Inflation in String Theory

Shamit Kachru (Stanford and SLAC)

I was asked by the organizers to cover inflation in string theory. Since there are O(1000) papers on this subject spread over a 20 year time span, my coverage will necessarily be both idiosyncratic and sporadic.

My goal is to make and illustrate three different points:

1. Inflation is sufficiently sensitive to UV physics that one needs a UV complete theory (like string theory) to do satisfactory model building.

2. Within string theory, there are many non-trivial constraints on building successful inflationary models. This particular UV completion does not seem to make it obvious that "anything goes."

3. Several of the most interesting ideas proposed in recent years involve novel dynamical mechanisms and (correlated) interesting observational signatures.

It is an idea going back to Guth that to explain the horizon and flatness problems of cosmology, a period of early universe **inflation**

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad a(t) \sim e^{Ht}$$

is a good idea.

Since we must eventually exit inflation, this expansion should be driven by a dynamical scalar field (not a false vacuum energy which is relaxed by a first order transition).

I. UV sensitivity in inflationary model building

The potentials which support slow-roll inflation are a little bit strange:

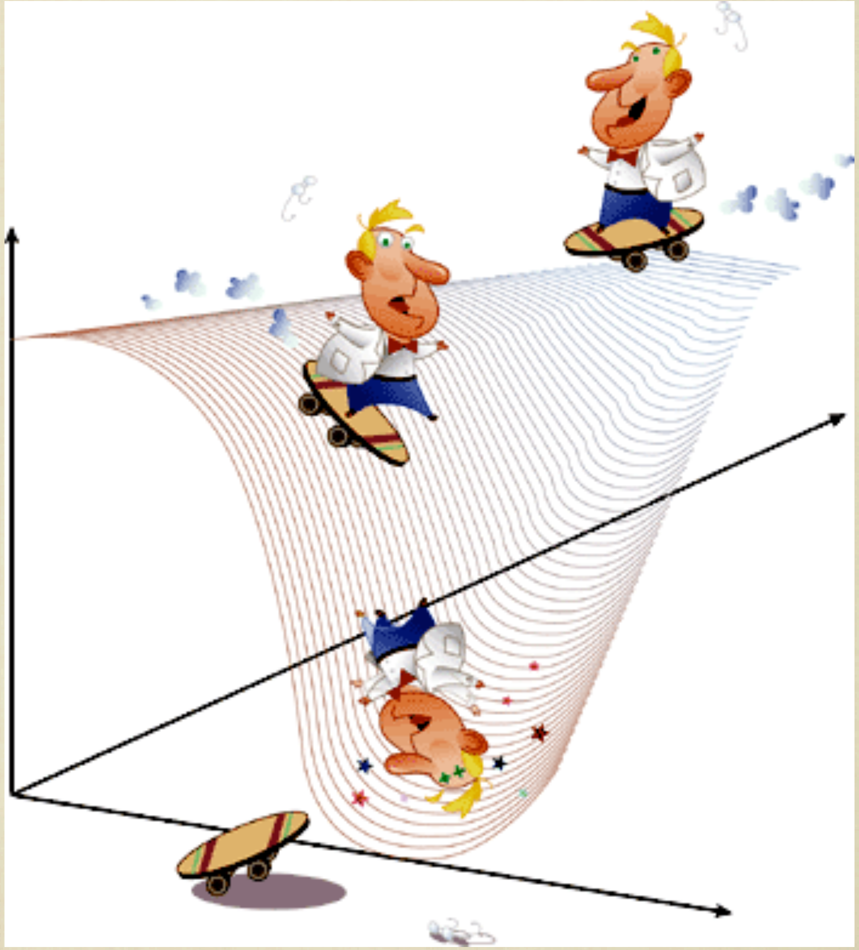
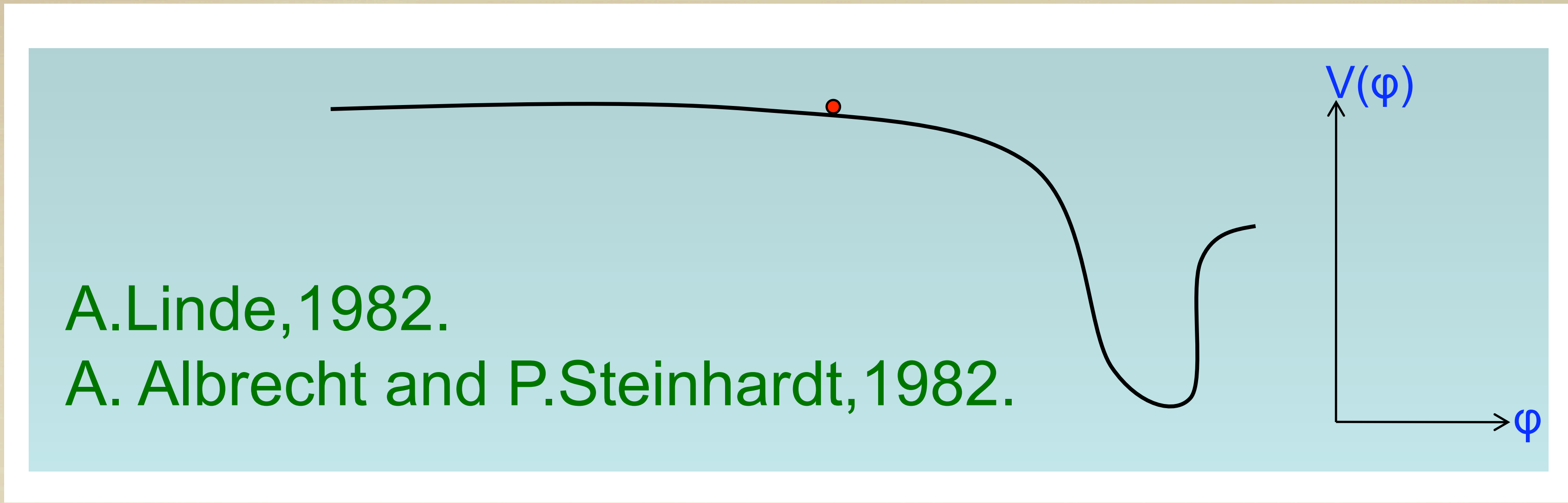


Figure stolen from A. Linde



The slow-roll conditions which ensure accelerated expansion of sufficient duration:

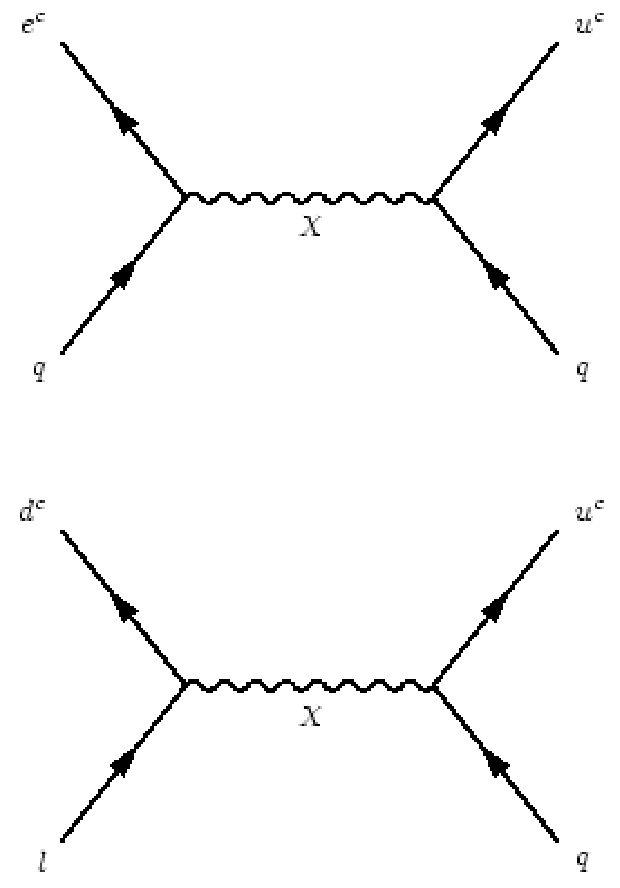
$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 << 1$$

$$\eta = M_P^2 \frac{V''}{V} << 1$$

are sensitive to dimension six, Planck suppressed corrections to the potential.

Therefore, one can only construct robust models of inflation in a theoretical framework that can determine the potential at least this accurately.

This degree of sensitivity to high scale physics is rare in model building. Even proton decay in GUTs only depends on dimension six, GUT scale suppressed operators:



Dimension 6 proton decay mediated by the X boson $(3,2)_{-\frac{5}{6}}$ in $SU(5)$ GUT.

In fact, in the models of most interest for the next generation of experiments designed to measure B-mode polarization, the situation is much more interesting.

E.g. the proposed CMBPol experiment can perhaps reach a primordial gravity wave signal down to $r \sim .01$

There is a famous bound which arises just from the definition of r as ratio of power in tensor modes to power in scalar modes:

$$r \sim .14 \frac{\mathcal{P}_g}{(\frac{\delta\rho}{\rho})^2}$$

This "Lyth bound" correlates the tensor mode signal with the distance the inflaton traversed in field space, during slow roll.

$$N_e = \frac{1}{M_P^2} \int d\phi \, \left(\frac{V}{V'}\right) \rightarrow$$

$$\frac{1}{M_P} \frac{\Delta\phi}{\Delta N} \;\sim\; M_P\left(\frac{V'}{V}\right) \;\sim\; \left(\frac{r}{7}\right)^{1/2}$$

Even in the handful of e-foldings closest to our present horizon, the total field excursion was roughly:

$$\frac{\Delta\Phi}{M_P} \sim \mathcal{O}(1) * \left(\frac{r}{.05}\right)^{1/2}$$

So basically, a CMBPol detection of r would imply that the inflaton traversed a super-Planckian distance in field space during inflation.

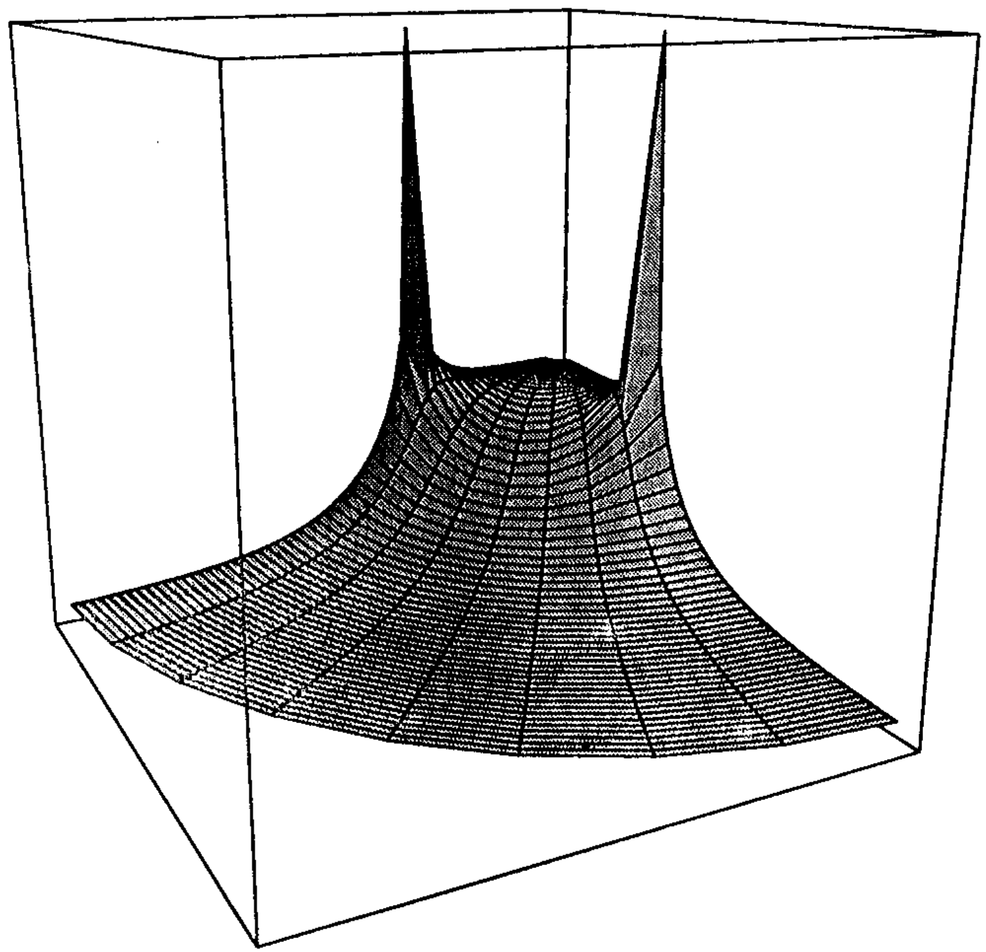
Why does this matter?

The candidate scalar inflatons in e.g. string theory typically arise as "moduli fields" parametrizing the size and shape of the extra dimensions, or the positions of branes within the extra dimensions.

As one changes the shape, typically the quantities in the low-energy effective field theory also vary. E.g., masses and couplings of other particles depend on the modulus.

In a patch of string scale size in field space, special points with extra light particles or enhanced symmetries often arise:

Figure 4.1: A plot of the metric $g_{\psi\bar{\psi}}$ against ψ for ψ in the fundamental region $0 \leq \arg\psi < \frac{2\pi}{5}$. The cusps correspond to the conifold at $\psi = 1$. As $\psi \to \infty$ the metric tends asymptotically to a metric of constant negative curvature.



Candelas, De la ossa, Green, Parkes

But if one wants a flat potential over a super-Planckian distance for the inflaton, even couplings to other super-massive modes (e.g. Kaluza-Klein modes):

$$V(\phi, \psi) = V_{inf}(\phi) + M_{KK}^2 \psi^2 + \alpha \phi^2 \psi^2 + ...$$

will be dangerous. Integrating out psi loops will produce a potential for the inflaton that typically has O(1) variation over distances of order M_{KK} in field space

The best idea to evade such corrections is to use a Nambu-Goldstone boson or an axion as the inflaton. The subleties and successes of this idea in the string context will be discussed in a later part of my talk.

A second potential observable that would indicate novel UV physics is Non-Gaussianity in the spectrum of scalar perturbations.

In a model of single field inflation where the inflaton itself generates the density perturbations:

* Slow-roll --> negligible non-Gaussianity

$$f_{NL} \sim \mathcal{O}(.01)$$

Acquaviva, Bartolono, Matarrese, Riotto; Maldacena

More general inflaton Lagrangians with higher derivatives

$$\mathcal{L} = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_P^2 R + 2P(X,\phi)\right)$$

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

can produce significant non-Gaussian fluctuations if the speed of sound

$$c_s^2 = \frac{dP}{dE} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

Creminelli; Alishahiha, Silverstein, Tong; Arkani-Hamed, Cheng, Mukohyama, Zaldarriaga; Chen, Huang, S.K., Shiu

is small. In any such models, higher derivative terms play a crucial role, so the UV completion is important by definition.

So our punch lines so far are:

1. To make any inflation model, one must control all terms which can impart a mass of order Hubble to the inflaton, which includes dimension six, Planck-suppressed corrections to the Lagrangian.

2. To make a model which gives measurable B-modes or Non-Gaussianity, one has to have an even better understanding, controlling the inflaton potential over super-Planckian distances in field space (i.e., to all orders in a polynomial expansion). This requires knowledge of all couplings to even very massive fields (at a given point in field space).

Of course there are other important issues that are UV sensitive in early cosmology:

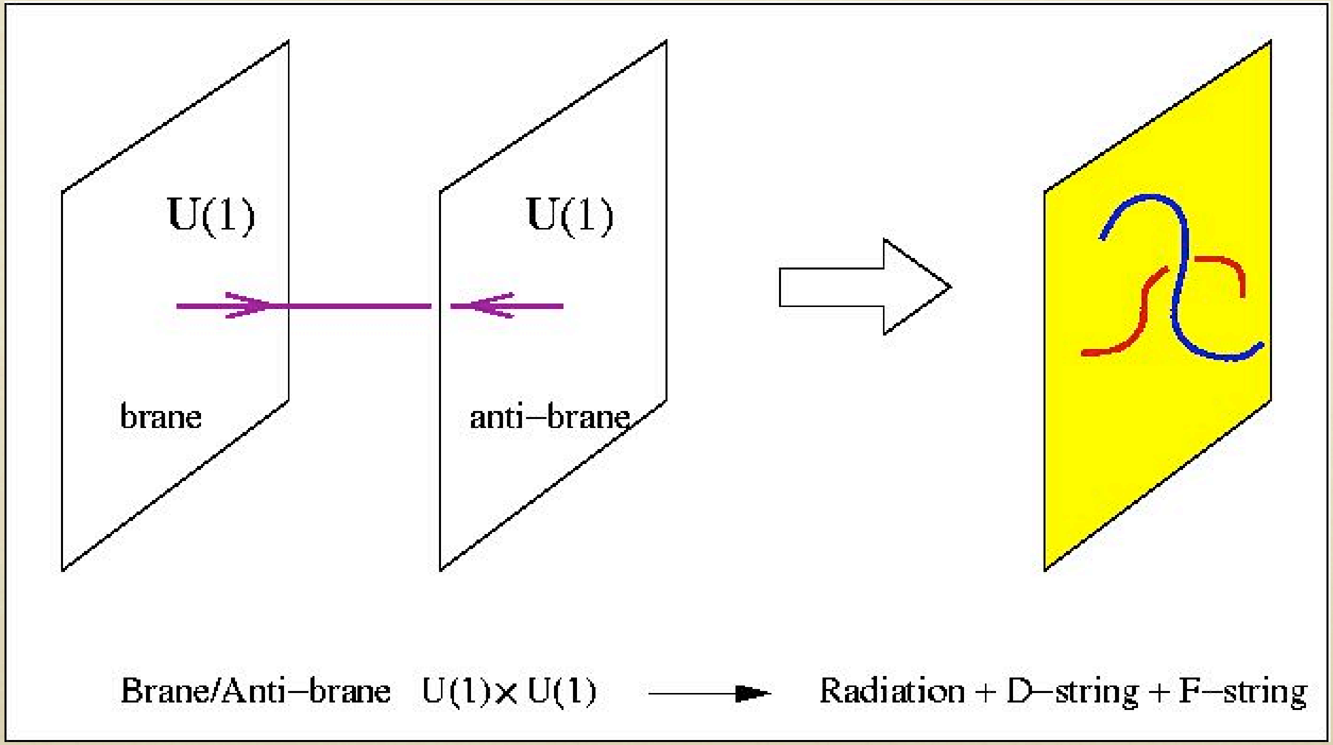
Inflation is not past eternal. What came before? How do we resolve the original singularities? Etc.

“Patch problem”: what gave rise to the relatively smooth Hubble-sized patch of space-time with homogeneous inflaton, that could inflate? For low-energy models, this is a particularly sharp question. But, tied up with measure issues.

We will IGNORE these issues and just talk about model building. Perhaps justified, perhaps not.

II. Building inflation with D-branes

One well studied class of scenarios postulates that the inflaton is the modulus controlling a brane / anti-brane separation. Its potential arises from Coulomb attraction between the oppositely charged branes:



Dvali, Tye

Burgess, Martineau, Quevedo, Rajesh, Zhang

In its earliest form, this idea suffers from the following problem. The Coulomb potential for branes separated by a distance d is

$$V(r) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3}{M_{10}^8 d^4}\right)$$

Or in terms of a canonically normalized field:

$$V(\phi) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3^3}{M_{10}^8 \phi^4}\right)$$

Then using the standard definition of the slow-roll parameters, we see that if the radius of the compactification manifold is L (this enters in determining the 4d Planck mass from the 10d one):

$$\eta = \mathcal{O}(1)(L/d)^6$$

So you run out of space in the extra dimensions, before you can separate enough to inflate!

A second problem, which is more serious, is that even if one did find a model where eta has no (very) negative eigenvalue, the Einstein-frame potential energy is basically

$$V \sim \frac{2T_3}{L^{12}}$$

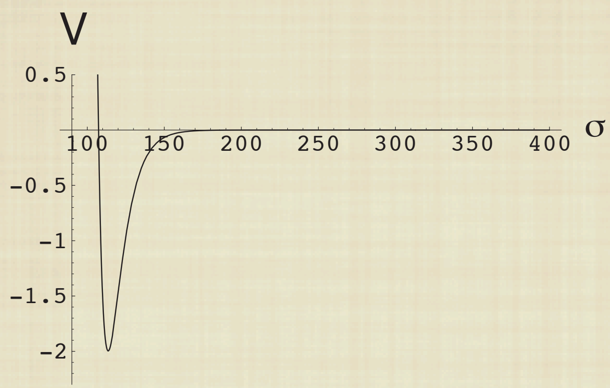
This sources rapid runaway to large L, not slow roll of the brane separation mode. A similar problem typically occurs with other compactification "moduli" (e.g. the dilaton).

So we learn a general lesson:

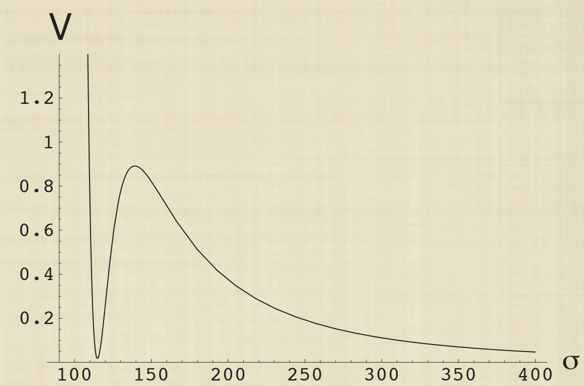
If one wishes to inflate at some hubble scale H in string theory, it is important to give dangerous moduli a mass which is larger than H.

In particular, high scale inflation (which can generate observable B-modes) requires moduli stabilization at a very high scale.

In any scenario where the SUSY-breaking scale and the moduli potential are simply correlated:



SUSY minimum



Potential after SUSY breaking

KKLT

this could lead to a bound relating the scale of inflation to the scale of SUSY breaking (so that high scale inflation requires higher scale SUSY breaking).

Kallosh, Linde

Various ways to solve the problems of the Brane/Anti-brane inflation model have been discovered.

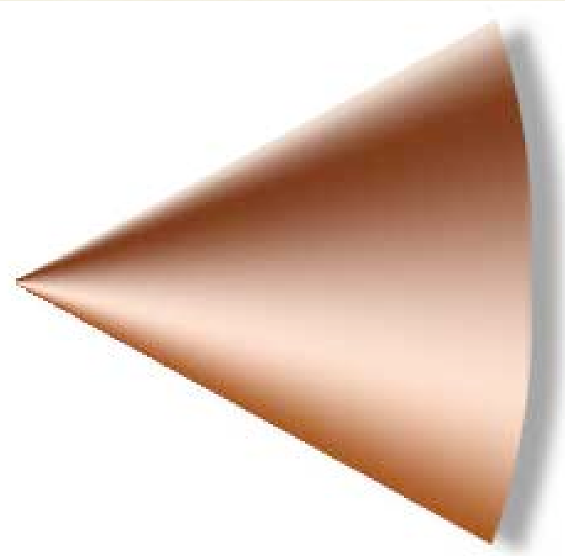
In one variant, one places the branes in a warped compactification geometry with approximate metric:

KKLMMT

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(dr^2 + r^2 ds^2_{T^{1,1}})$$

$$h(r) = \frac{27\pi}{4}\frac{1}{r^4}(\alpha')^2 g_s N$$

The picture you should have in mind for the extra dimensions is a cone with 5d base (compactified at some large value of the radial coordinate):



The tip is actually smoothed out in a way that will not be important for us.

This geometry arises in a canonical example of the AdS/CFT correspondence. It is called the (warped) conifold; the dual field theory is an N=1 supersymmetric conformally invariant gauge theory.

Klebanov, Witten; Klebanov, Strassler

The warping arises from backreaction of background gauge fluxes threading the extra dimensions. These help to stabilize the problematic moduli fields.

Gukov, Vafa, Witten; Dasgupta, Rajesh, Sethi; Giddings, S.K., Polchinski

$$W = \int_M (F - \tau H) \wedge \Omega)$$

The Coulomb attraction between a brane and an anti-brane in this warped geometry takes the form:

$$V = 2T_3 \frac{r_0^4}{R} \left(1 - \frac{1}{N} \frac{r_0^4}{r_1^4}\right) .$$

where:

r_0 is the anti-D location

r_1 is the D brane location

and the warped geometry naturally allows r_0 to be exponentially small.

This class of models then offers promise of evading the most basic problems outlined earlier:

* The fluxes together with other effects (e.g. non-perturbative dynamics to fix the volume modulus) allow one to fix the fast-rolling moduli.

* The warping softens the Coulomb potential enough so that at attainable brane separations, one can achieve slow roll.

Subtleties from inflaton/modulus mixing

Unfortunately, new more refined problems arise as old problems are solved.

Suppose we call the chiral multiplet containing the volume modulus

$$\rho \sim L^4.$$

Dimensional reduction on the compact Calabi-Yau manifold shows that the 4d effective theory has Kahler potential:

$$K(\rho,\bar{\rho},\phi,\bar{\phi}) = -3\log\left(\rho+\bar{\rho}-k(\phi,\bar{\phi})\right)$$

A stabilization mechanism that fixes ρ (or any combination of fields other than the one appearing in the argument of the log) then generically imparts a mass to the D-brane position modes!

You can see this because the resulting supergravity potential has the form:

$$V(r,\phi) = \frac{X(\rho)}{r^{\alpha}} = \frac{X(\rho)}{(\rho - \phi\bar{\phi}/2)^{\alpha}}$$

Expanding about some vacuum for ρ

$$V = V_0 \left(1 + \alpha \frac{\phi\bar{\phi}}{2r} + ...\right).$$

This is a Hubble-scale mass for the would-be inflaton. The problem here is very analogous to the SUGRA eta problem.

Copeland, Liddle, Lyth, Stewart, Wands

Options for solving it:

* Include additional brane dynamics in throat (e.g. D7s with Non-perturbative W):

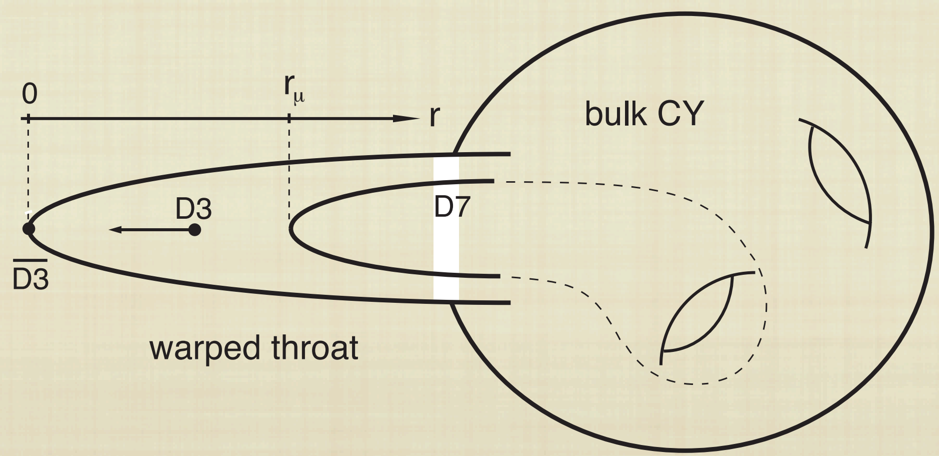
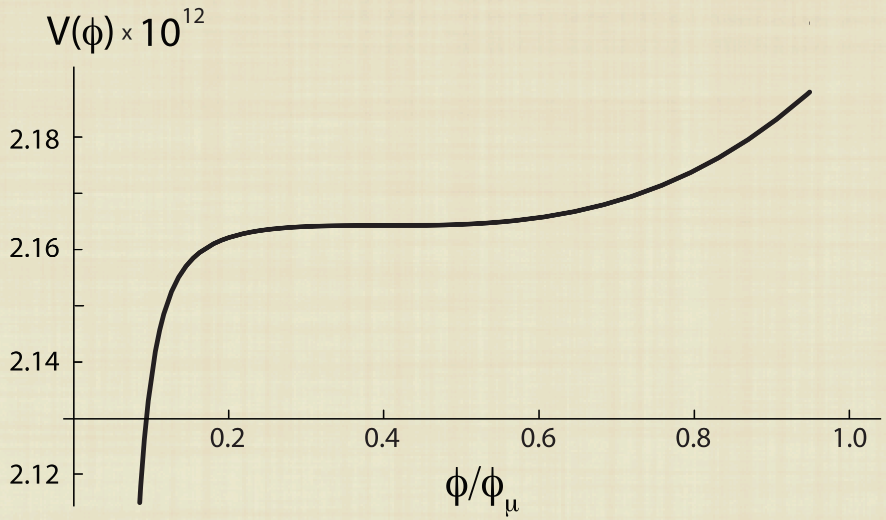


Figure 1: Cartoon of an embedded stack of D7-branes wrapping a four-cycle Σ_4, and a mobile D3-brane, in a warped throat region of a compact Calabi-Yau. The D3-brane feels a force from the D7-branes and from an anti-D3-brane at the tip of the throat.

Baumann, Dymarsky, Klebanov, McAllister, Steinhardt; Krause, Pajer; Burgess, Cline, Dasgupta, Firouzjahi

Then, for suitable D7 embedding and initial D3 location, one can find a very flat potential:



* Quite plausibly, perturbing the throat geometry by the equivalent of slightly irrelevant operators in the dual field theory, one can tune the inflaton potential.

Baumann, Dymarsky, SK, Klebanov, Mcallister (random discussions)

One can prove that all D3-brane inflation models based on such local throat geometries are small-field models.

Proof: By definition, one has

Baumann, McAllister

$$M_P^2 = M_{10}^8 V_6^w$$

where: $$V_6^w = \int d^6 y \sqrt{g} h$$

$$ds_{10}^2 = h^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y) g_{ij}(y) dy^i dy^j$$

We can break this into bulk & throat contributions:

$$V_6^w = (V_6^w)_{bulk} + (V_6^w)_{throat}$$

The throat contribution is easily computed for Sasaki-Einstein cone metrics:

$$(V_6^w)_{throat} = Vol(X_5) \int_0^{r_{UV}} dr\ r^5\ h(r)$$
$$= 2\pi^4 g_s N(\alpha')^2 r_{UV}^2$$

By neglecting the bulk contribution, we only decrease our estimate of the 4d Planck scale:

$$M_P^2 > M_{10}^8 (V_6^w)_{throat}$$

Now, it is easy to derive a bound. The canonical field corresponding to the D3 position is:

$$\phi^2 = T_3 r^2$$

$$T_3 = \frac{1}{(2\pi)^3}\frac{1}{g_s(\alpha')^2}$$

For a throat inflation model:

$$\Delta r \leq r_{UV}$$

So:

$$\left(\frac{\Delta\phi}{M_P}\right)^2 < T_3\frac{r_{UV}^2}{M_P^2} < T_3\frac{r_{UV}^2}{M_{10}^8(V_6^w)_{throat}}$$

And inserting our estimate for the throat volume, we find:

$$\left(\frac{\Delta\phi}{M_P}\right)^2 < \frac{4}{N}$$

which implies:

$$r \leq \frac{4}{N} \times .01$$

This is not good for a B-mode signal, since large N is required in these models to begin with.

Plausibly, by using wrapped higher p-branes instead of D3 branes, one can increase the B-mode signal to the verge of detectability.

Becker, Leblond, Shandera

possible signatures of brane models:

1. The end of brane inflation occurs when a brane and an anti-brane annihilate. This involves an abelian Higgs model (the tachyon which condenses to end inflation), and so cosmic string can form by the Kibble mechanism.

Sarangi, Tye;
Copeland, Myers, Polchinski

Because of the warped geometry, these cosmic D-strings or superstrings are naturally quite light:

$$10^{-12} \leq G_N\mu \leq 10^{-8}$$

They **may** be distinguishable from standard field theory strings, due to (p,q) flavor structure or differences in recombination probability (they can miss each other in extra dimensions!).

Jackson, Jones, Polchinski

2. Non-Gaussian density fluctuations

The kinetic structure of the D-brane action is actually a natural generalization of the relativistic point-particle:

$$\sqrt{1-\dot{x}^2} \quad \rightarrow$$

$$S = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\,\left[f(\phi)^{-1}\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} - f(\phi)^{-1} + V(\phi)\right]$$

where f is determined by the warp factor in the metric.

Expanding this out gives an action with a very nontrivial structure of higher derivative terms. In DBI inflation, one finds a solution for an inflaton trajectory where all terms are equally important -- this is a potentially UV completable D-brane example of K-inflation.

Silverstein, Tong

In these models, the brane has a large gamma factor

$$\gamma \sim \frac{1}{\sqrt{1-f\dot{\phi}^2}}$$

These theories generate a large non-Gaussianity, with:

$$f_{NL} \sim \gamma^2$$

and a characteristic shape in k-space peaked on equilateral triangles.

While it seems difficult to get compactified throats of the precise sort I've described that can support DBI inflation (very large N would be needed), it seems very likely that generalized throats which would support it do exist. Seeing a bispectrum with the appropriate shape would certainly be suggestive.

III. Stringy inflation with shift symmetries: LARGE field models

An obviously good idea for making slow roll inflation, is to use a field with a softly broken shift symmetry.

c.f. "natural inflation" of Freese, Frieman, Olinto

* symmetry under $a \longrightarrow a + c$ **forbids** radiative corrections of the form

$$\delta V \sim \frac{V}{M_P^2} a^2$$

(because the spurion of shift symmetry breaking must appear). So a major worry (the eta problem) is basically gone.

In many simple cases, the potential for an axion takes the form:

$$V(a) = \Lambda^4 \left(1 - cos(a/f_a)\right)$$

with: f = "axion decay constant"

Λ = "dynamical scale"

In e.g. heterotic string theory, there is a two-form B-field potential in 10 dimensions. KK reduction on a Calabi-Yau M results in an axion

$$a_i = \int_{C_i} B$$

for each generator C_i of $H_2(M)$.

This in general results in many axions in the 4d low-energy effective field theory.

The potential for these axions is in general generated entirely by "worldsheet instantons," string worldsheets that wrap C. As a result:

$$\Lambda = M_{string}\, \mathrm{Exp}(-\mathrm{Area}(C))$$

First question: Can one of these axions give us a good inflaton candidate that has $\Delta a > M_P$ and could explain a detection of gravity waves?

With the generic cos potential, one finds:

$$\epsilon \sim \left(\frac{M_P}{f_a}\right)^2,\ \ \eta \sim \left(\frac{M_P}{f_a}\right)^2$$

So to achieve robust slow-roll inflation, one would want to have:

$$f >> M_P$$

On the other hand in string theory, the axions we just discussed in the heterotic string have:

$$\frac{f^2}{M_P^2} \sim \frac{(\alpha')^2}{\mathrm{Vol(M)}} \times \mathrm{Area}(C)$$

and more generally, string axions have:

$$f_a \simeq \frac{M_P}{S_{inst}}$$

Svrcek, Witten

Does this mean that one cannot use shift symmetries to realize large-field inflation in string theory? NO.

1. A very reasonable idea which seems fairly generic, is the following. We will explain it in a fairly roundabout way, but the connection to the idea of shift-symmetry will become clear.

Silverstein, Westphal; + McAllister

Generically, in Calabi-Yau moduli space, there exist singular points around which submanifolds A,B undergo "monodromy" transformations. Around the generic conifold point in moduli space, for instance:

$$A \to A$$
$$B \to B + A$$

where A is a vanishing cycle at the conifold, and B is its homology dual.

Now, suppose one has a brane wrapping B (and filling space-time). As one goes around the conifold point, traversing a path of length f in field space, one adds to it a brane wrapping A, which costs extremely small energy near the conifold point, as A is a vanishing cycle.

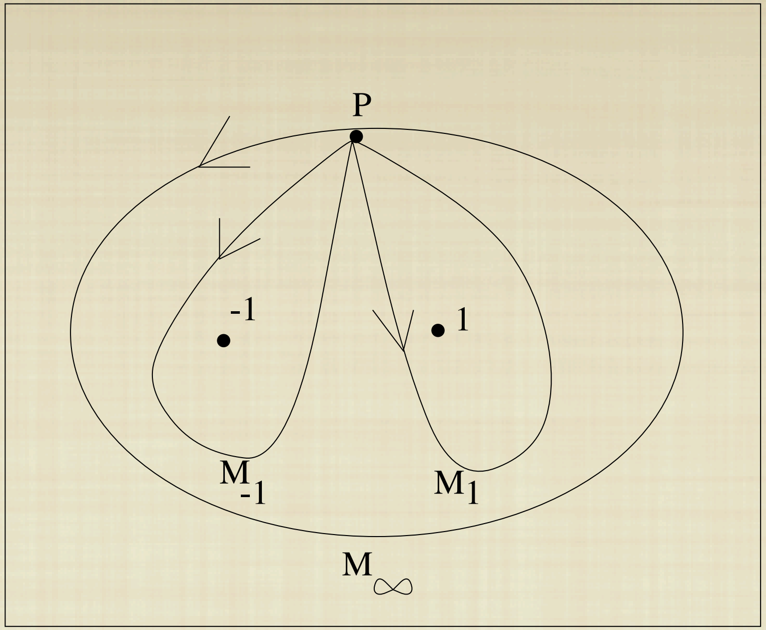


Fig. 1: The u plane with monodromies around 1, -1, and ∞. Note the choice of base point in the definition of the monodromies.

In this figure for instance, the conifold point in moduli space can be identified with -1, and the path is to be identified with M_{-1}

* Clearly, in this process,

$$\Delta V \sim T \ \mathrm{Vol}(A),$$
$$\Delta\Phi \sim F$$

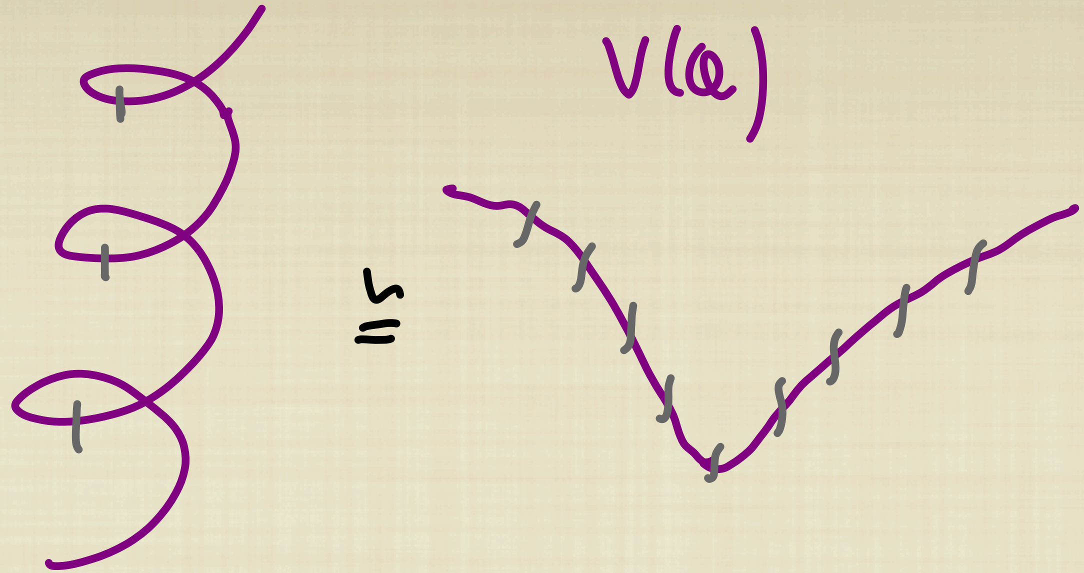
while V itself is given by

$$V \sim T \ \mathrm{Vol}(B)$$

Imagine an encircling path close to the singular point in moduli space (so A is small), but far enough away to keep F fixed and reasonable. Then one can get small slow-roll parameters on a path enacting the monodromy transformation many times.

Since A is a vanishing cycle, the energy cost of encircling A (and adding the brane wrapping a) is quite small, and the path around the singular point is almost one which enjoys a shift symmetry -- there is a softly broken shift symmetry. So, this "monodromy mechanism" is an example of inflation using a Nambu-Goldstone boson or axion.

Although F itself is sub-Planckian, the many traversals of the path lead to a super-Planckian field range. So one can construct inflationary models which give measurable gravitational waves by this mechanism.



In a specific example involving compactification on the "nilmanifold," one finds

$$V(\phi) \sim \phi^{2/3}$$

for the potential in a suitable field range.

2. A more brute-force idea, is to simply utilize many axions each of which enjoys the smaller field range.

Dimopoulos, SK, McGreevy, Wacker; Easther, McAllister

Typical Calabi-Yau spaces have large betti numbers, so they do give rise to a large number of axions.

Suppose:

$$\mathcal{L} = \sum_{i=1}^{N} \left(\frac{1}{2}(\partial a_i)^2 - \Lambda^4(1 - cos(a_i/f)) \right)$$

Since there are N independent shift symmetries, there will be no correction to the potential of the form:

$$\Delta V(a_i) \sim \frac{V}{M_P^2} a_i^2$$

Instead, any symmetry breaking term involving two distinct axions must scale like

$$\sim \frac{\Lambda_i^4 \Lambda_j^4}{M^4}$$

and so such terms are negligibly small.

Now naively, since the equations of motion are

$$\ddot{a}_i + 3H\dot{a}_i = -\frac{\partial V}{\partial a_i}$$

and V is basically the sum of N independent terms, one has:

$$\epsilon \sim \left(\frac{M_P}{f}\right)^2 \frac{1}{N^2}, \quad \eta \sim \left(\frac{M_P}{f}\right) \frac{1}{N}$$

This is an example of "assisted inflation"; it is radiatively stable only because of the N independent shift symmetries (otherwise, having the multiple fields doesn't help).

Liddle, Mazumdar, Schunk;
Kaloper et al.

At large N, it looks like generic initial condtions lead to:

$$(\Delta a)^2_{total} = \sum (\Delta a_i)^2 \sim N f^2$$

So if one can realize large N at the same time as the other assumptions that go into writing down the stringy axion potential (large volume, fixed decay constants, etc.), one can make large-field inflation models this way.

These assumptions seem hard to realize concretely at large volume, and may require a Calabi-Yau with e.g. many conifolds.

Grimm;
Kallosh, Sivanandam, Soroush

Conceptual Problem:

There are N dependent radiative corrections in quantum gravity.

$$\delta M_P^2 \sim \pm \frac{N}{16\pi^2}\Lambda_{UV}^2$$

Then there is an uncertainty in inflationary parameters:

$$\eta \sim \frac{1}{N}\left(\frac{M_P}{f}\right)^2 \left(1 \pm \frac{N\Lambda_{UV}^2}{16\pi^2 M_P^2}\right)$$

This implies that just for self-consistency, one can trust the most naive analysis only up to a maximal value of N

$$N_{max} \simeq 16\pi^2 \frac{M_P^2}{\Lambda_{UV}^2}$$

AND A CONSEQUENT MAXIMAL VALUE OF THE NUMBER OF E-FOLDINGS:

$$N_e \simeq N(\frac{f}{M_P})^2 \leq 16\pi^2(\frac{f}{M_P})^2\frac{M_P^2}{\Lambda_{UV}^2}$$

THE REAL NUMBER IS SENSITIVE TO THE UV CUTOFF. IN STRING THEORY, ACTUALLY, THE LEADING N-DEPENDENT CORRECTION IS AN ALPHA' CORRECTION:

$$\mathcal{L}_{10D} = M_{10}^8\left(\mathcal{R}_{10} + \zeta(3)(\alpha')^3\mathcal{R}_{10}^4 + ...\right)$$

Using

$$\int_M R \wedge R \wedge R = \frac{\chi(M)}{(2\pi)^3}$$

we can find an N-dependent correction to the 4D Einstein term, and hence estimate the stringy answer:

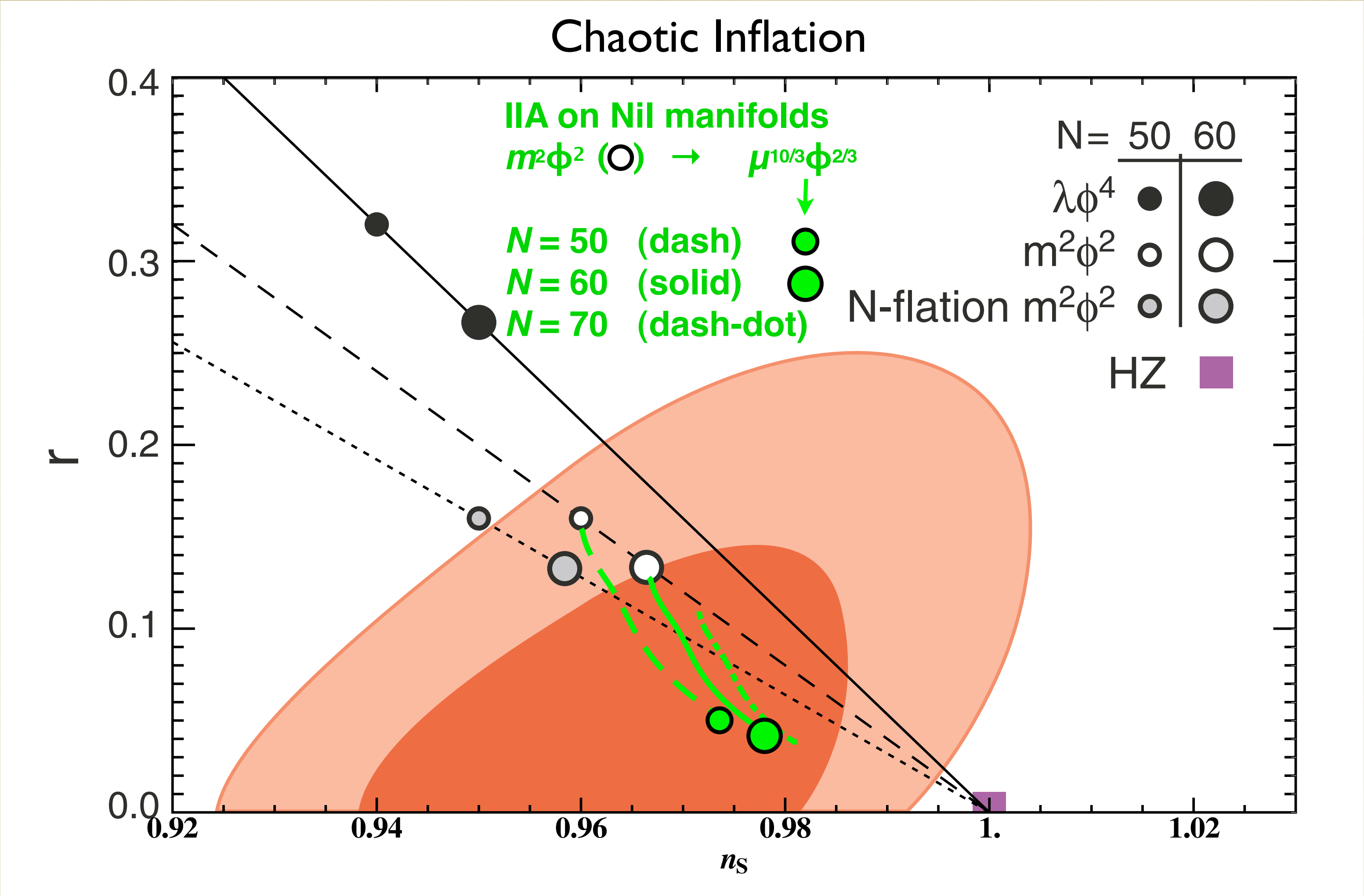
$$\delta M_P^2 = \frac{N \Lambda_{UV}^2}{16\pi^2} = \frac{\chi(M)}{8\pi^3} \zeta(3) \frac{(\alpha')^3}{V_6} M_P^2$$

Then we find:

$$N_e \leq \frac{2\pi^3}{\zeta(3)} \frac{N}{|\chi(M)|}$$

So one can, for a Calabi-Yau with a suitable topology that gives a cancellation in the Euler character, possibly get a sufficient number of e-foldins.

Of course, this “N-flation” model is only string motivated, not derived in any particular compactification. (The large number of fields involved make that seem very difficult in practice if not in principle).



String models now appear on the plots in papers like WMAP 5!

Summary:

* Many, many classes of models have been proposed.

* Some are approaching "rigorous" construction (by model-building standards).

* Most interesting development: possible experimental signatures! Cosmic strings, Non-Gaussianity, Tensor modes,...

* Imminent Planck launch together with the inherent UV sensitivity of the subject, promise to keep this area vibrant in the forseeable future!