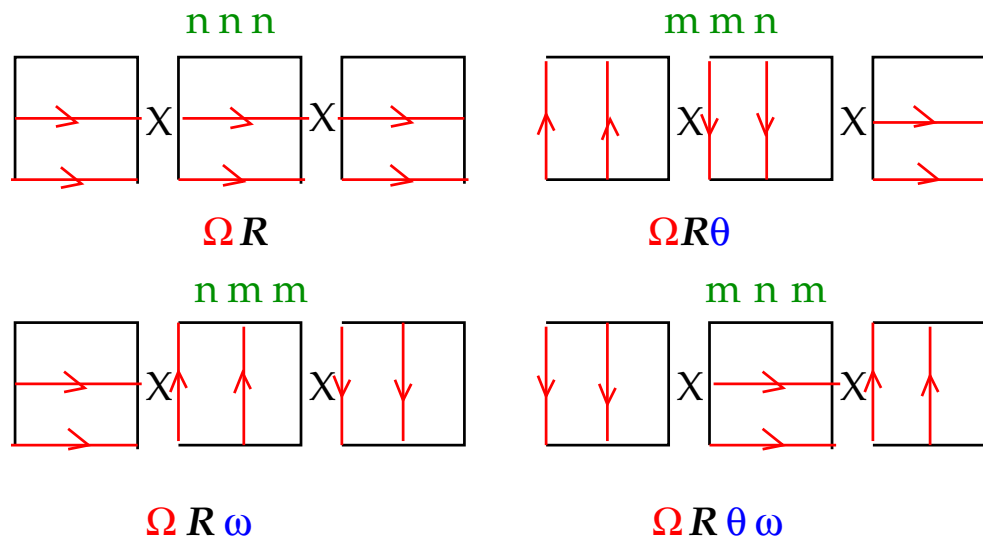
$$\theta : (z_1, z_2, z_3) \longrightarrow (-z_1, -z_2, z_3) \quad (40)$$

$$\omega : (z_1, z_2, z_3) \longrightarrow (z_1, -z_2, -z_3)$$

with  $z_i$  the 3 complexified coordinates in  $T^6$ . We made this theory under

$$\Omega\mathcal{R} : (z_1, z_2, z_3) \longrightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3) \quad (41)$$

- Now D6-branes have mirrors under  $\Omega\theta, \Omega\omega, \Omega\theta\omega, \Omega\mathcal{R}$ . 3 more sets of orientifold planes appear



- There are 8 orientifold planes of each type. They wrap 3-cycles in the 6-torus:

$$\begin{aligned}
 (1, 0)(1, 0)(1, 0) & \quad ; \quad (0, 1)(0, -1)(1, 0) & (42) \\
 (1, 0)(0, 1)(0, -1) & \quad ; \quad (0, -1)(1, 0)(0, 1)
 \end{aligned}$$



- The RR tadpole cancellation conditions are now modified (there are 8 orientifolds of each type).

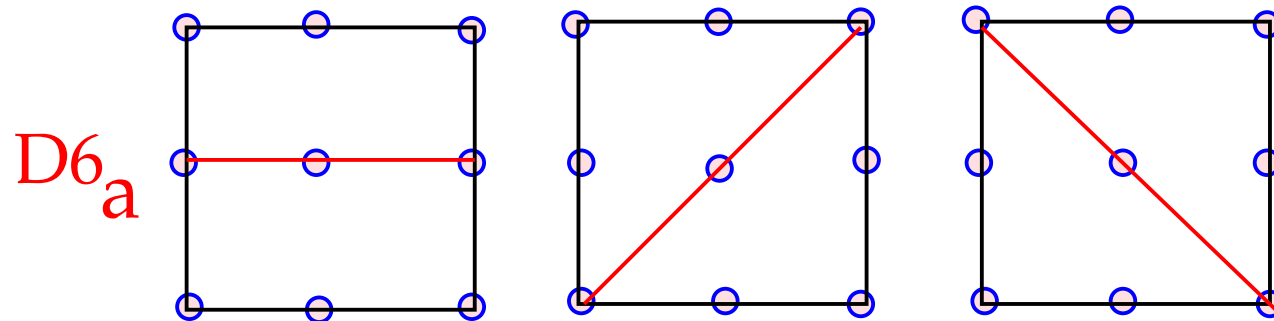
$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 \quad \sum_a N_a n_a^1 m_a^2 m_a^3 = -16\beta_2\beta_3$$

$$\sum_a N_a m_a^1 n_a^2 m_a^3 = -16\beta_1\beta_3 \quad \sum_a N_a m_a^1 m_a^2 n_a^3 = -16\beta_1\beta_2$$

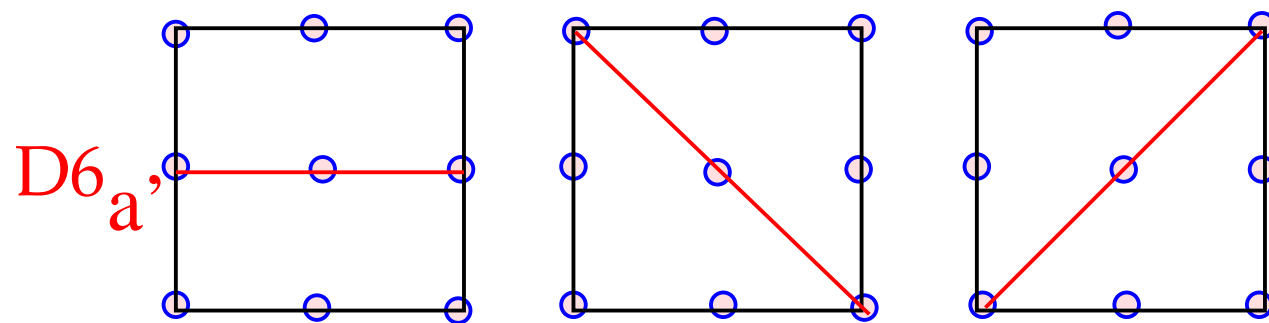
- Now one can find SUSY brane configurations consistent with tadpole conditions.

## ***D6-branes in Type IIA $Z_2 \times Z_2$ orientifold***

- Now D6-brane configurations must be **consistent with both the  $Z_2 \times Z_2$  and  $\Omega\mathcal{R}$  projections.**
- The  $Z_2 \times Z_2$  orbifold has  $4 \times 4 \times 4$  fixed points. One simple invariant configurations is **locating *D6-branes* passing through orbifold fixed points** (and adding the orientifold mirror  $D6_a^*$  branes):



○  $a = \theta a = \omega a = \theta\omega a$



$\Omega R a$

- If we have a **stack of  $2N$   $D6$ -branes** passing through fixed points the projections give rise to a gauge group

–  $\theta$  projection

$$U(2N) \longrightarrow U(N) \times U(N) \quad (43)$$

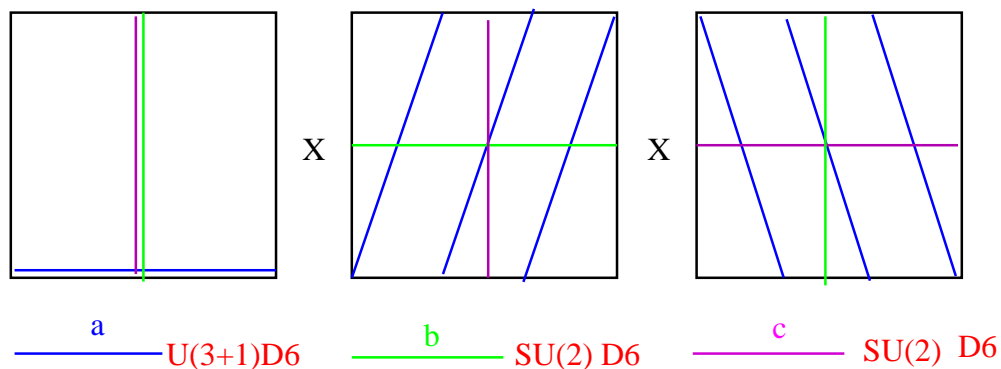
–  $\omega$  projection

$$U(N) \times U(N) \longrightarrow U(N) \quad (44)$$

- Chiral fermions, given by  $I_{ab}$  essentially unaffected by the  $Z_2 \times Z_2$  twist.  
Same rules as toroidal case apply in practice.

## A MSSM-LIKE EXAMPLE

$N_i$	$(n_i^1, m_i^1)$	$(n_i^2, m_i^2)$	$(n_i^3, m_i^3)$
$N_a = 6 + 2$	$(1, 0)$	$(3, 1)$	$(3, -1)$
$N_b = 2$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 2$	$(0, 1)$	$(0, -1)$	$(1, 0)$



$$I_{ab} = 3 \quad ; \quad I_{ac} = -3 \quad ; \quad I_{bc} = 1$$

$F_L$                        $F_R$                        $H$

$$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{3B+L} \quad (45)$$

- It contains 3 quark/lepton generations, one Higgs multiplet at intersections
- It is supersymmetric if  $U_2 = U_3$ , i.e. same angles on 2-nd and 3-d complex plane. The  $\mu$ -term is the distance of b,c branes in 1-st torus.
- This model may be embedded into a RR-tadpole free  $N = 1$   $Z_2 \times Z_2$  Type IIA orientifold if appropriate  $h_1, h_2, h_O$  D6-branes added :

$N_i$	$(n_i^1, m_i^1)$	$(n_i^2, m_i^2)$	$(n_i^3, m_i^3)$
$N_{h_1} = 2$	$(-2, 1)$	$(-3, 1)$	$(-4, 1)$
$N_{h_2} = 2$	$(-2, 1)$	$(-4, 1)$	$(-3, 1)$
$N_{h_O} = 40$	$(1, 0)$	$(1, 0)$	$(1, 0)$

The full spectrum is shown in the table....

Sector	$SU(4) \times SU(2) \times$ $\times SU(2) \times [USp(40)]$	$Q_a$	$Q_{h_1}$	$Q_{h_2}$	$Q'$
$(ab) F_L$	$3(4, 2, 1)$	1	0	0	1/3
$(ac) F_R$	$3(\bar{4}, 1, 2)$	-1	0	0	-1/3
$(bc) H$	$(1, 2, 2)$	0	0	0	0
$(ah'_1)$	$6(\bar{4}, 1, 1)$	-1	-1	0	5/3
$(ah_2)$	$6(4, 1, 1)$	1	0	-1	-5/3
$(bh_1)$	$8(1, 2, 1)$	0	-1	0	2
$(bh_2)$	$6(1, 2, 1)$	0	0	-1	-2
$(ch_1)$	$6(1, 1, 2)$	0	-1	0	2
$(ch_2)$	$8(1, 1, 2)$	0	0	-1	-2
$(h_1 h'_1)$	$23(1, 1, 1)$	0	-2	0	4
$(h_2 h'_2)$	$23(1, 1, 1)$	0	0	-2	-4
$(h_1 h'_2)$	$196(1, 1, 1)$	0	1	1	0
$(fh_1)$	$(1, 1, 1) \times [40]$	0	-1	0	2
$(fh_2)$	$(1, 1, 1) \times [40]$	0	0	-1	-2

- There is a single  $U(1)$  which remains massless after  $B \wedge F$  couplings operate (in addition to  $U(1)_{B-L}$ ):

$$U(1)' = \frac{1}{3}U(1)_a - 2[U(1)_{h_1} - U(1)_{h_2}] \quad (46)$$

- The singlets in the  $h_1 - h_2^*$  sector can get vevs along flat directions,  $U(1)'$  is broken and the charged spectrum is reduced to:

Sector	Matter	$SU(3+1) \times SU(2) \times SU(2) \times [USp(40)]$	$Q_a$	$Q_h$	$Q'$
(ab)	$F_L$	$3(4, 2, 1)$	1	0	1/3
(ac)	$F_R$	$3(\bar{4}, 1, 2)$	-1	0	-1/3
(bc)	$H$	$(1, 2, 2)$	0	0	0
(bh)		$2(1, 2, 1)$	0	1	2
(ch)		$2(1, 1, 2)$	0	-1	-2

- This spectrum is just a **left-right version of the MSSM with a single Higgs** and some **leptonic exotics which can become massive at the electroweak scale**.



- It is remarkable how close to the content of the MSSM one can get with such a simple brane configuration. We will compute the Yukawa couplings for this model later.
- The model has no gauge unification. Has also extra massless adjoints. One can construct models without adjoints in which D6-branes wrap rigid 3-cycles<sup>a</sup>
- Models based in other orientifold orbifolds (e.g.  $Z_6$ <sup>b</sup>) have been constructed with the low-energy spectrum of the MSSM.

---

<sup>a</sup>Blumenhagen, Cvetič, Marchesano, Shiu.

<sup>b</sup>G.Honecker, T. Ott, hep-th/0404055.