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Supersymmetric Grand Unification Lecture 3 : Heterotic String Orbifold GUTs

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Abstract

Three lectures on supersymmetric GUTs given at PITP 2008 "Strings and Phenomenology," July 14 - 25, 2008.

1 Heterotic String Orbifolds and Orbifold GUTs

2 Phenomenological guidelines

We use the following guidelines when searching for "realistic" string models [65, 66]. We want to:

1. Preserve gauge coupling unification;
2. Low energy SUSY as solution to the gauge hierarchy problem, i.e. why is $M_Z << M_G$;
3. Put quarks and leptons in **16** of SO(10);
4. Put Higgs in **10**, thus quarks and leptons are distinguished from Higgs by their SO(10) quantum numbers;
5. Preserve GUT relations for 3rd family Yukawa couplings;
6. Use the fact that GUTs accommodate a "Natural" See-Saw scale $\mathcal{O}(M_G)$;
7. Use intuition derived from Orbifold GUT constructions, [67, 68] and
8. Use local GUTs to enforce family structure [69–71].

It is the last two guidelines which are novel and characterizes our approach.

2.1 $E_8 \times E_8$ 10D heterotic string compactified on $\mathbb{Z}_3 \times \mathbb{Z}_2$ 6D orbifold

There are many reviews and books on string theory. I cannot go into great detail here, so I will confine my discussion to some basic points. We start with the 10d heterotic string theory, consisting of a 26d left-moving bosonic string and a 10d right-moving superstring. Modular invariance requires the momenta of the internal left-moving bosonic degrees of freedom (16 of them) to lie in a 16d Euclidean even self-dual lattice, we choose to be the $\mathrm{E}_8 \times \mathrm{E}_8$ root lattice.[1]

[1] For an orthonormal basis, the E_8 root lattice consists of following vectors, $(n_1, n_2, \cdots, n_8)$ and $(n_1 + \frac{1}{2}, n_2 + \frac{1}{2}, \cdots, n_8 + \frac{1}{2})$, where $n_1, n_2, \cdots n_8$ are integers and $\sum_{i=1}^{8} n_i = 0 \bmod 2$.

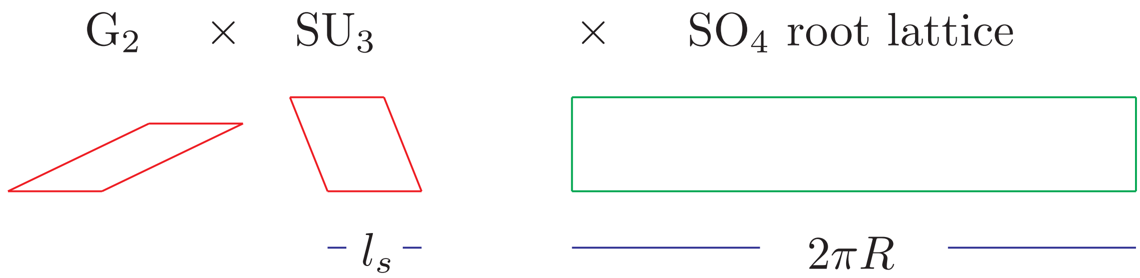

Figure 1: $G_2 \oplus SU(3) \oplus SO(4)$ lattice. Note, we have taken 5 directions with string scale length ℓ_s and one with length $2\pi R \gg \ell_s$. This will enable the analogy of an effective 5d orbifold field theory.

2.1.1 Heterotic string compactified on $T^6/\mathbb{Z}_6$

We first compactify the theory on 6d torus defined by the space group action of translations on a factorizable Lie algebra lattice $G_2 \oplus SU(3) \oplus SO(4)$ (see Fig. 5). Then we mod out by the $\mathbb{Z}_6$ action on the three complex compactified coordinates given by $Z^i \to e^{2\pi i \mathbf{r}_i \cdot \mathbf{v}_6} Z^i$, $i = 1, 2, 3$, where $\mathbf{v}_6 = \frac{1}{6}(1, 2, -3)$ is the twist vector, and $\mathbf{r}_1 = (1, 0, 0, 0)$, $\mathbf{r}_2 = (0, 1, 0, 0)$, $\mathbf{r}_3 = (0, 0, 1, 0)$.[2]

The $\mathbb{Z}_6$ orbifold is equivalent to a $\mathbb{Z}_2 \times \mathbb{Z}_3$ orbifold, where the two twist vectors are $\mathbf{v}_2 = 3\mathbf{v}_6 = \frac{1}{2}(1, 0, -1)$ and $\mathbf{v}_3 = 2\mathbf{v}_6 = \frac{1}{3}(1, -1, 0)$. The $\mathbb{Z}_2$ and $\mathbb{Z}_3$ sub-orbifold twists have the $SU(3)$ and $SO(4)$ planes as their fixed torii. In Abelian symmetric orbifolds, gauge embeddings of the point group elements and lattice translations are realized by shifts of the momentum vectors, $\mathbf{P}$, in the $E_8 \times E_8$ root lattice[3] [72], *i.e.*, $\mathbf{P} \to \mathbf{P} + k\mathbf{V} + l\mathbf{W}$, where k, l are some integers, and $\mathbf{V}$ and $\mathbf{W}$ are known as the gauge twists and Wilson lines [73]. These embeddings are subject to modular invariance requirements [74, 75]. The Wilson lines are also required to be consistent with the action of the point group. In the $\mathbb{Z}_6$ model, there are at most three consistent Wilson lines [76], one of degree 3 ($\mathbf{W}_3$), along the $SU(3)$ lattice, and two of degree 2 ($\mathbf{W}_2$, $\mathbf{W}_2'$), along the $SO(4)$ lattice.

[2]Together with $\mathbf{r}_4 = (0, 0, 0, 1)$, they form the set of positive weights of the $\mathbf{8}_v$ representation of the $SO(8)$, the little group in 10d. $\pm\mathbf{r}_4$ represent the two uncompactified dimensions in the light-cone gauge. Their space-time fermionic partners have weights $\mathbf{r} = (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$ with even numbers of positive signs; they are in the $\mathbf{8}_s$ representation of $SO(8)$. In this notation, the fourth component of $\mathbf{v}_6$ is zero.

[3]The E_8 root lattice is given by the set of states $\mathbf{P} = \{n_1, n_2, \cdots, n_8\}$, $\{n_1 + \frac{1}{2}, n_2 + \frac{1}{2}, \cdots, n_8 + \frac{1}{2}\}$ satisfying $n_i \in \mathbb{Z}$, $\sum_{i=1}^{8} n_i = 2\mathbb{Z}$.

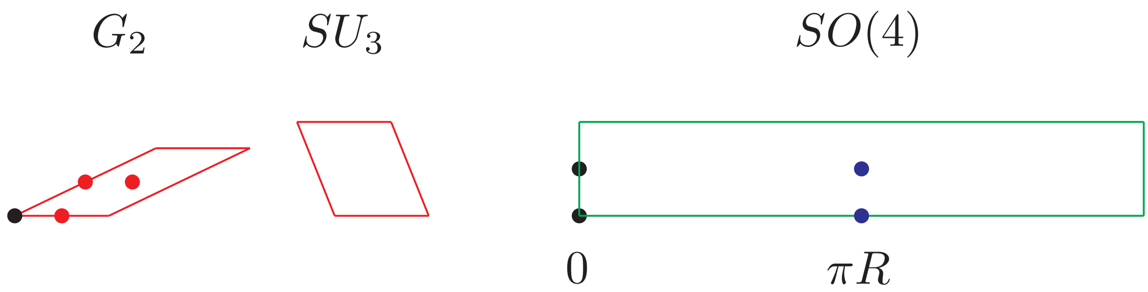

Figure 2: $G_2 \oplus SU(3) \oplus SO(4)$ lattice with $\mathbb{Z}_2$ fixed points. The T_3 twisted sector states sit at these fixed points. The fixed point at the origin and the symmetric linear combination of the red (grey) fixed points in the G_2 torus have $\gamma = 1$.

The $\mathbb{Z}_6$ model has three untwisted sectors (U_i, $i = 1, 2, 3$) and five twisted sectors (T_i, $i = 1, 2, \cdots, 5$). (The T_k and T_{6-k} sectors are CPT conjugates of each other.) The twisted sectors split further into sub-sectors when discrete Wilson lines are present. In the $SU(3)$ and $SO(4)$ directions, we can label these sub-sectors by their winding numbers, $n_3 = 0, 1, 2$ and n_2, $n_2' = 0, 1$, respectively. In the G_2 direction, where both the $\mathbb{Z}_2$ and $\mathbb{Z}_3$ sub-orbifold twists act, the situation is more complicated. There are four $\mathbb{Z}_2$ fixed points in the G_2 plane. Not all of them are invariant under the $\mathbb{Z}_3$ twist, in fact three of them are transformed into each other. Thus for the T_3 twisted-sector states one needs to find linear combinations of these fixed-point states such that they have definite eigenvalues, $\gamma = 1$ (with multiplicity 2), $e^{i2\pi/3}$, or $e^{i4\pi/3}$, under the orbifold twist [76, 77] (see Fig. 6). Similarly, for the $T_{2,4}$ twisted-sector states, $\gamma = 1$ (with multiplicity 2) and -1 (the fixed points of the $T_{2,4}$ twisted sectors in the G_2 torus are shown in Fig. 7). The T_1 twisted-sector states have only one fixed point in the G_2 plane, thus $\gamma = 1$ (see Fig. 8). The eigenvalues γ provide another piece of information to differentiate twisted sub-sectors.

Massless states in 4d string models consist of those momentum vectors $\mathbf{P}$ and $\mathbf{r}$ ($\mathbf{r}$ are in the $SO(8)$ weight lattice) which satisfy the following mass-shell equations [72, 74],

$$\frac{\alpha'}{2} m_R^2 = N_R^k + \frac{1}{2} \left|\mathbf{r} + k\mathbf{v}\right|^2 + a_R^k = 0 \,, \tag{1}$$

$$\frac{\alpha'}{2} m_L^2 = N_L^k + \frac{1}{2} \left|\mathbf{P} + k\mathbf{X}\right|^2 + a_L^k = 0 \,, \tag{2}$$

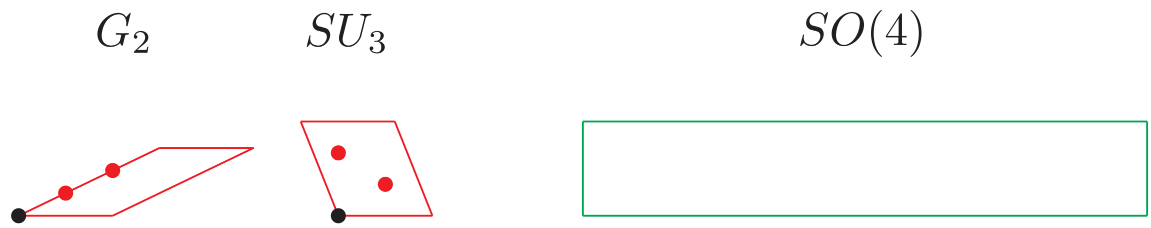

Figure 3: $G_2 \oplus SU(3) \oplus SO(4)$ lattice with $\mathbb{Z}_3$ fixed points for the T_2 twisted sector. The fixed point at the origin and the symmetric linear combination of the red (grey) fixed points in the G_2 torus have $\gamma = 1$.

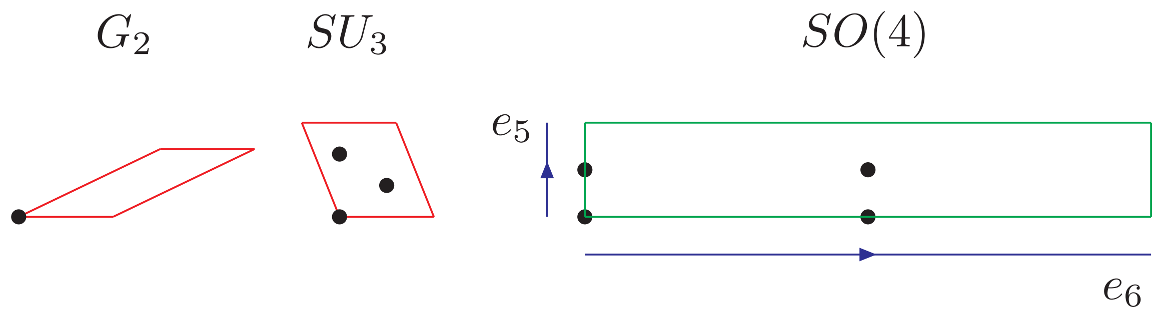

Figure 4: $G_2 \oplus SU(3) \oplus SO(4)$ lattice with $\mathbb{Z}_6$ fixed points. The T_1 twisted sector states sit at these fixed points.

where α' is the Regge slope, N_R^k and N_L^k are (fractional) numbers of the right- and left-moving (bosonic) oscillators, $\mathbf{X} = \mathbf{V} + n_3\mathbf{W}_3 + n_2\mathbf{W}_2 + n_2'\mathbf{W}_2'$, and a_R^k, a_L^k are the normal ordering constants,

$$\begin{aligned} a_R^k &= -\frac{1}{2} + \frac{1}{2}\sum_{i=1}^{3} |\widehat{kv_i}| \left(1 - |\widehat{kv_i}|\right) , \\ a_L^k &= -1 + \frac{1}{2}\sum_{i=1}^{3} |\widehat{kv_i}| \left(1 - |\widehat{kv_i}|\right) , \end{aligned} \tag{3}$$

with $\widehat{kv_i} = \mathrm{mod}(kv_i, 1)$.

These states are subject to a generalized Gliozzi-Scherk-Olive (GSO) projection $\mathcal{P} = \frac{1}{6}\sum_{\ell=0}^{5} \Delta^\ell$ [72]. For the simple case of the k-th twisted sector ($k = 0$ for the untwisted sectors) with no Wilson lines ($n_3 = n_2 = n_2' = 0$) we have

$$\Delta = \gamma\phi \exp\left\{ i\pi \left[(2\mathbf{P} + k\mathbf{X}) \cdot \mathbf{X} - (2\mathbf{r} + k\mathbf{v}) \cdot \mathbf{v} \right] \right\} , \tag{4}$$

where ϕ are phases from bosonic oscillators. However, in the $\mathbb{Z}_6$ model, the GSO projector must be modified for the untwisted-sector and $T_{2,4}$, T_3 twisted-sector states in the presence of Wilson lines [68]. The Wilson lines split each twisted sector into sub-sectors and there must be additional projections with respect to these sub-sectors. This modification in the projector gives the following projection conditions,

$$\mathbf{P} \cdot \mathbf{V} - \mathbf{r}_i \cdot \mathbf{v} = \mathbb{Z} \ \ (i = 1, 2, 3), \quad \mathbf{P} \cdot \mathbf{W}_3, \ \mathbf{P} \cdot \mathbf{W}_2, \ \mathbf{P} \cdot \mathbf{W}_2' = \mathbb{Z}, \tag{5}$$

for the untwisted-sector states, and

$$T_{2,4} : \ \mathbf{P} \cdot \mathbf{W}_2, \ \mathbf{P} \cdot \mathbf{W}_2' = \mathbb{Z} , \qquad T_3 : \ \mathbf{P} \cdot \mathbf{W}_3 = \mathbb{Z} , \tag{6}$$

for the $T_{2,3,4}$ sector states (since twists of these sectors have fixed torii). There is no additional condition for the T_1 sector states.

2.1.2 An orbifold GUT – heterotic string dictionary

We first implement the $\mathbb{Z}_3$ sub-orbifold twist, which acts only on the G_2 and $SU(3)$ lattices. The resulting model is a 6d gauge theory with $\mathcal{N} = 2$ hypermultiplet matter, from the untwisted and $T_{2,4}$ twisted sectors. This 6d theory is our starting point to reproduce the orbifold GUT models. The next step is to implement the $\mathbb{Z}_2$ sub-orbifold twist. The geometry of the extra dimensions closely resembles that of 6d orbifold GUTs. The $SO(4)$ lattice has four $\mathbb{Z}_2$ fixed points at 0, πR, $\pi R'$ and $\pi(R+R')$, where R and R' are on the e_5 and e_6 axes, respectively, of the lattice (see Figs. 6 and 8). When one varies the modulus parameter of the $SO(4)$ lattice such that the length of one axis (R) is much larger than the other (R') and the string length scale (ℓ_s), the lattice effectively becomes the $S^1/\mathbb{Z}_2$ orbi-circle in the 5d orbifold GUT, and the two fixed points at 0 and πR have degree-2 degeneracies. Furthermore, one may identify the states in the intermediate $\mathbb{Z}_3$ model, *i.e.* those of the untwisted and $T_{2,4}$ twisted sectors, as bulk states in the orbifold GUT.

Space-time supersymmetry and GUT breaking in string models work exactly as in the orbifold GUT models. First consider supersymmetry breaking. In the field theory, there are two gravitini in 4d, coming from the 5d (or 6d) gravitino. Only one linear combination is consistent with the space reversal, $y \to -y$; this breaks the $\mathcal{N}=2$ supersymmetry to that of $\mathcal{N}=1$. In string theory, the space-time supersymmetry currents are represented by those half-integral $SO(8)$ momenta.[4] The $\mathbb{Z}_3$ and $\mathbb{Z}_2$ projections remove all but two of them, $\mathbf{r} = \pm\frac{1}{2}(1,1,1,1)$; this gives $\mathcal{N}=1$ supersymmetry in 4d.

Now consider GUT symmetry breaking. As usual, the $\mathbb{Z}_2$ orbifold twist and the translational symmetry of the $SO(4)$ lattice are realized in the gauge degrees of freedom by degree-2 gauge twists and Wilson lines respectively. To mimic the 5d orbifold GUT example, we impose only one degree-2 Wilson line, $\mathbf{W}_2$, along the long direction of the $SO(4)$ lattice, $\mathbf{R}$.[5] The gauge embeddings generally break the 5d/6d (bulk) gauge group further down to its subgroups, and the symmetry breaking works exactly as in the orbifold GUT models. This can clearly be seen from the following string theoretical realizations of the orbifold parities

$$P = p\, e^{2\pi i\, [\mathbf{P}\cdot\mathbf{V}_2 - \mathbf{r}\cdot\mathbf{v}_2]}\,, \quad P' = p\, e^{2\pi i\, [\mathbf{P}\cdot(\mathbf{V}_2+\mathbf{W}_2) - \mathbf{r}\cdot\mathbf{v}_2]}\,, \tag{7}$$

[4]Together with $\mathbf{r}_4 = (0,0,0,1)$, they form the set of positive weights of the $\mathbf{8}_v$ representation of the $SO(8)$, the little group in 10d. $\pm\mathbf{r}_4$ represent the two uncompactified dimensions in the light-cone gauge. Their space-time fermionic partners have weights $\mathbf{r} = (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$ with even numbers of positive signs; they are in the $\mathbf{8}_s$ representation of $SO(8)$. In this notation, the fourth component of $\mathbf{v}_6$ is zero.

[5]Wilson lines can be used to reduce the number of chiral families. In all our models, we find it is sufficient to get three-generation models with two Wilson lines, one of degree 2 and one of degree 3. Note, however, that with two Wilson lines in the $SO(4)$ torus we can break $SO(10)$ directly to $SU(3)\times SU(2)\times U(1)_Y \times U(1)_X$ (see for example, Ref. [78]).

where $\mathbf{V}_2 = 3\mathbf{V}_6$, and $p = \gamma\phi$ can be identified with intrinsic parities in the field theory language.[6] Since $2(\mathbf{P}\cdot\mathbf{V}_2 - \mathbf{r}\cdot\mathbf{v}_2)$, $2\mathbf{P}\cdot\mathbf{W}_2 = \mathbb{Z}$, by properties of the $E_8 \times E_8$ and $SO(8)$ lattices, thus $P^2 = P'^2 = 1$, and Eq. (136) provides a representation of the orbifold parities. From the string theory point of view, $P = P' = +$ are nothing but the projection conditions, $\Delta = 1$, for the untwisted and $T_{2,4}$ twisted-sector states (see Eqs. (133), (134) and (135)).

To reaffirm this identification, we compare the masses of KK excitations derived from string theory with that of orbifold GUTs. The coordinates of the $SO(4)$ lattice are untwisted under the $\mathbb{Z}_3$ action, so their mode expansions are the same as that of toroidal coordinates. Concentrating on the $\mathbf{R}$ direction, the bosonic coordinate is $X_{L,R} = x_{L,R} + p_{L,R}(\tau \pm \sigma) +$ oscillator terms, with p_L, p_R given by

$$\begin{aligned} p_L =& \quad \tfrac{m}{2R} + \left(1 - \tfrac{1}{4}|\mathbf{W}_2|^2\right)\tfrac{n_2 R}{\ell_s^2} + \tfrac{\mathbf{P}\cdot\mathbf{W}_2}{2R} \quad , \\ p_R =& \qquad\qquad p_L - \tfrac{2n_2 R}{\ell_s^2} \, , \end{aligned} \tag{8}$$

where m (n_2) are KK levels (winding numbers). The $\mathbb{Z}_2$ action maps m to $-m$, n_2 to $-n_2$ and $\mathbf{W}_2$ to $-\mathbf{W}_2$, so physical states must contain linear combinations, $|m, n_2\rangle \pm |-m, -n_2\rangle$; the eigenvalues ± 1 correspond to the first $\mathbb{Z}_2$ parity, P, of orbifold GUT models. The second orbifold parity, P', induces a non-trivial degree-2 Wilson line; it shifts the KK level by $m \to m + \mathbf{P}\cdot\mathbf{W}_2$. Since $2\mathbf{W}_2$ is a vector of the (integral) $E_8 \times E_8$ lattice, the shift must be an integer or half-integer. When $R \gg R' \sim \ell_s$, the winding modes and the KK modes in the smaller dimension of $SO(4)$ decouple. Eq. (137) then gives four types of KK excitations, reproducing the field theoretical mass formula in Eq. (112).

2.2 MSSM with R parity

In this section we discuss just one "benchmark" model (Model 1) obtained via a "mini-landscape" search [65] of the $E_8 \times E_8$ heterotic string compactified

[6]For gauge and untwisted-sector states, p are trivial. For non-oscillator states in the $T_{2,4}$ twisted sectors, $p = \gamma$ are the eigenvalues of the G_2-plane fixed points under the $\mathbb{Z}_2$ twist. Note that $p = +$ and $-$ states have multiplicities 2 and 1 respectively since the corresponding numbers of fixed points in the G_2 plane are 2 and 1.

on the $\mathbb{Z}_6$ orbifold [66].[7] The model is defined by the shifts and Wilson lines

$$V = \left(\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0, 0\right) \left(\frac{1}{2}, -\frac{1}{6}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right) \tag{9a}$$

$$W_2 = \left(0, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 0, 0, 0\right) \left(4, -3, -\frac{7}{2}, -4, -3, -\frac{7}{2}, -\frac{9}{2}, \frac{7}{2}\right) \tag{9b}$$

$$W_3 = \left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right) \left(\frac{1}{3}, 0, 0, \frac{2}{3}, 0, \frac{5}{3}, -2, 0\right) . \tag{9c}$$

A possible second order 2 Wilson line is set to zero.

The shift V is defined to satisfy two criteria.

- The first criterion is the existence of a local $SO(10)$ GUT [8] at the T_1 fixed points at $x_6 = 0$ in the $SO(4)$ torus (Fig. 8).

$$P \cdot V = \mathbb{Z};\ P \in SO(10) \text{ momentum lattice.} \tag{10}$$

  Since the T_1 twisted sector has no invariant torus and only one Wilson line along the x_6 direction, all states located at these two fixed points must come in complete $SO(10)$ multiplets.

- The second criterion is that two massless spinor representations of $SO(10)$ are located at the $x_6 = 0$ fixed points.

Hence, the two complete families on the local $SO(10)$ GUT fixed points gives us an excellent starting point to find the MSSM. The Higgs doublets and third family of quarks and leptons must then come from elsewhere.

Let us now discuss the effective 5d orbifold GUT [81]. Consider the orbifold $(T^2)^3/\mathbb{Z}_3$ plus the Wilson line W_3 in the SU_3 torus. The $\mathbb{Z}_3$ twist does not act on the SO_4 torus, see Fig. 7. As a consequence of embedding the $\mathbb{Z}_3$ twist as a shift in the $E_8 \times E_8$ group lattice and taking into account the W_3 Wilson line, the first E_8 is broken to $SU(6)$. This gives the effective 5d orbifold gauge multiplet contained in the $\mathcal{N} = 1$ vector field V. In addition we find the massless states $\Sigma \in \mathbf{35}$, $\mathbf{20}+\mathbf{20}^c$ and 9 $(\mathbf{6}+\mathbf{6}^c)$ in the 6d untwisted sector and T_2, T_4 twisted sectors. Together these form a complete $\mathcal{N} = 2$ gauge multiplet $(V + \Sigma)$ and a $\mathbf{20}$ + 9 $(\mathbf{6})$ dimensional hypermultiplets. In

[7] For earlier work on MSSM models from $\mathbb{Z}_6$ orbifolds of the heterotic string, see [69,70].

[8] For more discussion on local GUTs, see [69,71]

fact the massless states in this sector can all be viewed as "bulk" states moving around in a large 5d space-time.

Now consider the $\mathbb{Z}_2$ twist and the Wilson line W_2 along the x_6 axis in the SO_4 torus. The action of the $\mathbb{Z}_2$ twist breaks the gauge group to $SU(5)$, while W_2 breaks $SU(5)$ further to the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Let us now consider those MSSM states located in the bulk. From two of the pairs of $\mathcal{N}=1$ chiral multiplets $\mathbf{6}+\mathbf{6^c}$, which decompose as

$$\begin{aligned} 2\times(\mathbf{6}+\mathbf{6^c}) \quad \supset \quad & \left[(1,\mathbf{2})^{--}_{\mathbf{1,1}} + (\mathbf{3},\mathbf{1})^{-+}_{\mathbf{-2/3,1/3}}\right] + \left[(1,\mathbf{2})^{++}_{\mathbf{-1,-1}} + (\overline{\mathbf{3}},\mathbf{1})^{--}_{\mathbf{2/3,-1/3}}\right] \\ & + \left[(1,\mathbf{2})^{-+}_{\mathbf{1,1}} + (\mathbf{3},\mathbf{1})^{--}_{\mathbf{-2/3,1/3}}\right] + \left[(1,\mathbf{2})^{+-}_{\mathbf{-1,-1}} + (\overline{\mathbf{3}},\mathbf{1})^{++}_{\mathbf{2/3,-1/3}}\right], \end{aligned} \tag{11}$$

we obtain the third family b^c and lepton doublet, l. The rest of the third family comes from the $\mathbf{10}+\mathbf{10}^c$ of $SU(5)$ contained in the $\mathbf{20}+\mathbf{20}^c$ of $SU(6)$, in the untwisted sector.

Now consider the Higgs bosons. The bulk gauge symmetry is $SU(6)$. Under $SU(5)\times U(1)$, the adjoint decomposes as

$$\mathbf{35} \to \mathbf{24}_0 + \mathbf{5}_{+1} + \mathbf{5}^c_{-1} + 1_0. \tag{12}$$

Thus the MSSM Higgs sector emerges from the breaking of the $SU(6)$ adjoint by the orbifold and the model satisfies the property of "gauge-Higgs unification."

In the models with gauge-Higgs unification, the Higgs multiplets come from the 5d *vector* multiplet (V,Σ), both in the adjoint representation of $SU(6)$. V is the 4d gauge multiplet and the 4d chiral multiplet Σ contains the Higgs doublets. These states transform as follows under the orbifold parities $(P\ P')$:

$$V: \quad \left(\begin{array}{ccc|cc|c} (++) & (++) & (++) & (+-) & (+-) & (-+) \\ (++) & (++) & (++) & (+-) & (+-) & (-+) \\ (++) & (++) & (++) & (+-) & (+-) & (-+) \\ \hline (+-) & (+-) & (+-) & (++) & (++) & (--) \\ (+-) & (+-) & (+-) & (++) & (++) & (--) \\ \hline (-+) & (-+) & (-+) & (--) & (--) & (++) \end{array}\right) \tag{13}$$

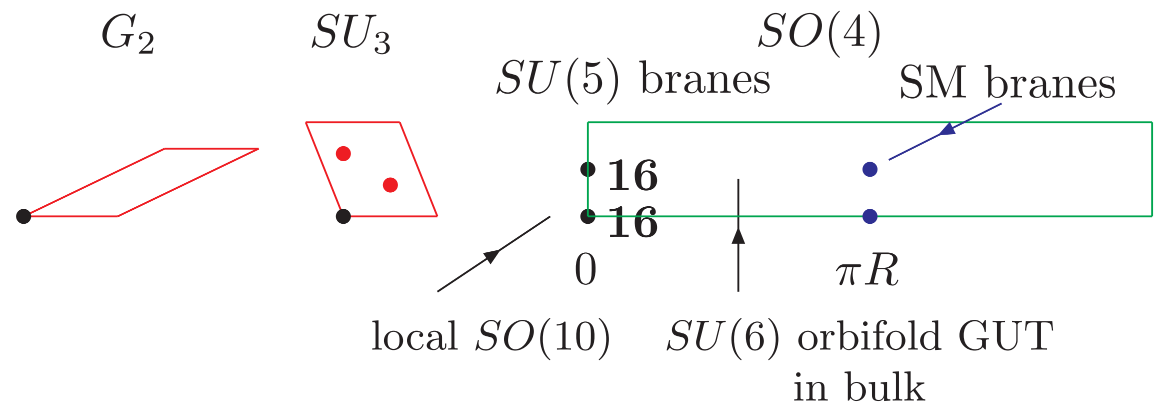

Figure 5: The two families in the T_1 twisted sector.

$$\Phi : \left(\begin{array}{ccc|cc|c} (--) & (--) & (--) & (-+) & (-+) & (+-) \\ (--) & (--) & (--) & (-+) & (-+) & (+-) \\ (--) & (--) & (--) & (-+) & (-+) & (+-) \\ \hline (-+) & (-+) & (-+) & (--) & (--) & (++) \\ (-+) & (-+) & (-+) & (--) & (--) & (++) \\ \hline (+-) & (+-) & (+-) & (++) & (++) & (--) \end{array} \right). \tag{14}$$

Hence, we have obtained doublet-triplet splitting via orbifolding.

2.3 D_4 Family Symmetry

Consider the $\mathbb{Z}_2$ fixed points. We have four fixed points, separated into an $SU(5)$ and SM invariant pair by the W_2 Wilson line (see Fig. 9). We find two complete families, one on each of the SO_{10} fixed points and a small set of vector-like exotics (with fractional electric charge) on the other fixed points. Since W_2 is in the direction orthogonal to the two families, we find a non-trivial D_4 family symmetry. This will affect a possible hierarchy of fermion masses. We will discuss the family symmetry and the exotics in more detail next.

The discrete group D_4 is a non-abelian discrete subgroup of SU_2 of order 8. It is generated by the set of 2×2 Pauli matrices

$$D_4 = \{\pm 1, \pm\sigma_1, \pm\sigma_3, \mp i\sigma_2\}. \tag{15}$$

In our case, the action of the transformation $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ takes $F_1 \leftrightarrow F_2$, while the action of $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ takes $F_2 \rightarrow -F_2$. These are symmetries of the string. The first is an unbroken part of the translation group in the direction orthogonal to W_2 in the SO_4 torus and the latter is a stringy selection rule resulting from $\mathbb{Z}_2$ space group invariance. Under D_4 the three families of quarks and leptons transform as a doublet, $(F_1,\ F_2)$, and a singlet, F_3. Only the third family can have a tree level Yukawa coupling to the Higgs (which is also a D_4 singlet). In summary:

- Since the top quarks and the Higgs are derived from the $SU(6)$ chiral adjoint and **20** hypermultiplet in the 5D bulk, they have a tree level Yukawa interaction given by

$$\frac{g_5}{\sqrt{\pi R}} \int_0^{\pi R} dy \mathbf{20^c}\ \Sigma\ \mathbf{20} = g_G\ q\ H_u\ t^c \tag{16}$$

  where g_5 (g_G) is the 5d (4d) $SU(6)$ gauge coupling constant evaluated at the string scale.

- The first two families reside at the $\mathbb{Z}_2$ fixed points, resulting in a D_4 family symmetry. Hence family symmetry breaking may be used to generate a hierarchy of fermion masses.[9]

2.4 More details of "Benchmark" Model 1 [66]

Let us now consider the spectrum, exotics, R parity, Yukawa couplings, and neutrino masses. In Table 9 we list the states of the model. In addition to the three families of quarks and leptons and one pair of Higgs doublets, we have vector-like exotics (states which can obtain mass without breaking any SM symmetry) and SM singlets. The SM singlets enter the superpotential in several important ways. They can give mass to the vector-like exotics via effective mass terms of the form

$$E E^c \tilde{S}^n \tag{17}$$

where E, E^c ($\tilde{S}$) represent the vector-like exotics and SM singlets respectively. We have checked that all vector-like exotics obtain mass at supersymmetric

[9]For a discussion of D_4 family symmetry and phenomenology, see Ref. [79]. For a general discussion of discrete non-Abelian family symmetries from orbifold compactifications of the heterotic string, see [80].

points in moduli space with $F = D = 0$. The SM singlets also generate effective Yukawa matrices for quarks and leptons, including neutrinos. In addition, the SM singlets give Majorana mass to the 16 right-handed neutrinos n_i^c, 13 conjugate neutrinos n_i and Dirac mass mixing the two. We have checked that the theory has only 3 light left-handed neutrinos.

However, one of the most important constraints in this construction is the existence of an exact low energy R parity. In this model we identified a generalized $B-L$ (see Table 9) which is standard for the SM states and vector-like on the vector-like exotics. This $B-L$ naturally distinguishes the Higgs and lepton doublets. Moreover we found SM singlet states

$$\tilde{S} = \{h_i,\ \chi_i,\ s_i^0\} \tag{18}$$

which can get vacuum expectation values preserving a matter parity $\mathbb{Z}_2^{\mathcal{M}}$ subgroup of $U(1)_{B\text{–}L}$. It is this set of SM singlets which give vector-like exotics mass and effective Yukawa matrices for quarks and leptons. In addition, the states χ_i give Majorana mass to neutrinos.

2.5 Gauge Coupling Unification and Proton Decay

We have checked whether the SM gauge couplings unify at the string scale in the class of models similar to Model 1 above [81]. All of the 15 MSSM-like models of Ref. [66] have 3 families of quarks and leptons and one or more pairs of Higgs doublets. They all admit an $SU(6)$ orbifold GUT with gauge-Higgs unification and the third family in the bulk. They differ, however, in other bulk and brane exotic states. We show that the KK modes of the model, including only those of the third family and the gauge sector, are <u>not</u> consistent with gauge coupling unification at the string scale. Nevertheless, we show that it is possible to obtain unification if one adjusts the spectrum of vector-like exotics below the compactification scale. As an example, see Fig. 10. Note, the compactification scale is less than the 4d GUT scale and some exotics have mass two orders of magnitude less than M_c, while all others are taken to have mass at M_{STRING}. In addition, the value of the GUT coupling at the string scale, $\alpha_G(M_{STRING}) \equiv \alpha_{string}$, satisfies the weakly coupled heterotic string relation

$$G_N = \frac{1}{8}\,\alpha_{string}\,\alpha' \tag{19}$$

#	irrep	label		#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/3,1/3)}$	q_i		3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-4/3,-1/3)}$	u_i^c
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(2,1)}$	e_i^c		8	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(2/3,-1/3)}$	d_i^c		1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,1/3)}$	d_i
4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1,-1)}$	ℓ_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1,1)}$	ℓ_i^c
1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1,0)}$	ϕ_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1,0)}$	ϕ_i^c
6	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(2/3,2/3)}$	δ_i^c		6	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-2/3)}$	δ_i
14	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,*)}$	s_i^+		14	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1,*)}$	s_i^-
16	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	n_i^c		13	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
5	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	η_i^c		5	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
10	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,0)}$	h_i		2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{(0,0)}$	y_i
6	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{(0,*)}$	f_i		6	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{(0,*)}$	f_i^c
2	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{(-1,-1)}$	f_i^-		2	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{(1,1)}$	f_i^{c+}
4	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i		32	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
2	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,2/3)}$	v_i^c		2	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-2/3)}$	v_i

Table 1: Spectrum. The quantum numbers under SU(3) $\times$ SU(2) $\times$ [SU(4) $\times$ SU(2)$'$] are shown in boldface; hypercharge and $B-L$ charge appear as subscripts. Note that the states $s_i^{\pm}$, f_i, $\bar{f}_i$ and m_i have different $B-L$ charges for different i, which we do not explicitly list [66].

or

$$\alpha_{string}^{-1} = \frac{1}{8}\,(\frac{M_{Pl}}{M_{\rm STRING}})^2. \tag{20}$$

In Fig. 11 we plot the distribution of solutions with different choices of light exotics. On the same plot we give the proton lifetime due to dimension 6 operators. Recall in these models the two light families are located on the $SU(5)$ branes, thus the proton decay rate is only suppressed by M_c^{-2}. Note, 90% of the models are already excluded by the Super-Kamiokande bounds on the proton lifetime. The remaining models may be tested at a next generation megaton water čerenkov detector.

3 Conclusion

We have discussed an evolution of SUSY GUT model building in these lectures. We saw that 4d SUSY GUTs have many virtues. However there are

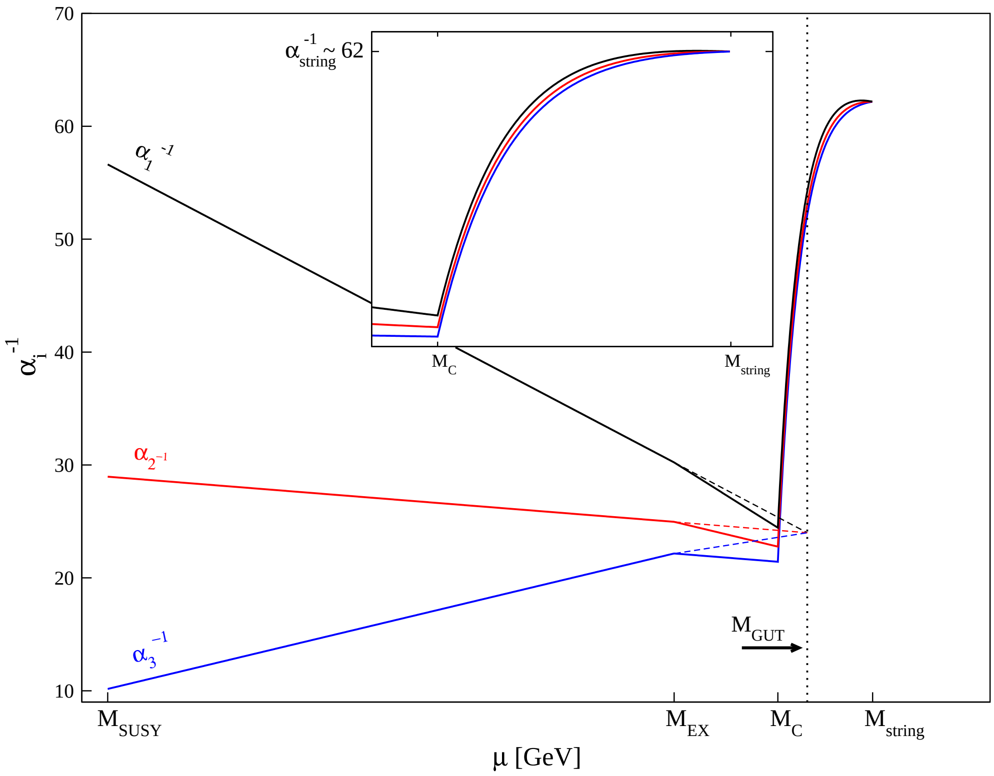

Figure 6: An example of the type of gauge coupling evolution we see in these models, versus the typical behavior in the MSSM. The "tail" is due to the power law running of the couplings when towers of Kaluza-Klein modes are involved. Unification in this model occurs at $M_{\rm STRING} \simeq 5.5 \times 10^{17}$ GeV, with a compactification scale of $M_c \simeq 8.2 \times 10^{15}$ GeV, and an exotic mass scale of $M_{\rm EX} \simeq 8.2 \times 10^{13}$ GeV.

some problems which suggest that these model may be difficult to derive from a more fundamental theory, i.e. string theory. We then discussed orbifold GUT field theories which solve two of the most difficult problems of 4d GUTs, i.e. GUT symmetry breaking and Higgs doublet-triplet splitting. We then showed how some orbifold GUTs can find an ultra-violet completion within the context of heterotic string theory.

The flood gates are now wide open. In recent work [66] we have obtained many models with features like the MSSM: SM gauge group with 3 families and vector-like exotics which can, in principle, obtain large mass. The

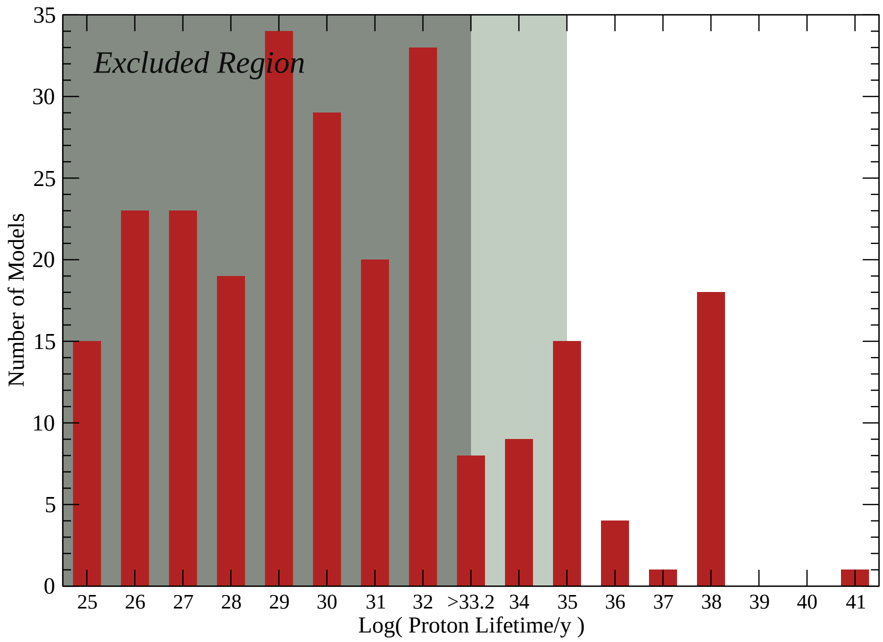

Figure 7: Histogram of solutions with $M_{\rm STRING} > M_c \gtrsim M_{\rm EX}$, showing the models which are excluded by Super-K bounds (darker green) and those which are potentially accessible in a next generation proton decay experiment (lighter green). Of 252 total solutions, 48 are not experimentally ruled out by the current experimental bound, and most of the remaining parameter space can be eliminated in the next generation of proposed proton decay searches.

models have an exact R-parity and non-trivial Yukawa matrices for quarks and leptons. In addition, neutrinos obtain mass via the See-Saw mechanism. We showed that gauge coupling unification can be accommodated [81]. Recently, another MSSM-like model has been obtained with the heterotic string compactified on a $T^6/\mathbb{Z}_{12}$ orbifold [82].

Of course, this is not the end of the story. It is just the beginning. We must still obtain predictions for the LHC. This requires stabilizing the moduli and breaking supersymmetry. In fact, these two conditions are <u>not</u> independent, since once SUSY is broken, the moduli will be stabilized. The scary fact is that the moduli have to be stabilized at just the right values to be consistent with low energy phenomenology.

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