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PITP '08 Lecture I S Kachru

Where are there principled excuses to talk about strings & phenomenology? After all, in most scenarios, strings are relevant at $M_s \sim 10^{17}$ GeV $>>$ any $E_{\text{experiment}}$.

$\boxed{\text{Excuse 1}}$: The relevant phenomenology is dominated by M_s or M_P suppressed operators. This is true in many cases of central interest:

a) SUSY mediation mechanisms where

$$X = x + \theta\psi_x + \theta^2 F_x$$

$$\langle F_x \rangle \gtrsim 10^{11}\text{ GeV} \quad \Big\} \begin{array}{l}\text{gravity,}\\ \text{anomaly,}\\ \text{mirage}\\ \text{mediation}\end{array}$$

$\Rightarrow$ terms $K \supset \int d^4\theta \, \frac{X^\dagger X}{M_P^2} \phi_{SM}^\dagger \phi_{SM}$

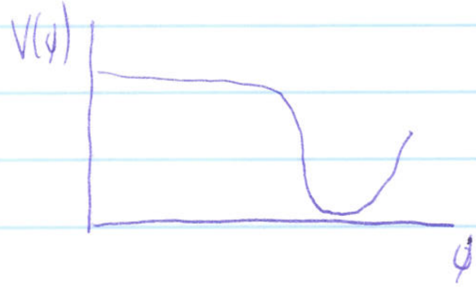
dominate (or $O(1)$ contribute to) soft masses, and need to be:

- <u>suppressed</u>, in anomaly mediation
- <u>flavor blind</u>, in gravity mediation

These are dim 6 M_P suppressed $\rightarrow$ sensitive to UV physics.

b) Proton decay

c) Inflation



$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2$$

$$\eta = M_P^2 \, V''/V$$

So <u>dim 6</u> operators can $\Rightarrow$ $\mathcal{O}(1)$ η;

but inflation <u>requires</u> $\epsilon, \eta \ll 1$ for slow roll (suitable generalization for non slow-roll also $\rightarrow$ sensitivity to M_P suppressed ops).

<u>Some</u> ("large field", $\Delta\phi > M_P$) models are

sensitive to operators of dim >> 6

$$V = V_{ren} + \sum_{n=1}^{\infty} \phi^4 \, c_n \left(\frac{\phi}{M_P} \right)^n \Bigg\} \begin{array}{l} \text{any } \mathcal{O}(1) \\ c_n \text{ can} \\ \text{ruin large} \\ \text{field infl.} \end{array}$$

We may talk about a) & c) a bit more later. But first ...

$\boxed{\text{Excuse 2}}$: The field theory dynamics involved is strongly coupled, in a way that is amenable to a weakly coupled <u>dual</u> <u>gravity</u> description.

a) DSB, needed to explain why

$$F_X \ll M_{pl}$$

in SUSY models, is often a strong coupling phenomenon. Some DSB models have weakly coupled gravity descriptions.

b) Composite models (SUSY or not) often have a useful gravity dual description. Then phenomena like $\left\{ \begin{array}{l} \text{Yukawa hierarchies} \\ \text{soft mass hierarchies} \end{array} \right.$ are geometrically explained.

I'll start by lecturing about 2 a) & b) in an EQFT & string context, then move to 1 a) & (c) as time permits.

I. Hierarchies from a slice of AdS_5.

(c.f. Randall/Sundrum papers)

a. Trapping gravity in AdS_5

We can obviously live in a higher-D (say 5D) world if the extra dims are compact.

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E.g.

$$dS^2 = \eta_{\mu\nu} dX^\mu dX^\nu + R^2 dX_5^2$$

5D Einstein action

$$S_5 = \int d^5x \sqrt{-G}\, M_5^3 \mathcal{R}_5$$

Integrating out "extra" X_5 dim $\Rightarrow$

$$\left.M_4^2 \sim M_5^3 R\ \right\} \rightarrow \text{4D gravity w/ } G_N \text{ fixed by this}$$

For small R, clearly compatible w/ experiment.

[At this level -- could worry about modulus, etc.].

<u>More general</u>: Metric can be <u>warped</u>.

Consider:

$$S = \int d^5x \sqrt{-G}\,(\mathcal{R} - \Lambda) +$$

$$\int d^4x \sqrt{-g}\,(-V_{brane})$$

$$g_{\mu\nu} = \delta^M_\mu \delta^N_\nu G_{MN}(X_5 = 0) \qquad \begin{array}{l}\mu = 1 \cdots 4\\ M = 1 \cdots 5\end{array}$$

Following Randall + Sundrum, we take the most general $SO(3,1)$ symmetric ansatz:

$$ds^2 = e^{2A(x_5)} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$$

Then, Einstein's equations $\Rightarrow$

$$(\star) \quad 6(A')^2 + \tfrac{1}{2}\Lambda = 0 \qquad ' \equiv \frac{d}{dx_5}$$

$$(\surd) \quad 3A'' + \tfrac{1}{2} V \delta(x_5) = 0$$

Choosing $\Lambda < 0$, can solve $(\star) \Rightarrow$

$$A = \pm k x_5 \qquad k = \sqrt{\frac{-\Lambda}{12}}$$

Then integrating $(\surd)$ from $x_5 = -\epsilon \to \epsilon$ to pick up δ-function contribution $\Rightarrow$

$$3\Delta(A') = -\tfrac{1}{2} V$$

discontinuity across $x_5 = 0$

So, to solve the EOM w/our ansatz for the metric, we need to take:

$$A = \begin{cases} -kX_5 & X_5 > 0 \\ kX_5 & X_5 < 0 \end{cases}$$

Then we must tune brane tension V in terms of Λ:

$$\boxed{V = 12k = 12\sqrt{\frac{-\Lambda}{12}}} \quad \Bigg\} \text{ tune of 4D c.c.}$$

This yields a solution where

$$ds^2 = e^{-2k|X_5|}\eta_{\mu\nu}\,dX^\mu dX^\nu + dX_5^2$$

- The warp factor is sharply peaked at $X_5 = 0$, where the "Planck brane" is located.
- X_5 is noncompact, but $\exists$ 4D gravity!

$$M_4^2 = M_5^3 \int dX_5 \, e^{-2k|X_5|} < \infty$$

This is finite! Such a solution has 4D gravity. (A Planck brane observer would see our Newton's law).

This is just a slice of the AdS_5 metric.

b. Relation to D3 metrics

The solution for a D3-brane stack (N of 'em) in IIB supergravity, is

$$ds^2 = h^{-1/2}\, dX_{\parallel}^2 + h^{1/2}\left(dr^2 + r^2\, d\Omega_5^2\right)$$

$$\left.h(r) = 1 + \frac{4\pi g N}{r^4}(\alpha')^2 \right\} \left(\begin{matrix}+ \text{ 5-form} \\ \text{flux}\end{matrix}\right)$$

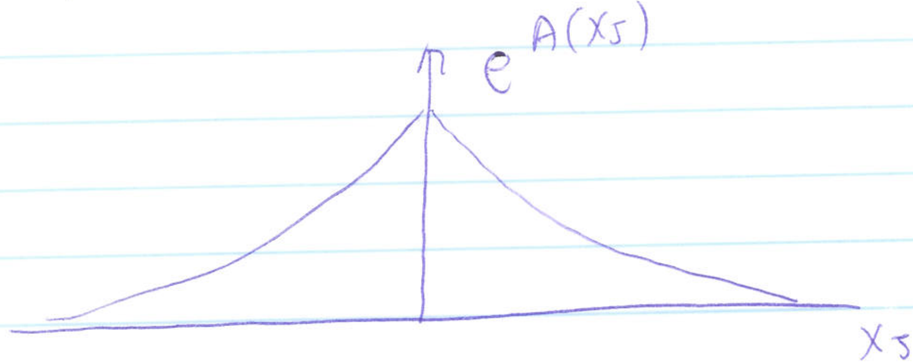
Defining $U = \frac{r}{\alpha'}$, and taking $\alpha' \to 0$ w/ fixed U $\Rightarrow$

$$ds^2 = \alpha' \left[\frac{U^2}{\sqrt{4\pi g N}}\, dX_{\parallel}^2 + \sqrt{4\pi g N}\, \frac{dU^2}{U^2} + \sqrt{4\pi g N}\, d\Omega_5^2 \right]$$

This is $AdS_5 \times S^5$ with

$$R_{AdS}^2 = R_{S^5}^2 = \sqrt{4\pi g N}\, \alpha'.$$

Claim: The RS metric is the same as the 5D truncation of this, with some cutoff at $U = U_{max}$, & a $\mathbb{Z}_2$ copy of the sol'n inserted at U_{max}:



This is HW problem 1.

C. Hierarchies from IR branes

Consider now a case w/ 2 branes, located at $x_5 = 0$ & $x_5 = \pi$. $x_5 \in [0, \pi]$ now.

Then:

$$S = \int d^5x \sqrt{-G}\,(R_5 - \Lambda) + \underbrace{S_{IR}}_{x_5 = \pi} + \underbrace{S_{UV}}_{x_5 = 0}$$

Here:

$$S_{IR} = \int d^4x \sqrt{-g_{IR}} \left(\mathcal{L}_{IR} - V_{IR} \right)$$

$$S_{UV} = \int d^4x \sqrt{-g_{UV}} \left(\mathcal{L}_{UV} - V_{UV} \right).$$

This $S \rightarrow$ hierarchies of scales in a natural way!

Agan, consider

$$ds^2 = e^{-2A(x_5)} \eta_{\mu\nu} dx^\mu dx^\nu + r^2 dx_5^2$$

($\rightarrow$ size of x_5 interval is πr).

Einstein eqns:

$$(\square) \quad 6 \frac{(A')^2}{r^2} + \frac{1}{2} \Lambda = 0$$

$$(\triangle) \quad 3 \frac{A''}{r^2} + \frac{1}{2} \frac{V_{UV}}{r} \delta(x_5) + \frac{1}{2} \frac{V_{IR}}{r} \delta(x_5 - \pi) = 0$$

Defining $k = \sqrt{\frac{-\Lambda}{12}} \Rightarrow A(x_5) = kr|x_5|$ from $(\square)$.

- Conistent w/ a $\mathbb{Z}_2 \rightarrow$ find a $\mathbb{Z}_2$ invt sol'n with $-\pi < x_5 < \pi$, then quotient by $\mathbb{Z}_2$.

Now $A = kr|x_5| \rightarrow$

$$A'' = 2kr\,[\,\delta(x_5) - \delta(x_5 - \pi)\,]$$

Then to solve (Δ) need

$$\left. V_{UV} = -V_{IR} = 12k \quad \right\} \text{ tune of CC}$$

[This is to get Poincaré invt metric].

Now

$$ds^2 = e^{-2kr x_5} \eta_{\mu\nu}\, dx^\mu dx^\nu + r^2 dx_5^{\,2}$$

$$0 \le x_5 \le \pi$$

Computing M_4 :

$$M_4^{\,2} = \frac{M_5^{\,3}}{k}\left(1 - e^{-2kr\pi}\right)$$

$\Rightarrow$ M_4 depends <u>weakly</u> on r; the 4D graviton must be localized @ UV-brane.

Now also notice

$$g^{UV}_{\mu\nu} = \eta_{\mu\nu}$$

$$g^{IR}_{\mu\nu} = \eta_{\mu\nu}\, e^{-2kr\pi}$$

In particular, a scalar on IR brane w/ cut-off scale mass M_* has

$$\mathcal{L} \sim \int d^4x \left(g^{\mu\nu}_{IR}\, \partial_\mu \varphi\, \partial_\nu \varphi - \tfrac{1}{2} M_*^2 \varphi^2 \right) \sqrt{-g_{IR}}$$

$$\sim \int d^4x \left(e^{-2k\pi r}\, \partial_\mu \varphi\, \partial_\nu \varphi - \tfrac{1}{2} e^{-4\pi kr} \varphi^2 \cdot M_*^2 \right)$$

And letting $\tilde{\varphi} = e^{-k\pi r}\varphi \rightarrow$

$$\mathcal{L} \sim \int d^4x \left(\partial_\mu \tilde{\varphi}\, \partial^\mu \tilde{\varphi} - \tfrac{1}{2} e^{-2\pi kr} \tilde{\varphi}^2 \cdot M_*^2 \right)$$

$$\Rightarrow \boxed{\tilde{M} = e^{-\pi kr} M_*} \quad (☺)$$

So the "natural" energy scale @ IR brane is $\tilde{M} \ll M_* \Rightarrow$ light scalars are <u>natural</u> for $kr \sim$ <u>few</u>.

This is just the gravity dual of dimensional transmutation...

d. AdS / CFT duality

IIB string theory on $AdS_5 \times S^5$ $\cong$ $\mathcal{N}=4$ $SU(N)$ gauge theory

with $\frac{R_{AdS}^4}{l_s^4} \sim 4\pi g_{ym}^2 N$

Precise map (GKP, W):

- Bulk field $\Phi \in AdS \leftrightarrow$ operator $\mathcal{O} \in CFT$

$$ds^2 = e^{-2ky} dX^\mu dX^\nu + dy^2$$

$$k = 1/R_{AdS}$$

- Given a boundary value

$$\Phi(x^\mu, y=-\infty) \equiv \Phi_0(X^\mu)$$

We had been using UV brane to "chop off" $y<0$; not here.

then

$$\left\langle e^{-\int d^4x\, \Phi_0\, \mathcal{O}} \right\rangle_{CFT} = e^{-\Gamma(\Phi_0)}$$

$\Gamma(\Phi_0)$ = SUGRA (string) action of sol'n
w/ boundary value Φ_0.

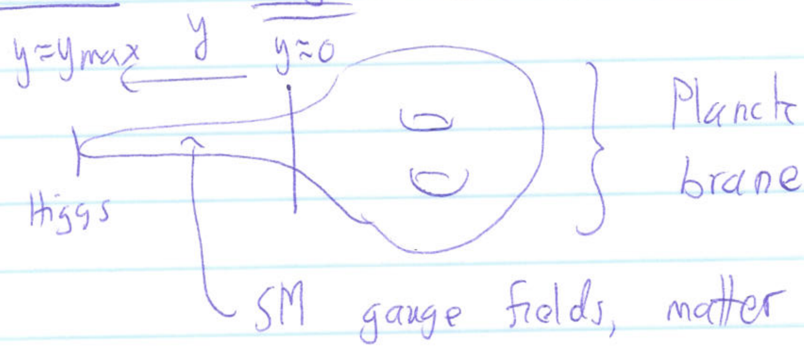
What do the cutoffs at UV & IR brane correspond to?

We saw in that energies <u>redshift</u> like e^{-ky}.

- Cutoff at y_{max} (IR) $\Rightarrow$ minimal energy scale $\sim e^{-ky_{max}}$ $\Rightarrow$ CFT develops a mass gap at y_{max}!
- Cutoff at $y_{min} = 0$ $\Rightarrow$ <u>maximal</u> energy scale; CFT is cutoff in UV here & coupled to strings/quantum gravity.

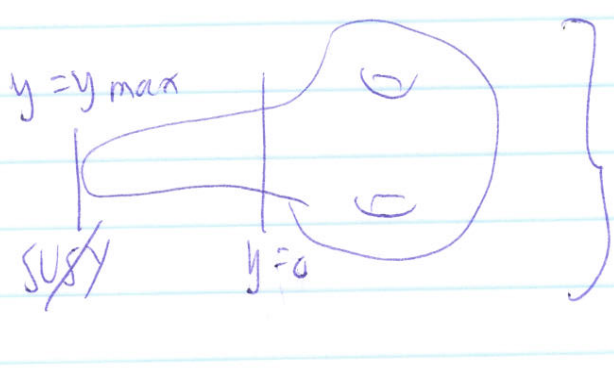
How can we <u>use</u> this?

<u>Ambitious</u>: <u>Forget</u> SUSY.



We'll see that varying matter field properties moderately can explain Yukawa hierarchies.
[Will explain why matter on IR brane is bad next time].

<u>Still interesting</u>:



Use gauge/gravity to geometrize DSB!

$\Lambda_{SUSY} \ll M_P$ via warping. We'll see that:

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- Can make simple string realizations
- Can still explain Yukawas if you wish

The Yukawa explanations in the 2 cases are opposite:

	Non-SUSY	SUSY
top	composite	point-like
1st 2 gens	point-like	composite

We'll focus on the SUSY models:

- → models of single-sector $\cancel{\text{S}}$USY
- → (in a limit) general gauge mediation