PITP '08 Lecture III

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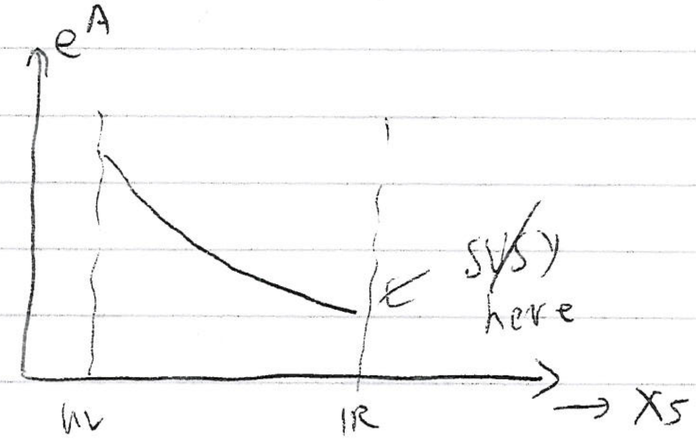
At this stage, we have a rough idea of how to build string models with



$$ds^2 = e^{2A}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dx_5^2$$

<u>Slogan</u>: IR ~~SUSY~~ in AdS/CFT (when its a <u>state</u> in the SUSY gravity sol'n) $\longleftrightarrow$ gravity dual of DSB.

(can see our $\overline{D3}$ state in arXiv: 0801.1520.

What can we use this for? We'll revert to 5D cartoons + describe applications to strongly coupled model building

I. Bulk wavefunctions & Yukawa hierarchies

[Good reference for formulae: hep-ph/0601213]

A. Scalars

You saw on the HW that if

$$S_5 = \int \sqrt{-G}\left[|\partial\phi|^2 - M_\phi^2|\phi|^2\right]$$

with $\qquad M_\phi^2 = ak^2 \in \qquad ds^2 = e^{-2ky}\eta\, dx\, dx + dy^2$

then $\exists$ a zero mode <u>iff</u> you add

appropriate ∂ terms @ the UV & IR branes.

Defining $\quad \alpha \equiv \sqrt{4+a}$

and writing the ∂ term

$$S_\partial = -\int d^4x\, dy\, \sqrt{-G}\; 2bk\left[\delta(y) - \delta(y-\pi R)\right]|\phi|^2$$

$\Rightarrow$ get zero modes

$$\phi^{(0)}(y) \simeq e^{bky} \qquad b = 2 \pm \alpha$$

Where does the scalar <u>live</u> in the interval?

$$\int d^5x \sqrt{-G}\; G^{mn}\, \partial_m \phi^* \partial_n \phi \quad + \cdots$$

$$\sim \int d^5x \; e^{2(b-1)ky}\, \eta^{\mu\nu}\, \partial_\mu \varphi^* \partial_\nu \varphi$$

where $\quad \phi(x,y) = \varphi(x)\, e^{bky}$

So: compared to a 5d flat metric, the

y profile is $\quad \tilde{\phi}^{(0)}(y) = e^{(b-1)ky}$

$$= e^{(1 \pm \sqrt{4+a})ky}$$

$b > 1$	localized towards <u>IR</u>
$b = 1$	<u>flat</u>
$b < 1$	localized towards <u>UV</u>

[Can easily check that massive KK modes have $m_n \sim nk\, e^{-\pi kR}$ -- localized in IR].

B. Fermions

A similar analysis for fermions on $S^1/\mathbb{Z}_2 \Rightarrow$

- Each 5D fermion is 4-component (Dirac).
- $\mathbb{Z}_2$ flips <u>one</u> of $\Psi_\pm = \pm \gamma_5 \Psi_\pm$.

- The guy that is projected in, say ψ_+, has
$$\tilde{\psi}_+^{(0)}(y) = e^{(\frac{1}{2}-c)ky}$$
where the bulk mass was $\sim ck$.

$c > 1/2$ localized towards UV

$c < 1/2$ localized towards IR

Via AdS/CFT each field would have a dual $\mathcal{O}$ if we remove the UV brane:

Field	Wavefcn	$\Delta(\mathcal{O})$
$\phi^{(0)}(y)$	$e^{(1\pm\sqrt{4+a})ky}$	$2+\sqrt{4+a}$
$\psi_+^{(0)}(y)$	$e^{(\frac{1}{2}-c)ky}$	$\frac{3}{2}+\lvert c+\frac{1}{2}\rvert$

C. Yukawa couplings

Now, suppose you want to "explain" why the range of $|\lambda_{ijk}|$ in SM $\sim \underset{e}{\underset{\uparrow}{10^{-6}}}$ to $\underset{top}{\underset{\uparrow}{1}}$.

1. In RS models

Their idea is to have the Higgs on the IR brane.

- 4D Fermion masses will arise from couplings like

i = Flavor index

$$\int d^4x \int dy \sqrt{-G}\, \lambda_{ij}^{(5)} \left[\overline{\Psi}_{iL}(x,y)\, \Psi_{jR}(x,y) + h.c. \right]$$

$$\times H(x)\, \delta(y - \pi R) \equiv \int d^4x \left(\lambda_{ij}\, \overline{\Psi}^{(0)}_{iL+}\, \Psi^{(0)}_{jR}\, H + \cdots \right)$$

Now since the zero mode profile is

$$\tilde{\Psi}^{(0)}_{iL,R} \sim e^{\left(\frac{1}{2} - c_{iL,R}\right) ky}$$

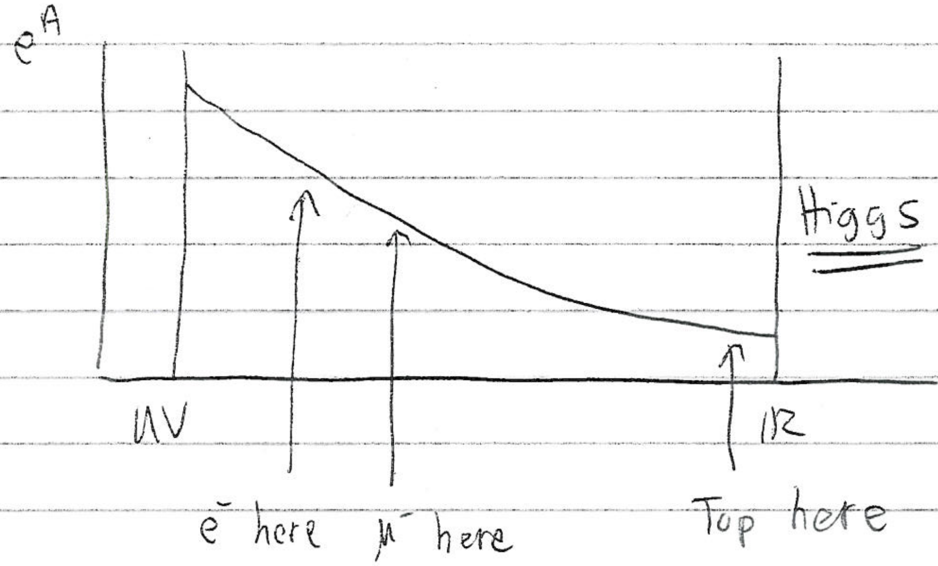
we'll find

$$\lambda_{ij} \simeq \lambda_{ij}^{(5)}\, e^{(1 - c_{iL} - c_{jR})\, k\pi R}$$

for $c_{iL}, c_{jR} > \frac{1}{2}$.

So eg the e^- Yukawa $\sim 10^{-6}$ could be obtained with $c_e \simeq 2/3$ ($\in$ very $O(1)$).

Picture in RS :



Slogan (can be made more precise using AdS/CFT):

"e^- & μ^- are <u>elementary</u> states, Top & Higgs
or point-like
are <u>composites</u>."

<u>Slight</u> elaboration :

$$\mathcal{L}_{4D} = \mathcal{L}_{CFT} + \bar{\Psi}_L^{(0)} i \gamma^\mu \partial_\mu \Psi_L^{(0)} + \epsilon \left(\bar{\Psi}_L^{(0)} \mathcal{O}_R + h.c. \right)$$

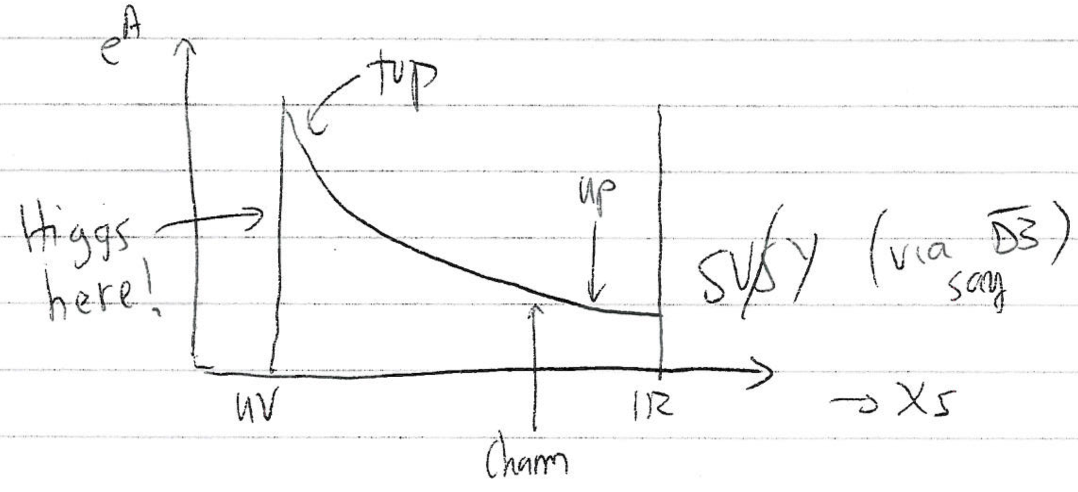
<u>Point-like</u> $(c > \frac{1}{2}) \Rightarrow \Delta(\mathcal{O}) = \frac{3}{2} + |c + \frac{1}{2}|$

$> \frac{5}{2} \Rightarrow \Delta(\bar{\Psi}\mathcal{O}) > 4$; the operator coupling Ψ to the CFT is <u>irrelevant</u>.

- The coupling of pt-like states to the CFT <u>decreases</u> in the IR.
- But they only couple to (composite) Higgs through these $\psi\mathcal{O}$ type couplings.
- The exponential suppression of ϵ via running $\Rightarrow$ the tiny Yukawa for pt-like states.

2. Yukawas in models w/ IR DSB

Our picture from lecture II was actually



This <u>totally</u> <u>reverses</u> the picture one would want for explaining Yukawas.

Higgs is now @ UV-brane $\Rightarrow$

- For large top Yukawa, top = pt-like
- Small 1^{st} 2 gen Yukawas $\Rightarrow$ 1^{st} 2 gens should be composite.

But now that will correlate with ~~SUSY~~ :

- composites are closer to ~~SUSY~~ $\rightarrow$ expect that the 1^{st} 2 generations will have larger spartner masses

- Note that heavy stop $\rightarrow$ severe fine-tuning to keep H light; small Yukawas $\rightarrow$ no such issue for 1^{st} 2 generations (but FCNC issues exist & are serious).

$\uparrow$

Arkani-Hamed, Murayama: FCNC $\rightarrow$ 1^{st} 2 gens > 22 TeV
hep-ph/9703259

then, 2 loop RG $\rightarrow$ − stop mass^2
unless M_{stop} > few TeV

II. Towards string models of strongly coupled ~~SUSY~~ with/without composites

(Work in progress w/ Franco, Simic, Verlinde)

We've seen that in the warped conifold with p $\overline{D3}$s, M D5s, N D3s $(M = kN)$:

- $\exists$ ~~SUSY~~ metastable configs
- $V \sim p\, e^{-\frac{8\pi}{3}\frac{k}{g_s M}}$ } exp small $\rightarrow$ gravity dual of DSB

A. ~~SUSY~~ solution

The metric away from the tip in the SUSY theory with M D5s + N D3s was determined by Klebanov-Tseytlin:

$$ds^2 = e^{2A(r)} \eta_{\mu\nu}\, dX^\mu dX^\nu + e^{-2A(r)}\left(dr^2 + r^2 e_\psi^2 + r^2 \sum_{i=1}^{2}\left[(e_{\theta i})^2 + (e_{\phi i})^2\right]\right)$$

$$e_\psi = \frac{1}{3}\left(d\psi + \sum_i \cos\theta_i\, d\phi_i\right)$$

$$e_{\theta i} = \frac{1}{\sqrt{6}}\, d\theta_i \qquad e_{\phi i} = \frac{1}{\sqrt{6}} \sin\theta_i\, d\phi_i$$

and the warp factor is

$$e^{-4A} = \frac{27\pi g_s}{4r^4}\left(N + \frac{3g_s M^2}{2\pi}\left(\ln\left(\frac{r}{r_{uv}}\right) + \frac{1}{4}\right)\right).$$

τ_{IIB} = constant

$$F_5 = (1+*) \times -27\pi\left(N + \frac{3g_s M^2}{2\pi}\ln\left(\frac{r}{r_{uv}}\right)\right)$$
$$\times \ \mathrm{Vol}(T^{11}) \leftarrow e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} \wedge e_{\theta_2} \wedge e_{\phi_2}$$

+ 3-form fluxes:

$$F_3 = \frac{9M}{2}\, e_\psi \wedge (e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2})$$

$$B_2 = \frac{9g_s M}{2}(e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2}) \ln(r/r_{uv})$$

The solution preserves the global $SU(2) \times SU(2)$ of the conifold.

~~SUSY~~ State:

- Open string probe analysis → embiggened $\overline{D3}$s @ the IR end of geometry

(11)

- Smeared gravity sol'n (DeWolfe, Sk, Mulligan):

- New metric

$$ds^2 = r^2 e^{2a(r)} \eta_{\mu\nu} dX^\mu dX^\nu +$$

$$e^{-2a(r)} \left[\frac{dr^2}{r^2} + e^{2b(r)} e_\psi^2 + e^{2c(r)} \sum_i (e_{\theta_i}^2 + e_{\phi_i}^2) \right]$$

where - to leading order in M/N

- to leading order in $S = p e^{-\frac{8\pi k}{3 g M}}$

$$(\star) \quad \boxed{e^{-2a} = \left(\frac{1}{2} + \frac{S}{32 r^4}\right) \sqrt{27\pi g_s N}}$$

$$e^{2b} = 1 + \frac{S}{r^4}$$

Dilaton $\left\{ \Phi = \log g_s + \frac{1}{r^4} \left[-3 S \log r \right] \right.$

$+ \underbrace{\cdots}_{\Delta(\text{flux})}$ UPSHOT:

- $T^{1,1}$ "squashed", (Non-ISD (1,2) flux sourced, and Φ now runs -- all normalizable parts →

a ~~SUSY~~ state in the SUSY KT field theory.

B. "Imagining" the Standard Model

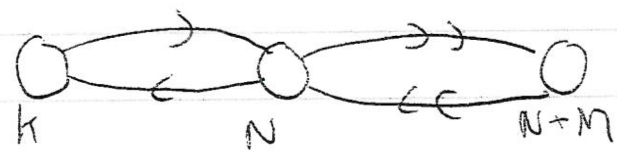
Now, we want to add "SM" $\Rightarrow$ need to slightly change the CFT to get a new $SU(5)$ global symmetry (whose gauging $\Rightarrow$ the SM gauge group).

So: Add D7s stretching down the throat. "Kuperstein embedding"

Conifold: $\sum z_i^2 = \epsilon^2$

(5) D7 WV: $z_4 = \mu$

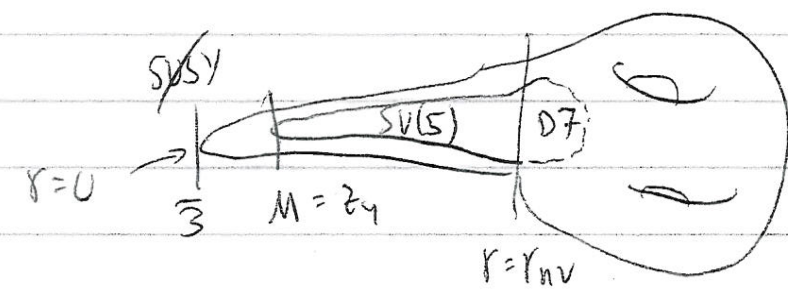
Quiver:



$$W_{dual} = \epsilon^{ij}\epsilon^{kl} A_i B_k A_j B_l + \tilde{q}\,(A_1 B_1 + A_2 B_2)\, q$$
$$+ \mu\, q\, \tilde{q}$$

[<u>Important</u>: Breaks $U(1)_R$ of dual QFT @ tree level]

So we now have an <u>SU(5)</u> gauge symmetry, in the bulk, going down to M:



What we <u>want</u> to do (will sketch idea, then discuss 5d results):

- Add SM gens "at" r_α $\alpha = 1,2,3$ via intersections or bundles $\subset$ D7:

  a) $r_{1,2} \ll r_3 = r_{uv}$ 1^{st} 2 gens <u>composite</u>
  top pt-like

  b) $r_1 = r_2 = r_3 = r_{uv}$ All 3 pt-like
  ($\to$ limit of Meade-Seiberg-Shih)
  I think

- $\bar{3}$-7 strings $\Rightarrow$ "Messengers" of $\cancel{SUSY}$ to $SU(5)$ gaugino.
- Gaugino has $c = 1/2$ wavefcn $\Rightarrow$ transmits splittings as in gauge/gaugino mediation, but couples to generation α with strength $\underline{\alpha_5(r_\alpha)}$ . . .

$$\boxed{\frac{1}{g_5^2(r_\alpha)} = \frac{1}{g_{5,0}^2} + \mathrm{Vol}\,(\Sigma_4 \mid \text{down to } r_\alpha)}$$

$\leftarrow$ GUT coupling strength

- "Compositeness" contribution to splittings: Generations which are composite feel directly the $\cancel{SUSY}$ sector. We saw

$$ds^2 = r^2 e^{2a} \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{-2a} \frac{dr^2}{r^2}$$

$$e^{2a} = \left(\frac{1}{2} + \frac{S}{32 r^4}\right) \sqrt{g_s \cdot 27\pi N}$$

But after coordinate change, this is exactly the $A^2(z)$ in HW problem 4

$$A^2(z) \propto \frac{1}{z^2}\left(1 - \frac{1}{(2\pi g N)^2} \times \frac{3}{5} \underbrace{S}_{\text{susy scale}} z^4\right)$$

This contributes a non-vanishing bit to sfermion mass for highly composite gens.

[Dies rapidly with $z \sim 1/r$].

So this mechanism of $\cancel{\text{SUSY}}$ → competition between gaugino mediation + strong coupling "compositeness" contributions to masses.

[Cf Gherghetta et al hep-ph/0704.3571 for a 5D model that is similar but distinct in UV origin]

See also: Luty + Terning '99; Nomura et al '04

AdS/CFT ⇒ good playground for strongly coupled model building!