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Supersymmetric Grand Unification Lecture 2 : 5d Orbifold GUTs

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Abstract

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1 Orbifold GUTs

1.1 GUTs on a Circle

As the first example of an orbifold GUT consider a pure $SO(3)$ gauge theory in 5 dimensions [56]. The gauge field is

$$A_M \equiv A_M^a\ T^a,\ a = 1, 2, 3;\ M, N = \{0, 1, 2, 3, 5\}. \tag{1}$$

The gauge field strength is given by

$$F_{MN} \equiv F_{MN}^a\ T^a = \partial_M A_N - \partial_N A_M + i[A_M, A_N] \tag{2}$$

where T^a are $SO(3)$ generators. The Lagrangian is

$$\mathcal{L}_5 = -\frac{1}{4 g_5^2 k}\ Tr(F_{MN} F^{MN}) \tag{3}$$

and we have $Tr(T^a\ T^b) \equiv k \delta^{ab}$. The inverse gauge coupling squared has mass dimensions one.

Let us first compactify the theory on $\mathcal{M}_4 \times S^1$ with coordinates $\{x_\mu, y\}$ and $y = [0, 2\pi R)$. The theory is invariant under the local gauge transformation

$$A_M(x_\mu, y) \to U\ A_M(x_\mu, y)\ U^\dagger - iU\ \partial_M\ U^\dagger,\ \ U = exp(i\theta^a(x_\mu, y)\ T^a). \tag{4}$$

Consider the possibility $\partial_5 A_\mu \equiv 0$. We have

$$F_{\mu 5} = \partial_\mu A_5 + i[A_\mu, A_5] \equiv D_\mu\ A_5. \tag{5}$$

We can then define

$$\tilde{\Phi} \equiv A_5\ \frac{\sqrt{2\pi R}}{g_5} \equiv A_5/g \tag{6}$$

where $g_5 \equiv \sqrt{2\pi R}\ g$ and g is the dimensionless 4d gauge coupling. The 5d Lagrangian reduces to the Lagrangian for a 4d $SO(3)$ gauge theory with massless scalar matter in the adjoint representation, i.e.

$$\mathcal{L}_5 = \frac{1}{2\pi R}[-\frac{1}{4 g^2 k}\ Tr(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{2k}\ Tr(D_\mu \tilde{\Phi}\ D^\mu \tilde{\Phi})]. \tag{7}$$

In general we have the mode expansion

$$A_M(x_\mu, y) = \sum_n [a_M^n \cos n\frac{y}{R} + b_M^n \sin n\frac{y}{R}] \tag{8}$$

where only the cosine modes with $n = 0$ have zero mass. Otherwise the 5d Laplacian $\partial_M \partial^M = \partial_\mu \partial^\mu + \partial_y \partial^y$ leads to Kaluza-Klein [KK] modes with effective 4d mass

$$m_n^2 = \frac{n^2}{R^2}. \tag{9}$$

1.2 Fermions in 5d

The Dirac algebra in 5d is given in terms of the 4×4 gamma matrices $\gamma_M,\ M = 0, 1, 2, 3, 5$ satisfying $\{\gamma_M, \gamma_N\} = 2g_{MN}$. A four component massless Dirac spinor $\Psi(x_\mu, y)$ satisfies the Dirac equation

$$i\gamma_M \partial^M \Psi = 0 = i(\gamma_\mu \partial^\mu - \gamma_5 \partial_y)\Psi. \tag{10}$$

We can obtain a chiral theory in 4d with the following parity operation

$$\mathcal{P} : \Psi(x_\mu, y) \to \Psi(x_\mu, -y) = P\Psi(x_\mu, y) \tag{11}$$

with $P = -\gamma_5$. We then have

$$\begin{aligned} \Psi_L &\sim \cos n\frac{y}{R} \\ \Psi_R &\sim \sin n\frac{y}{R}. \end{aligned} \tag{12}$$

Finally, in 4d the four component Dirac spinor decomposes into two Weyl spinors with

$$\Psi = \begin{pmatrix} \psi_1 \\ i\sigma_2 \psi_2^* \end{pmatrix} \tag{13}$$

where $\psi_{1,2}$ are two left-handed Weyl spinors.

1.3 GUTs on an Orbi-Circle

Let us briefly review the geometric picture of orbifold GUT models compactified on an orbi-circle $\mathrm{S}^1/\mathbb{Z}_2$. The circle $S^1 \equiv \mathbb{R}^1/\mathcal{T}$ where $\mathcal{T}$ is the action of translations by $2\pi R$. All fields Φ are thus periodic functions of y (up to a finite gauge transformation), i.e.

$$\mathcal{T} : \Phi(x_\mu, y) \to \Phi(x_\mu, y + 2\pi R) = T\ \Phi(x_\mu, y) \tag{14}$$

where $T \in SO(3)$ satisfies $T^2 = 1$. This corresponds to the translation $\mathcal{T}$ being realized non-trivially by a degree-2 Wilson line (i.e., background gauge

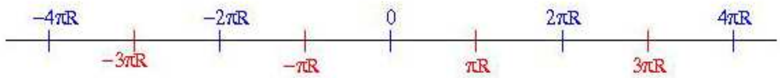

Figure 1: The real line moded out by the space group of translations, $\mathcal{T}$, and a $\mathbb{Z}_2$ parity, $\mathcal{P}$.

field - $\langle A_5 \rangle \neq 0$ with $T \equiv \exp(i \oint \langle A_5 \rangle dy))$. Hence the space group of $\mathrm{S}^1/\mathbb{Z}_2$ is composed of two actions, a translation, $\mathcal{T}\,:\, y \to y + 2\pi R$, and a space reversal, $\mathcal{P}\,:\, y \to -y$. There are two (conjugacy) classes of fixed points, $y = (2n)\pi R$ and $(2n+1)\pi R$, where $n \in \mathbb{Z}$.

The space group multiplication rules imply $\mathcal{T}\mathcal{P}\mathcal{T} = \mathcal{P}$, so we can replace the translation by a composite $\mathbb{Z}_2$ action $\mathcal{P}' = \mathcal{P}\mathcal{T}\,:\, y \to -y + 2\pi R$. The orbicircle $\mathrm{S}^1/\mathbb{Z}_2$ is equivalent to an $\mathbb{R}/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold, whose fundamental domain is the interval $[0,\, \pi R]$, and the two ends $y = 0$ and $y = \pi R$ are fixed points of the $\mathbb{Z}_2$ and $\mathbb{Z}_2'$ actions respectively.

A generic 5d field Φ has the following transformation properties under the $\mathbb{Z}_2$ and $\mathbb{Z}_2'$ orbifoldings (the 4d space-time coordinates are suppressed),

$$\mathcal{P}\,:\, \Phi(y) \to \Phi(-y) = P\Phi(y)\,, \qquad \mathcal{P}'\,:\, \Phi(y) \to \Phi(-y + 2\pi R) = P'\Phi(y)\,, \tag{15}$$

where P, $P' \equiv PT = \pm$ are *orbifold parities* acting on the field Φ in the appropriate group representation.[1] The four combinations of orbifold parities give four types of states, with wavefunctions

$$\begin{aligned}
\zeta_m(++) &\sim \cos(my/R), \\
\zeta_m(+-) &\sim \cos[(2m+1)y/2R], \\
\zeta_m(-+) &\sim \sin[(2m+1)y/2R], \\
\zeta_m(--) &\sim \sin[(m+1)y/R],
\end{aligned} \tag{16}$$

where $m \in \mathbb{Z}$. The corresponding KK towers have masses

$$M_{\mathrm{KK}} = \begin{cases} m/R & \text{for } (PP') = (++)\,, \\ (2m+1)/2R & \text{for } (PP') = (+-) \text{ and } (-+)\,, \\ (m+1)/R & \text{for } (PP') = (--)\,. \end{cases} \tag{17}$$

Note that only the Φ_{++} field possesses a massless zero mode.

[1]Where it is assumed that $[P, T] = 0$.

For example, consider the Wilson line $T = \exp(i\pi T^3) = \mathrm{diag}(-1,-1,1)$. Let $A_\mu(y)$ ($A_5(y)$) have parities $P = +(-)$, respectively. Then only A^3_μ has orbifold parity $(++)$ and A^3_5 has orbifold parity $(--)$.[2] Define the fields

$$W^\pm = \frac{1}{\sqrt{2}}(A^1 \mp iA^2) \tag{18}$$

with $T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$ and $[T^3, T^\pm] = \pm T^\pm$. Then $W^\pm_\mu$ $[W^\pm_5]$ have orbifold parity $(+-)$ $[(-+)]$, respectively. Thus the $SO(3)$ gauge group is broken to $SO(2) \approx U(1)$ in 4d. The local gauge parameters preserve the (P,T) parity/holonomy, i.e.

$$\begin{aligned}\theta^3(x_\mu, y) &= \theta^3_m(x_\mu)\zeta_m(++)\\ \theta^{1,2}(x_\mu, y) &= \theta^{1,2}_m(x_\mu)\zeta_m(+-).\end{aligned} \tag{19}$$

Therefore $SO(3)$ is not the symmetry at $y = \pi R$.

1.4 A Supersymmetric $SU(5)$ orbifold GUT

Consider the 5d orbifold GUT model of ref. [57]. The model has an $SU(5)$ symmetry broken by orbifold parities to the SM gauge group in 4d. The compactification scale $M_c = R^{-1}$ is assumed to be much less than the cutoff scale.

The gauge field is a 5d vector multiplet $\mathcal{V} = (A_M, \lambda, \lambda', \sigma)$, where A_M, σ (and their fermionic partners λ, λ') are in the adjoint representation (**24**) of $SU(5)$. This multiplet consists of one 4d $\mathcal{N} = 1$ supersymmetric vector multiplet $V = (A_\mu, \lambda)$ and one 4d chiral multiplet $\Sigma = ((\sigma + iA_5)/\sqrt{2}, \lambda')$. We also add two 5d hypermultiplets containing the Higgs doublets, $\mathcal{H} = (H_5, {H_5}^c)$, $\overline{\mathcal{H}} = (\bar{H}_{\bar{5}}, \bar{H}^c_{\bar{5}})$. The 5d gravitino $\Psi_M = (\psi^1_M, \psi^2_M)$ decomposes into two 4d gravitini ψ^1_μ, ψ^2_μ and two dilatini ψ^1_5, ψ^2_5. To be consistent with the 5d supersymmetry transformations one can assign positive parities to $\psi^1_\mu + \psi^2_\mu$, $\psi^1_5 - \psi^2_5$ and negative parities to $\psi^1_\mu - \psi^2_\mu$, $\psi^1_5 + \psi^2_5$; this assignment partially breaks $\mathcal{N} = 2$ to $\mathcal{N} = 1$ in 4d.

The orbifold parities for various states in the vector and hyper multiplets are chosen as follows [57] (where we have decomposed all the fields into SM irreducible representations and under $SU(5)$ we have taken $P = (++++$

[2] Note, $A^3_5(-y) = -A^3_5(y) + \frac{1}{R}$.

$+), \; P' = (- - - + +))$

States	P	P'	States	P	P'
$V(\mathbf{8},\mathbf{1},\mathbf{0})$	$+$	$+$	$\Sigma(\mathbf{8},\mathbf{1},\mathbf{0})$	$-$	$-$
$V(\mathbf{1},\mathbf{3},\mathbf{0})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{3},\mathbf{0})$	$-$	$-$
$V(\mathbf{1},\mathbf{1},\mathbf{0})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{1},\mathbf{0})$	$-$	$-$
$V(\bar{\mathbf{3}},\mathbf{2},\mathbf{5/3})$	$+$	$-$	$\Sigma(\mathbf{3},\mathbf{2},-\mathbf{5/3})$	$-$	$+$
$V(\mathbf{3},\mathbf{2},-\mathbf{5/3})$	$+$	$-$	$\Sigma(\bar{\mathbf{3}},\mathbf{2},\mathbf{5/3})$	$-$	$+$
$T(\mathbf{3},\mathbf{1},-\mathbf{2/3})$	$+$	$-$	$T^c(\bar{\mathbf{3}},\mathbf{1},\mathbf{2/3})$	$-$	$+$
$H(\mathbf{1},\mathbf{2},+\mathbf{1})$	$+$	$+$	$H^c(\bar{\mathbf{1}},\mathbf{2},-\mathbf{1})$	$-$	$-$
$\bar{T}(\bar{\mathbf{3}},\mathbf{1},+\mathbf{2/3})$	$+$	$-$	$\bar{T}^c(\mathbf{3},\mathbf{1},-\mathbf{2/3})$	$-$	$+$
$\bar{H}(\mathbf{1},\mathbf{2},-\mathbf{1})$	$+$	$+$	$\bar{H}^c(\mathbf{1},\mathbf{2},+\mathbf{1})$	$-$	$-$

.(20)

We see the fields supported at the orbifold fixed points $y = 0$ and πR have parities $P = +$ and $P' = +$ respectively. They form complete representations under the $SU(5)$ and SM groups; the corresponding fixed points are called $SU(5)$ and SM "branes." In a 4d effective theory one would integrate out all the massive states, leaving only massless modes of the $P = P' = +$ states. With the above choices of orbifold parities, the SM gauge fields and the H and $\bar{H}$ chiral multiplet are the only surviving states in 4d. We thus have an $\mathcal{N} = 1$ SUSY in 4d. In addition, the $T + \bar{T}$ and $T^c + \bar{T}^c$ color-triplet states are projected out, solving the doublet-triplet splitting problem that plagues conventional 4d GUTs.

1.5 Gauge Coupling Unification

We follow the field theoretical analysis in ref. [58] (see also [59, 60]). It has been shown there the correction to a generic gauge coupling due to a tower of KK states with masses $M_{\mathrm{KK}} = m/R$ is

$$\alpha^{-1}(\Lambda) = \alpha^{-1}(\mu_0) + \frac{b}{4\pi} \int_{r\Lambda^{-2}}^{r\mu_0^{-2}} \frac{\mathrm{d}t}{t}\, \theta_3\left(\frac{\mathrm{i}t}{\pi R^2}\right) , \tag{21}$$

where the integration is over the Schwinger parameter t, μ_0 and Λ are the IR and UV cut-offs, and $r = \pi/4$ is a numerical factor. θ_3 is the Jacobi theta function, $\theta_3(t) = \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\pi m^2 t}$, representing the summation over KK states.

For our $S^1/\mathbb{Z}_2$ orbifold there is one modification in the calculation. There are four sets of KK towers, with mass $M_{\mathrm{KK}} = m/R$ (for $P = P' = +$), $(m+1)/R$ (for $P = P' = -$) and $(m+1/2)/R$ (for $P = +$, $P' = -$ and

$P = -$, $P' = +$), where $m \geq 0$. The summations over KK states give respectively $\frac{1}{2}\left(\theta_3(\mathrm{i}t/\pi R^2) - 1\right)$ for the first two cases and $\frac{1}{2}\theta_2(\mathrm{i}t/\pi R^2)$ for the last two (where $\theta_2(t) = \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\pi(m+1/2)^2 t}$), and we have separated out the zero modes in the $P = P' = +$ case.

Tracing the renormalization group evolution from low energy scales, we are first in the realm of the MSSM, and the beta function coefficients are $\mathbf{b}^{MSSM} = (-\frac{33}{5}, -1, 3)$. The next energy threshold is the compactification scale M_c. From this scale to the cut-off scale, M_*, we have the four sets of KK states.

Collecting these facts, and using $\theta_2(\mathrm{i}t/\pi R^2) \simeq \theta_3(\mathrm{i}t/\pi R^2) \simeq \sqrt{\frac{\pi}{t}}R$ for $t/R^2 \ll 1$, we find the RG equations,

$$\alpha_i^{-1}(M_Z) = \alpha_*^{-1} - \frac{b_i^{MSSM}}{2\pi}\log\frac{M_*}{M_Z} + \frac{1}{4\pi}\left(b_i^{++} + b_i^{--}\right)\log\frac{M_*}{M_c} - \frac{b^{\mathcal{G}}}{2\pi}\left(\frac{M_*}{M_c} - 1\right) + \delta_i^2 + \delta_i^l \tag{22}$$

for $i = 1, 2, 3$, where $\alpha_*^{-1} = \frac{8\pi^2 R}{g_5^2}$ and we have taken the cut-off scales, $\mu_0 = M_c = \frac{1}{R}$ and $\Lambda = M_*$. (Note, this 5d orbifold GUT is a non-renormalizable theory with a cut-off. In string theory, the cut-off will be replaced by the physical string scale, M_{STRING}.) $b^{\mathcal{G}} = \sum_{P=\pm, P'=\pm} b^{\mathcal{G}}_{PP'}$, so in fact it is the beta function coefficient of the orbifold GUT gauge group, $\mathcal{G} = SU(5)$. The beta function coefficients in the last two terms have an $\mathcal{N} = 2$ nature, since the massive KK states enjoy a larger supersymmetry. In general we have $b^{\mathcal{G}} = 2C_2(\mathcal{G}) - 2N_{hyper}T_R$. The first term (in Eqn. 22) on the right is the 5d gauge coupling defined at the cut-off scale, the second term accounts for the one loop RG running in the MSSM from the weak scale to the cut-off, the third and fourth terms take into account the KK modes in loops above the compactification scale and the last two terms account for the corrections due to two loop RG running and weak scale threshold corrections.

It should be clear that there is a simple correspondence to the 4d analysis. We have

$$\begin{aligned} \alpha_G^{-1}\ (4d) \quad &\leftrightarrow \quad \alpha_*^{-1} - \frac{b^{\mathcal{G}}}{2\pi}\left(\frac{M_*}{M_c} - 1\right)\ (5d) \\ \delta_i^h\ (4d) \quad &\leftrightarrow \quad \frac{1}{4\pi}\left(b_i^{++} + b_i^{--}\right)\log\frac{M_*}{M_c} - \frac{b_i^{MSSM}}{2\pi}\log\frac{M_*}{M_G}\ (5d). \end{aligned} \tag{23}$$

Thus in 5d the GUT scale threshold corrections determine the ratio M_*/M_c (note the second term in Eqn. 23 does not contribute to δ_s^h). For $SU(5)$ we

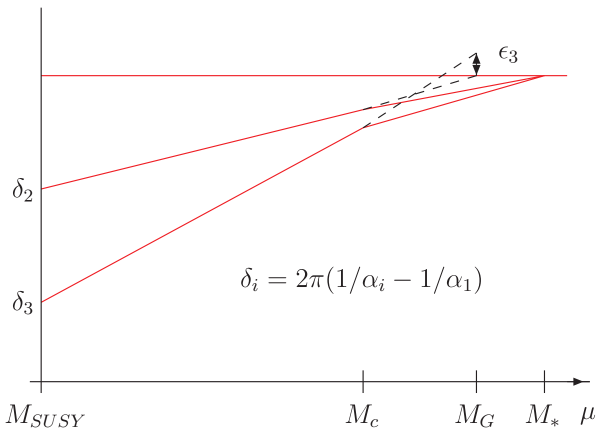

Figure 2: The differences $\delta_i = 2\pi(1/\alpha_i - 1/\alpha_1)$ are plotted as a function of energy scale μ. The threshold correction ϵ_3 defined in the 4d GUT scale is used to fix the threshold correction in the 5d orbifold GUT.

have $\mathbf{b}^{++} + \mathbf{b}^{--} = (-6/5, 2, 6)$ and given δ_s^h (Eqn. **??**) we have

$$\delta_s^h = \frac{12}{28\pi} \log \frac{M_*}{M_c} \approx +0.94 \tag{24}$$

or

$$\frac{M_*}{M_c} \approx 10^3. \tag{25}$$

If the GUT scale is defined at the point where $\alpha_1 = \alpha_2$, then we have $\delta_1^h = \delta_2^h$ or $\log \frac{M_*}{M_G} \approx 2$. In 5d orbifold GUTs, nothing in particular happens at the 4d GUT scale. However, since the gauge bosons affecting the dimension 6 operators for proton decay obtain their mass at the compactification scale, it is important to realize that the compactification scale is typically lower than the 4d GUT scale and the cut-off is higher (see Figure 2).

1.6 Quarks and Leptons in 5d Orbifold GUTs

Quarks and lepton fields can be put on either of the orbifold "branes" or in the 5d bulk. If they are placed on the $SU(5)$ "brane" at $y = 0$, then they come in complete $SU(5)$ multiplets. As a consequence a coupling of the type

$$W \supset \int d^2\theta \int dy \; \delta(y) \; \bar{H} \; 10 \; \bar{5} \tag{26}$$

will lead to bottom - tau Yukawa unification. This relation is good for the third generation and so it suggests that the third family should reside on the $SU(5)$ brane. Since this relation does not work for the first two families, they might be placed in the bulk or on the SM brane at $y = \pi\, R$. Without further discussion of quark and lepton masses (see [8, 61–63] for complete $SU(5)$ or $SO(10)$ orbifold GUT models), let us consider proton decay in orbifold GUTs.

1.7 Proton Decay

1.7.1 Dimension 6 Operators

The interactions contributing to proton decay are those between the so-called X gauge bosons $A_\mu^{(+-)} \in V(+-)$ (where ${A_\mu^{(+-)}}^{ai}(x_\mu, y)$ is the five dimensional gauge boson with quantum numbers $(\bar{3}, 2, +5/3)$ under SU(3)× SU(2) × U(1), a and i are color and SU(2) indices respectively) and the $\mathcal{N} = 1$ chiral multiplets on the $SU(5)$ brane at $y = 0$. Assuming all quarks and leptons reside on this brane we obtain the $\Delta B \neq 0$ interactions given by

$$\mathcal{S}_{\Delta B \neq 0} = -\frac{g_5}{\sqrt{2}} \int d^4x {A_\mu^{(+-)}}^{ai}(x_\mu, 0) J^\mu_{ai}(x_\mu) + h.c. \quad . \tag{27}$$

The currents J^μ_{ai} are given by:

$$\begin{aligned} J^\mu_{ai} &= \epsilon_{abc}\, \epsilon_{ij} (u^c)^*_b\, \bar{\sigma}^\mu\, q^{cj} + q^*_{ai}\, \bar{\sigma}^\mu\, e^c - \tilde{l}^*_i\, \bar{\sigma}^\mu\, (d^c)_a \\ &= (u^c)^*\, \bar{\sigma}^\mu\, q + q^*\, \bar{\sigma}^\mu\, e^c - \tilde{l}^*\, \bar{\sigma}^\mu\, d^c \quad , \end{aligned} \tag{28}$$

Upon integrating out the X gauge bosons we obtain the effective lagrangian for proton decay

$$\mathcal{L} = -\frac{g_G^2}{2M_X^2} \sum_{i,j} \left[(q_i^* \bar{\sigma}^\mu u_i^c)\, (\tilde{l}_j^* \bar{\sigma}_\mu d_j^c) \; + \; (q_i^* \bar{\sigma}^\mu e_i^c)\, (q_j^* \bar{\sigma}_\mu u_j^c) \right] \quad , \tag{29}$$

where all fermions are weak interaction eigenstates and $i, j, k = 1, 2, 3$ are family indices. The dimensionless quantity

$$g_G \equiv g_5 \frac{1}{\sqrt{2\pi R}} \tag{30}$$

is the four-dimensional gauge coupling of the gauge bosons zero modes. The combination

$$M_X = \frac{2M_c}{\pi} \quad , \tag{31}$$

proportional to the compactification scale

$$M_c \equiv \frac{1}{R} \quad , \tag{32}$$

is an effective gauge vector boson mass arising from the sum over all the Kaluza-Klein levels:

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2 M_c^2} = \frac{1}{2M_X^2} \quad . \tag{33}$$

Before one can evaluate the proton decay rate one must first rotate the quark and lepton fields to a mass eigenstate basis. This will bring in both left- and right-handed quark and lepton mixing angles. However, since the compactification scale is typically lower than the 4d GUT scale, it is clear that proton decay via dimension 6 operators is likely to be enhanced.

1.7.2 Dimension 5 Operators

The dimension 5 operators for proton decay result from integrating out color triplet Higgs fermions. However in this simplest $SU(5)$ 5d model the color triplet mass is of the form [64]

$$W \supset \int d^2\theta \; dy \; (T(-+)^c \; \partial_y \; T(+-) + \bar{T}(-+)^c \; \partial_y \; \bar{T}(+-)) \tag{34}$$

where a sum over massive KK modes is understood. Since only $T, \; \bar{T}$ couple directly to quarks and leptons, no dimension 5 operators are obtained when integrating out the color triplet Higgs fermions.

1.7.3 Dimension 4 baryon and lepton violating operators

If the theory is constructed with an R parity or family reflection symmetry, then no such operators will be generated.

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