# D-BRANE MODEL BUILDING, PART II


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# III The Mirror : magnetized IIB branes

- Mirror symmetry exchanges IIA and IIB compactifications
- In Type IIB we have $D9$, $D7$, $D5$ and $D3$ branes wrapping 6-,4-, 2- and 0-cycles in the CY and subject to magnetic fluxes.
- We will consider 2 general classes of IIB models:
  - 1) Magnetized $Dp$ IIB branes in toroidal/orbifold settings
  - 2) IIB D3-branes at singularities
- The first class [a] is T-dual (equivalent) to the intersecting D6-brane models already discussed.
- The second class may be considered also as a limiting class of magnetized branes wrapping cycles which are collapsed at a CY singularity.

[a] Bachas hep-th/9503030; Angelantonj et al. hep-th/0007090.

## Magnetized toroidal IIB branes

- One considers $N_a$ D9-branes wrapped $m_a^i$ times on the i-th 2-torus in $T^6$ and with $n_a^i$ units of $U(1)_a$ magnetic flux:

$$m_a^i \frac{1}{2\pi} \int_{T_i^2} F_a^i \; = \; n_a^i \tag{1}$$

- ($n_a^i$,$m_a^i$) are now the dual of the D6-brane wrapping numbers.
- The relative angle of $D6_a - D6_b$ branes is mapped to:

$$\theta_{ab} \; = \; arctg(F_b^i) \; - \; arctg(F_a^i) \tag{2}$$

with

$$F_a^i \; = \; \frac{n_a^i}{m_a^i R_{x_i} R_{y_i}} \tag{3}$$

- In the presence of a magnetic flux $F$ in a IIB brane wrapping $T^2$ open string boundary conditions get modified:

$$\partial_\sigma X \; - \; F\partial_\tau Y \; = \; 0 \tag{4}$$

$$\partial_\sigma Y \; + \; F\partial_\tau X \; = \; 0 \tag{5}$$

- Note that $F$ interpolates between N and D boundary conditions. At formally infinite flux they are purely D.

- This allows us also to describe lower dimensional branes:

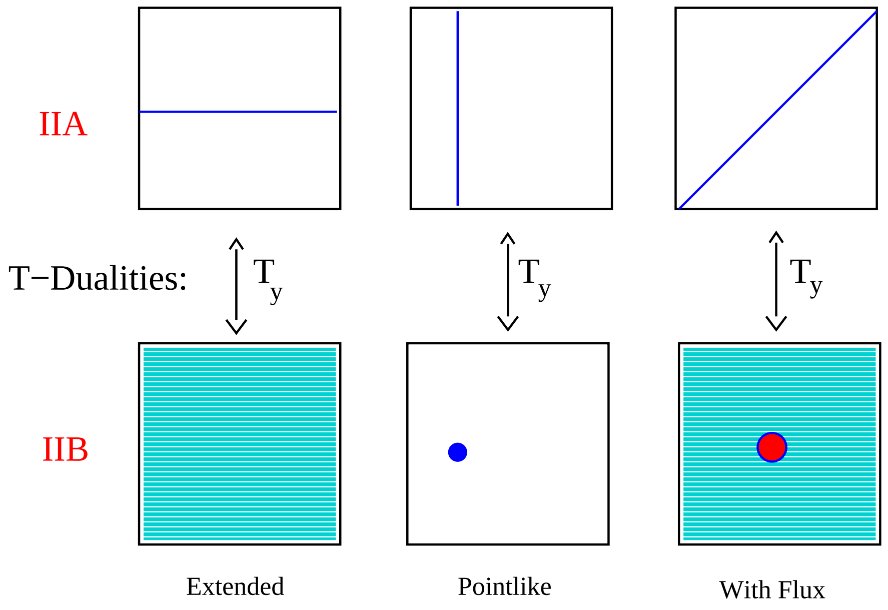
$$D9 \; \rightarrow (n_a^1, m_a^1)(n_a^2, m_a^2)(n_a^3, m_a^3) \tag{6}$$

$$D7_1 \; \rightarrow (1,0)(n_a^2, m_a^2)(n_a^3, m_a^3) \tag{7}$$

$$D5_1 \; \rightarrow (n_a^1, m_a^1)(1,0)(1,0) \tag{8}$$

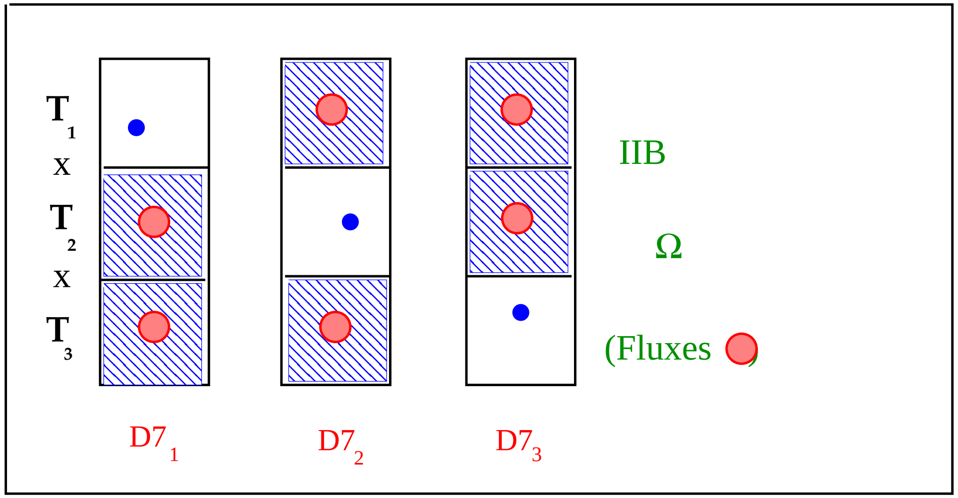
$$D3 \; \rightarrow (1,0)(1,0)(1,0) \tag{9}$$

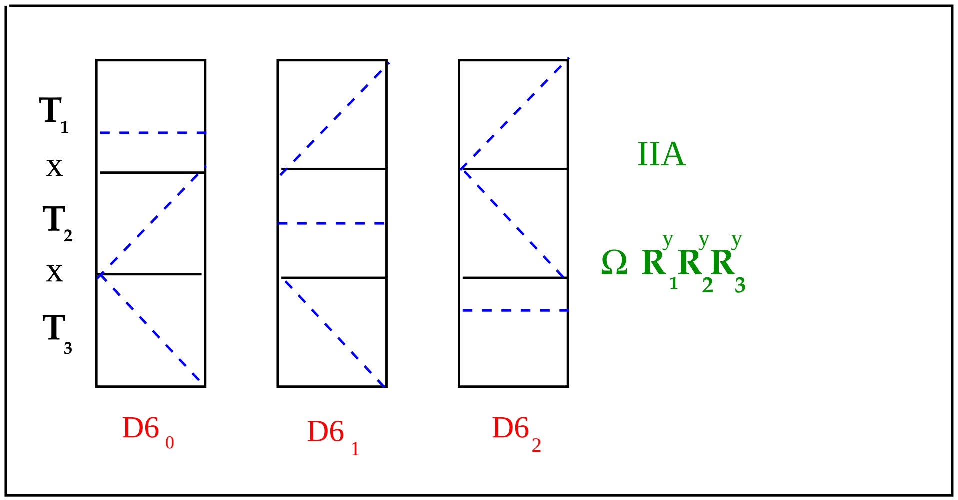
- Any intersecting D6-brane model may be converted into a magnetized $D9$-brane model with apropriate fluxes.
- A T-duality exchanges Neumann and Dirichlet boundary conditions alog the duality direction:



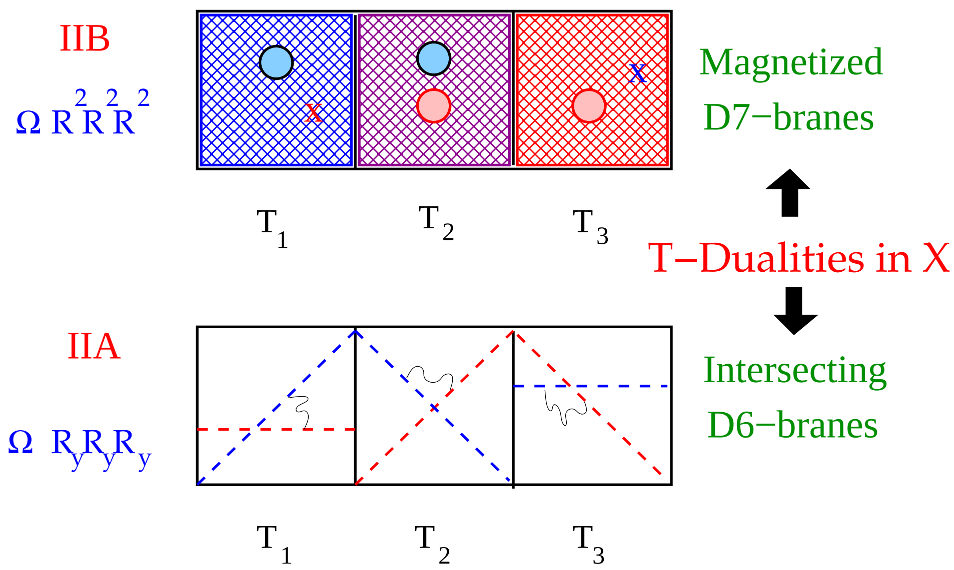

- E.g. The $Z_2 \times Z_2$ IIA orientifold example has now the MSSM residing at $D7_i$ branes. RR-tadpoles cancelled by additional magnetized $D9$ branes.

T-duality in 3 x-tori directions exchanges D7-models models with fluxes and intersecting D6 IIA models



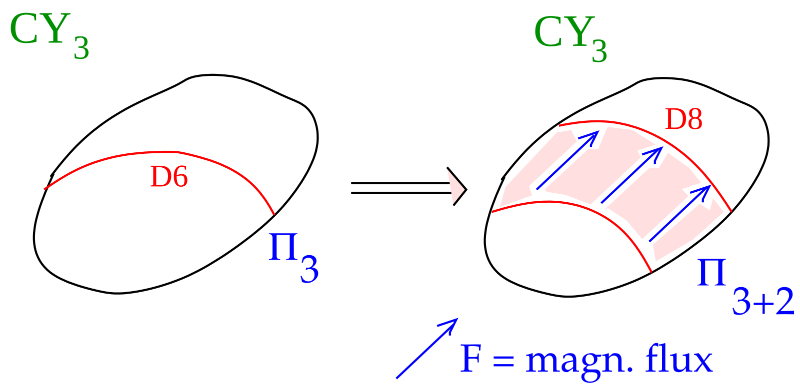
In this case a T-duality along 3 horizontal directions exchanges a magnetized D7 IIB model and an intersecting D6 IIA model.




- Chirality arises from the missmatch of L- and R-handed fermions in compact dimensions in the presence of a magnetic flux.

# Back to Type IIA: D8-branes with fluxes

- It turns out that in order to recover full mirror symmetry one has to consider in the IIA side new possibilities. D8-branes wrapping 5-cycles with magnetic fluxes.



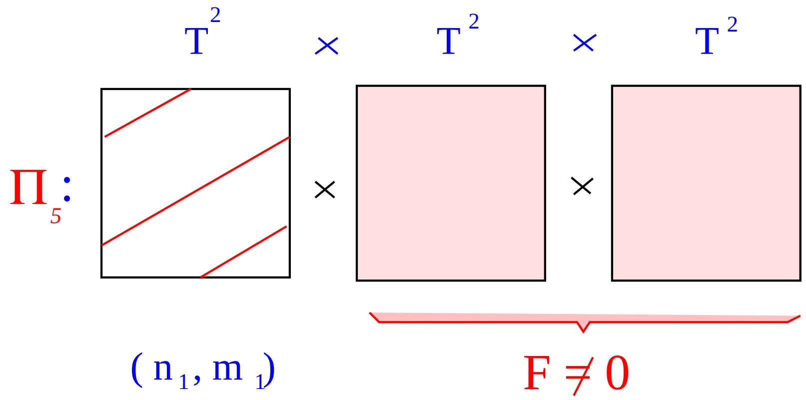

- Naively one would say $D6$-branes exhaust all the possibilities for constructing space-filling BPS D-branes. Other Type IIA options $D4, D8$ would wrap homologically trivial cycles in a CY.

- This is not quite true for $D8$'s because they can carry non-trivial magnetic flux $F$ in their worldvolume. The flux $F$ induces $D6$ charge in the worldvolume of the $D8$ rendering them stable BPS objects:

$$D6 - charge : \int_{M_4 \times \Pi_5} \mathcal{F} \wedge C_7 \qquad (10)$$

- The $D8$'s wrap 5-cycles $\Pi_5$ which are 'coisotropic' submanifolds in the CY.
- The $D8$ carries $D6$ charge corresponding to the 3-cycle $\Pi_3^F$ Poincare dual to $F$ inside $\Pi_5$.

- A simple example is a $D8$-brane wrapping $T^2 \times T^2$ and a 1-cycle on the other $T^2$




- There is quantized magnetic flux inside $T^2 \times T^2$:

$$\begin{aligned} F &= n_{xx} dx_2 \wedge dx_3 + n_{xy} dx_2 \wedge dy_3 + n_{yx} dy_2 \wedge dx_3 \\ &+ n_{yy} dy_2 \wedge dy_3 + \tilde{n}_2 dx_2 \wedge dy_2 + \tilde{n}_3 dx_3 \wedge dy_3 \end{aligned}$$

- There is a D-term condition analogous to that of D6's. However in addition there is an F-term condition:

$$(F + J_c)^2|_{\Pi_5} = 0 \tag{11}$$

- since ( the $T_i$ are the Kahler moduli)

$$\begin{aligned} F^2 &= (n_{xy}n_{yx} - n_{xx}n_{yy} + \tilde{n}_2\tilde{n}_3)\, dx_2 \wedge dy_2 \wedge dx_3 \wedge dy_3 \\ (J_c)^2|_{\Pi_5} &= -T_2T_3 dx_2 \wedge dy_2 \wedge dx_3 \wedge dy_3 \\ F \wedge J_c|_{\Pi_5} &= -i(\tilde{n}_2T_3 + \tilde{n}_3T_2) dx_2 \wedge dy_2 \wedge dx_3 \wedge dy_3 \end{aligned}$$

- one gets the F-term constraint

$$(T_2 + i\tilde{n}_2)(T_3 + i\tilde{n}_3) = n_{xy}n_{yx} - n_{xx}n_{yy} \tag{12}$$

- The F-term condition may be understood as coming from a superpotential

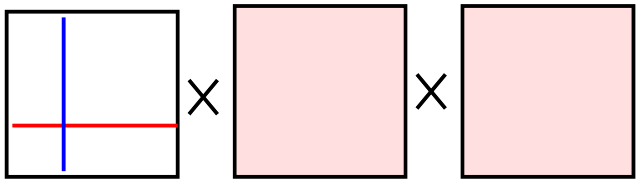
$$W_1 = \Phi_1\,(T_2T_3 - f_1) \quad ; \quad f_1 = n_{xy}n_{yx} - n_{xx}n_{yy} \tag{13}$$

where $\Phi_1$ is an open string modulus along the first torus ($D8$ location + Wilson line).

## Chirality

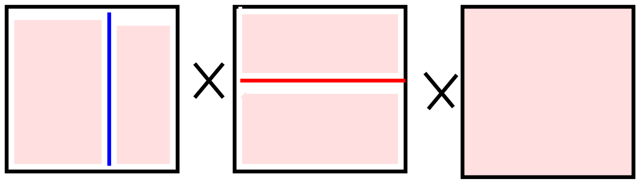
- Chirality arises from mixture of intersections and magnetized branes. Chiral fermions arise at overlaps of both $D6$ and $D8$ 's .

$D8_a - D8_b$



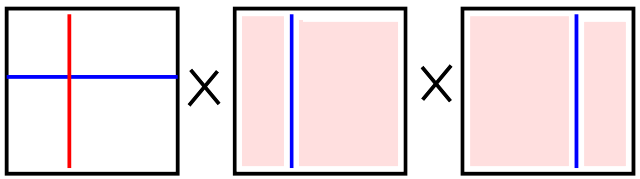
$$I_{ab} = (n_1^a m_1^b - m_1^a n_1^b) \int_{T^2 \times T^2} (F_a - F_b)^2$$

$D8_a - D8_b$



$$I_{ab} = \int_{C^2 \times T^2} (F_a - F_b)^2$$

$D8_a - D6_b$



$$I_{ab} = (n_1^a m_1^b - m_1^a n_1^b) \int_{C^2} (F_a - F_b)$$

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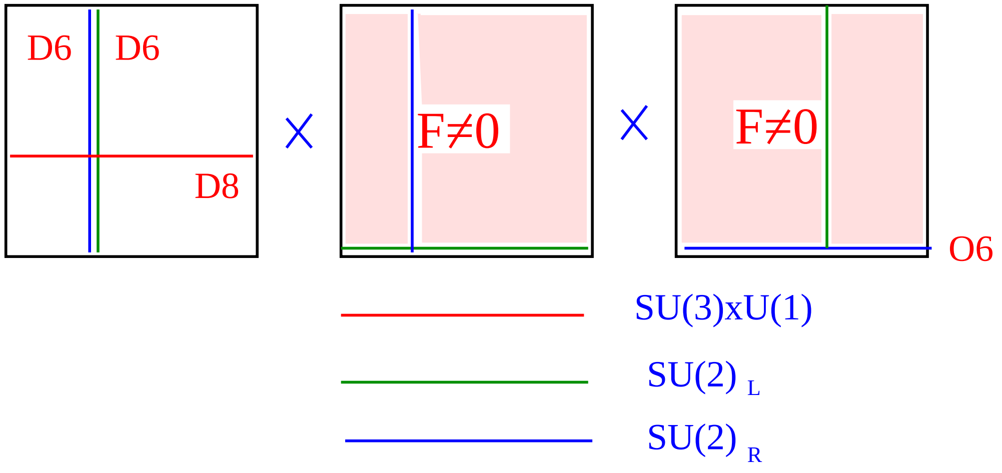
## Model building applications

- These magnetized $D8$-branes may be used to construct semirealistic compactifications in $Z_2 \times Z_2$ orientifold with MSSM-like spectrum and 3 generations [a].
- They have a couple of advantages over $D6$-brane models:
  - One can fix the Kahler moduli without the addition of closed string fluxes (which would require using the supergravity aproximation).
  - Their $D6$-induced charges correspond to non-factorized cycles. This makes the model-building more flexible.
- The model building possibilities of this new tool are still to be explored. We will content ourselves with an example here.

[a] A.Font, L.E.I.,F. Marchesano, hep-th/0607219.

# A MSSM-like $D8$-$D6$ example

- Consider the $Z_2 \times Z_2$ Type IIA orientifold with $D8$'s and $D6$'s as follows:




| $N_i \;\; Dp_i$ | $D8: \; (n,m)_i \times (n_{xx}, n_{xy}, n_{yx}, n_{yy})_{(jk)}$ <br> $D6: \; (n_1, m_1)(n_2, m_2)(n_3, m_3)$ |
|---|---|
| $N_a = 6 + 2 \; D8_a$ | $(1,0)_1 \times (-1, 3, -3, 10)_{(23)}$ |
| $N_b = 2 \; D6_b$ | $(0,1)(1,0)(0,-1)$ |
| $N_c = 2 \;\; D6_c$ | $(0,1)(0,-1)(1,0)$ |

- Stacks a,b,c give rise to the SM sector.

- Additional branes to cancel all RR-tadpoles and fix the $T_i$ (in collaboration with brane a)).

| $N_i$ $Dp_i$ | $D8: (n,m)_i \times (n_{xx}, n_{xy}, n_{yx}, n_{yy})_{(jk)}$ <br> $D6: (n_1,m_1)(n_2,m_2)(n_3,m_3)$ |
|---|---|
| $N_M = 4$ $D6_M$ | $(-2,1)_1 \times (-3,1) \times (-3,1)$ |
| $N_X = 2$ $D8_X$ | $(1,0)_2 \times (-1,0,0,2)_{(13)}$ |
| $N_Y = 2$ $D8_Y$ | $(1,0)_3 \times (-1,0,0,2)_{(12)}$ |

- The gauge group is

$$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{3B+L} \times G_{hidden} \quad (14)$$

- The chiral spectrum with MSSM quantum numbers is:

| Intersection | Matter fields | Rep. | $Q_{3B+L}$ |
|---|---|---|---|
| $a-b$ | $Q_L + L$ | $3(3+1, 2_L)$ | 1 |
| $a-c$ | $U_R + D_R \;;\, E_R + \nu_R$ | $3(\bar{3}+1, 2_R)$ | -1 |
| $b-c$ | $H + \overline{H}$ | $(2_L, 2_R)$ | 0 |
| $b-M$ | $L'$ | $6(2_M, 2_L)$ | 0 |
| $c-M$ | $R'$ | $6(2_M, 2_R)$ | 0 |

- This corresponds to 3 quark/lepton generations plus a minimal Higgs set. In addition there are some extra exotic leptons which can become massive at the electroweak scale.

- The D-term conditions give

$$\tau_2 = \tau_3 \quad ; \quad \tau_1\tau_2\tau_3 = 9\tau_1 + 6\tau_2 + 6\tau_3 \tag{15}$$

- The F-term conditions fix the 3 untwisted Kahler moduli $T_i$:

$$\begin{aligned} D8_a &\longrightarrow T_2T_3 = F_a^2 = 1 \\ D8_X &\longrightarrow T_1T_3 = F_X^2 = 2 \\ D8_Y &\longrightarrow T_1T_2 = F_Y^2 = 2 \end{aligned} \tag{16}$$

- one thus has

$$ReT_1 = 2 \ ; \ ReT_2 = ReT_3 = 1 \ ; \ ImT_i = 0 \tag{17}$$

- The MSSM gauge kinetic functions are given by

$$\begin{aligned} f_{SU(3+1)} &= n_1 n_{yy} S - n_1 n_{xx} U_1 = 10S + U_1 \\ f_{SU(2)_L} &= \frac{1}{2} U_2 \\ f_{SU(2)_R} &= \frac{1}{2} U_3 \end{aligned}$$

- A variety of MSSM-like models may be built..

## RCFT Type IIB orientifold MSSM-like models

- A large class of Type IIB Rational Conformal Field Theory (RCFT) orientifold models, of order 180000 with MSSM-like spectra were constructed in 2004 by Schellekens and colaborators [a].

- These are non-geometrical compactifications in which the CY geometry is replaced by RCFT Gepner models with total central charge $c = 9$ .

- This $c = 9$ system is obtained by tensoring $N = 2$ Minimal SCFT models each one with central charge

$$c = \frac{3k}{k+2} \quad , \quad k = 1, ..., \infty \tag{18}$$

- There are 168 ways of solving $\sum_i c_{k_i} = 9$. In addition one can obtain new compactifications by moding by the discrete symmetries, a $Z_{k+2}$ symmetry per minimal factor : 5403 models.

[a] Dijkstra,Huiszoon,Schellekens hep-th/0411129.

- Altogether there are 49304 possibilities for orientifold operations.
- The role of branes played by certain boundary states. For a given model the number of boundary states is finite, typically $10^2 - 10^3$ .
- Search for MSSM spectra coming from a,b,c,d SM boundary states ('Madrid quiver' structure) plus possible hidden sector boundaries. 179520 MSSM-like models found with different spectra in 2004 sample.
- The general structure of SM gauge group (there is also in general a hidden sector)

| Type | SM Gauge group | B-L |
|---|---|---|
| 0 | $U(3) \times Sp(2) \times U(1) \times U(1)$ | massless |
| 1 | $U(3) \times U(2) \times U(1) \times U(1)$ | massless |
| 2 | $U(3) \times Sp(2) \times O(2) \times U(1)$ | massless |
| 3 | $U(3) \times U(2) \times O(2) \times U(1)$ | massless |
| 4 | $U(3) \times Sp(2) \times Sp(2) \times U(1)$ | massless |
| 5 | $U(3) \times U(2) \times Sp(2) \times U(1)$ | massless |
| 6 | $U(3) \times Sp(2) \times U(1) \times U(1)$ | massive |
| 7 | $U(3) \times U(2) \times U(1) \times U(1)$ | massive |

- The geometric interpretation of these models corresponds to Type IIB orientifolds with magnetized D7-branes. The D7's would wrap 4-cycles in the CY.
- They have just the spectrum of the MSSM and vectorlike matter.
- No exotics. Some have gauge coupling unification (but extra vector-like matter).
- Limitation: correspond to particular points in CY space. Do not know yet how to extract the effective acion, feasible in principle.
- But the biggest set of MSSM-like solutions in the literature!

## IV-THE LOW ENERGY EFFECTIVE ACTION

- Kahler potential
- Gauge kinetic function
- Yukava couplings

## Closed string moduli in CY orientifolds

- Type II orientifolds have massless Kahler $T_i$ and complex structure $U_m$ moduli. They are defined in terms of geometric quantities as follows [a]
  - Type IIA:

$$T^i \;=\; Vol(\Sigma_2^{(i)}) + i\, B_2^{(i)}\, ;\; U^m \;=\; e^{-\phi} Vol(\Sigma_3^{(m)}) + i\, C_3^{(m)} \quad (19)$$

  - Type IIB (D3/D7)

$$U^m \;=\; \int_{\Sigma^{(3)}} \Omega\, ;\; T^i \;=\; e^{-\phi} Vol(\Sigma_4^{(i)}) \;+\; i\, C_4^{(i)} \quad (20)$$

- Here $\Sigma_n$ denotes a cycle of dimension=n in the CY, $C_n$ are RR n-forms integrated over those cycles, $\Omega$ denotes the holomorphic 3.form in the CY and $\phi$ is the dilaton.

[a] T.Grimm, J. Louis hep-th/0412277.

# Closed string moduli Kahler potential in CY orientifolds

- The Kahler potential for Type IIA orientifolds may be obtained by dimensional reduction from $D = 10$ and is given by

$$K_{IIA} \;=\; -log(Vol_{CY}) \;-\; 2log\int_{CY} Re(C\Omega)\wedge *Re(\overline{C\Omega}) \qquad (21)$$

where

$$C = e^{-\phi_4}\left[\frac{1}{8i}\int \Omega\wedge\Omega^*\right]^{-1/2} \;\; ; \;\; e^{\phi_4} = e^{\phi}/(Vol)^{1/2} \qquad (22)$$

- For the case of the Kahler potential for IIB (D3/D7) orientifolds one gets

$$K_{IIB} \;=\; -2log(Vol_{CY}) \;-\; log\, i\int_{CY}\Omega\wedge\Omega \;-\; log(S+S^*) \qquad (23)$$

where

$$S \;=\; \frac{1}{\phi} \;+\; i\, C_0 \qquad (24)$$

is the complex dilaton field.

- In both cases one can check that the dependence on Kahler moduli and complex structure field is separated in the Kahler potential.
- In Type IIA the perturbative superpotential depends on the Kahler moduli but not on the complex structure.
- In Type IIB the perturbative superpotential depends on the complex structure moduli but not on the Kahler moduli.
- This separation may have phenomenological relevance (flavour problem).

# Closed string moduli in IIA toroidal orientifolds

- There are IIA orientifold closed string moduli scalars:
  - The complex structure moduli. They are governed by the dilaton $\lambda = e^{\phi}$ and the shape of each tori. The $\eta^I$ are the RR scalars with a role in $U(1)$ anomalies.

$$S = \frac{M_s^3}{\lambda} R_x^{(1)} R_x^{(2)} R_x^{(3)} + i\eta^0 \tag{25}$$

$$U^{(i)} = \frac{M_s^3}{\lambda} R_x^{(i)} R_y^{(j)} R_y^{(k)} + i\eta^i \ , \quad i \neq j \neq k \tag{26}$$

(27)

  - The Kahler moduli . The real part controls the size of the tori.

$$T^{(i)} = M_s^2 R_x^{(i)} R_y^{(i)} + i\,(B_2(i)) \tag{28}$$

## Closed string moduli in IIB toroidal orientifolds

- For the $\Omega R^2 R^2 R^2$ IIB orientifold ($D7$, $D3$ branes ).
  - The dilaton

$$S = \frac{1}{\lambda} + iC^0 \tag{29}$$

  - The Kahler moduli . The real part is the 4-volume transverse to the i-th torus. The $C^i$ are the RR scalars playing a role in $U(1)$ anomalies.

$$T^{(i)} = \frac{M_s^4}{\lambda} R_x^{(j)} R_y^{(j)} R_x^{(k)} R_y^{(k)} + iC^i \quad , \quad i \neq j \neq k \tag{30}$$

  - The complex structure moduli.

$$U^{(i)} = \tau_i \tag{31}$$

- Note that the real parts may be obtained from those of Type IIA through 3 T-dualities in the x-directions:

$$R_x^{(i)} \longrightarrow \frac{\alpha'}{R_x^{(i)}} \tag{32}$$

- Kahler and complex structure moduli are exchanged

$$IIA, D6 \ U^{(i)} \ , \ T^{(i)} \ \longleftrightarrow \ IIB, D7, D3 \ T^{(i)} \ , \ U^{(i)} \tag{33}$$

## The toroidal moduli Kahler potential

- It has the typical log structure.

$$K \; = \; -log(S+S^*) - log\,(\Pi_i\,(U^{(i)}+U^{(i)^*})) - log\,(\Pi_i\,(T^{(i)}+T^{(i)^*})) \tag{34}$$

- It has the same structure for IIA and IIB although the moduli have different meaning.
- These diagonal toroidal moduli are the untwisted moduli of the $Z_2 \times Z_2$ orientifold. Other orbifolds may have additional off-diagonal moduli.

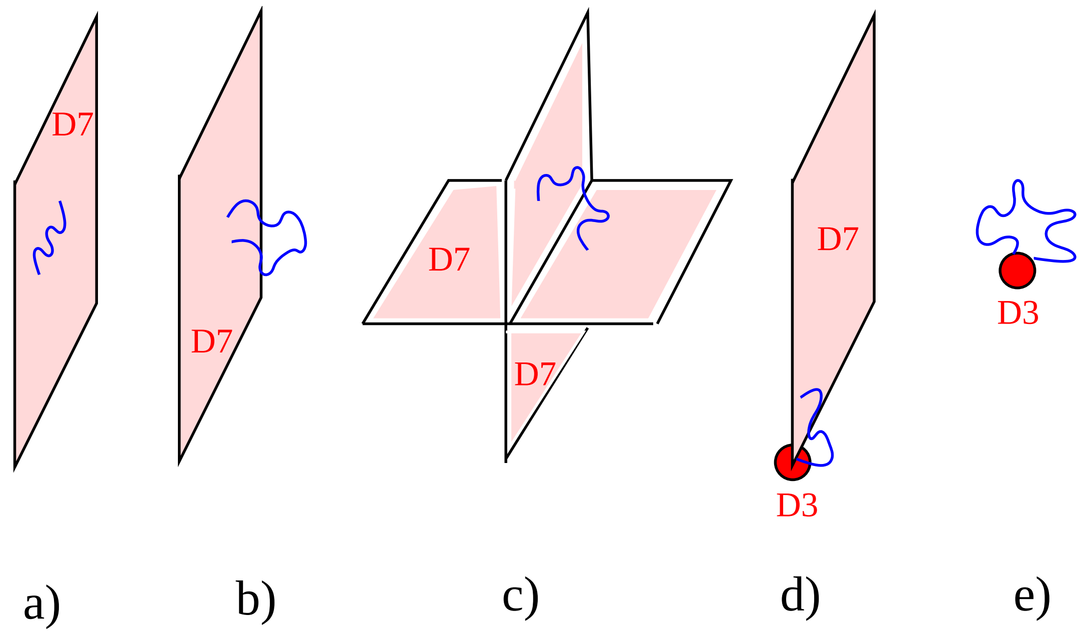
## Kahler metrics of matter fields (IIB)

- To compute low-energy physical quantities (like physical Yukawa couplings, SUSY-breaking soft terms) it is important to know the Kahler metrics of the matter fields

$$K_{ab}\Phi_a\Phi_b^* \tag{35}$$

- $K_{ab}$ are non-holomorphic functions of the closed string moduli. Their dependence on the moduli is dictated by the geometric origin of the field.
- These metrics have been computed for simple cases either by dimensional reduction or explicit string correlators [a].
- Different origin of chiral matter fields in IIB-D3/D7 toroidal orientifolds. The classification still applies to general CY orientifolds. (No magnetic fluxes).

[a] L.E.I., C. Muñoz, S. Rigolin hep-ph/9812397; Lust et al. hep-th/0406092.




– a) Fields from $(7^i7^i)_j, j \neq i$. Come from dimensional reduction of $D = 8$ vector multiplets in the brane. They give rise to adjoints in toroidal example but may lead to more interesting matter in more general cases. The metric is ($t_i = 2ReT_i, u_j = 2ReU_j$)

$$K_{(7^i7^i)_j} = \frac{1}{t_k u_j} \quad (36)$$

– b) Fields from $(7^i7^i)_i, i = 1, 2, 3$. Come from dimensional reduction of $D = 8$ scalar multiplets which parametrize the position of $7^i$-brane in transverse dimensions. Adjoints in toroidal case.

$$K_{(7^i7^i)_i} = \frac{1}{su_i} \tag{37}$$

– c) Fields from two intersecting $D7$-branes.

$$K_{(7^i7^j)} = \frac{1}{t_k^{1/2}s^{1/2}(u_iu_j)^{1/2}} \quad i \neq j \neq k \tag{38}$$

– d) Fields from open strings between $D3$ and $D7$-branes.

$$K_{(37^i)} = \frac{1}{t_j^{1/2}t_k^{1/2}(u_ju_k)^{1/2}} \quad i \neq j \neq k \tag{39}$$

– e) Fields from open strings in $D3$-branes.

$$K_{(33)_i} = \frac{1}{t_i} \tag{40}$$

- The results for intersecting $D6$-branes may be obtained from T-duality.

## Effect of magnetic fluxes on Kahler metrics

- The above metrics corresponded to the case with no magnetic fluxes on the $D7$-branes. But chirality typically requires magnetic fluxes. Consider the presence of magnetic fluxes through i-th torus $F_i$

$$F_i \; = \; n_i \left( \frac{s t_i}{t_j t_k} \right)^{1/2} \tag{41}$$

- Then for fields of types a), b) one gets [a]

$$K_{(7^i 7^i)_j} \; = \; \frac{1}{t^k u_j} \left| \frac{1 + iF^k}{1 + iF^j} \right| \; ; \; K_{(7^i 7^i)_i} \; = \; \frac{1}{s u_i} \left( 1 + \left| F^j F^k \right| \right) , \tag{42}$$

where $i \neq j \neq k$ label the 3 2-tori

- Kahler metric for matter fields coming from intersecting magnetized $D7$'s one

[a] Lust et al. hep-th/0404134.

has (Lust et al.)

$$K_{ab} \;=\; \frac{1}{(st_1t_2t_3)^{1/4}}(\Pi_{i=1}^{3} u_i^{-\theta_{ab}})\sqrt{\frac{\Gamma(\theta^i_{ab})}{\Gamma(1-\theta^i_{ab})}} \qquad (43)$$

where $s = ReS, u_i = Re(U^{(i)}), t_i = Re(T^{(i)})$.

- For dilute fluxes (large t) this behaves with $t = t_1 = t_2 = t_3$ like

$$K_{ab} \;\simeq\; \frac{1}{s^{1/2}t^{1/2}} \qquad (44)$$

- In the Type IIA case the same result applies exchanging Kahler and c.s. moduli and interpreting the magnetic fluxes in terms of angles of intersecting $D6$ branes.
- The Kahler metric is important to compute SUSY breaking soft terms.

## The gauge kinetic function

- The gauge coupling constant in IIA orientifolds may be obtained from the Dirac-Born-Infeld (DBI) action of the D6-branes

$$S_{DBI} = \int_{\Pi_{D6}} \frac{1}{\lambda} \sqrt{\det(G + F)} + S_{CS} \tag{45}$$

- Expanding to quadratic order in the gauge field strength $F$ :

$$\frac{1}{g_a^2} = \frac{Vol(\Pi_3)}{\lambda} = \frac{M_s^3}{\lambda} \sqrt{\Pi_{i=1}^3((n_a^i R_x^{(i)})^2 + (m_a^i R_y^{(i)})^2)} \tag{46}$$

- If the $D6$-brane preserves same SUSY as orientifold plane, the expression simplifies a lot. Indeed, using the trigonometric expresion

$$\Pi_{i=1}^3 \,(1 + tan^2\theta_i)^{1/2} = 1 - \sum_{i \neq j} tan\theta_i tan\theta_j \tag{47}$$

one finds

$$\frac{1}{M_s^3 g_a^2} = n_a^1 n_a^2 n_3^3 R_x^1 R_x^2 R_x^3 - \sum_{i\neq j\neq k} n_a^i m_a^j m_a^k R_x^i R_y^j R_y^k \quad (48)$$

$$= n_a^1 n_a^2 n_3^3 Re(S) - \sum_{i\neq j\neq k} n_a^i m_a^j m_a^k Re(U^i) \quad (49)$$

- and hence by holomorphicity of kinetic function

$$f_a = n_a^1 n_a^2 n_3^3 S - \sum_{i\neq j\neq k} n_a^i m_a^j m_a^k U^i \quad (50)$$

- In the Type IIB case one also expands the DBI action. The $m_a^i m_a^j$ pieces come from the insertion of $F^2$ backgrounds in the expansion. One then finds the same expression exchanging $U^i \leftrightarrow T^i$.
- Note that in general coupling constants would not unify in a MSSM model. The Fields $S, U^i$ contain RR scalars participating in GS mechanism.

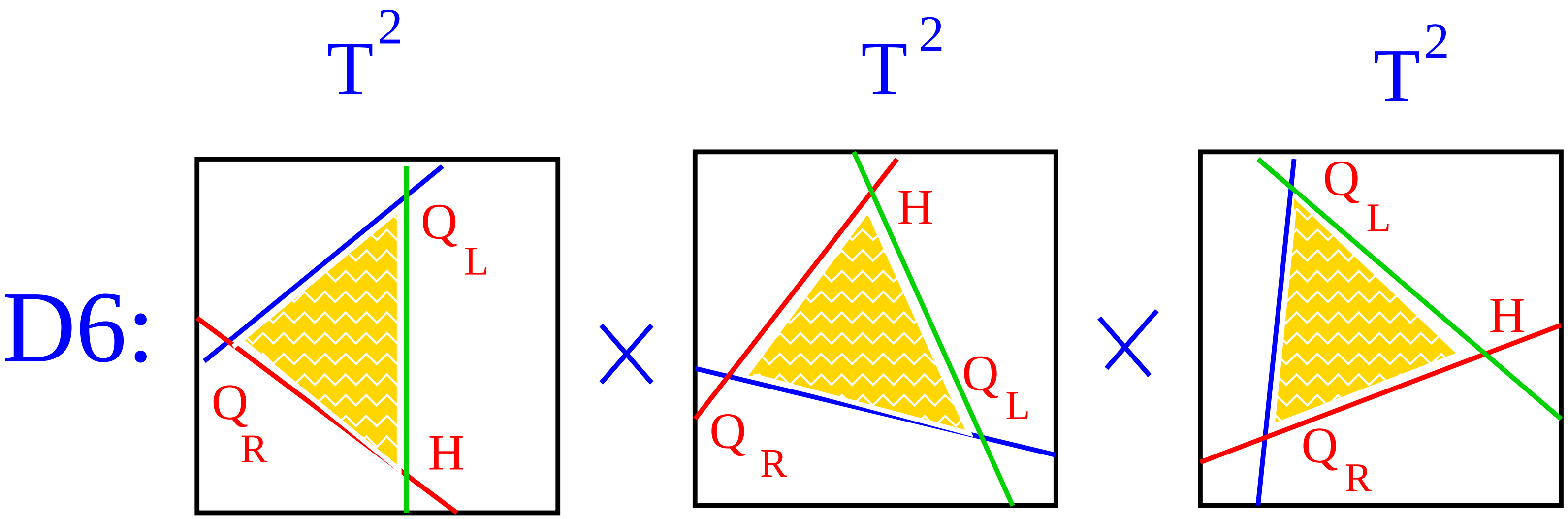
- In the MSSM-like model we described before one finds:

$$f_{SU(3+1)} = U_1 + 9S \; ; \; f_{SU(2)_L} = \frac{1}{2}U_2 \; ; \; f_{SU(2)_R} = \frac{1}{2}U_3 \quad (51)$$

- SUSY condition implies $ReU_2 = ReU_3 = ReU$ so that $SU(2)_L$ and $SU(2)_L$ are unified due to SUSY.
- One can tune $S$, $U(1)$ to get unification.
- There are more complicated models in which gauge coupling unification is more easily achieved.

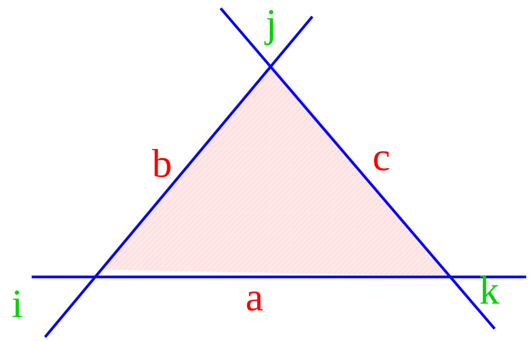
## Yukawa couplings in IIA toroidal orientifolds

- One of the quantities of most phenomenological interest are the Yukawa couplings among fermions and Higgs multiplets in SM-like compactifications.
- They may be computed in Type IIA from string correlators wich involve world-sheet instanton contributions.




- Consider first one subtorus and three D-branes labeled a,b,c wrapping $T^2$ with intersecting numbers $I_{ab}, I_{bc}, I_{ca}$.

- Yukawa couplings come from worldsheet instanton contributions [a]




- If $i, j, k = 0, 1, ...$ label the chiral fields at intersections One expects a semiclassical contribution

$$Y_{ijk} \propto \sum exp\left(-\frac{A_{ijk}}{2\pi\alpha'}\right) \tag{52}$$

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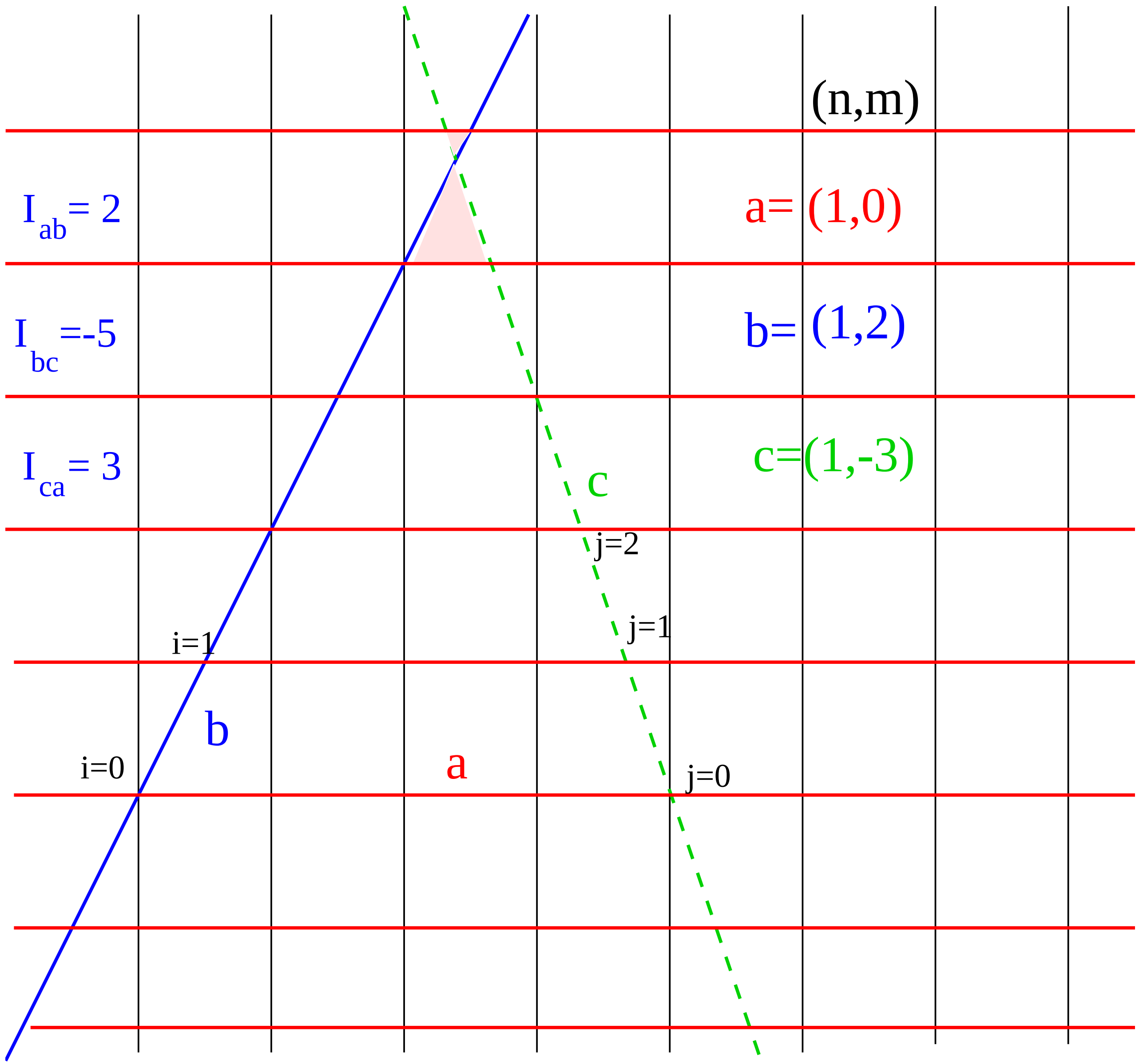
[a] Aldazabal et al.(2000);D.Cremades,L.I.,F. Marchesano (2003,2004)


(n,m)
I ab = 2
a= (1,0)
I bc =-5
b= (1,2)
I ca = 3
c=(1,-3)
c
j=2
j=1
i=1
b
i=0
a
j=0

- One finds in the simple $T^2$ case

$$A_{ijk}(l) \propto \sum_l \frac{A}{2}|I_{ab}I_{bc}I_{ca}|\left(\frac{i}{I_{ab}} + \frac{j}{I_{bc}} + \frac{k}{I_{ca}} + \tilde{\epsilon} + l\right)^2 \quad (53)$$

- $\tilde{\epsilon}$ parametrizes relative positions of branes (open string moduli).

$$Y_{ijk} \propto \vartheta \begin{bmatrix} \delta \\ \phi \end{bmatrix} (t) = \sum_{l\in\mathbf{Z}} q^{\frac{1}{2}(\delta+l)^2} e^{2\pi i(\delta+l)\phi}, \quad q = e^{-2\pi t}. \quad (54)$$

$$\delta = \frac{i}{I_{ab}} + \frac{j}{I_{bc}} + \frac{k}{I_{ca}} + \tilde{\epsilon} \quad (55)$$

$$\phi = 0, \quad (56)$$

$$t = \frac{A}{\alpha'}|I_{ab}I_{bc}I_{ca}|.$$

where $\theta$ is the standard Jacobi theta function with characteristics..

- This is easily generalized to include complex Kahler moduli $J^a$, $a = 1, 2, 3,$ Wilson lines $\theta_i^a$ on the D6-branes and the full $T^2 \times T^2 \times T^2$ .

$$Y_{ijk} = h_{\rm qu} \cdot h_{\rm cl} = h_{\rm qu} \prod_{r=1}^{n} \vartheta \left[ \begin{array}{c} \delta^{(r)} \\ \phi^{(r)} \end{array} \right] (0, \kappa^{(r)}) \qquad (57)$$

$h_{qu}$ is flavor-independent [a]. Here the $\vartheta$-function parameters are given by

$$\delta^{(r)} = \frac{i^{(r)}}{I_{ab}^{(r)}} + \frac{j^{(r)}}{I_{ca}^{(r)}} + \frac{k^{(r)}}{I_{bc}^{(r)}} + \frac{I_{ab}^{(r)}\epsilon_c^{(r)} + I_{ca}^{(r)}\epsilon_b^{(r)} + I_{bc}^{(r)}\epsilon_a^{(r)}}{I_{ab}^{(r)}I_{bc}^{(r)}I_{ca}^{(r)}}, \qquad (58)$$

$$\phi^{(r)} = I_{ab}^{(r)}\theta_c^{(r)} + I_{ca}^{(r)}\theta_b^{(r)} + I_{bc}^{(r)}\theta_a^{(r)}, \qquad (59)$$

$$\kappa^{(r)} = |I_{ab}^{(r)}I_{bc}^{(r)}I_{ca}^{(r)}|\, J^{(r)} \qquad (60)$$

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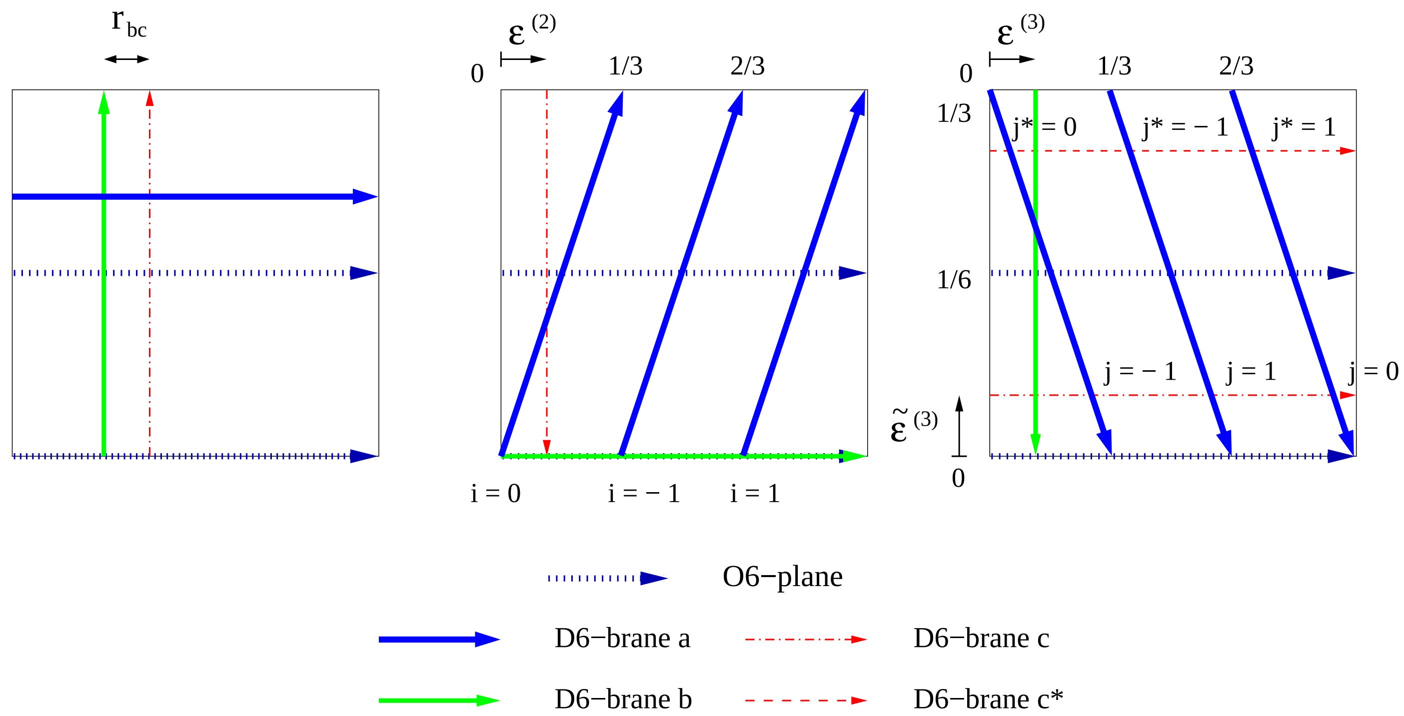
[a] Cvetic,Papadimitriou;Abel,Owen (2003);Lust, Mayr,Richter,Stieberger (2004)

# Application to the MSSM-like model




$$W_{Yuk} = Y^U_{ij} Q^i_L H_u U^j_R \quad + \quad Y^D_{ij*} Q^i_L H_d D^{j*}_R \tag{61}$$

- with Yukawa matrices

$$
\begin{aligned}
{Y_{ij}}^{U} &\sim \vartheta\begin{bmatrix} \frac{i}{3}+\epsilon^{(2)} \\ \theta^{(2)} \end{bmatrix}\left(\frac{3J^{(2)}}{\alpha'}\right)\times\vartheta\begin{bmatrix} \frac{j}{3}+\epsilon^{(3)}+\tilde{\epsilon}^{(3)} \\ \theta^{(3)}+\tilde{\theta}^{(3)} \end{bmatrix}\left(\frac{3J^{(3)}}{\alpha'}\right), \\
{Y_{ij*}}^{D} &\sim \vartheta\begin{bmatrix} \frac{i}{3}+\epsilon^{(2)} \\ \theta^{(2)} \end{bmatrix}\left(\frac{3J^{(2)}}{\alpha'}\right)\times\vartheta\begin{bmatrix} \frac{j*}{3}+\epsilon^{(3)}-\tilde{\epsilon}^{(3)} \\ \theta^{(3)}-\tilde{\theta}^{(3)} \end{bmatrix}\left(\frac{3J^{(3)}}{\alpha'}\right).
\end{aligned}
\tag{62}
$$

Then, the Yukawa matrices can be expressed as ($A, B, \tilde{B}$ diagonal matrices bilinear in $\theta$-functions)

$$
Y^U \sim A\cdot\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\cdot B, \qquad Y^D \sim A\cdot\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\cdot\tilde{B}. \tag{63}
$$

- In this model only the third generation becomes massive. One can obtain:

$$\frac{h_t}{h_b} \sim \sqrt{\frac{\mathrm{Tr}\,(B \cdot \bar{B})}{\mathrm{Tr}\,(\tilde{B} \cdot \bar{\tilde{B}})}} \simeq e^{2ImJ^{(3)}}$$

- Thus one can understand the smallness of $h_b$ in geometric terms.
- This structure could be a good starting point. The masslessness of other generations is due to factorization of family dependence. E.g, slight departure from factorization would lead to smaller but non-vanishing masses for rest of families.
- Notice the presence of complex phases (origin of SM CP-violation) are phases from Kahler moduli and Wilson lines.

# Yukawa couplings: the IIB mirror

- The computation of Yukawa couplings in the Type IIB mirror is quite different [a]. Consider the case of magnetized $D9$-branes. One has to do a KK-reduction and compute the massles spectrum from the zero modes of Dirac and K-G equations in extra dimensions.

- The initial gauge group may be e.g. $U(n)$ ( or e.g. $SO(32)$, it will not be crucial for our purposses), in D=10.

$$L = -\frac{1}{4}\mathrm{Tr}\left\{F^{MN}F_{MN}\right\} + \frac{i}{2}\mathrm{Tr}\left\{\bar{\Psi}\Gamma^{M}D_{M}\Psi\right\}$$

- We then compactify the theory down to $D=4$. The $D=10$ fields can then be expanded:

$$\begin{aligned} \Psi(w) &= \sum_n \chi_n(x) \otimes \psi_n(y) \\ A_i(w) &= \sum_n \varphi_{n\ i}(x) \otimes \phi_{n\ i}(y) \end{aligned}$$

[a] D. Cremades,L.E.I., F. Marchesano hep-th/0404229.

- Here $x^\mu, \mu = 0, .., 3$ and $y^m, m = 4, .., 9$ . The internal wave functions verify:

$$\begin{aligned} i\,\tilde{\not{D}}_6\ \psi_n^{ab} &= m_n \psi_n^{ab}, \qquad m_n = 0 \\ \Delta_6\,\phi_n^{ab} &= M_n^2 \phi_n^{ab}, \qquad \text{smallest } M_n^2 \end{aligned}$$

- The initial gauge group $U(n)$ is broken to $U(p_a) \times U(p_b) \times$ ... by adding constant fluxes $F_a$, $F_b$ etc...

$$F = \begin{pmatrix} F_a & & \\ & F_b & \\ & & \ddots \end{pmatrix}, \quad \Rightarrow \quad A = \begin{pmatrix} A_a & & \\ & A_b & \\ & & \ddots \end{pmatrix},$$

- The $D = 10$ gaugino field has now $D = 4$ zero modes including gauginos and chiral fermions in bifundamentals:

$$\Psi = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \begin{matrix} D = 4\, U(p_a) \\ \text{gaugino} \end{matrix} & \begin{matrix} D = 4\ \text{bif.}(p_a, \bar{p}_b) \\ \text{chiral fermion} \end{matrix} \\ \text{CPT conj.} & \begin{matrix} D = 4\, U(p_b) \\ \text{gaugino} \end{matrix} \end{pmatrix}$$

$$\text{Dirac eq.} \begin{cases} \not{D}\ \Psi = 0, \\ D = \partial - iA \end{cases}$$

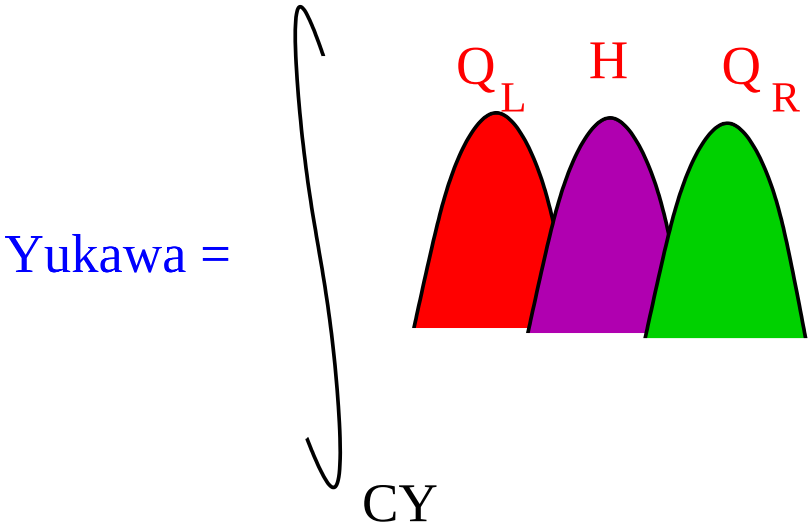
- In general $\not{D}\Psi = 0$ has several independent chiral fermion solutions $\longrightarrow$ family replication.

- From the $D = 10$ kinetic +gauge vertex term:

$$\mathrm{Tr}\ \{\bar{\Psi}\Gamma^M D_M \Psi\} \quad \longrightarrow \quad A \cdot \Psi \cdot \Psi \text{ coupling}$$

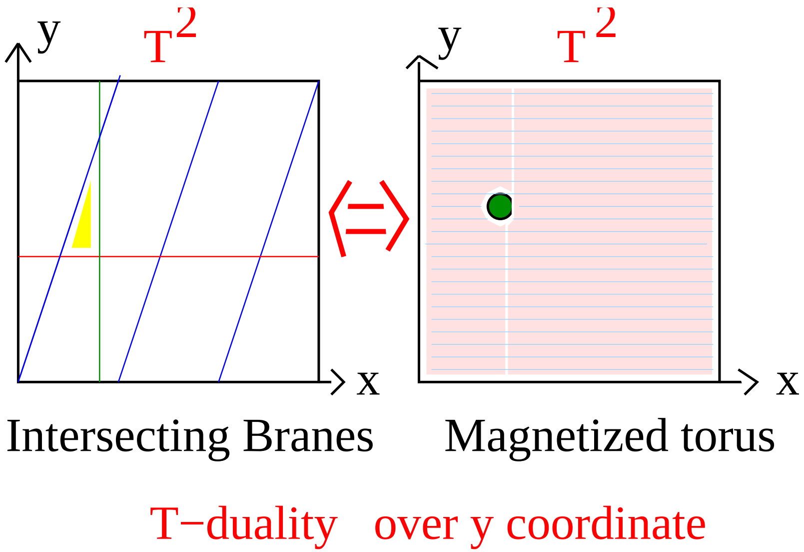
KK reduction: $\downarrow$

$$Y_{IJK} = \frac{g}{2}\int_{\mathbf{X_6}} \psi_I^{\alpha\dagger}\Gamma^i\, \psi_J^{\beta}\phi_{K\ i}^{\gamma}\, f_{\alpha\beta\gamma} \tag{64}$$



- The Yukawa coupling constant are thus obtained as overlap integrals of the three wave functions in the extra six dimensions.
- One can explicitely compute the wave-functions $\psi_J^\beta$ and $\phi_{K\ i}^\gamma$ for the case of $T^{2n}$ compactifications with magnetic fluxes. They turn out to be proportional to Jacobi theta functions depending on compact corrdinates.
- The results should be equivalent to the results found for intersecting $D6$ branes after the appropriate replacements.

# Matching with intersecting D6-branes




- With T-dualities along three y-directions we go from intersecting D6-branes to Type I string theory (IIB orientifold under $\Omega$ with $D9$-branes).

- Yukawa couplings in intersecting D6-brane models may be rewritten

$$Y_{ijk}^{\rm int} = h_{\rm qu} \prod_{r=1}^{n} e^{H_{\rm int}^{(r)}/2} \vartheta \begin{bmatrix} \delta_{ijk}^{(r)} \\ 0 \end{bmatrix} \left( \nu^{(r)}, J^{(r)} (I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}) \right) \qquad (65)$$

Here $H_{int}$ is a known function of the open string moduli $\nu^{(r)}$ (brane locations $\epsilon^i$ and W.L.) , $J^{(r)}$ are the Kahler moduli of the 3-tori.

- Yukawa couplings in magnetized T-dual obtained upon explicit integration:

$$Y_{ijk} = \frac{g_{10}}{2} \prod_{r=1}^{3} \left( \frac{2{\rm Im}\, \tau^{(r)}}{(\mathcal{A}^{(r)})} \right)^{1/4} \left| \frac{\mu_{ab}^{(r)} \mu_{ca}^{(r)}}{\mu_{bc}^{(r)}} \right|^{1/4} e^{H_{magn}^{(r)}/2} \times \qquad (66)$$

$$\times \, \vartheta \begin{bmatrix} \delta_{ijk}^{(r)} \\ 0 \end{bmatrix} \left( \zeta^{(r)}, \tau^{(r)} |I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}| \right) \qquad (67)$$

where $\mu_{ab} = \theta_{ab}/\alpha'$, $\zeta^{(r)}$ are open string moduli (actually W.L.), $\mathcal{A}^{(r)}$ are the areas of the 3 tori, and $\tau^{(r)}$ are the complex structure of the tori.

- One can check that both expressions agree in the large volume (small angle) limits as long as:

$$h_{qu} = g_{10}\alpha'^{-3/4}\prod_{r=1}^{3}\left|\frac{\mu_{ab}^{(r)}\mu_{ca}^{(r)}}{\mu_{bc}^{(r)}}\right|^{1/4} = e^{\phi_{10}/2}\left|\frac{\theta_{ab}^{(r)}\theta_{ca}^{(r)}}{\theta_{bc}^{(r)}}\right|^{1/4}$$

- This agrees with the string computation of $h_{qu}$ [a] for small angle :

$$h_{qu} = e^{\phi_{10}/2}\prod_{r=1}^{3}\left[\frac{\Gamma(1-\theta_{ab}^{(r)})\Gamma(1-\theta_{ca}^{(r)})\Gamma(\theta_{ab}^{(r)}+\theta_{ca}^{(r)})}{\Gamma(\theta_{ab}^{(r)})\Gamma(\theta_{ca}^{(r)})\Gamma(1-\theta_{ab}{}^{(r)}-\theta_{ca}^{(r)})}\right]^{1/4} \quad (68)$$

$$\longrightarrow e^{\phi_{10}/2}\prod_{r=1}^{3}\left[\frac{\theta_{ab}^{(r)}\theta_{ca}^{(r)}}{\theta_{bc}^{(r)}}\right]^{1/4} \quad (69)$$

[a] Cvetic,Papadimitriou;Abel,Owen (2003);Lust, Mayr,Richter,Stieberger (2004)

- In $N = 1$ supergravity the normalized Yukawa couplings are obtained from the Kahler metrics $K_{mn}$ of the bifundamentals and from the SUPERPOTENTIAL $W_{ijk}$:

$$Y_{ijk} = (K_{ab}K_{bc}K_{ca})^{-1/2}\, e^{K/2} W_{ijk}$$

- Matching is obtained if:

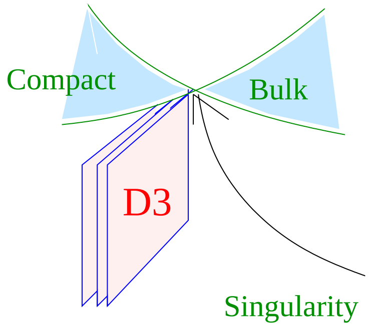
$$W_{ijk} = \prod_{r=1}^{3} \vartheta \begin{bmatrix} \delta_{ijk}^{(r)} \\ 0 \end{bmatrix} \left( \zeta^{(r)}, \tau^{(r)} |I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}| \right)$$

$$(K_{ab}K_{bc}K_{ca})^{-1/2}\, e^{K/2} = \frac{g_{10}}{2} \prod_{r=1}^{3} \left( \frac{2\mathrm{Im}\, \tau^{(r)}}{\mathcal{A}^{(r)}} \right)^{1/4} \left| \frac{\mu_{ab}^{(r)} \mu_{ca}^{(r)}}{\mu_{bc}^{(r)}} \right|^{1/4} e^{H_{mag}^{(r)}/2}$$

- This agrees with the computation of toroidal Kahler potential and metrics previously discussed.

- Note that Type IIA computation of Yukawa couplings requires a stringy computation summing worldsheet instanton contributions.
- Type IIB computation just requires a purely field-theoretical Kaluza-Klein reduction. No string computation involved.
- It is a nice check of Mirror Symmetry that both computations nicely match.

# V- BRANES AT SINGULARITIES





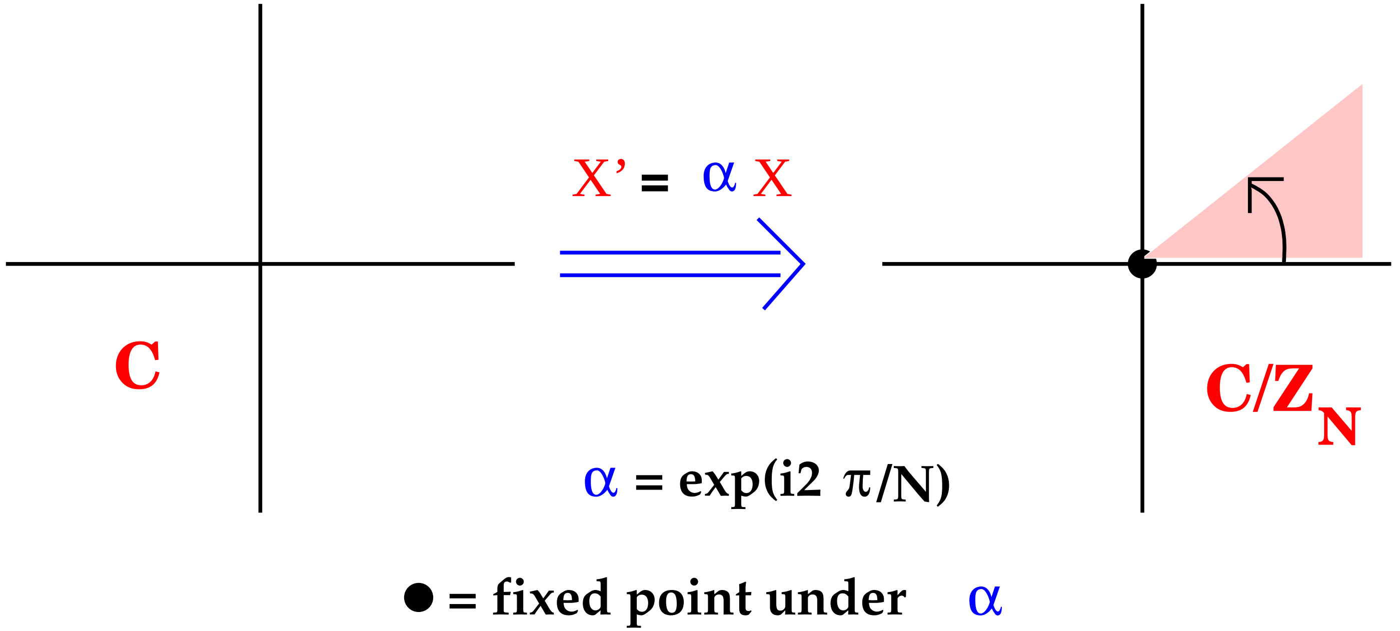
- G. Aldazabal, L. E. Ibáñez, F. Quevedo and A. M. Uranga, "D-branes at singularities: A bottom-up approach to the string embedding of the standard model," JHEP **0008**, 002 (2000) [arXiv:hep-th/0005067].
- D. Malyshev, H. Verlinde, 'D-branes at Singularities and String Phenomenology', hep-th/0711.2451.
- H. Verlinde and M. Wijnholt, "Building the standard model on a D3-brane," JHEP **0701** (2007) 106 [arXiv:hep-th/0508089]
- D. Berenstein, V. Jejjala and R. G. Leigh, "The standard model on a D-brane," Phys. Rev. Lett. **88** (2002) 071602 [arXiv:hep-ph/0105042].

## Bottom-up embedding of the SM in string theory

- Top-down approach to the embedding of the SM. Start with e.g a large gauge group (e.g. $E_8 \times E_8$) in $D = 10$ and break down the symmetry untill we find the SM.
- Bottom-up approach:
  - Look for local configurations of Dp-branes resembling as much as possible the SM.
  - This local configuration will in general be part of a global compact model. (Most likely myriads of CY may contain such local configuration).
  - Most relevant phenomenological properties depend only in the local configuration
- Most local branes filling Minkowski space: Stacks of D3-branes at singularities in the CY (required for chirality).

## D3-branes at a $Z_N$ singularity

- Complex coordinate X rotated by a discrete $\alpha = e^{i2\pi/N}$




- There is a fixed singular point at origin. A $Z_N$ orbifold singularity

- Consider the local complex coordinates $x_1, x_2, x_3$ in the CY. Consider the $Z_N$ twist generated by $\theta$:

$$\theta\,(x_1, x_2, x_3) \longrightarrow (\alpha^{l_1} x_1, \alpha^{l_2} x_2, \alpha^{l_3} x_3) \tag{70}$$

with $\theta^N = 1$, $l_a \in \mathbf{Z}$. One has $N = 1$ SUSY for $l_1 + l_2 + l_3 = 0$ mod $N$.

- Consider M D3-branes located on top of singularity. The open string spectrum must be invariant under $\theta$ and a simultaneous action on the CP factor degrees of freedom:

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_{n_0}, e^{2\pi i/N}\mathrm{I}_{n_1}, \ldots, e^{2\pi i(N-1)/N}\mathrm{I}_{n_{N-1}}) \tag{71}$$

where $\mathrm{I}_{n_i}$ is the $n_i \times n_i$ unit matrix, and $\sum_i n_i = M$.

- The projection for massless states:

Gauge bosons:

$$\lambda\psi^{\mu}_{-1/2}|0> \longrightarrow \lambda = \gamma_{\theta,3}\lambda\gamma^{-1}_{\theta,3} \tag{72}$$

- Then the gauge symmetry is broken to (at most) N factors:

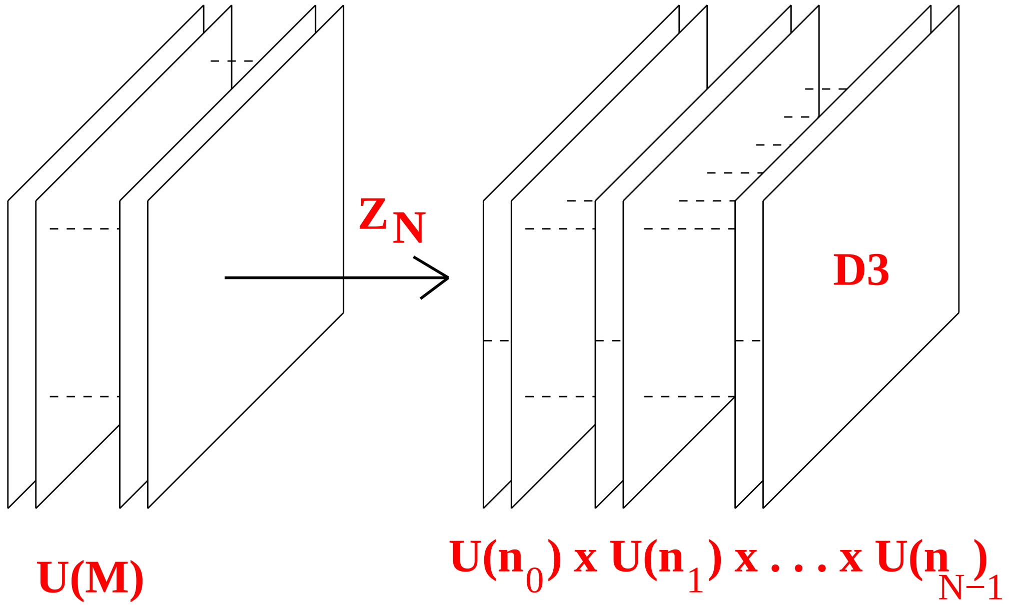
$$U(M) \longrightarrow U(n_0)\times U(n_1)\times ... \times U(n_{N-1}) \tag{73}$$

Chiral multiplets:

$$\lambda\psi^{r}_{-1/2}|0> \longrightarrow \lambda = e^{i2\pi l_r/N}\gamma_{\theta,3}\lambda\gamma^{-1}_{\theta,3} \quad , \quad r = 1,2,3 \tag{74}$$

Then there are chiral matter in bifundamentals:

$$\sum_{l_r}\sum_{i=0}^{N-1}(n_i,\bar{n}_{i+l_r}) \tag{75}$$



Z N
D3
U(M)
U(n0) x U(n1) x . . . x U(nN−1)

## Searching for a 3 generation SM

- We can start with D3 branes at a $Z_N$ singularity with twist $v = 1/N(l_1, l_2, l_3)$

$$U(3) \times U(2) \times U(n_2) \times .... \times U(n_{N-1}) \quad (76)$$

and then quarks would come from

$$\sum_r (n_0, \bar{n}_{0+l_r}) \quad (77)$$

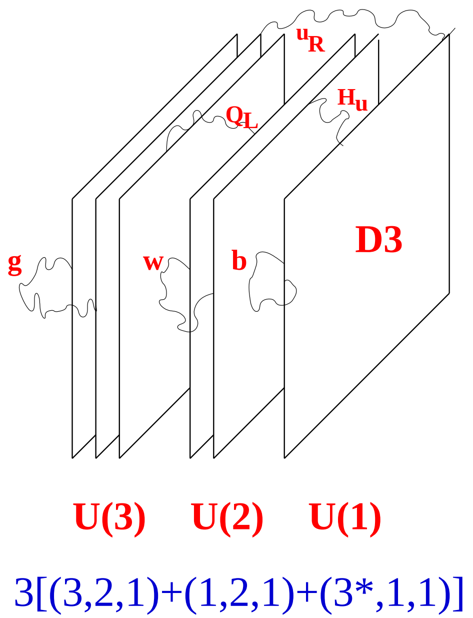
- Note that there are at most 3 left-handed quarks, and 3 generations are obtained for $l_1 = l_2 = l_3 = 1$ which corresponds to the $Z_3$ orbifold in the SUSY case.
- Thus only for $Z_3$ 3 generations appear(also for non-Abelian discrte groups containing $Z_3$).

## The SM from D3-branes at a $Z_3$ singularity

- At $Z_3$ we have gauge group $U(n_0) \times U(n_1) \times U(n_2)$ . Only 3 gauge factor possible . We want $n_0 = 3, n_1 = 2, n_2 = 1$ so we have CP matrix:

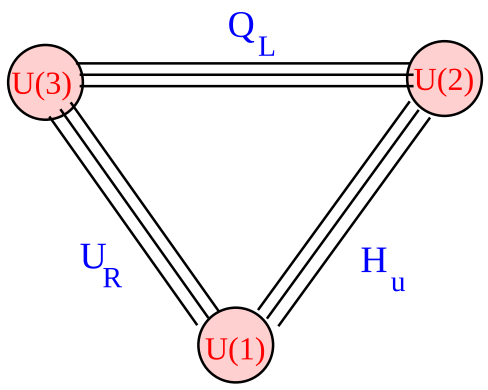
$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_3, e^{2\pi i/3}\mathrm{I}_2, e^{4\pi i/3}\mathrm{I}_1) \tag{78}$$

- This leads to gauge group and particle content:



## The SM $Z_3$ QUIVER

- Gauge group, chiral content and Yukawa couplings of $D3$-branes at singularities may be expressed in terms of graphs called Quivers.




- $Z_N$ quivers have N nodes representing gauge groups and bifundamental chiral fields are represented by the links. Closed triangles correspond to Yukawa couplings.
- More complicated singularities also admit a quiver rep. (ask Herman).
- But in this example the chiral spectrum is anomalous!.

## Local RR twisted tadpole cancellation

- Overall RR charge from the 4-form should vanish:

$$Tr\,\gamma_{\theta,3} \;=\; 0 \qquad (79)$$

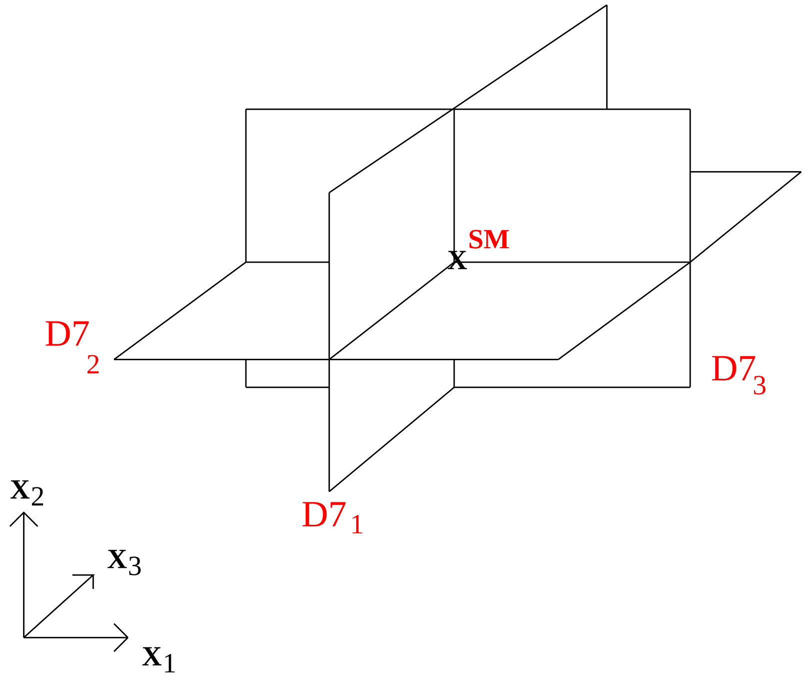
For our case with $\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_3, \alpha\mathrm{I}_2, \alpha^2\mathrm{I}_1)$ one has:

$$Tr\,\gamma_{\theta,3} \;=\; 3 \,+\, 2\alpha \,+\alpha^2 \;\neq\; 0 \qquad (80)$$

- Need to new source of RR charge: D7-branes. They should wrap the CY and pass through the D3-branes. RR tadpole conditions modified. For general $Z_N$ singularities:

$$[\prod_{r=1}^{3} 2\sin(\pi k l_r/N)\,]\;\mathrm{Tr}\,\gamma_{\theta^k,3} + \sum_{r=1}^{3} 2\sin(\pi k l_r/N)\;\mathrm{Tr}\,\gamma_{\theta^k,7_r} \;=\; 0 \qquad (81)$$

- There is one condition for each of the $N-1$ twisted sectors. This is because there are $N-1$ RR twisted charges.
- One can show that these conditions guarantee cancellation of non-Abelian anomalies .
- Here the $D7_r$ are transverse to the local $x_r$ complex coordinate. Since they are (before compactification) infinitely extended, D7-branes give rise only to global symetries. (Large wrapped volume corresponds to $g_7 = 0$).




- $D7_r$ sector. With CP twist matrix (for e.g. $r = 3$)

$$\gamma_{\theta,7_3} = \text{diag}\,(\,\mathrm{I}_{u_0}\,,\, e^{2\pi i/N}\mathrm{I}_{u_1}\,,\,\ldots,\, e^{2\pi i(N-1)/N}\mathrm{I}_{u_{N-1}}\,) \qquad (82)$$

one has (global) D7 symmetry: $U(u_0) \times ... \times U(u_{N-1})$ and matter fields:

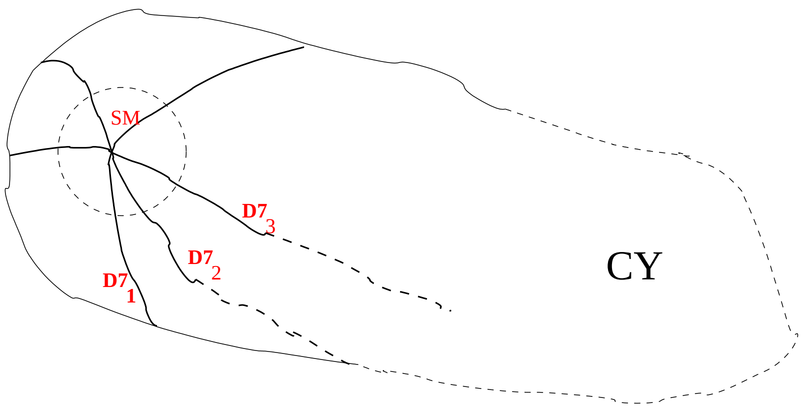
- $D3 - D7_3$ chiral multiplets. (Fermions $\lambda|s_3, -1/2>$)

$$\lambda_{37_3} = e^{i\pi l_3/N} \gamma_{\theta,3} \lambda \gamma_{\theta,7_3}^{-1} \rightarrow \sum_{i=0}^{N-1} (n_i, \bar{u}_{i+l_3}) \quad (83)$$

- $D7_3 - D3$ chiral multiplets.

$$\lambda_{7_3 3} = e^{i\pi l_3/N} \gamma_{\theta,7_3} \lambda \gamma_{\theta,3}^{-1} \rightarrow \sum_{i=0}^{N-1} (u_i, \bar{n}_{i+l_3}) \quad (84)$$

- These new multiplets precisely cancel the non-Abelian gauge anomalies from the D3-brane sector.




- In the case of $Z_3$ one has

$$3\mathrm{Tr}\,\gamma_{\theta,3} + \sum_{r=1}^{3} \mathrm{Tr}\,\gamma_{\theta,7_r} \; = \; 0 \tag{85}$$

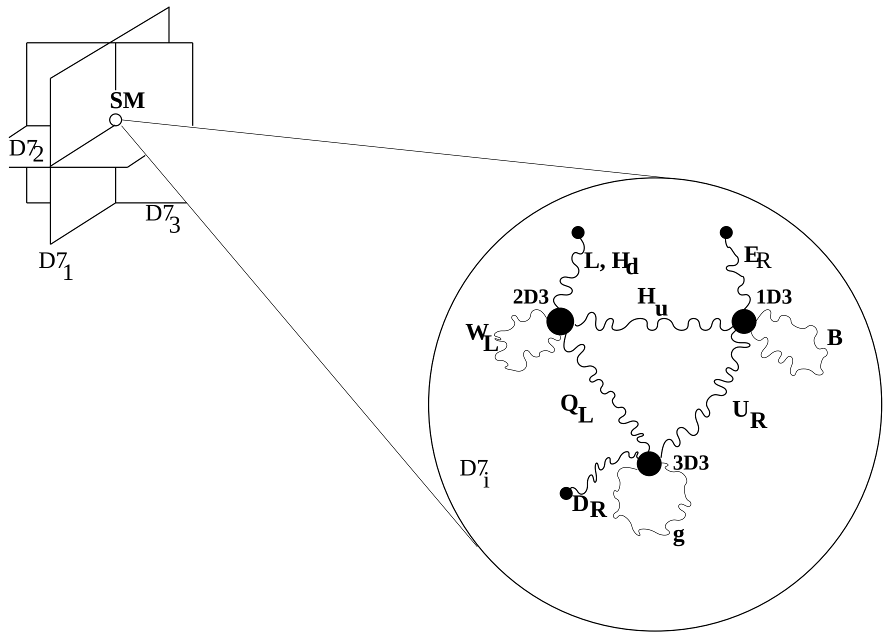
- Then adding 3 sets of $D7_r$ branes each transverse to the $x_r$ plane and with CP twist matrices ( $u_0 = 0, u_1 = 1, u_2 = 2$)

$$\gamma^r_{\theta,7} = \mathrm{diag}\,(e^{i2\pi/3}, e^{4\pi i/3}\mathrm{I}_2) \tag{86}$$

one gets (here $\alpha = exp(i2\pi/3)$)

$$\sum_r Tr\gamma_{\theta,7} + 3Tr\gamma_{\theta,3} \; = \; 3(\alpha + 2\alpha^2) \; + \; 3(3 + 2\alpha + \alpha^2) \; = \; 0 \tag{87}$$

and tadpoles cancel. There is a $(U(1) \times U(2))^3$ global symmetry.


SM
D7 2
D7 3
D7 1
L, H d
E R
2D3
H u
1D3
W L
B
Q L
U R
D7 i
3D3
D R
g

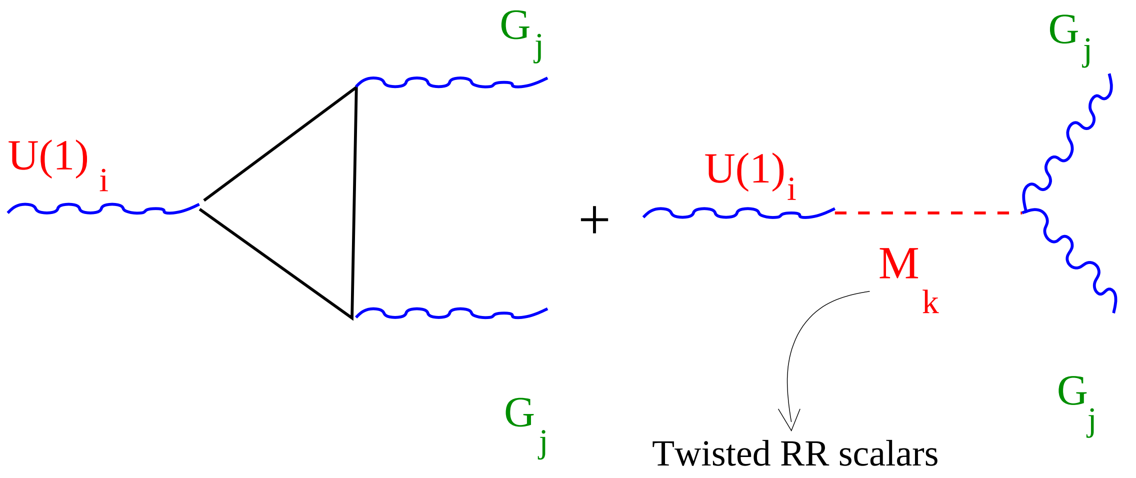
# $U(1)$ anomalies

- In a $Z_N$ singularity there are up to $N$ $U(1)$ factors:

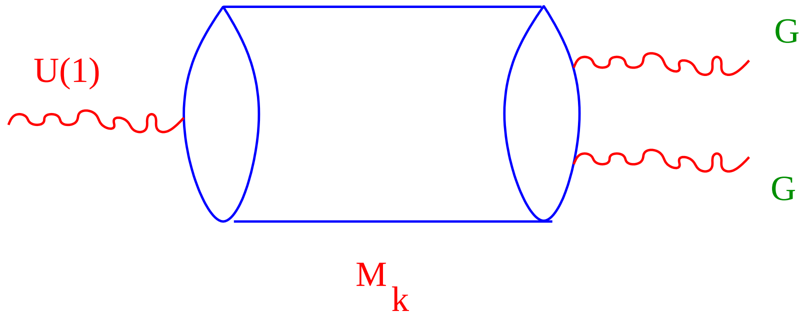
$$Z_N \quad : \quad U(M) \;\rightarrow\; U(n_0) \times U(n_1) \times ... \times U(n_{N-1}) \qquad (88)$$

- Most $U(1)$'s have triangle anomalies which are cancelled by a generalized GS mechanism, analogous to the one we discussed already.

- There is an important difference though: the scalars involved in the cancellation are the $N-1$ twisted RR fields in the singularity.



- The general expression for the mixed $U(1) - G^2$ anomaly may be written




$$A_{jl} = \frac{-i}{2N} \sum_{k=0}^{N-1} (n_j e^{i2\pi \frac{kj}{N}}) \times (\Pi_{r=1}^{3} 2sin(k\pi l_r/N)) \times (e^{i2\pi \frac{-kl}{N}}) \quad (89)$$

- One can consider a a general combination of $U(1)$'s

$$Q = \sum_{j=0}^{N-1} c_j \frac{Q_{n_j}}{n_j} \quad (90)$$

(we take $c_j = 0$ if $n_j = 0$).

- Then one can check that weighted diagonal generator with all $c_j$ equal

$$Q_{diag} = \sum_{j=0}^{N-1} \frac{Q_{n_j}}{n_j} \tag{91}$$

  is always anomaly free and massless. However it must be $n_j \neq 0$ for all j.

- This is generic. For some particular types of twists other anomaly-free $U(1)$'s may appear.

## Hypercharge

- In our case we have three $U(1)$'s from $U(3) \times U(2) \times U(1)$ and we have there is only one anomaly free $U(1)$: hypercharge

$$Y = Q_{diag} = -(\frac{Q_3}{3} + \frac{Q_2}{2} + Q_1) \quad (92)$$

- It automatically gives the correct asignements of hypercharge for SM particles
- Massless chiral spectrum:

| Matter fields | $Q_3$ | $Q_2$ | $Q_1$ | $Q_{u_1^r}$ | $Q_{u_2^r}$ | $Y$ |
|---|---|---|---|---|---|---|
| **33** sector | | | | | | |
| $3(3,2)$ | 1 | -1 | 0 | 0 | 0 | 1/6 |
| $3(\bar{3},1)$ | -1 | 0 | 1 | 0 | 0 | -2/3 |
| $3(1,2)$ | 0 | 1 | -1 | 0 | 0 | 1/2 |
| **37**$_r$ sector | | | | | | |
| $(3,1)$ | 1 | 0 | 0 | -1 | 0 | -1/3 |
| $(\bar{3},1;2')$ | -1 | 0 | 0 | 0 | 1 | 1/3 |
| $(1,2;2')$ | 0 | 1 | 0 | 0 | -1 | -1/2 |
| $(1,1;1')$ | 0 | 0 | -1 | 1 | 0 | 1 |
| **7**$_r$**7**$_r$ sector | | | | | | |
| $3(1;2)'$ | 0 | 0 | 0 | 1 | -1 | 0 |

Table 1: Spectrum of $SU(3) \times SU(2) \times U(1)$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s come from the D3-brane sector. The next two come from the D7$_r$-brane sectors, written as a single column (Aldazabal et al. hep-th/0005067).

## $sin^2\theta_W$ from D3-branes at $Z_N$ singularities

- For a general embedding of SM in $U(3)\times U(2)\times U(1)^{N-2}$ the hypercharge is given by

$$-Y = Q_{diag} = (\frac{Q_3}{3} + \frac{Q_2}{2} + \sum_{i=2}^{N-1} Q_i) \tag{93}$$

- $k_1$ = relative normalization of Y compared to non-Abelian generators:

$$k_1 = 2\sum_{j=1}^{N-1}\frac{1}{n_j} = 2(\frac{1}{3}+\frac{1}{2}+N-2) = \frac{5}{3}+2(N-2) \tag{94}$$

and then for $Z_N$ singularities one has

$$sin^2\theta_W = \frac{1}{1+k_1} = \frac{3}{6N-4} \tag{95}$$

- Thus for the $Z_3$ singularityone has at the string scale $sin^2\theta_W = 3/14$.
- In the SM construction one has at low energies (after one turns on vevs for $(1, 2')\ 7_r - 7_r$ ) the MSSM content with 3 sets of Higgs multiplets. Doing the running one finds no gauge coupling unification for $sin^2\theta_W = 3/14$.
- Blowing up the singularity, i.e. $< M_k > \neq 0$ may correct for this since the gauge kinetic functions for D3's are given by:

$$f_a \ = \ S \ + \sum_k \ d_k M_k \tag{96}$$

where $M_k$ are the twisted moduli at the singularity and $d_k$ computable coefficients. (The shift of the $M_k$ operates in GS mechanism)

- In fact gauge coupling unification nicely occurs in a left-right symmetric version of the model.

## A left-right symmetric $Z_3$ model

- We can construct a model with gauge group $U(3) \times U(2)_L \times U(2)_R$ by taking seven $D3$ branes with CP twist matrix:

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_3, e^{2\pi i/3}\mathrm{I}_2, e^{4\pi i/3}\mathrm{I}_2) \quad (97)$$

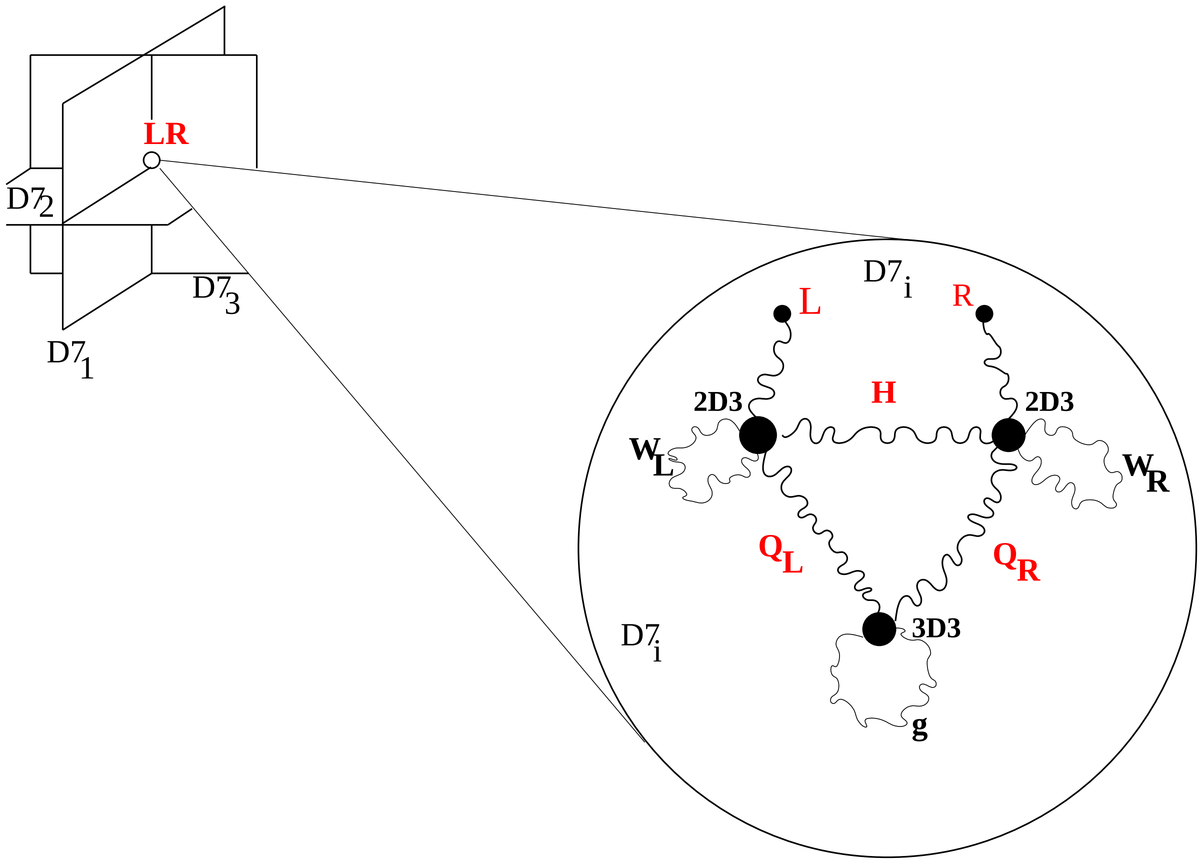
- The set of $D7_r$-branes required is quite simple. It is just 3 sets of 2 $D7_r$ branes with twist matrix

$$\gamma_{\theta,7^r} = \mathrm{diag}\,(e^{2\pi i/3}, e^{4\pi i/3}), \quad (98)$$

tadpoles cancel

$$\sum_r Tr\gamma_{\theta,7} + 3Tr\gamma_{\theta,3} = 3(\alpha+\alpha^2) + 3(3+2\alpha+2\alpha^2) = 0 \quad (99)$$

- This leads to gauge group and particle content:


LR
D7_2
D7_3
D7_1
D7_i
L
R
H
2D3
2D3
W_L
W_R
Q_L
Q_R
3D3
D7_i
g

| Matter fields | $Q_3$ | $Q_L$ | $Q_R$ | $Q_{U_1^i}$ | $Q_{U_2^i}$ | $B-L$ |
|---|---|---|---|---|---|---|
| **33** sector | | | | | | |
| $3(3,2,1)$ | 1 | -1 | 0 | 0 | 0 | 1/3 |
| $3(\bar{3},1,2)$ | -1 | 0 | 1 | 0 | 0 | -1/3 |
| $3(1,2,2)$ | 0 | 1 | -1 | 0 | 0 | 0 |
| **37**$_r$ sector | | | | | | |
| $(3,1,1)$ | 1 | 0 | 0 | -1 | 0 | -2/3 |
| $(\bar{3},1,1)$ | -1 | 0 | 0 | 0 | 1 | 2/3 |
| $(1,2,1)$ | 0 | 1 | 0 | 0 | -1 | -1 |
| $(1,1,2)$ | 0 | 0 | -1 | 1 | 0 | 1 |
| **7**$_r$**7**$_r$ sector | | | | | | |
| $3(1)'$ | 0 | 0 | 0 | 1 | -1 | 0 |

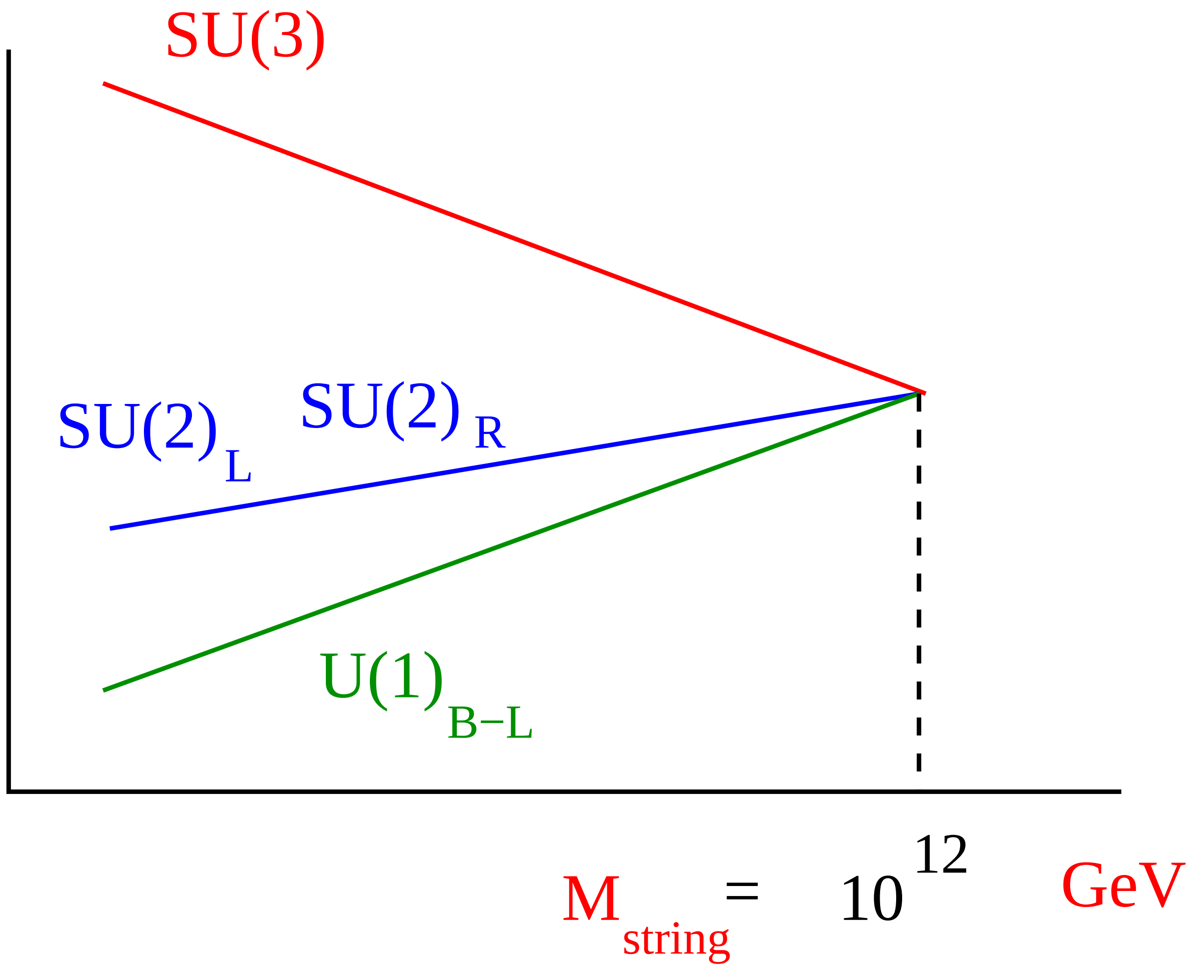
Table 2: Spectrum of $SU(3) \times SU(2)_L \times SU(2)_R$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s arise from the D3-brane sector. The next two come from the D7$_r$-brane sectors (Aldazabal et al. hep-th/0001083; hep-th/0005067).

- The extra triplets from $(7^r - 3) + (3 - 7^r)$ sectors are generically massive. Thus the low energy content is a minimal LR model with 3 EW Higgs sets. (This is another example of the necessity of 3 Higgs sets to cancel U(2) anomalies!).

- There are 2 anomalous $U(1)$'s which disappear from the low energy spectrum and one massless anomaly free $U(1)_{B-L}$:

$$Q_{B-L} = -2(\frac{Q_3}{3} + \frac{Q_L}{2} + \frac{Q_R}{2}) \tag{100}$$

- Recall: $Y = -T_R^3 + \frac{1}{2} Q_{B-L}$ and $k_{B-L} = 32/3$. One then finds that for $M_{W_R} \simeq 1$ TeV couplings nicely unify at $10^{12}$ GeV.




- The breaking $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ may be achieved if three extra D7-branes added leading to chiral fields transforming like

$(3,1,1)+(1,2,1)+(1,1,2)+h.c,)$ which do not modify the running at one loop.

- There are two regions for the running $M_R < Q < M_{string}$ with Left-Right gauge group and $M_Z < Q < M_R$ with SM content.

$$\sin^2\theta_W(M_Z) = \frac{3}{14}\left(1+\frac{11\alpha_e(M_Z)}{6\pi}\left[\left(B_L-\frac{3}{11}B_1'\right)\log\left(\frac{M_s}{M_R}\right)\right.\right.$$
$$\left.\left. + \left(b_2-\frac{3}{11}b_1\right)\log\left(\frac{M_R}{M_Z}\right)\right]\right) \quad (101)$$

$$\frac{1}{\alpha_e(M_Z)} - \frac{14}{3\alpha_3(M_Z)} = \frac{1}{2\pi}\left[\left(b_1+b_2-\frac{14}{3}b_3\right)\log\left(\frac{M_R}{M_Z}\right)\right.$$
$$\left. + \left(B_1'+B_L-\frac{14}{3}B_3\right)\log\left(\frac{M_s}{M_R}\right)\right] \quad (102)$$

where one has

$$B_1' = B_R + \frac{1}{4}B_{B-L} \ ; \ B_3 = -3, \ B_L = B_R = +3\,, \ B_{B-L} = 16 \quad (103)$$

- With $M_R = 1$ TeV one finds

$$sin^2 W(M_Z) \ = \ 0.231 \, ; \ M_{strings} \ = \ 9 \times 10^{11} \ GeV \tag{104}$$

- A right-handed $W_R$ gauge boson with $M_R = 1$ TeV would be accesible at LHC. It could be the signal of string unification at an intermediate scale $M_{string} = 10^{12}$ GeV.

- If $M_{string} = 10^{12}$ GeV and SUSY breaking scale is also of that order then one expects SUSY breaking soft terms of order

$$m_{soft} \ = \ \frac{M_{string}^2}{M_{Planck}} \ \simeq \ 10 TeV \tag{105}$$

which is of the required order of magnitude.

- Concerning Yukawa couplings, They are either of order 1 or zero. There are quark Yukawa couplings $\epsilon_{ijk} Q_L^i Q_R^j H^k$ from $(33)^3$ couplings.
- Lepton Yukawa couplings $L^i R^j H^k$ are perturbatively forbidden by (anomalous $U(1)$) gauge symmetries but may be induced by string instanton effects.
- Masses for right handed neutrinos may arise from non-renormalizable couplings involving the fields doing the $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ breaking. (In any event those masses should be $<$ 1 TeV).
- This minimal L-R configuration can be embedded in a compact scheme like F-theory. By the way....

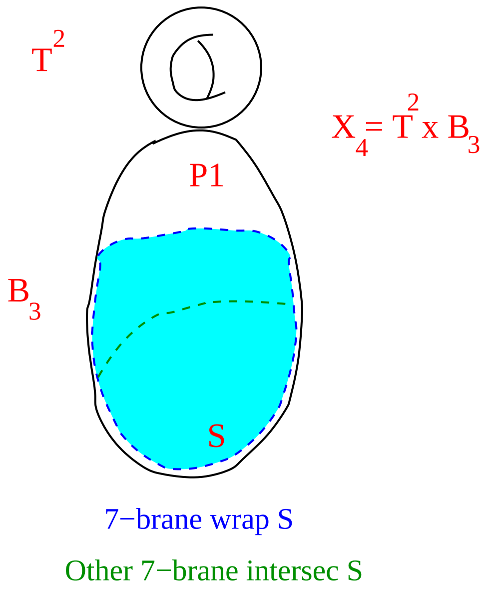
## Some comments about F-theory

- F-theory may be considered as a non-perturbative version of Type IIB orientifolds.
- Type IIB string theory in $D = 10$ has a non-perturbative $SL(2, \mathbf{Z})$ S-duality symmetry under which $\tau = \frac{1}{g_s} + iC_0$ transforms. The idea is to identify locally this $\tau$ with the complex structure of a 2-torus living in extra 11-th and 12-th dimensions.
- Thus F-theory gives a geometric description of the S-duality symmetry in compactifications of Type IIB theory.
- One considers compactifications of this 12-dimensional theory on a CY complex 4-fold $X_4$ down to $D = 4$.
- The CY 4-fold must be elliptically fibered over a complex 3-dimensional CY $B_3$, meaning that locally one can write

$$X_4 \simeq T^2 \times B_3 \simeq K3 \times S \tag{106}$$

with the complex structure modulus of the $T^2$ identified with $\tau$.

- These are clearly non-perturbative vacua since e.g. the $SL(2,\mathbf{Z})$ symmetry includes transformations under which $g_s \to 1/g_s$.




- The theory contains F-theory $(p,q)$ 7-branes which wrap the complex 2-fold $S$. Inside the 3-fold $B_3$ these 7-branes correspond to complex codimension 1 singularities. Depending on the canonical ADE classification of the singularities the gauge groups are $SU(n+1)$, $SO(2n)$ and $E_6, E_7, E_8$.

- Thus the gauge group in F-theoretical 7-branes goes beyond what one can get in perturbative Type IIB orientifold $D7$-branes in which only $SU(n+1)$ and $SO(2n)$ gauge groups may be obtained.
- Furthermore in F-theory the matter content in models with $SO(2n)$ gauge symmetry may include spinorial representations which are not present in perturbative IIB orientifold compactifications.
- The $D = 4$ chiral matter fields in F-theory have the same qualitative origin as in perturbative IIB orientifolds.
- In general addition of magnetic fluxes on 7-branes is required to get chirality.
- There are 3 general classes of chiral matter fields from 7-branes, analogous to the ones in IIB orientifolds, $A$, $\phi$ , $I$. Fields $I$ live at intersections of two 7-branes in a complex curve $\Sigma$ inside $S$.

- Recently Beasley, Heckmann and Vafa have constructed LOCAL F-theory models with a GUT gauge group and $S = dP_n$ surfaces. Gauge group broken to SM through $U(1)$ magnetic fluxes on $S$.
- They are local brane models which are consistent with gauge coupling unification.
- Many qualitative features of magnetized IIB D3/D7 orientifolds apply to the F-theory effective action. (E.g. the structure of SUSY breaking soft terms to be discussed later).