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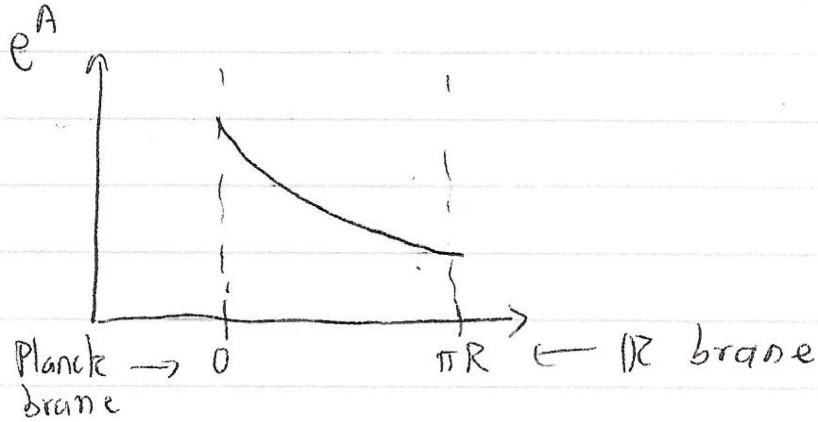
PITP '08 Lecture II S. Kachru

Today, we want to discuss how this picture:



$$ds^2 = e^{-2kx_5}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dx_5^2$$

can be roughly realized in IIB strings.

- Bulk will be SUSY
- We'll put $\cancel{\text{SUSY}}$ at IR brane, eventually.

I. A "confining" gravity dual

To realize this picture, we need to deal with both the IR cutoff + the UV cutoff.

A. AdS/CFT at the conifold

We already know that

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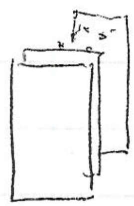


$\Big\}$ N D3s at a smooth pt $\xrightarrow[\text{@ low E}]{\text{open strings}}$ $\underline{\mathcal{N}=4 \text{ SYM}}$

$\Updownarrow$ $\underline{\underline{\text{DUAL}}}$

$\xrightarrow{\text{near horizon}}$ $\underline{AdS_5 \times S^5}$

How to get other examples? Any smooth point looks the same in near-horizon limit $\rightarrow$ put D3s @ singular pt!

A simple & canonical example: Conifold

$$\sum z_i^2 = 0 \quad \subset \mathbb{C}^4$$

$\uparrow$ cone over $S^3 \times S^2$

Arises in many compact CYs.

Change of variables $\Rightarrow$

$$(\checkmark) \qquad z_1 z_2 - z_3 z_4 = 0$$

Can solve $(\checkmark)$:

$$z_1 = A_1 B_1 \qquad z_2 = A_2 B_2 \qquad z_3 = A_1 B_2 \qquad z_4 = A_2 B_1$$

But you get same z_i if you act by:

$$A_k \to \lambda A_k \qquad B_\ell \to \lambda^{-1} B_\ell \qquad k,\ell = 1,2$$

with any $\lambda \in \mathbb{C}^*$.

- Writing $\lambda = s e^{i\alpha}$ $s \in \mathbb{R}^+$, s can be chosen to set (<u>away</u> from singularity $z_i = 0$)

$$(D) \qquad |A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2$$

To get the conifold, must // by the $U(1)$

$$(O) \qquad A_k \to e^{i\alpha} A_k \qquad B_\ell \to e^{-i\alpha} B_\ell$$

- $\exists$ an $SU(2) \times SU(2)$ symmetry: one acts on A_i, one on B_ℓ

<u>Dual field theory to N D3s @ conifold</u> : (Klebanov, Witten)

Consider the $U(1)$ gauge theory with $\mathcal{N}=1$ SUSY &

Field	Charge
$A_{1,2}$	$+1$
$B_{1,2}$	-1

D-term eqn $\Rightarrow$ $|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2$
(in absence of F-I term)

same as

$\mathcal{M}_{vacua}$: $\{D=0\}/_{U(1)}$ $\qquad$ $(\Box)$ & $(\circ)$!

So this gauge theory gives the conifold as its moduli space of vacua.

- This $U(1)$ is broken, but a D3 would have a worldvolume $U(1)$ $\Rightarrow$ want

	$U(1)$	$\times$	$U(1)$
$A_{1,2}$	1		-1
$B_{1,2}$	-1		1

COM $U(1)$ decouples; difference as above ✓.

This gives a D3 moduli space as expected.

- N D3s? Natural guess

	$U(N)$	$U(N)$
$A_{1,2}$	N	$\bar{N}$
$B_{1,2}$	$\bar{N}$	N

$\Bigg\}$ has the $SU(2) \times SU(2)$ symmetry

- No renormalizable W possible $\overset{??}{\Rightarrow} W = 0$?

- Diagonalize A, B w/ distinct eigenvalues $\rightarrow$ family of vacua w/ N D3s @ distinct pts on conifold; and, $G = U(1)^N$. <u>BUT</u>, $\exists$ massless charged chirals

- So, need a W. lowest order guess:

$$\boxed{W = \frac{\lambda}{2} \epsilon^{ij} \epsilon^{kl} \mathrm{Tr}\, A_i B_k A_j B_l}$$

This <u>does</u> <u>the</u> <u>job</u>.

CFT is strongly coupled: $R_A = R_B = 1/2 \Rightarrow$

$$\Delta(A) = \Delta(B) = 3/4, \quad \underline{\text{not}}\ 1 \quad \left(\text{large anomalous dim}\right).$$

<u>Moduli</u>: Gauge theory: 2 of $\Lambda_1, \Lambda_2, \lambda$

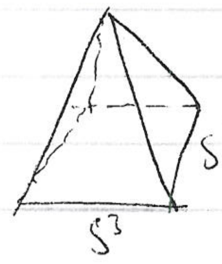
String Theory: $\tau_{IIB} \leftarrow$ axio-dilaton

Conifold = cone over $S^3 \times S^2 \leftarrow$ periods of B_2, C_2

(Klebanov, Strassler)

B. 'Perturbing' to get a confining theory

Since the conifold is a cone over $S^3 \times S^2$:



can consider having N D3s, M D5s on S^2!

Resulting gauge theory:

	$\underline{SU(N+M)}$	$\underline{SU(N)}$
$A_{1,2}$	$N+M$	$\overline{N}$
$B_{1,2}$	$\overline{N+M}$	N

(W as before in conifold theory.)

Dynamics?

<u>QFT Side</u>: "Cascade of Seiberg dualities"

Seiberg duality:

$SU(N_c)$ N_f flavors $\xrightarrow[\text{coupling}]{\text{strong}}$ $SU(N_f - N_c)$ N_f flavors + "mesons"

⑦

Here: gauge factor that runs to strong coupling is the $SU(N+M)$ [relatively less N_f vs N_c].

$$\text{"}N_c\text{"} = N+M \qquad \text{"}N_f\text{"} = 2N$$

$\Rightarrow$ dual group $= SU(2N-(N+M)) =$

$$SU(N-M)$$

So $SU(N+M) \times SU(N) \to SU(N) \times SU(N-M)$

Can check that the field content + W is self-similar w/ $N \to N-M \Rightarrow$ cascade!

<u>End</u> of cascade? Say $\underline{\underline{N = kM}}$.

Then eventually, you reach a step:

$$SU(2M) \times SU(M) \to SU(M)$$

where final $N_F = 0 \Rightarrow$ pure gauge theory!

Pure $SU(M)$ $\mathcal{N}=1$ QFT $\to$

- $W = \Lambda^3_{SU(M)}$
- M vacua (Witten index $= M$), related by phase rotations of Λ

<u>Gravity side:</u>

Klebanov/Strassler worked out metric etc, but essential physics is as follows.

Conifold: $\sum_{i=1}^{4} z_i^2 = 0$

<u>Deformed</u> conifold: $\sum_{i=1}^{4} z_i^2 = \epsilon^2$

(move in moduli space of CY to smooth the singularity)

M D5s $\Rightarrow$ $\int_{S^3 \text{ around D5s}} F_3 = M$

"Geometric transition" $\to$

$$\int_A F_3 = M \quad \text{where}$$

$A: \quad z_i = \bar{z}_i \Rightarrow \sum (\mathrm{Re}(z_i))^2 = \epsilon^2$

Also, $\exists$ N units of D3 charge.

$$F_5 = dC_4 + B_2 \wedge F_3 - C_2 \wedge H_3$$

$$\Rightarrow \quad dF_5 \simeq N_{D3} + \underline{\underline{H_3 \wedge F_3}}$$

So if we let $B \sim S^2 \times$ radial direction of conifold <u>cone</u>

$$\int_A F_3 = M \Rightarrow \text{if } \int_B H_3 = k \qquad (N = kM)$$

we also see D3 charge matches our expectations.

Physics of $\int_A F_3 = M$, $\int_B H_3 = k$?

Fluxes $\Rightarrow$ superpotential for moduli of the Calabi-Yau.

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Suppose Ω = holomorphic 3-form

$$\int_A \Omega = z \qquad \int_B \Omega = \frac{z}{2\pi i}\log(z) + \text{regular}$$

[$\rightarrow$ monodromy: as $z \rightarrow e^{2\pi i} z$, one has

$A \rightarrow A, \quad B \rightarrow B + A$]

Here $z \sim \epsilon^{\text{power}}$; A is <u>vanishing cycle</u> as one approaches conifold pt $\epsilon \rightarrow 0$, $z \rightarrow 0$.

Then: $W = \int (F - \tau H_3) \wedge \Omega \implies$

$$W = K \tau z + \frac{M}{2\pi i}\left[z \log(z) + \cdots \right]$$

$D_\tau W = 0$ (using $K \sim -\log\left(\int \Omega \wedge \bar{\Omega}\right) + \cdots$

$\implies$ $\boxed{z \sim \exp\left[-\frac{2\pi K}{g_s M} \right]}$, } M vacua, related by phase of z

z = A-cycle volume $\rightarrow$ near conifold pt in moduli space!

The fluxes backreact on metric & $\Rightarrow$ warping. How much?

If $ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A} \tilde{g}_{mn}(y) dy^m dy^n$

N D3s $\Rightarrow$ $e^{-4A} = \frac{4\pi g_s N}{\tilde{r}^4}$ distance to D3s in $\tilde{g}$ metric

A fact about the conifold geometry is that the distance from tip $\tilde{r}$ on the cone satisfies

$$\tilde{r}_{min} \simeq z^{1/3}$$ } Candelas, de la Ossa

$$\sim \exp\left[-2\pi k/3gM\right]$$

$$\Rightarrow \quad e^A\big|_{min} \sim e^{-2\pi k/3g_s M}$$

The warped energy scale $\equiv$ IR scale of the gluino condensate in $\mathcal{N}=1$ pure YM!

C. IR geometry

So, what is the IR SUGRA sol'n?

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- Conifold is <u>deformed</u> by fluxes:

$$\sum z_i^2 = \epsilon^2$$

- $$ds^2 \simeq a_0^2 \, dx_\mu dx^\mu + g_s M \, b_0^2 \left(\tfrac{1}{2} dr^2 + d\Omega_3^2 + r^2 d\tilde{\Omega}_2^2 \right)$$

S^2 collapses at <u>tip</u>

- $$a_0^2 = \frac{\epsilon^{4/3}}{g_s M} \qquad b_0^2 \sim O(1)$$

And, $\int_{S^3} F_3 = M \Rightarrow$

$$F_3 \sim f \, \epsilon_{ijk}$$

Vol element of S^3

$$f \simeq \frac{2}{\sqrt{g_s^3 M} \cdot b_0^3}$$

II. $\cancel{\text{SUSY}}$ solutions (cf Sk, Pearson, Verlinde)

Can we use the warping in the KS solution to break SUSY at a scale $\sim e^A|_{min} \sim e^{-2\pi k/3g_sM}$? <u>Sure</u>.

Consider a $\overline{D3}$ in the KS solution, with $N = kM$.

- No (probe) D3s in background $\to$ 3-$\bar{3}$ annihilation impossible, despite the presence of D3 charge.

- Dynamics?

$$S_{\overline{D3}} = -\frac{T_3}{g_s}\int d^4x \ \mathrm{Tr}\sqrt{\det(G_{11})\det(Q)}$$

$$- T_3 \int \mathrm{Tr}\left(2\pi i \, \iota_\Phi \iota_\Phi B_6 + C_4\right)$$

where:

$$Q^i{}_j = \delta^i{}_j + \frac{2\pi i}{g_s}[\Phi^i, \Phi^k](G_{kj} + g_s C_{kj})$$

and $\iota_\Phi \iota_\Phi B_6 = \Phi^n \Phi^m B_{mnpqrs} \dfrac{dy^p \cdots dy^s}{4!}$.

Note: $dB_6 = \dfrac{1}{g_s^2} *_{10} H_3 = \dfrac{-1}{g_s} \underbrace{dV_4}_{= \alpha_0^4 d^4x} \wedge F_3$

Follows from $DW = 0$.

where we used fact $G_3 = \underline{ISD} \Leftarrow *_6 G = iG$

Plugging in $\Rightarrow$ just from non-commutator terms

$$S_{D3} \sim \frac{T_3}{g_s} \int d^4x \sqrt{g_4}\, \mathrm{Tr}\, e^{4A} \left[2 + \frac{1}{2} e^{-2A} \partial^\mu \Phi^i \partial_\mu \Phi^j g_{ij} \right]$$

So $\exists$ potential $\sim e^{4A} \Rightarrow$

$$\boxed{\overline{D3}\text{s are attracted to the tip of the warped throat!}}$$

What happens to p $\overline{D3}$s at the tip?

They feel:

$$V_{eff} = \frac{T_3}{g_s} \left(p - \frac{i 4\pi^2}{3} f\, \epsilon_{ijk} \mathrm{Tr} [\Phi_i, \Phi_j] \Phi_k - \frac{\pi^2}{g_s^2} \mathrm{Tr}\left([\Phi^i, \Phi^j]^2 \right) + \cdots \right)$$

Where are the extrema? (HW!)

It is easy to see $\exists$ extrema where

$$[\Phi^i, \Phi^j] = -i g_s^2 f \epsilon_{ijk} \Phi^k$$

Up to rescaling, these are the commutation relations of SU(2) generators!

- $\exists$ critical pts for <u>each</u> p dim'l irrep.
- Minimal V $\Rightarrow$ p dim'l irrep
- Can see that the <u>radius</u> of the blob the $\overline{D3}$s blow up to is

$$R^2 \simeq 4\pi^2 \frac{p^2}{M^2} \times R_{S^3}^2$$

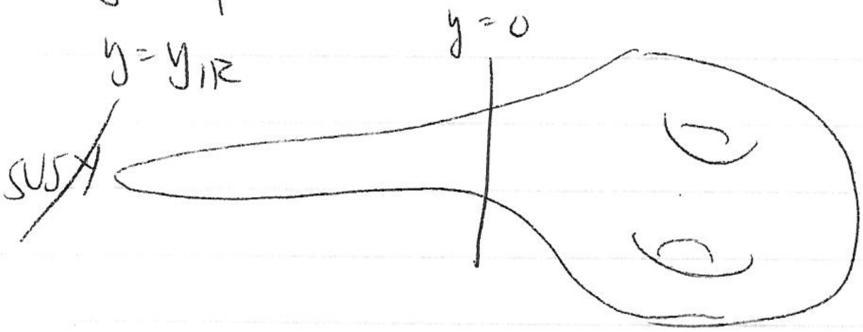
$\Rightarrow$ only <u>reliable</u> if $p \ll M$; otherwise, blob $\gtrsim$ size of space!

So the big picture now is:

- N D3s @ conifold + M D5s on S^2 +

p $\overline{D3}$s $\Longrightarrow$

Since the conifold is a generic singularity of compact CYs, can easily promote this to:



C.f. Giddings, SK, Polchinski

<u>Next time:</u> Add toy SM, discuss $\cancel{SUSY}$ transmission.