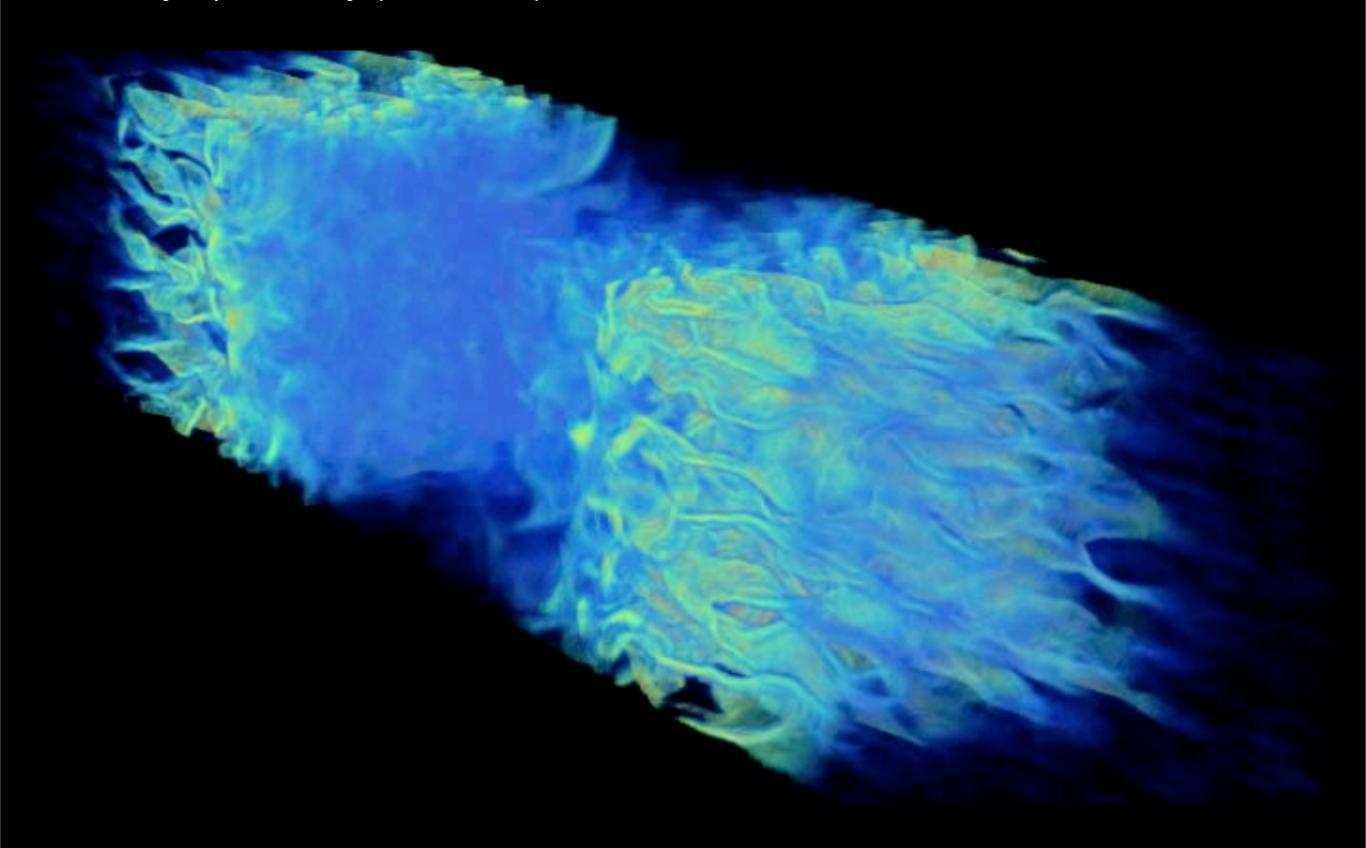
Kinetic Simulations of Astrophysical Plasmas Anatoly Spitkovsky (Princeton)



Contents

Plasma physics on computers

How PIC works

Electrostatic codes

Charge assignment and shape factors

Discretization effects

Electromagnetic codes

FDTD and Yee mesh

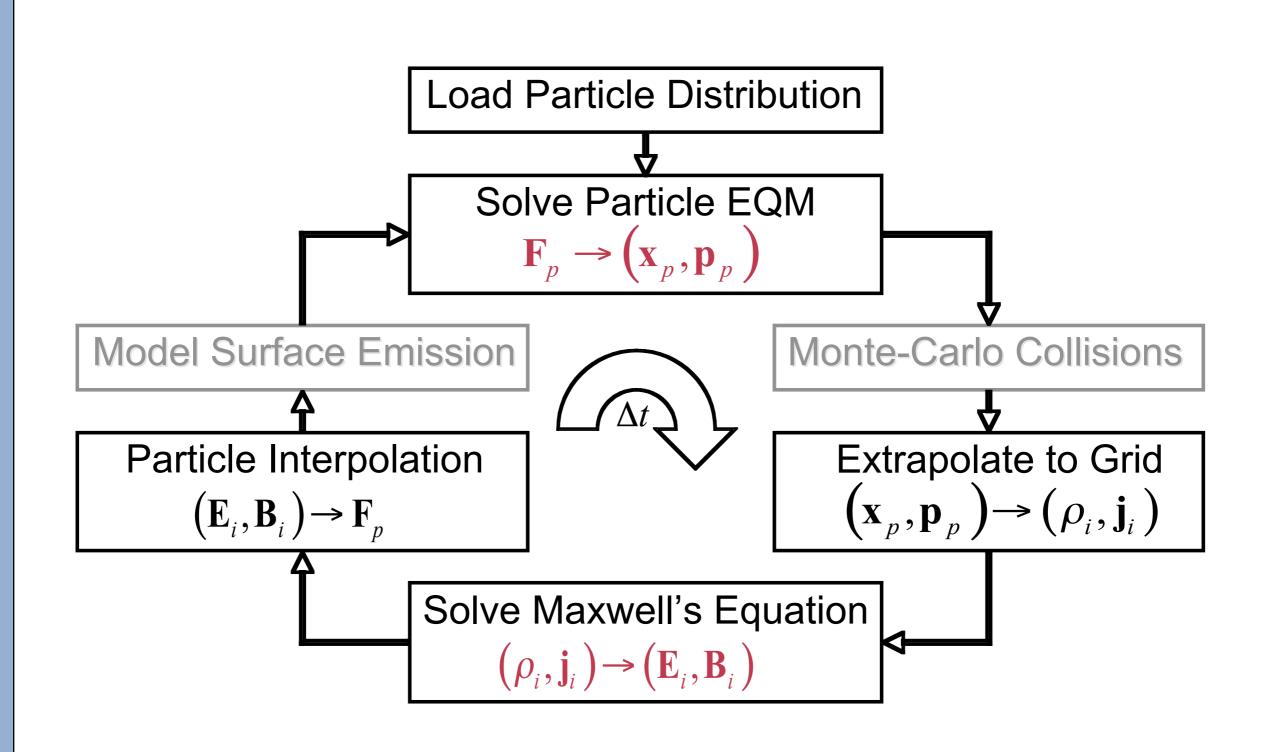
Particle movers: Boris' algorithm

Conservative charge deposition

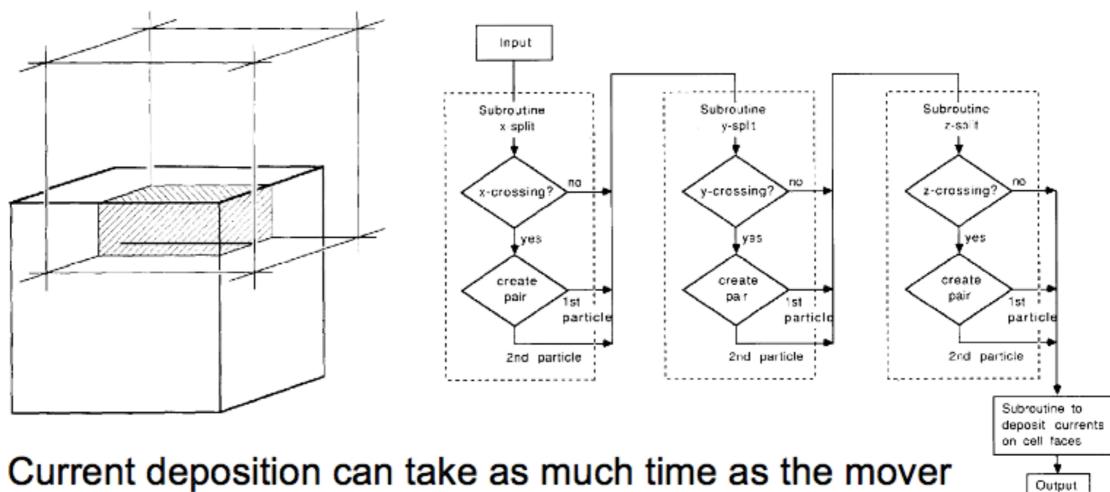
Boundary conditions

Applications and examples

PIC cycle



Charge and current deposition



Current deposition can take as much time as the mover (sometimes more). More optimized deposits exist (Umeda 2003).

Charge conservation makes the whole Maxwell solver local and hyperbolic (like nature intended!). Static fields can be established dynamically.

Special sauce

Particle shape should be smoothed to reduce noise. We use current filtering after deposition to reduce high frequency aliases.

Higher order FDTD schemes (4th spatial order) work better at reducing unphysical Cherenkov instability.

Boundary conditions

Periodic is simple -- just copy ghost zones and loop particles. Should not forget particle charge on the other side of the grid!

Conducting BCs: set E field parallel to boundary to 0. Boundary has to lie along the grid.

Outgoing BCs: match an outgoing wave to E, B fields at boundary (Lindman 1975).

Boundary conditions

Perfectly matched layer (Berenger 1994) -- works like absorbing material with different conductivity for E and B fields)

Moving window: simulation can fly at c to follow a fast beam. Outgoing plasma requires no conditions.

Injection: particles can be injected from boundary, or created in pairs throughout the domain. We implemented moving injectors and expanding domains for shock problems.

Parallelization

We use domain decomposition with ghost zones that are communicated via MPI. In 3D we decompose in slabs in y-z plane, so all x-s are on each processor (useful for shocks).

Optimization

Main time expense: the mover and the deposition.

Both involve moving data to and from memory, hence cache optimization is essential.

Single precision vs double.

See Kevin Bowers talk on Thursday at 4pm

Notes on PIC

There is no "subscale" physics with PIC -- we resolve the smallest scales! Converse is expense...

Usually deal with non-clumped flows, hence AMR usually not used. Some exceptions -- reconnection simulations.

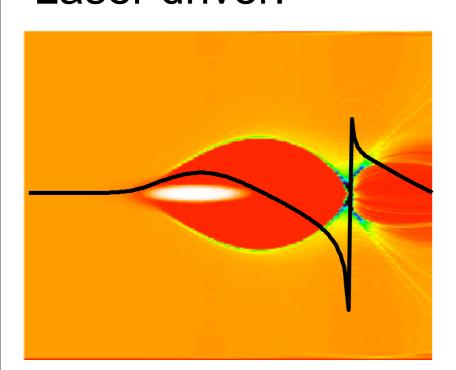
FDTD conserves divergence of B to machine precisio

PIC issues:

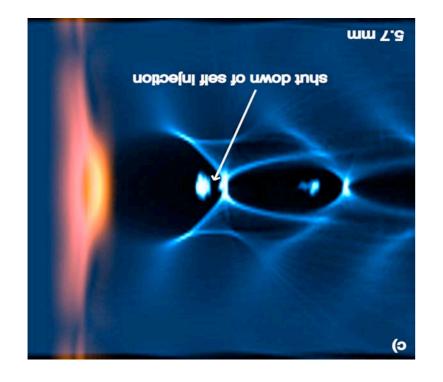
- Particle discretization error
- Smoothing error (finite size particles)
- Statistical noise (granular force)
- Grid aliasing (grid assignment)
- Deterioration of quadrature in time integration
- Short-range forces (collisions) neglected
- Analysis of large-scale simulations is nontrivial

but the alternative is 6D Vlasov integration...

Laser-plasma interaction and plasma based accelerators Laser driver:



Beam driver:

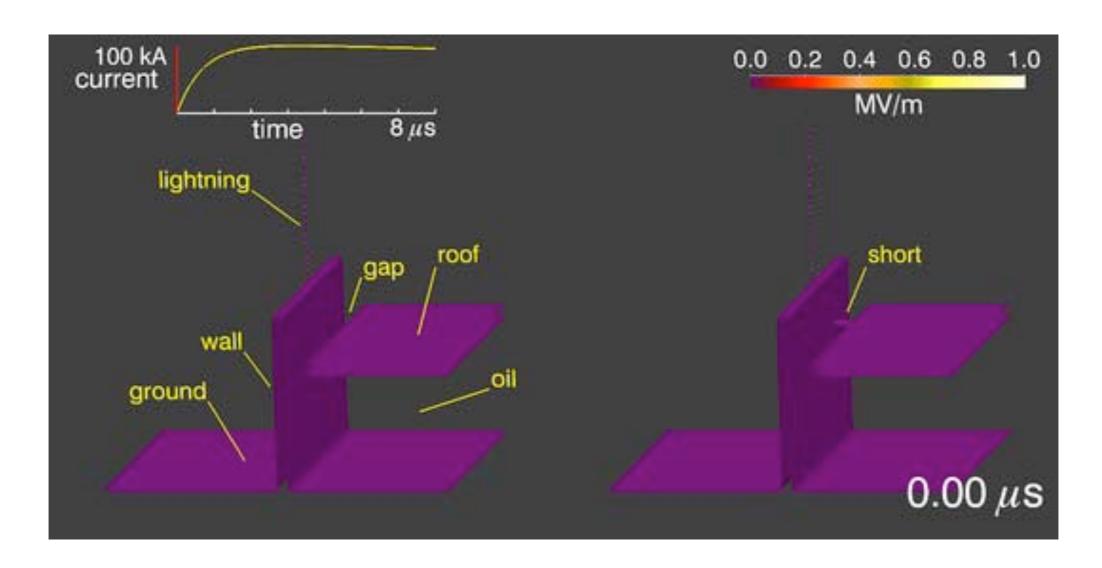




Engineering:

Gas discharges, plasma processing, film deposition. PIC with Monte-Carlo collisions and external circuit driving.

Lightning-oil tank interaction!



Astrophysics:

Any problem with multivalued, anisotropic or otherwise strange distribution function.

Collisionless shocks (solar wind, interstellar medium, relativistic jets): structure and the physics of shock mediation

Particle acceleration: when, where, how?

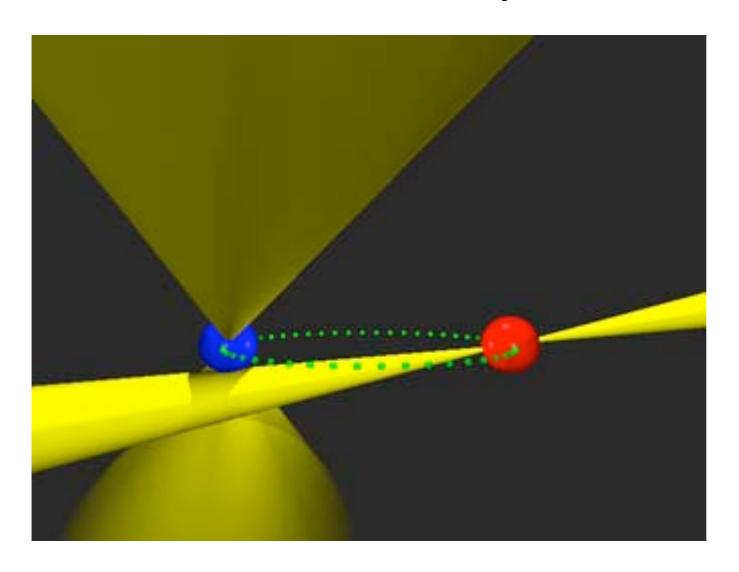
Cosmic ray propagation and field generation

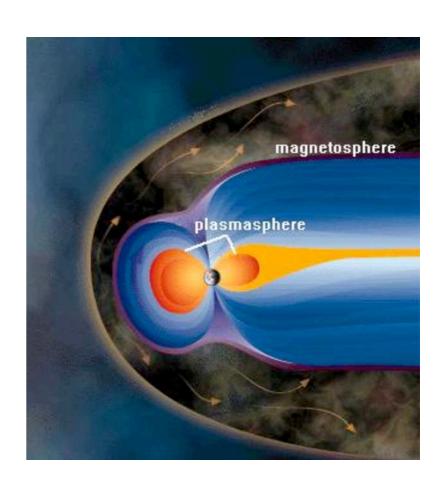
Reconnection

Dissipation of turbulence

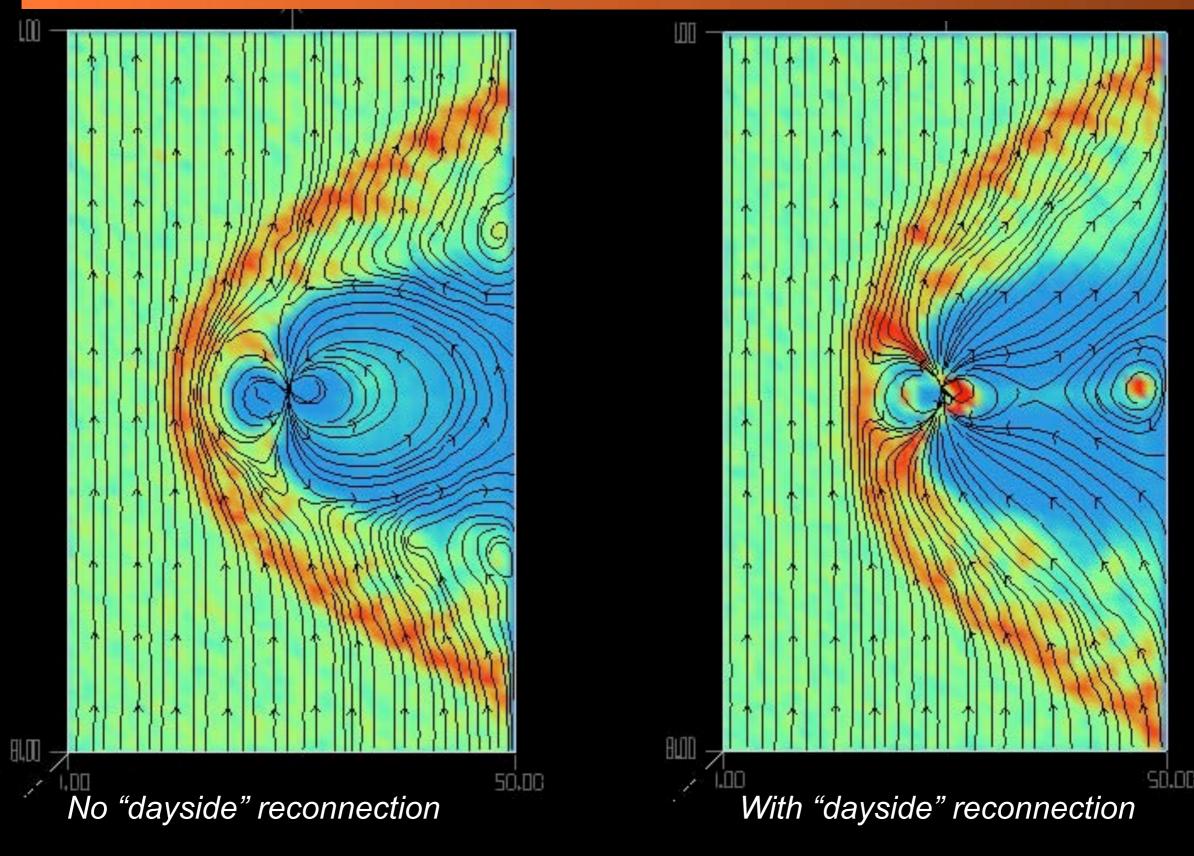
Case study: Wind-magnetosphere interaction in double pulsar binary J0737.

Simulation of a macroscopic system with PIC (AS & Arons 2004). Possible if the size of the system is > 50 skindepths.



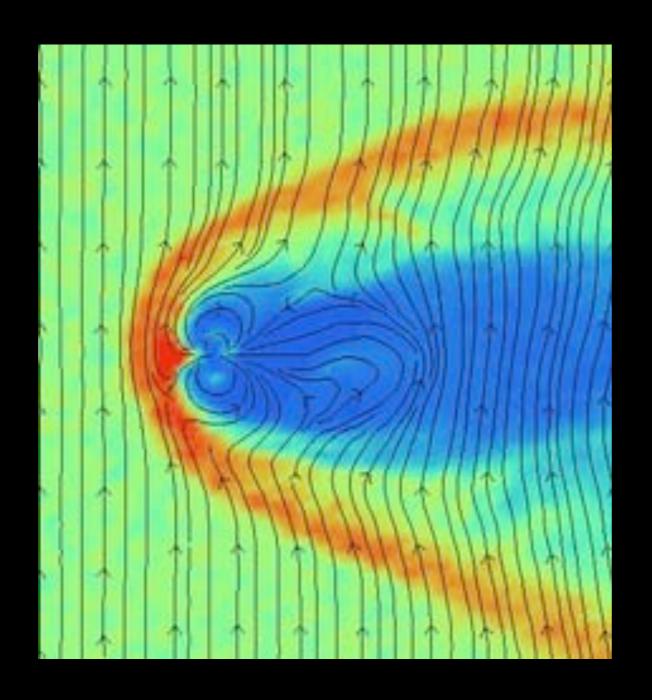


Shock and magnetosheath of pulsar B



Similar to the interaction between Earth magnetosphere and solar wind.

Shock and magnetosheath of pulsar B: effects of rotation



Shock modulated at 2Ω

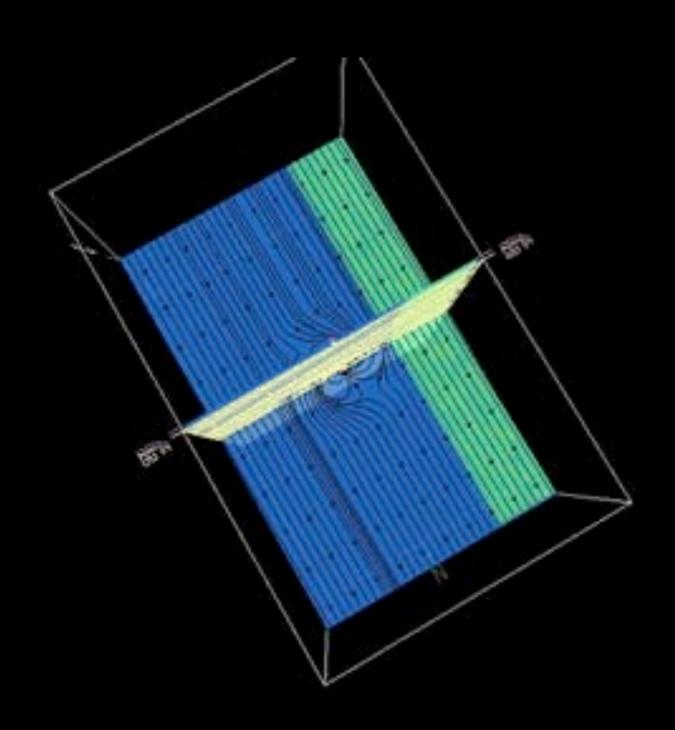
Reconnection once per period

Cusp filling on downwind side

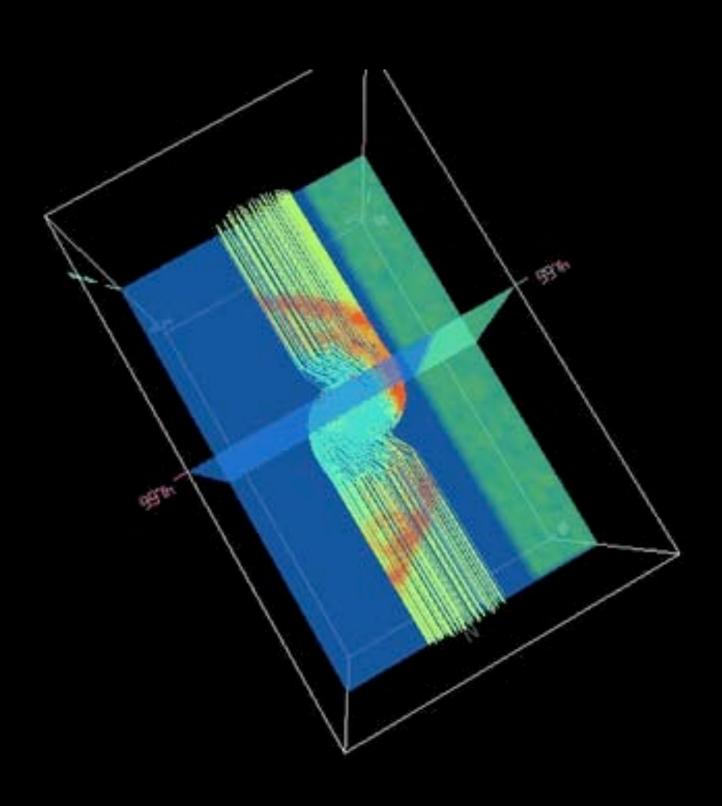
Density asymmetries

R_m~50000 km

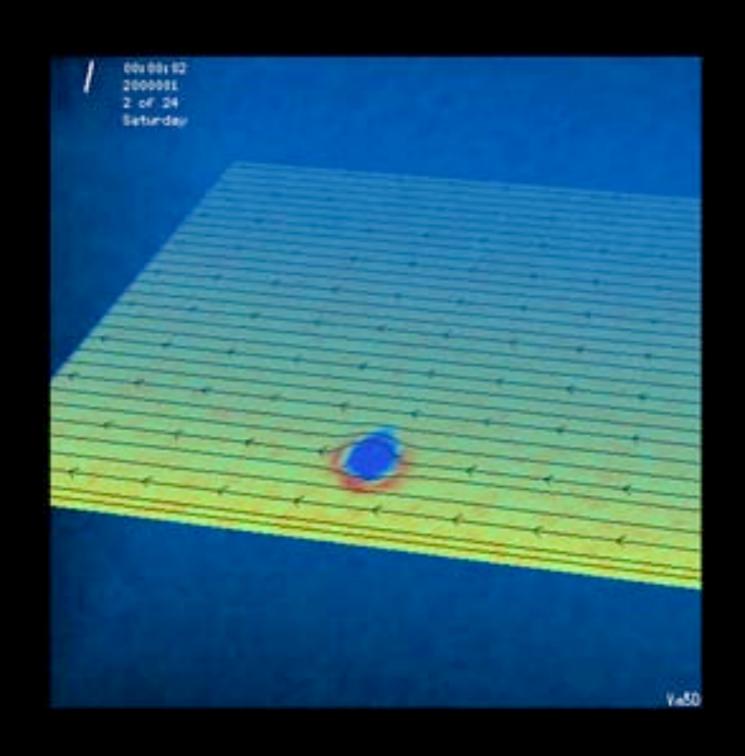
3D magnetosphere

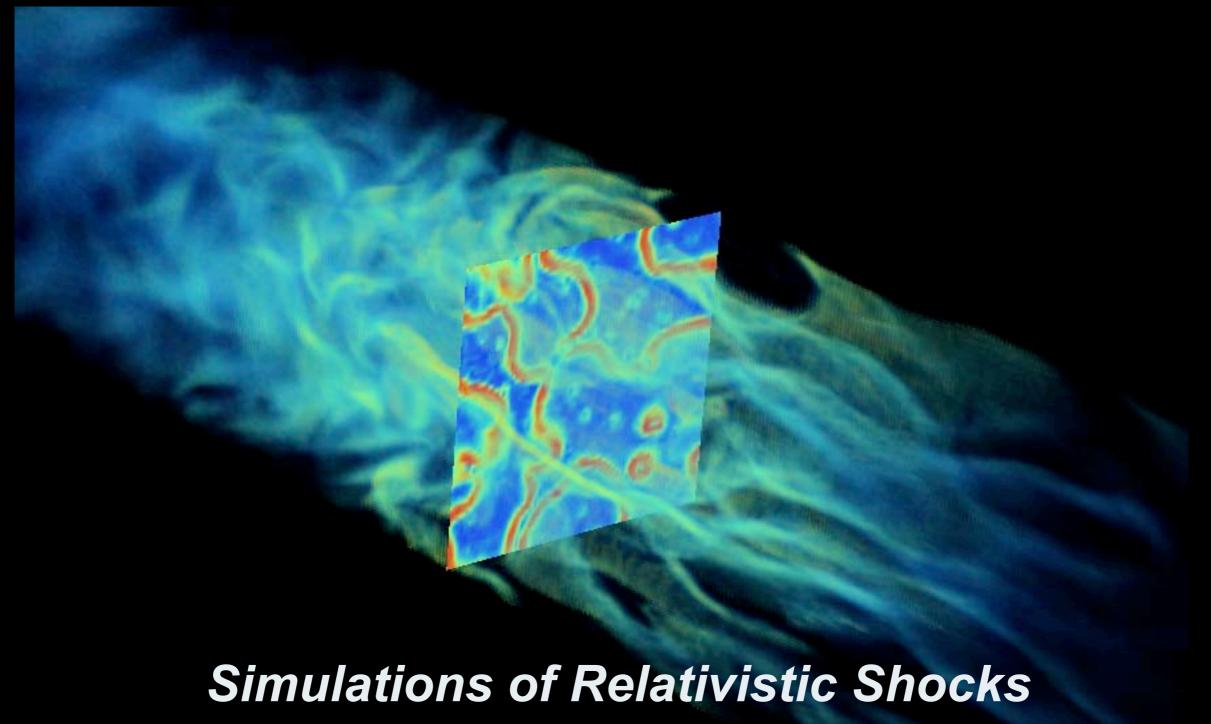


3D magnetosphere



3D magnetosphere





Anatoly Spitkovsky

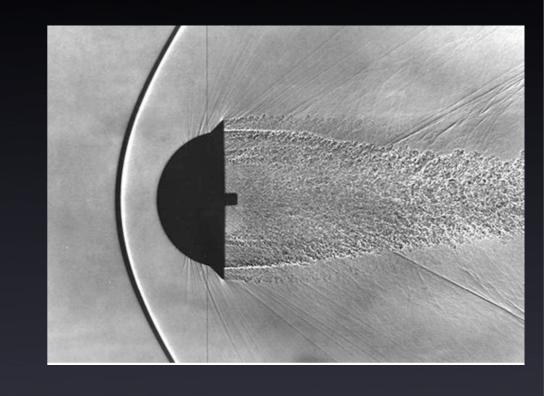
In collaboration with: Jon Arons & Phil Chang (Berkeley),
Uri Keshet (IAS), Boaz Katz (Weizmann),
Lorenzo Sironi & Mario Riquelme (Princeton)

The physics of collisionless shocks

Shock: sudden change in density, temperature, pressure that decelerates supersonic flow

Thickness ~mean free path in air: mean free path ~micron

On Earth, most shocks are mediated by collisions





Astro: Mean free path to Coulomb collisions in enormous: 1000pc in supernova remnants, ~Mpc in galaxy clusters

Mean free path > scales of interest

shocks must be mediated without direct collision, but through interaction with collective fields

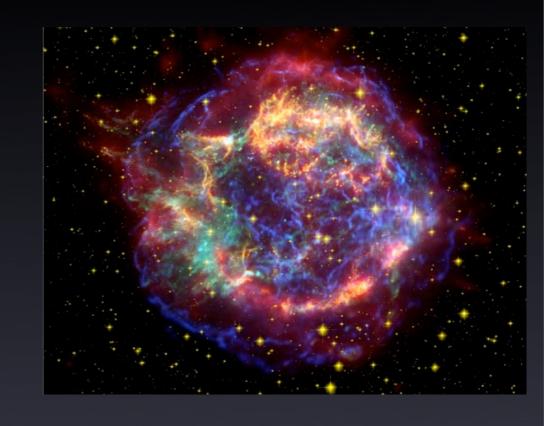
collisionless shocks

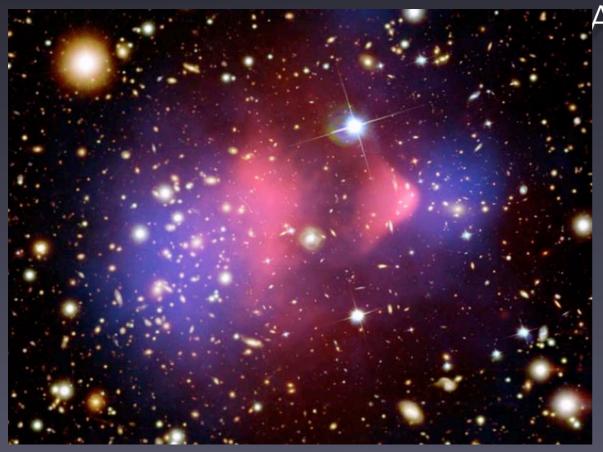
The physics of collisionless shocks

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collisionless shocks

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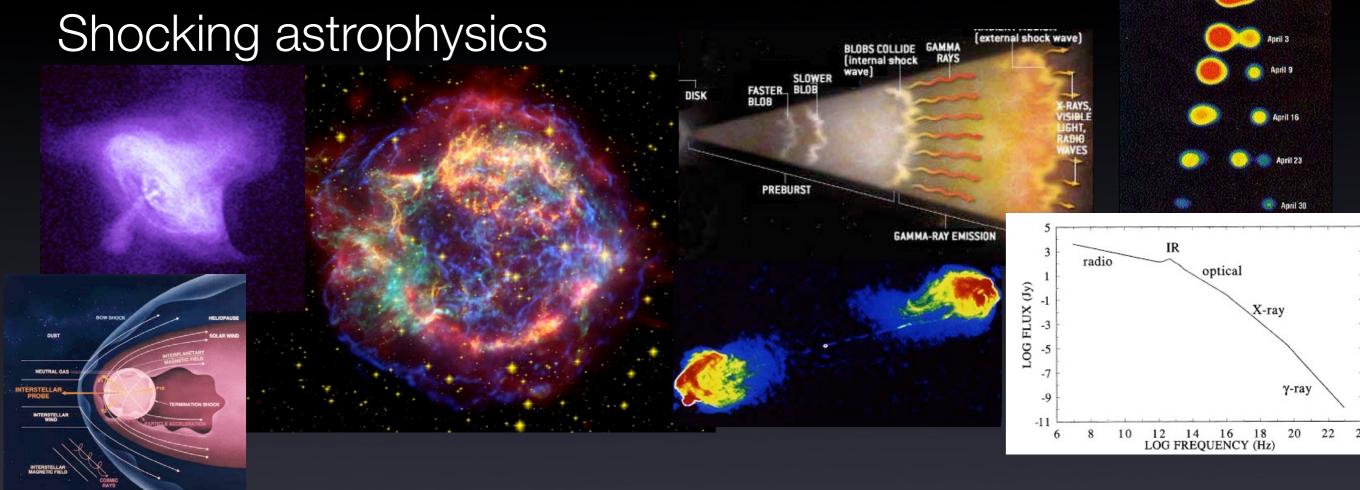


Astro: Mean free path to Coulomb collisions enormous: 1000pc in supernova remnants, ~Mpc in galaxy clusters

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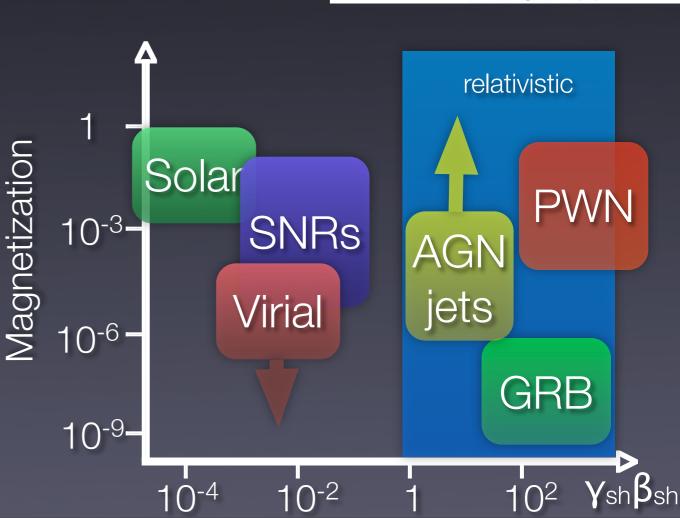
$$l_{Coul} = \frac{m_e^2 v^4}{8\pi n Z^2 e^4 ln\Lambda} \approx 1.4 \times 10^4 (\frac{T}{K})^2 (\frac{n}{cm^{-3}})^{-1} cm$$



Shocks span a range of parameters: nonrelativistic to relativistic flows

magnetization (magnetic/kinetic energy ratio)

composition (pairs/e-ions/pairs + ions)



Supernova Remnants

Crab Nebula

Age 954 yr

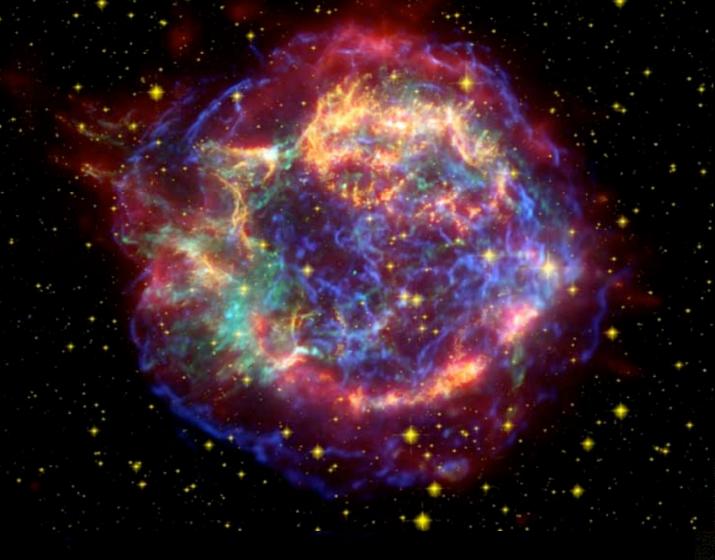
(1054 AD)



SN 1006; age 1002 yrs



Chandra X-ray observatory

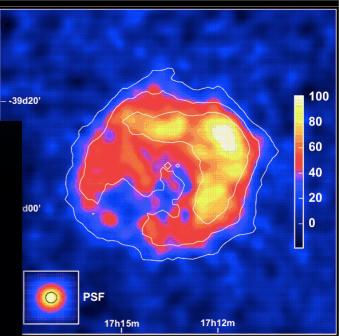


Casiopea A

Age 300 yr (1670 AD)

Age 436 yr (1572 AD)

Tycho

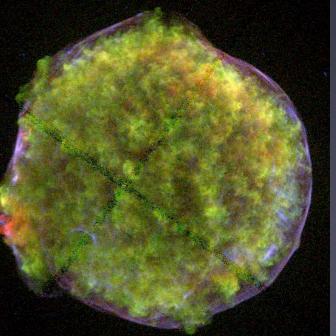


G347.3 TeV γ-rays

Explosions release 10⁵¹ ergs of energy

X-ray luminosity: 3.8x10³⁶ erg/s

Sun: 10³³ erg/s in optical



Supernova Remnants

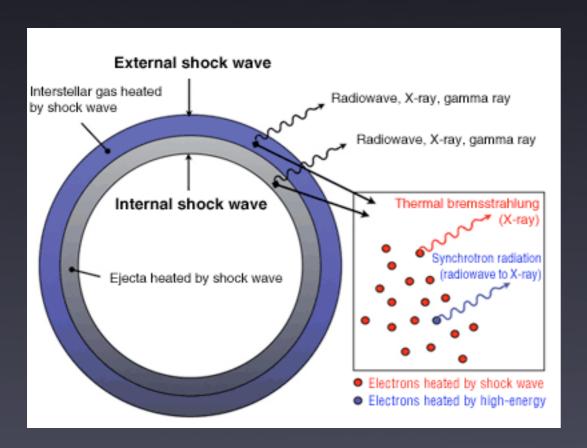
$$E_{SN} \sim 10^{51} ergs \quad E_{SN} \sim \frac{1}{2} M_{ej} v_{ej}^2 \quad v_{ej} \sim 10^4 km/s$$

Stages of evolution of supernova remnants

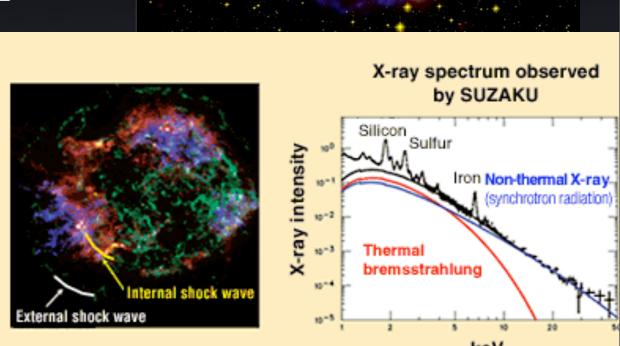
Free expansion ~ 200 yrs
Blast wave -- Sedov-Taylor E=const solution 10^6 K

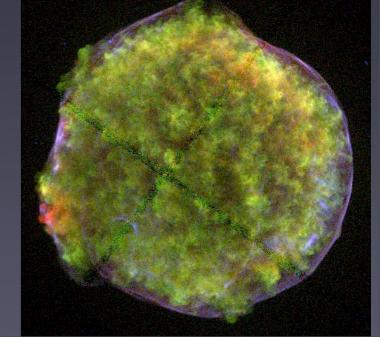
Radiative shock -- momentum conserving

Merge with ISM



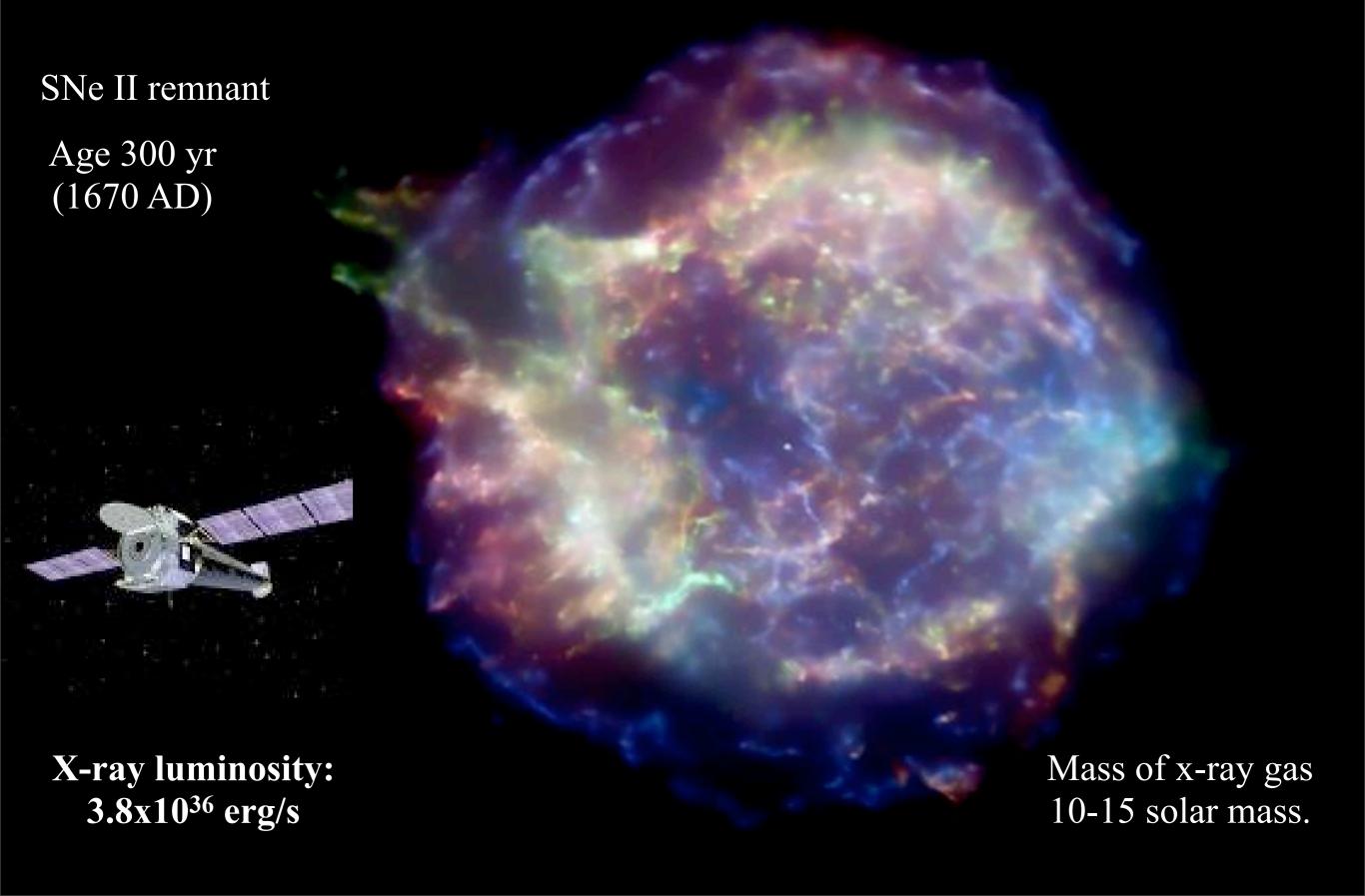
3% of the SNR energy is enough to explain energy density of galactic cosmic rays.



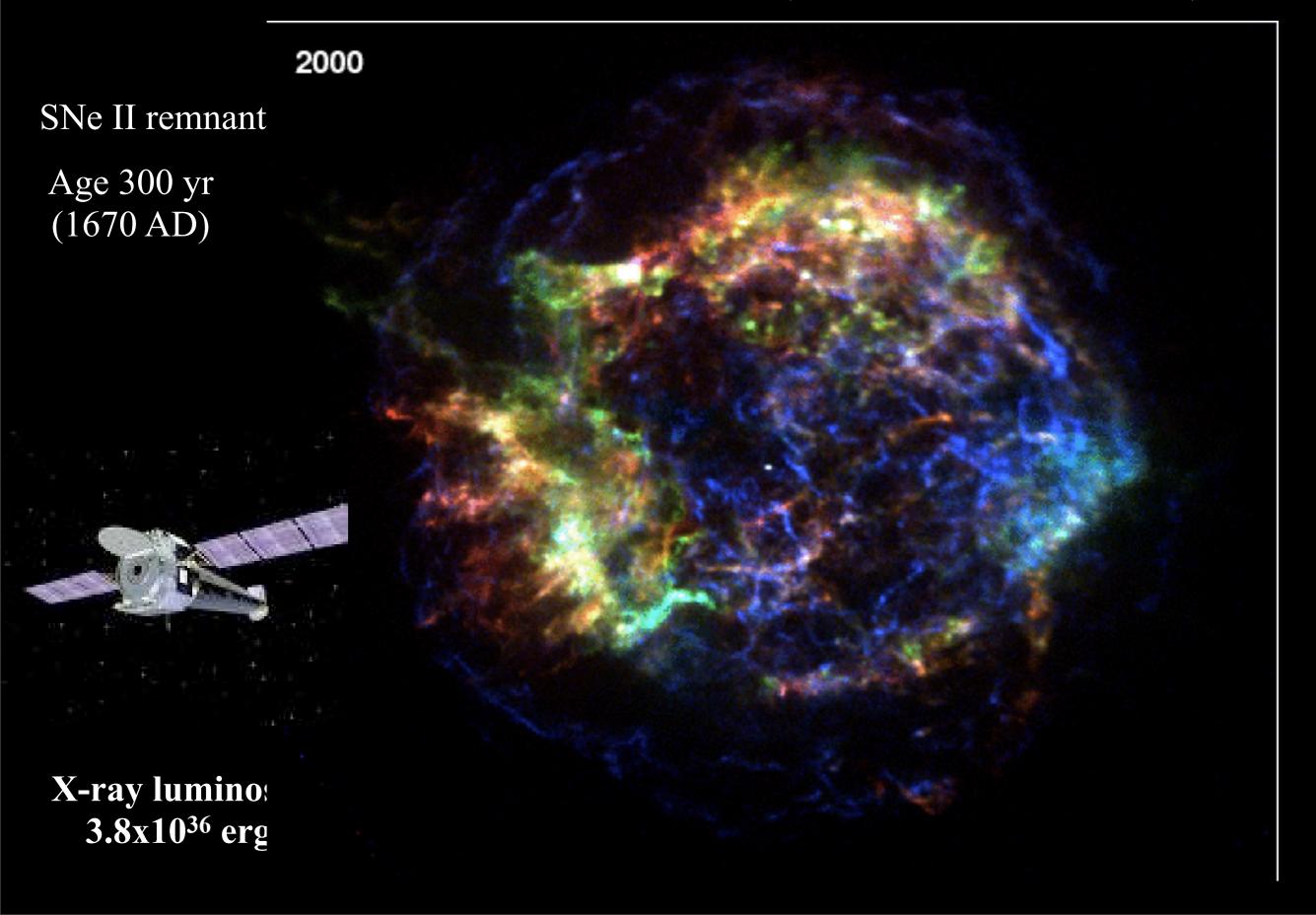


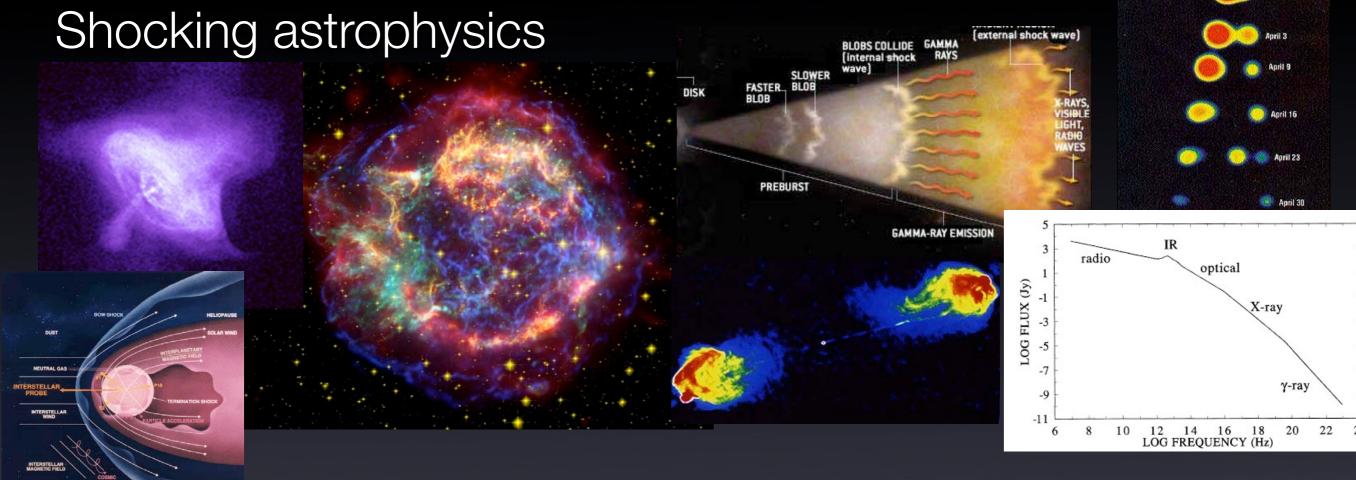
Tycho

SN remnant: Cas A (3-70 kev; Chandra)



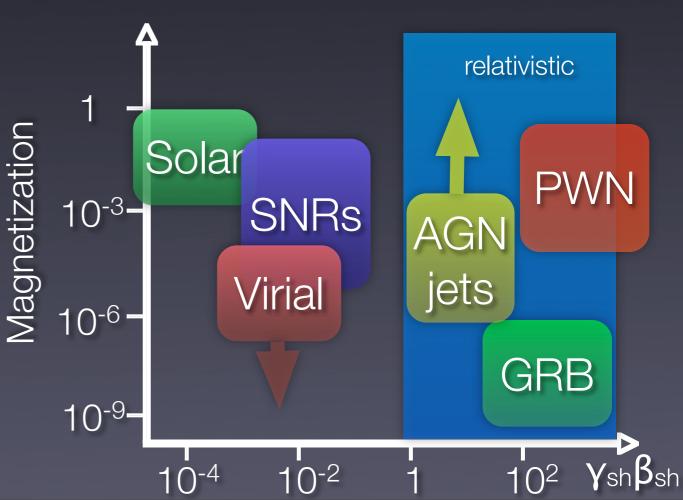
SN remnant: Cas A (3-70 kev; Chandra)



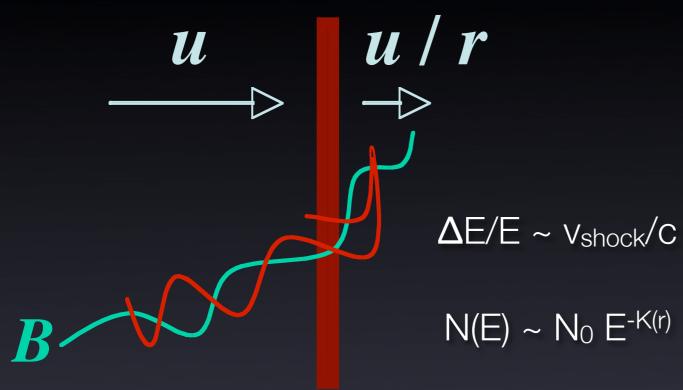


Astrophysical collisonless shocks can:

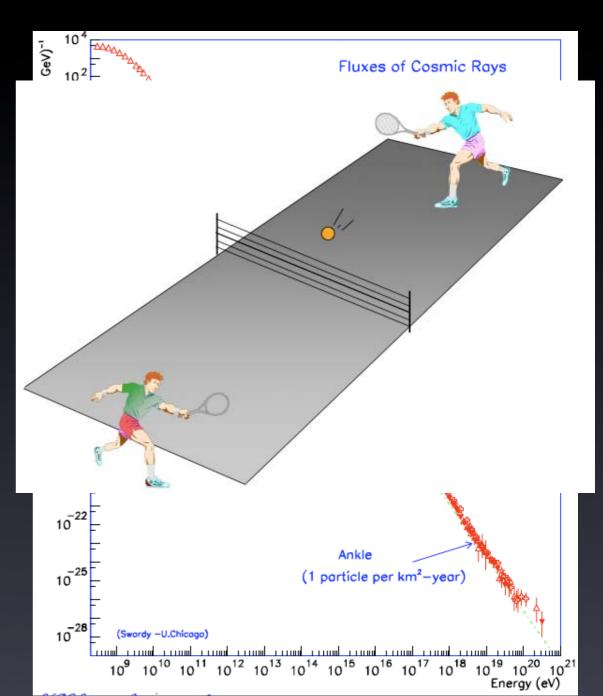
- 1. accelerate particles
- 2. amplify magnetic fields (or generate them from scratch)
- 3. exchange energy between electrons and ions

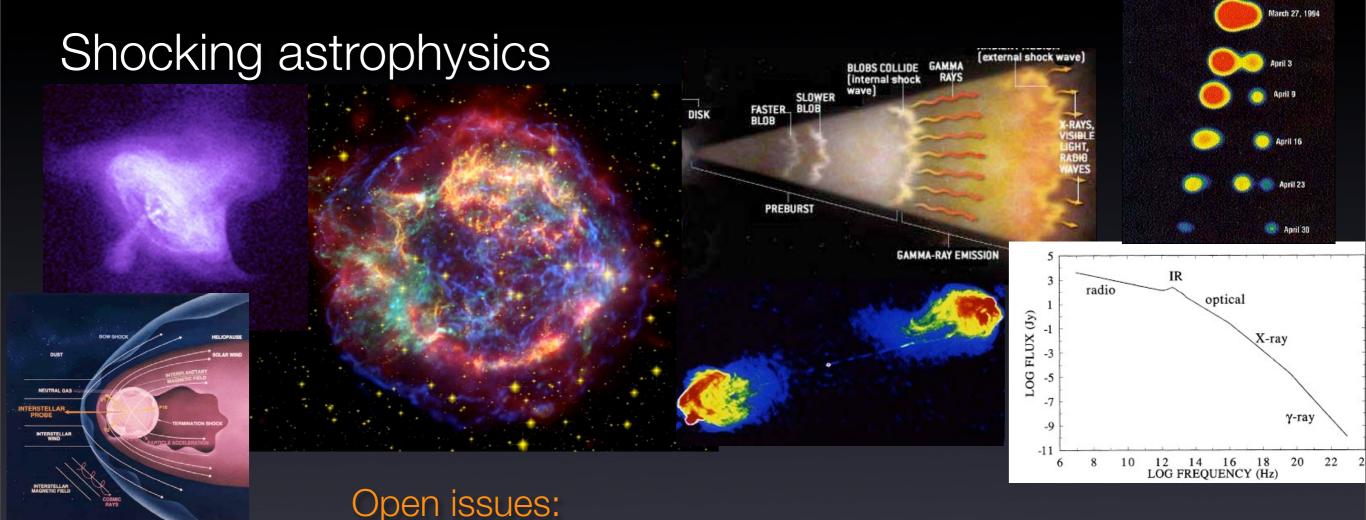


Particle acceleration:



- Original idea -- Fermi (1949) -- scattering off moving clouds. Too slow (second order in v/c) to explain CR spectrum, because clouds both approach and recede.
- In shocks, acceleration is first order in v/c, because flows are always converging (Blandford & Ostriker 78,Bell 78, Krymsky 77)
- Efficient scattering of particles is required.
 Particles diffuse around the shock. Monte
 Carlo simulations show that this implies very high level of turbulence. Is this realistic? Are there specific conditions?





What is the structure of collisionless shocks? Do they exist? How do you collide without collisions?

Particle acceleration -- Fermi mechanism? Other? Efficiency?

Generation of magnetic fields? GRB/SNR shocks, primordial fields?

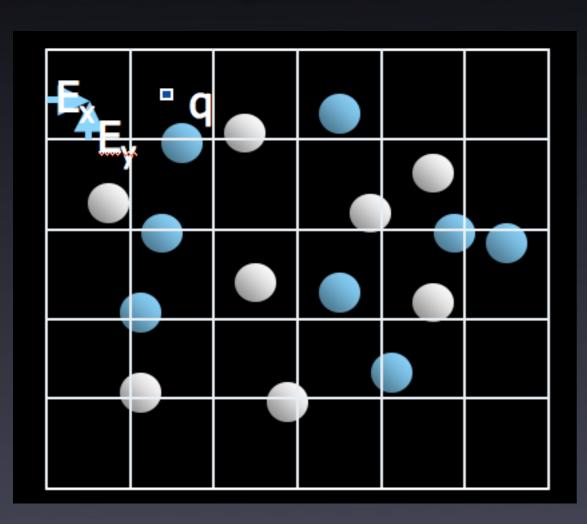
Equilibration between ions and electrons?

Turns out that all questions are related, and particle acceleration is the crucial link

Understanding conditions when particles are accelerated can constrain astrophysical models

Particle-in-Cell (PIC) method

Most fundamental way to treat plasma physics without (m)any approximations price: have to resolve tiny and fast scales (plasma skin depth and plasma freq.) to be interesting, simulations have to be large



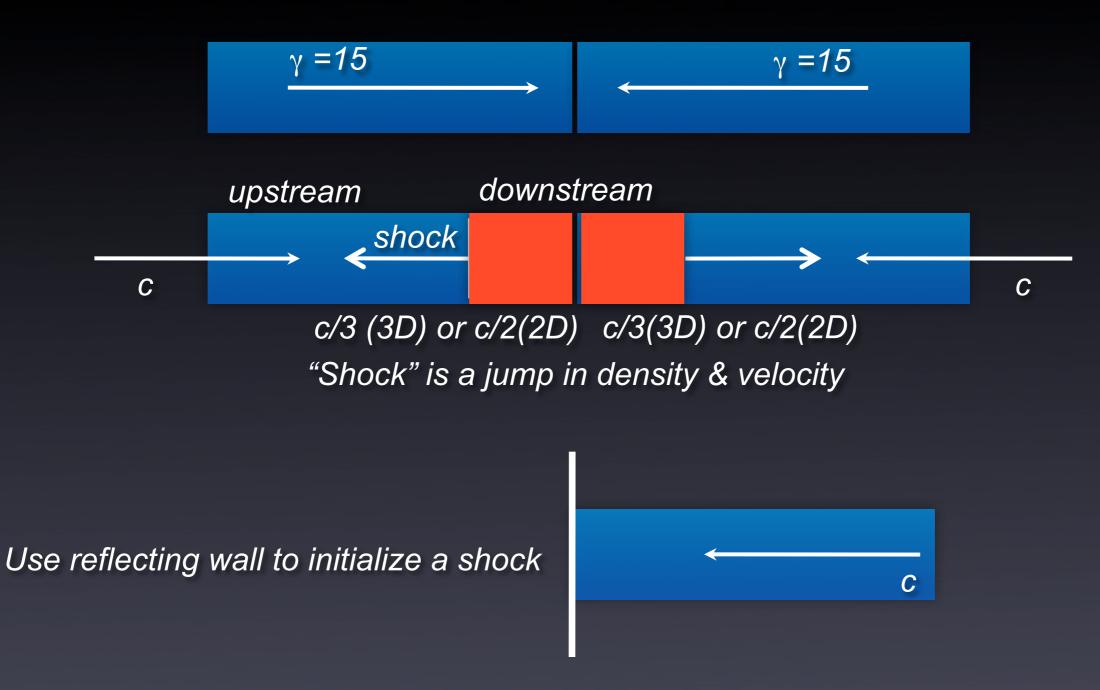
PIC method (aka PM method):

- Collect currents at cell edges
- Solve fields on the mesh (Maxwell's eqs)
- Interpolate fields to particle positions
- Move particles under Lorentz force

Commonly used in accelerator/plasma physics, and now starting to be accepted in astrophysics

The code: relativistic 3D EM PIC code TRISTAN-MP; grids up to $1024^2x10000$ Optimized for large-scale simulations with more than 20e9 particles. 100x100x1000 c/ ω_p Noise reduction, improved treatment of ultra-relativistic flows. Works in both 3D and 2D configurations. Most of the physics is captured in 2D Most of our results are now starting to be confirmed by independent groups

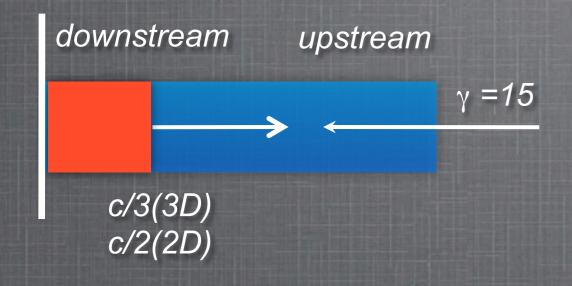
Problem setup



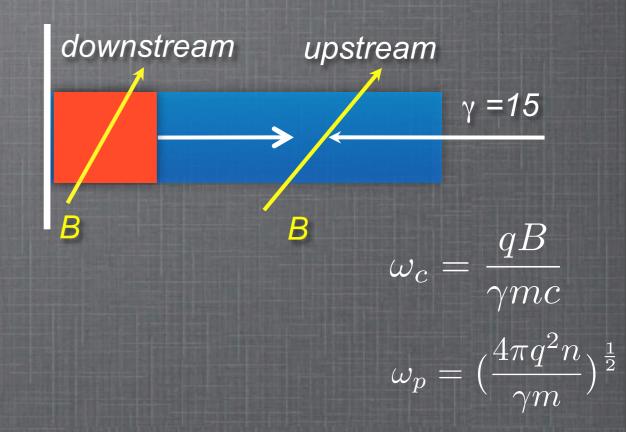
Simulation is in the downstream frame. If we understand how shocks work in this simple frame, we can boost the result to any frame to construct astrophysically interesting models. (in these simulations we do not model the formation of contact discontinuity)

We verified that the wall plays no adverse effect by comparing with a two-shell collision.

Setup



"Shock" is a jump in density & velocity



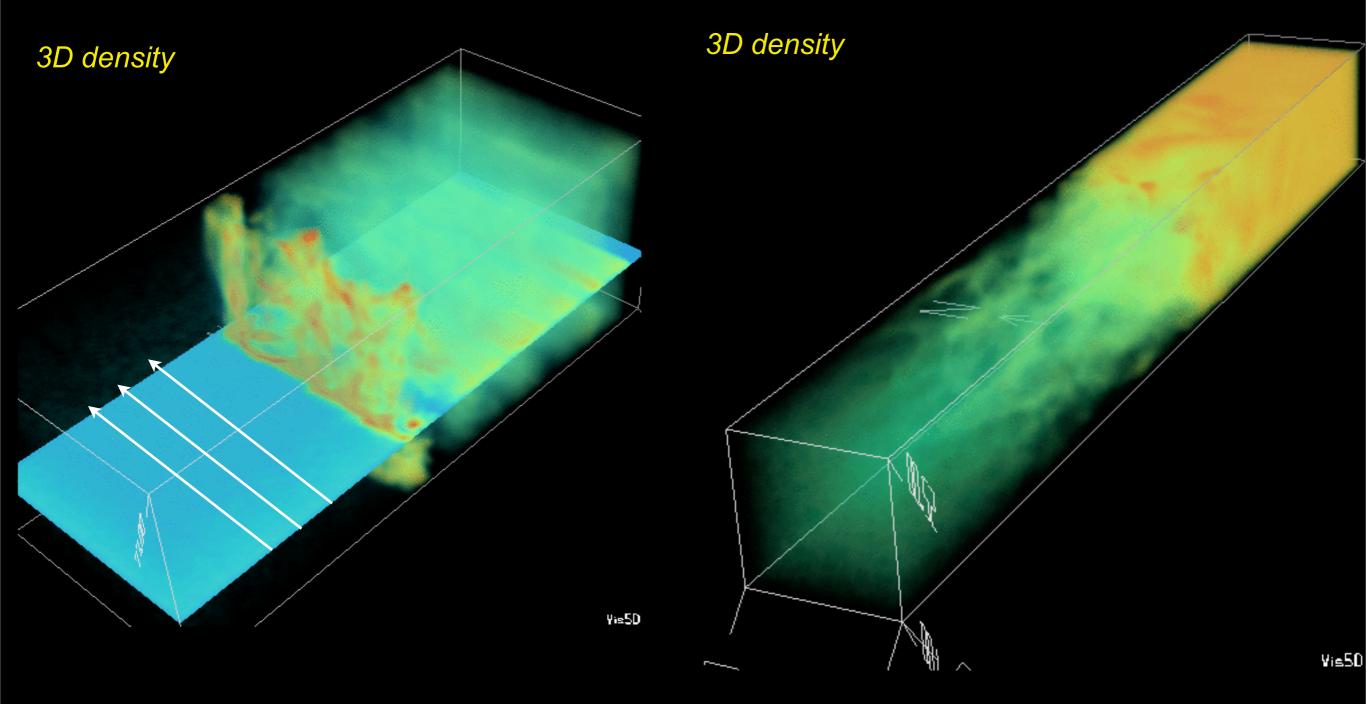
$$\sigma \equiv \frac{B^2/4\pi}{(\gamma - 1)nmc^2} = \frac{1}{M_A^2} = \left(\frac{\omega_c}{\omega_p}\right)^2 \left(\frac{c}{v}\right)^2 = \left[\frac{c/\omega_p}{R_L}\right]^2$$

Simulation is done in the "downstream" frame, where a shock is moving on the grid Vary: B field and orientation, speed of the flow, composition

Relativistic pair shocks

Shock structure for σ =0.1

Shock structure for σ =0

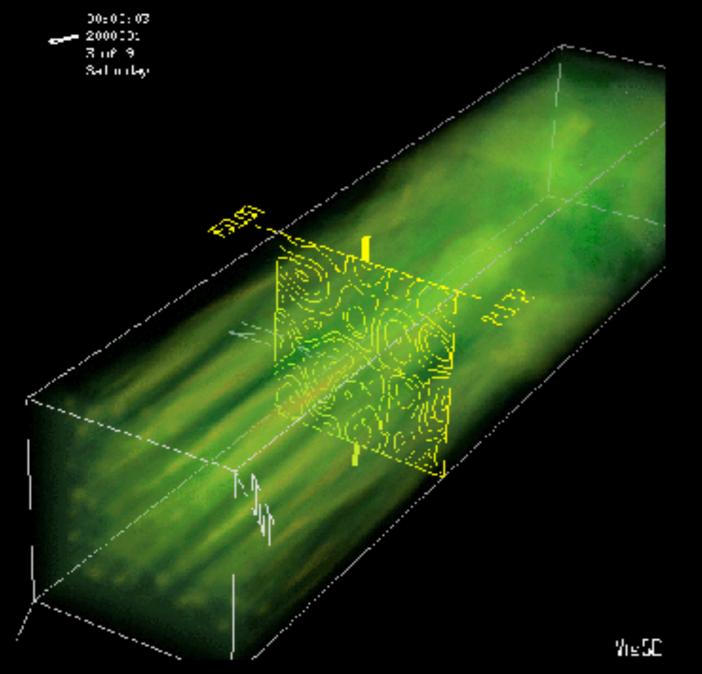


Magnetized shock is mediated by magnetic reflection, while the unmagnetized shock -- by field generation from filamentation instability. Transition is near σ =1e-3 (A.S. 2005)

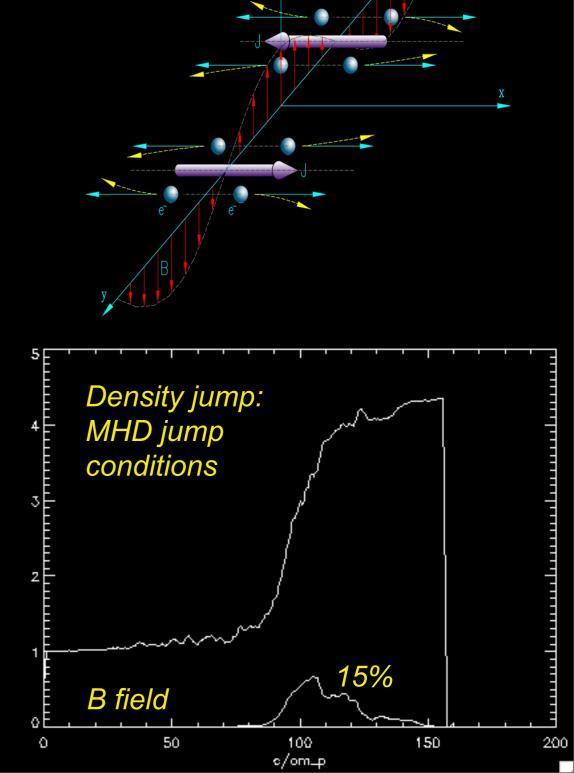
Unmagnetized pair shock

Magnetic field generation: Weibel instability

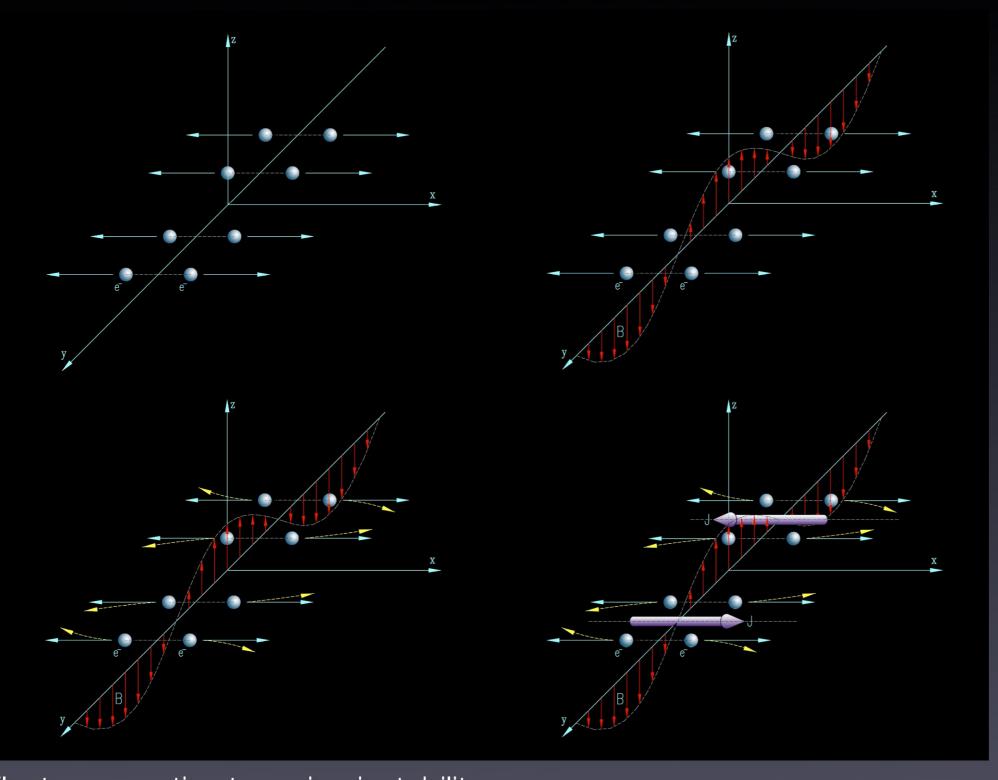
Field cascades from c/ω_p scale to larger scale due to current filament merging



Weibel instability generates subequipartition B fields that decay. Is asymptotic value nonzero? Competition between decay and inverse cascade (Chang, AS, Arons 08).



Weibel instability



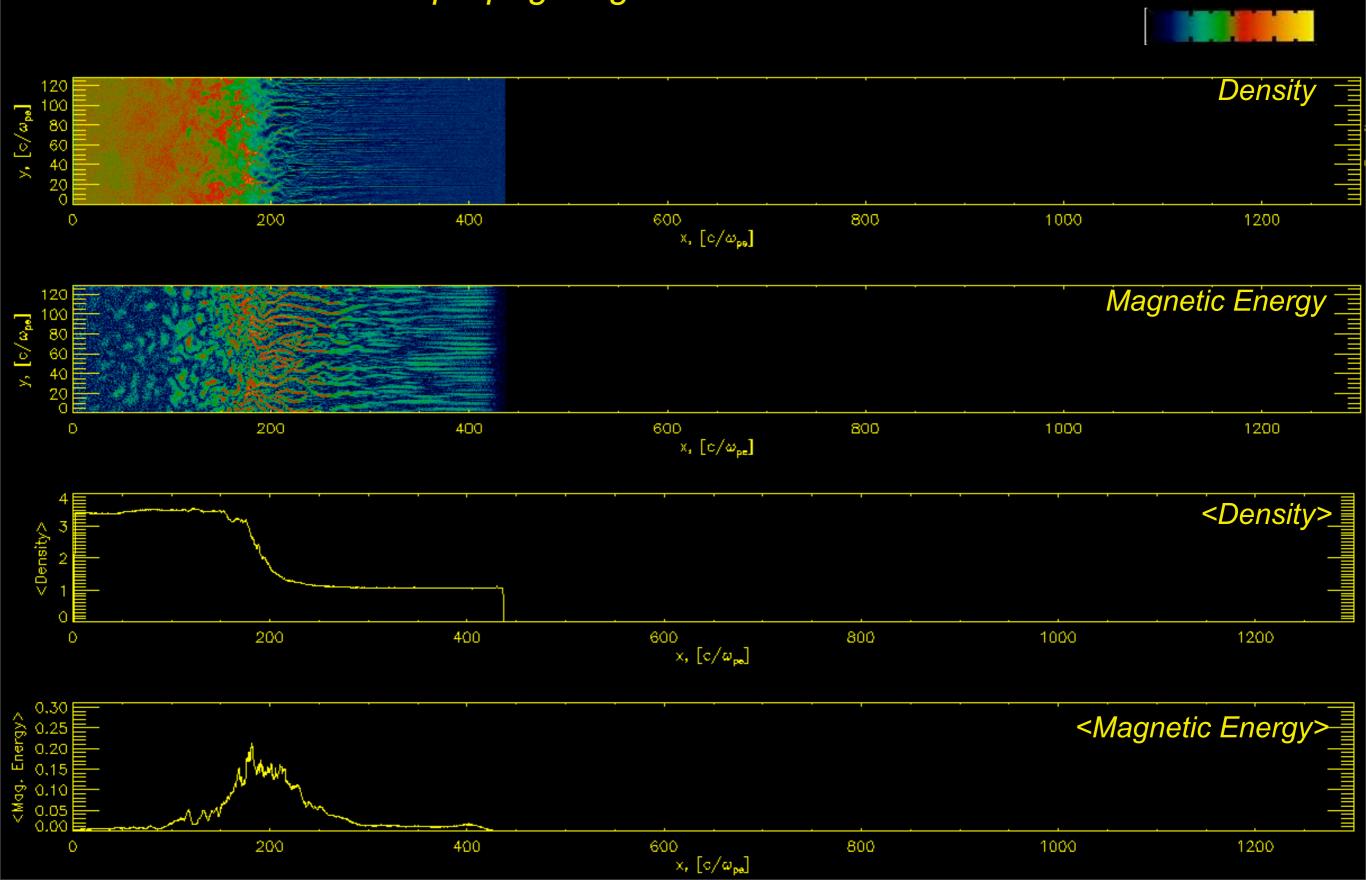
Weibel (1959) Moiseev & Sagdeev (1963) Medvedev & Loeb (1999)

Electromagnetic streaming instability.
Works by filamentation of plasma
Spatial growth scale -- skin depth,
time scale -- plasma frequency

$$L \approx c/\omega_{pe} = 10 \text{ km } \sqrt{\gamma/n_0[\text{cm}^{-3}]}$$
$$T \approx 1/\omega_p = 30 \text{ } \mu\text{s } \sqrt{\gamma/n_0[\text{cm}^{-3}]}$$

Relativistic pair shocks: no initial B field

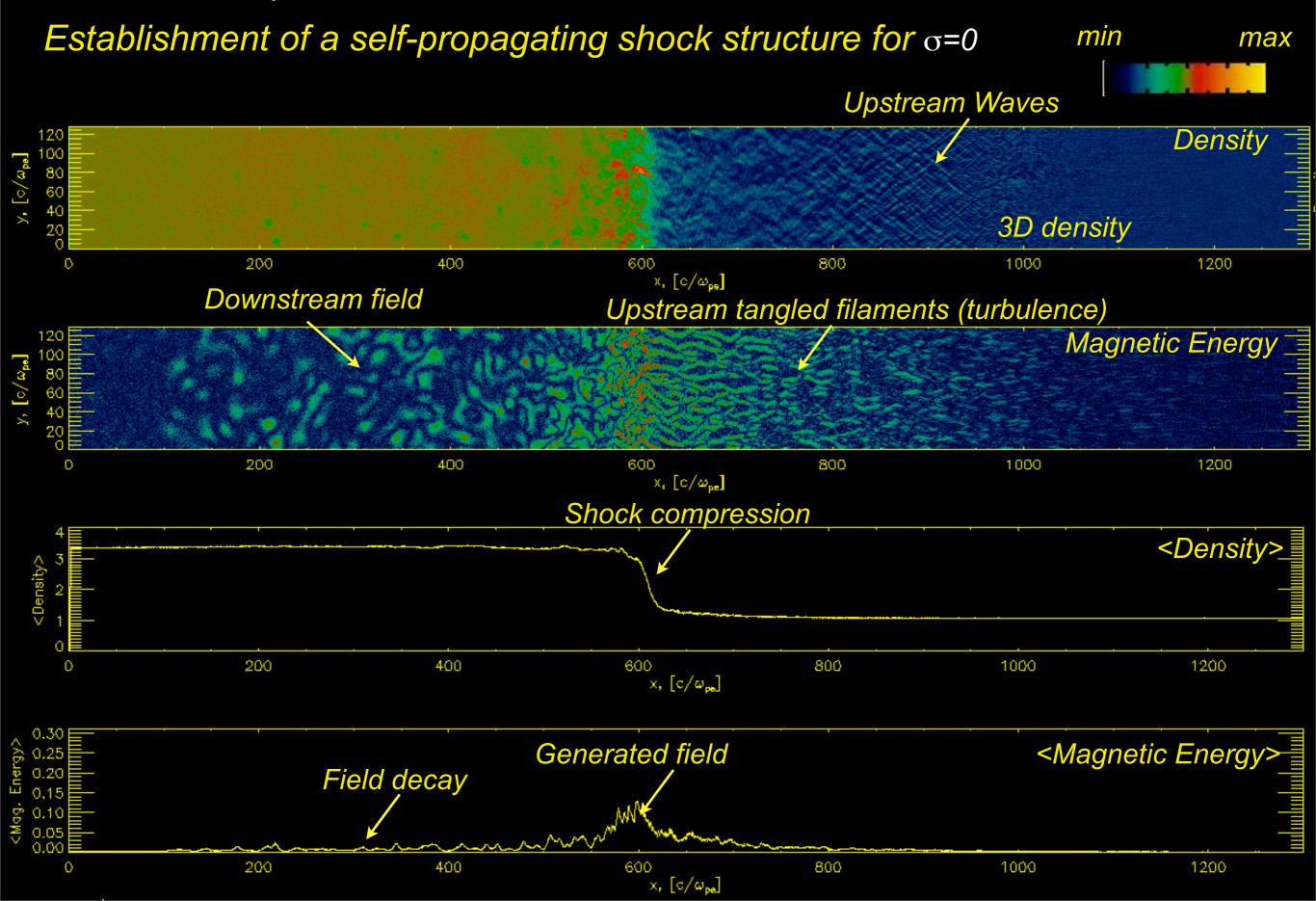
Establishment of a self-propagating shock structure for σ =0



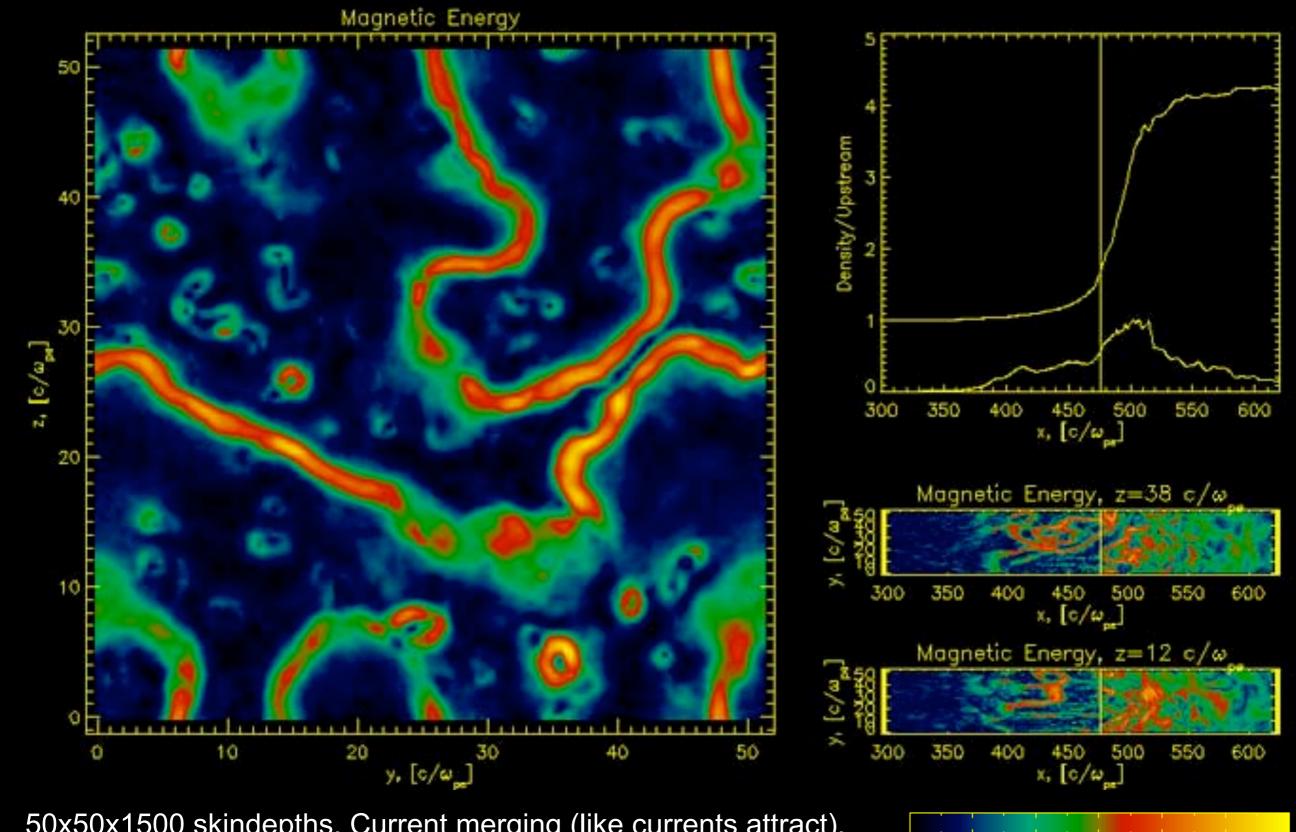
min

max

Relativistic pair shocks: no initial B field



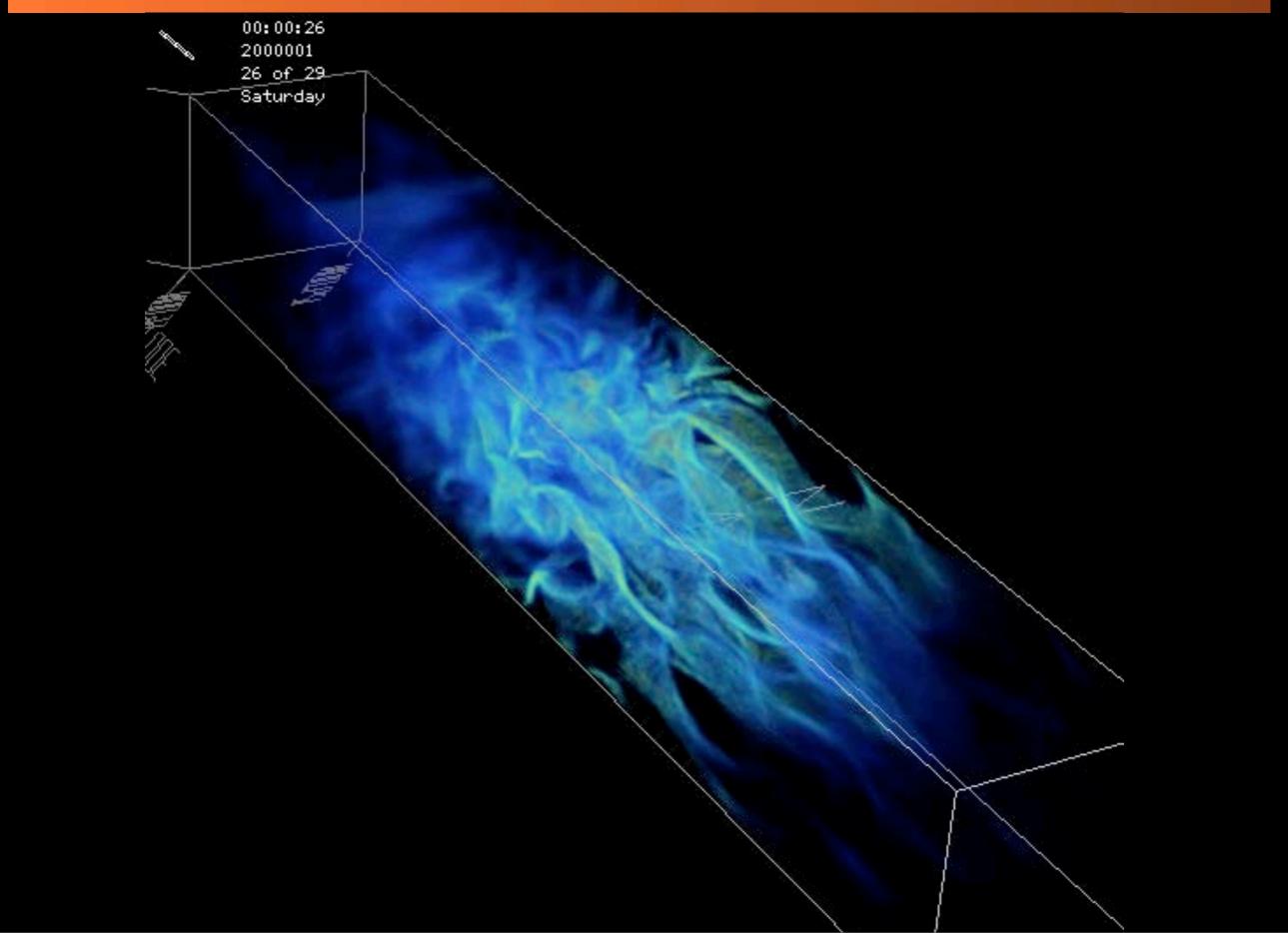
3D shock structure: long term



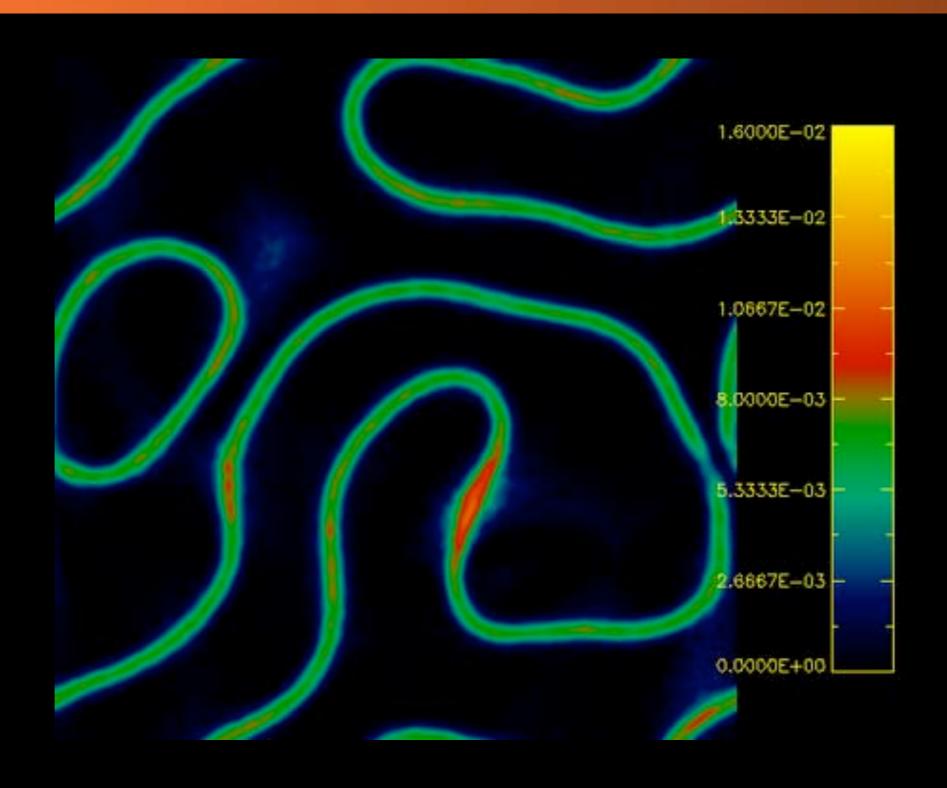
50x50x1500 skindepths. Current merging (like currents attract).

Secondary Weibel instability stops the bulk of the plasma. Pinching leads to randomization.

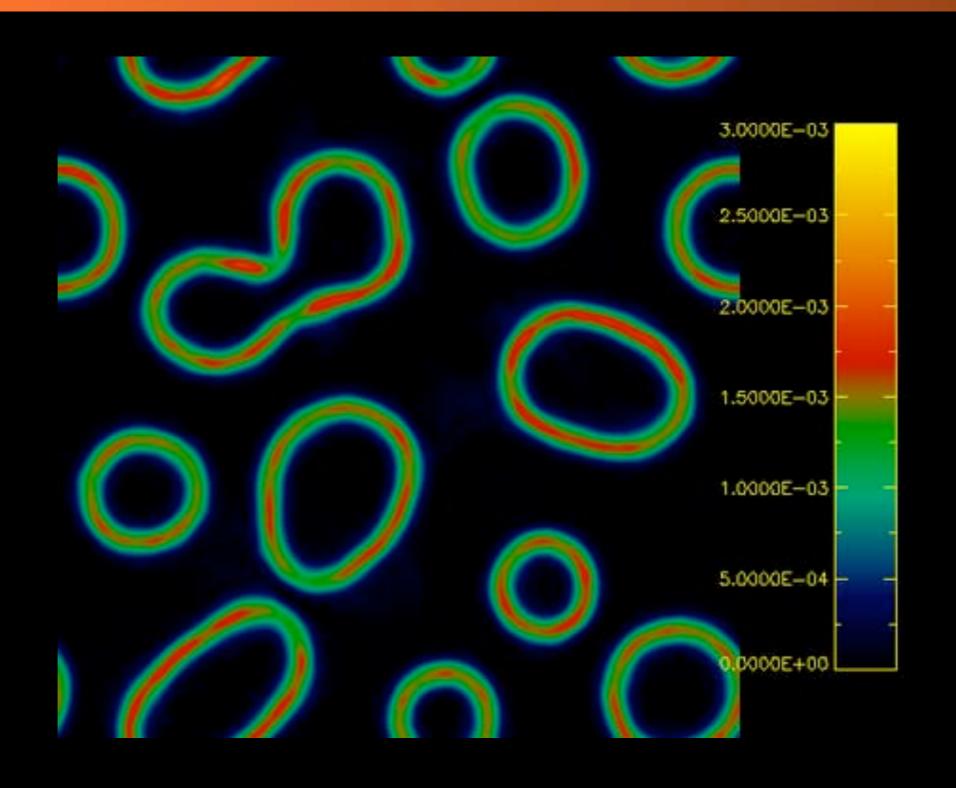
3D unmagnetized pair shock: magnetic energy



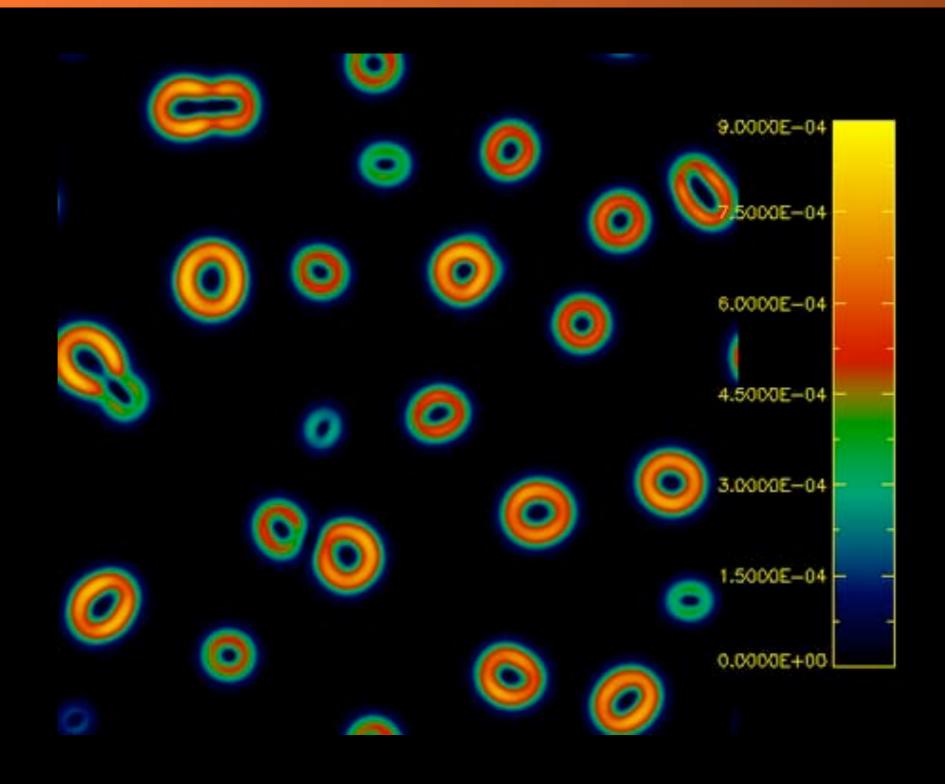
Counterstreaming instabilities



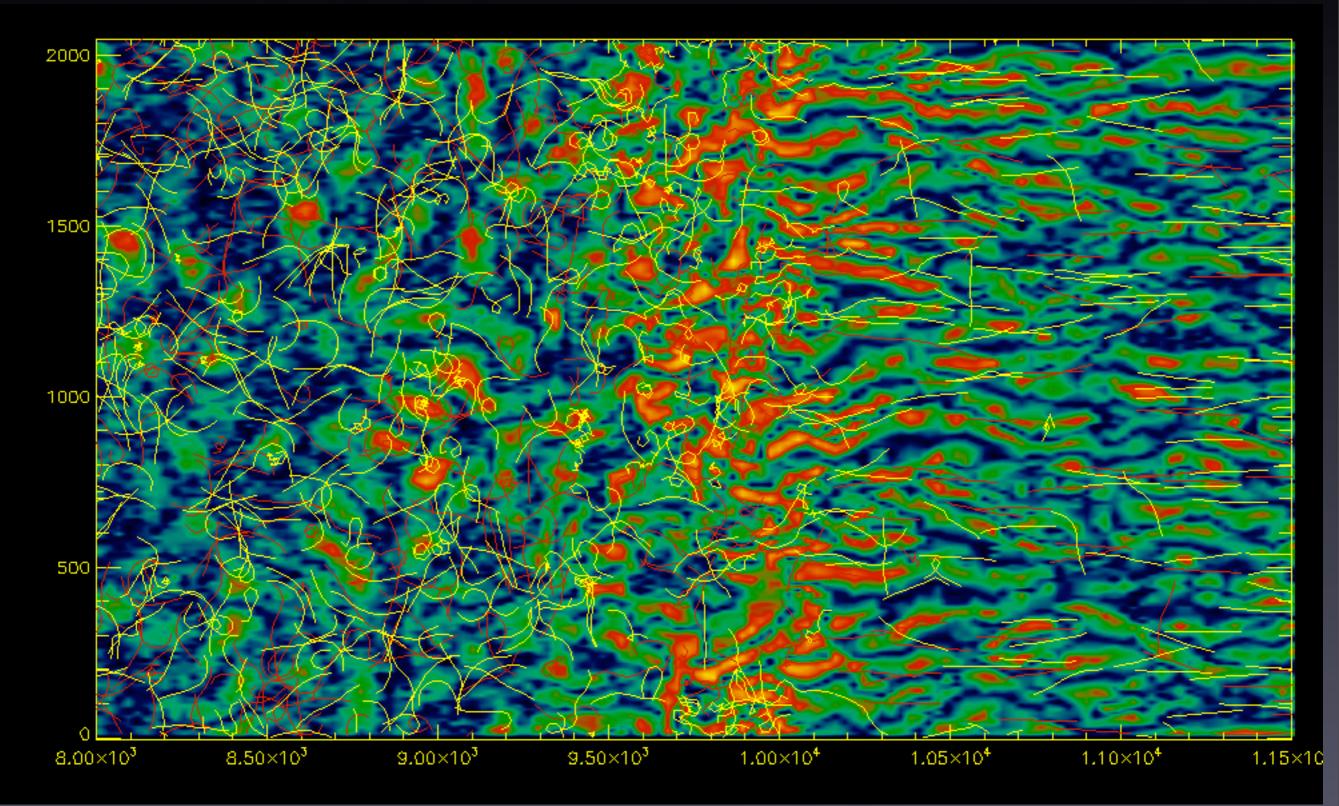
Counterstreaming instabilities



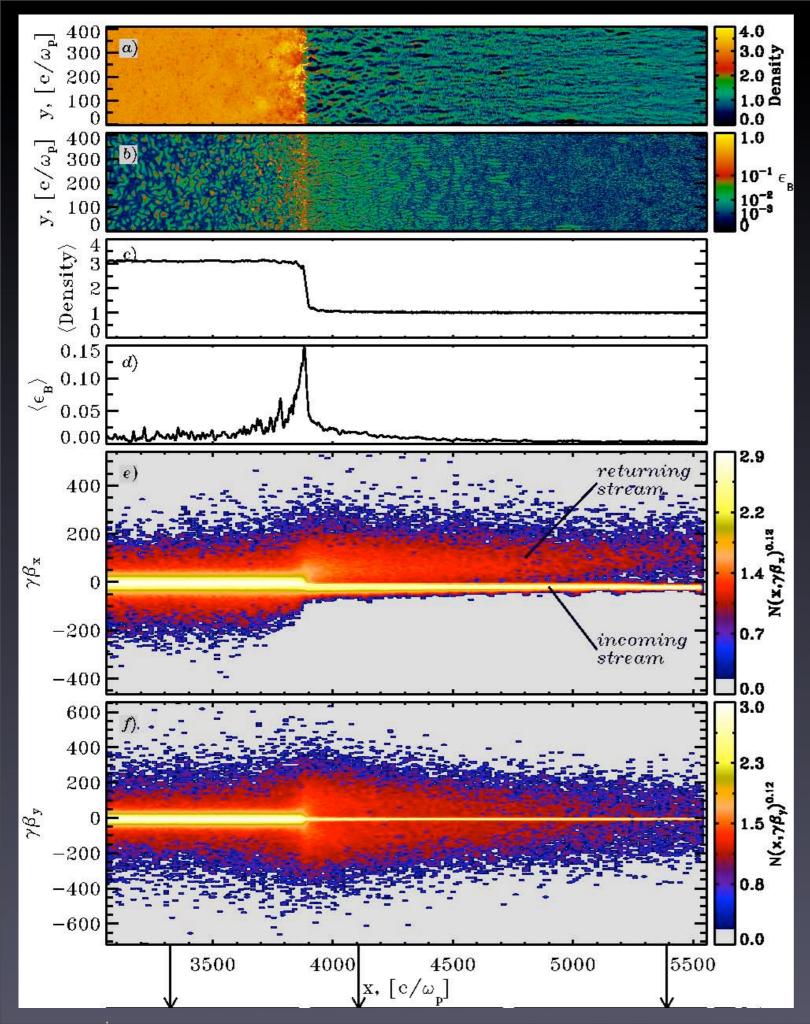
Counterstreaming instabilities



Unmagnetized pair shock: particle trajectories



color: magnetic energy density



Unmagnetized shock: shock is driven by returning particle precursor (CR!)

Steady counterstreaming leads to self-replicating shock structure

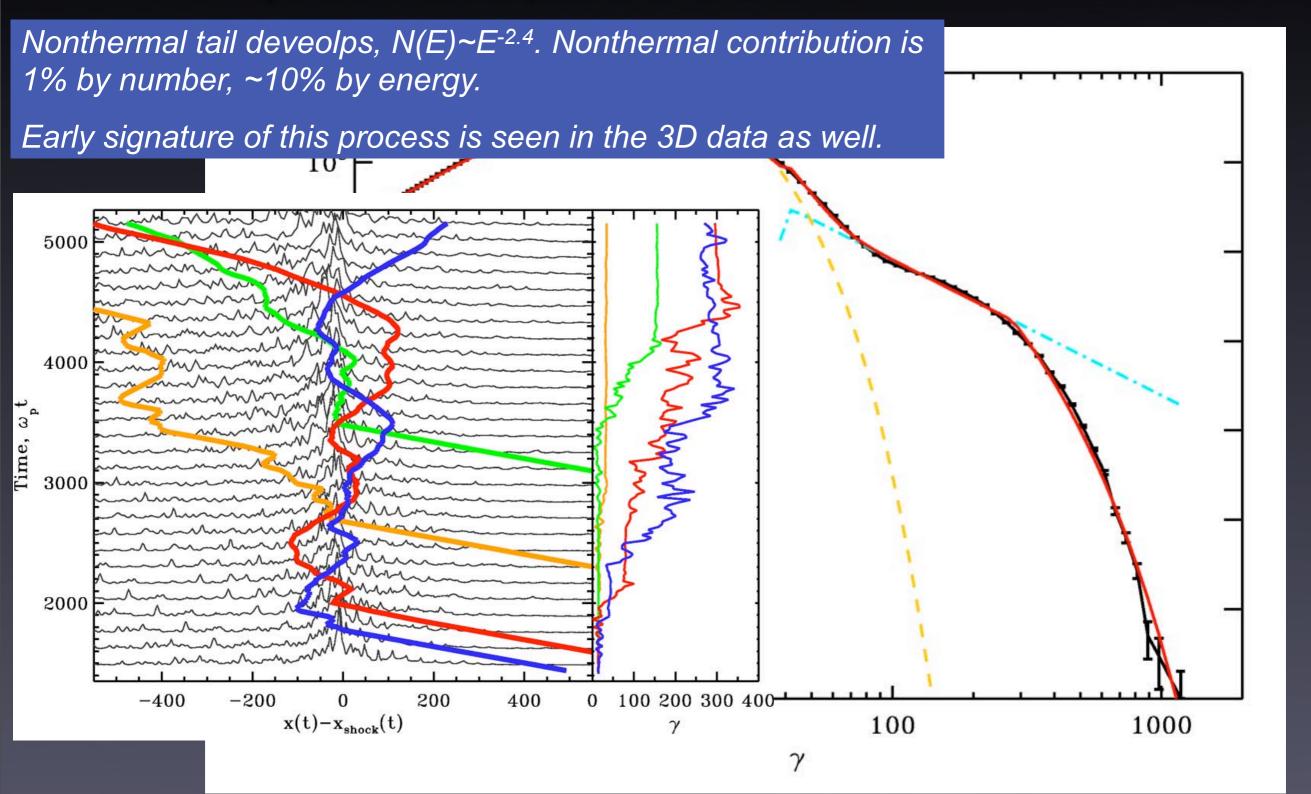
x- px momentum space

x- py momentum space

Shock structure for $\sigma=0$ (AS '08)

Unmagnetized pair shock:

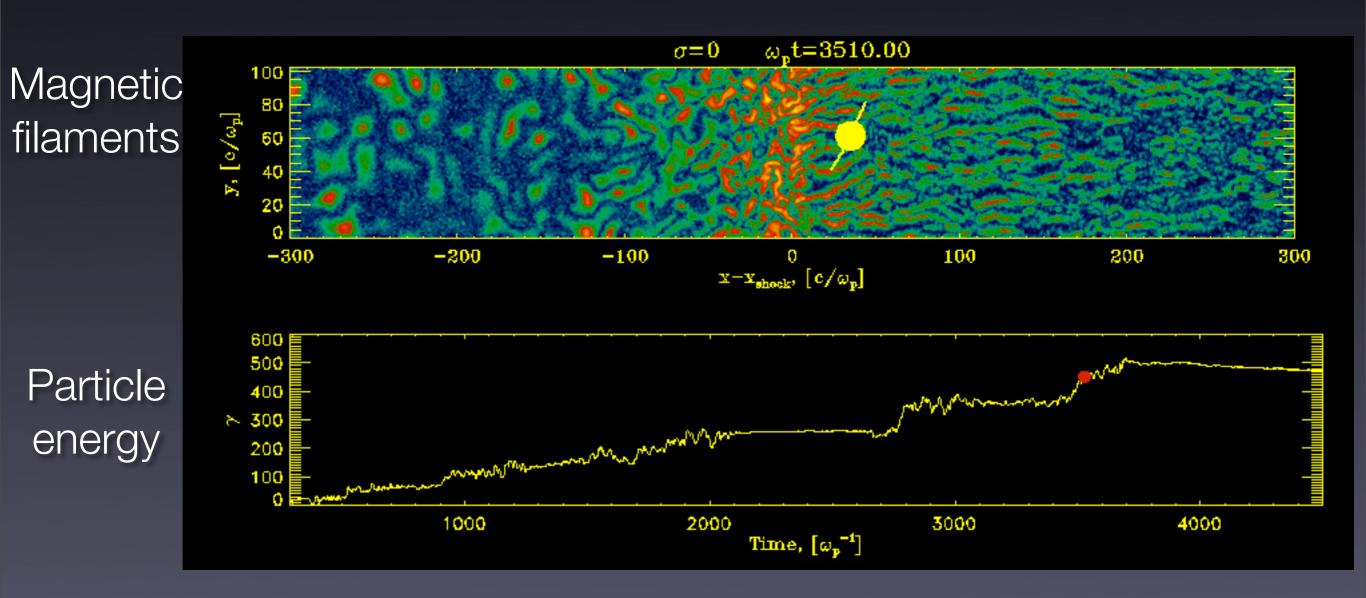
downstream spectrum: development of nonthermal tail!



Unmagnetized pair shock: particle trajectories

Nonthermal tail develops, N(E)~E-2.4. Nonthermal contribution is 1% by number, ~10% by energy. Well fit by low energy Maxwellian + power law with cutoff.

Same process is seen in the 3D data as well. Easy to have $\Delta B/B >> 1$ when B=0! Injection works self-consistently from the thermal distribution.

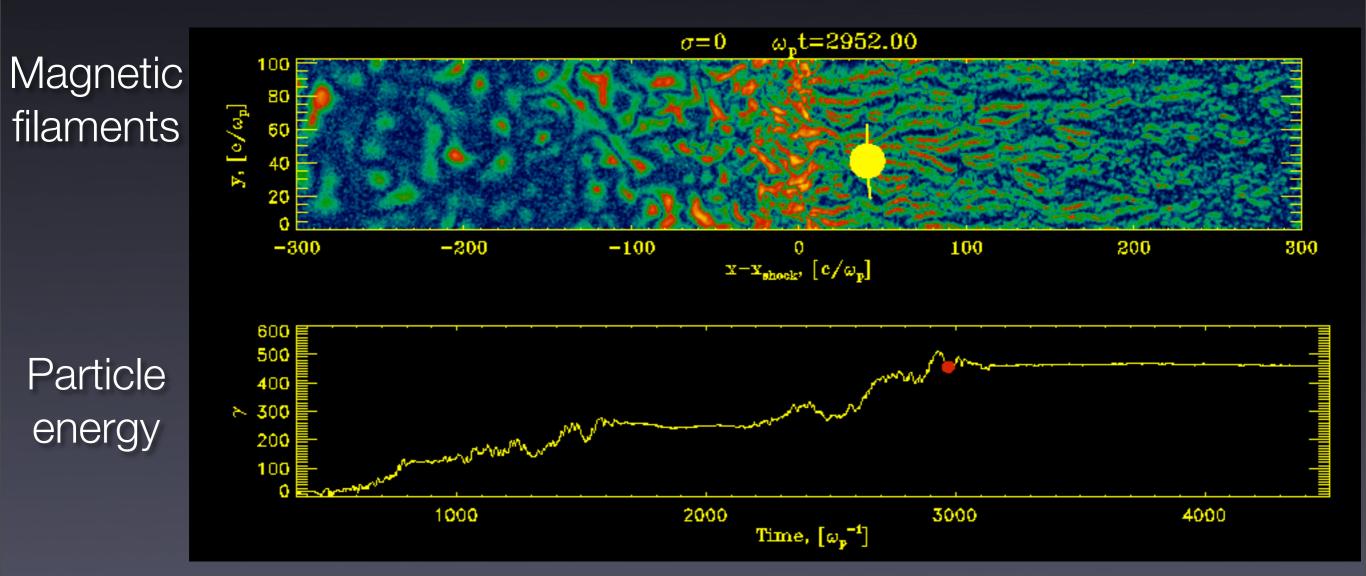


Particles that are accelerated the most, graze the shock surface

Unmagnetized pair shock: particle trajectories

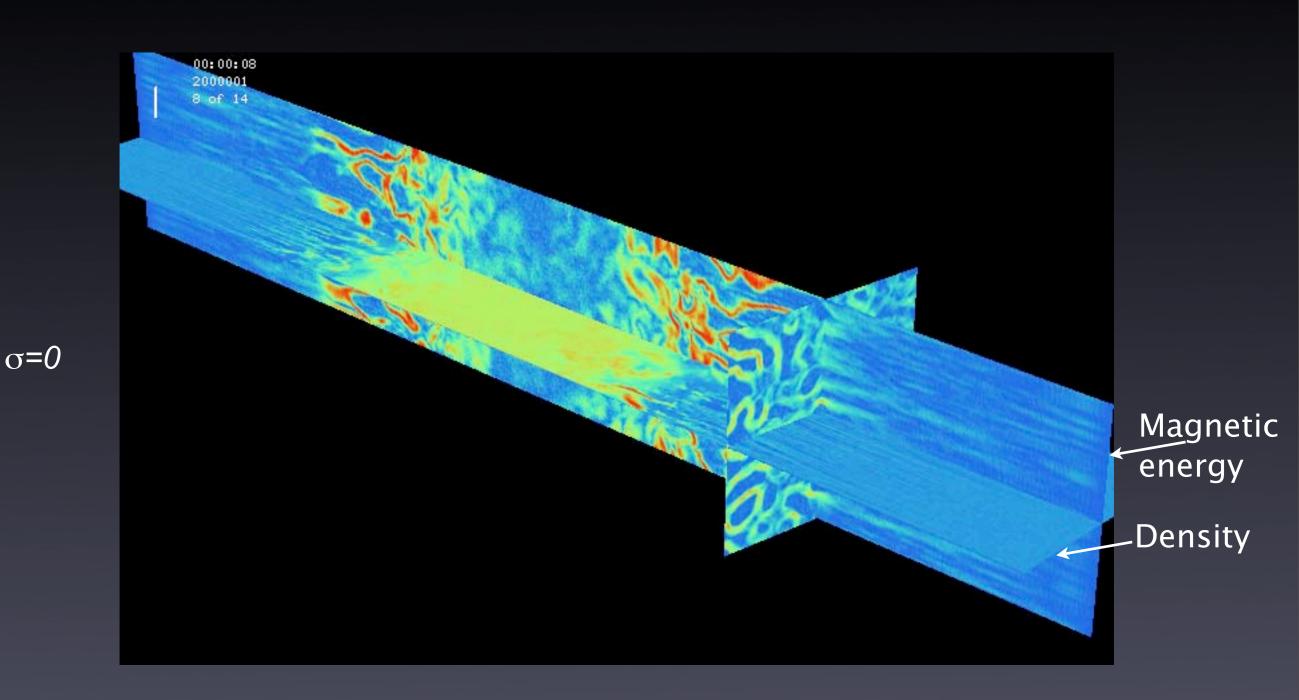
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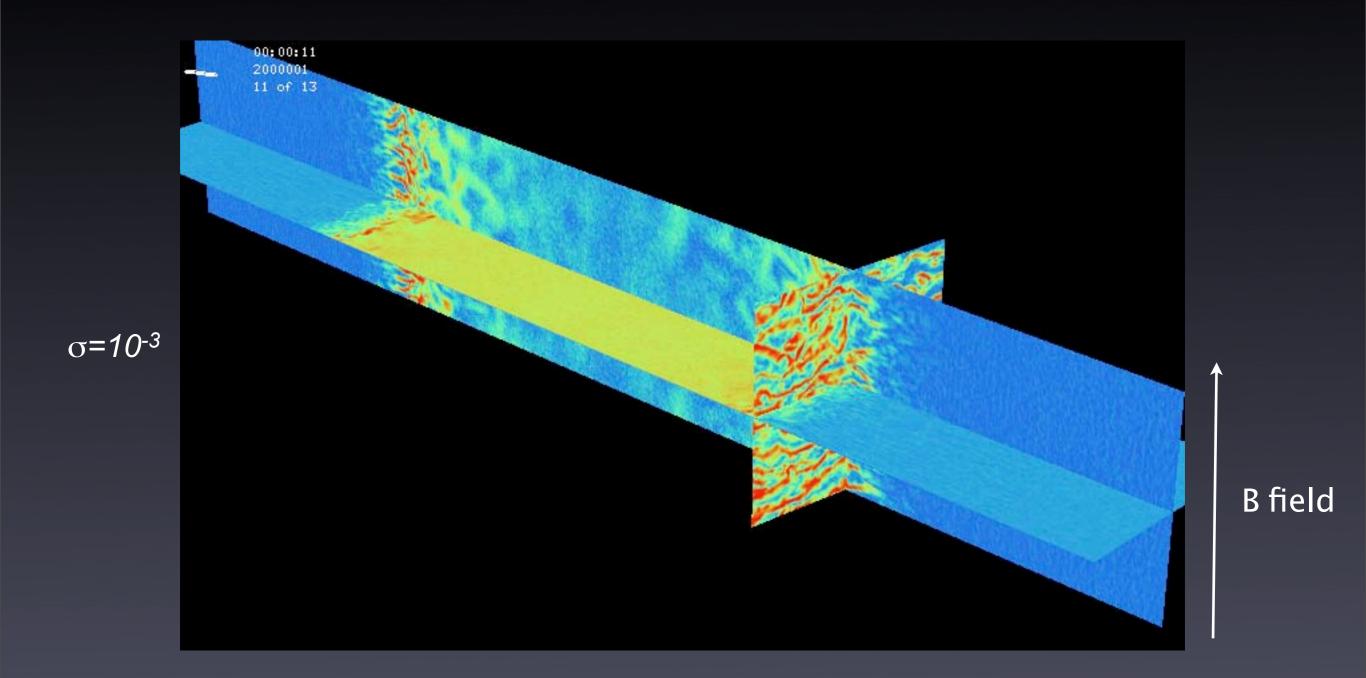


Particles that are accelerated the most, graze the shock surface

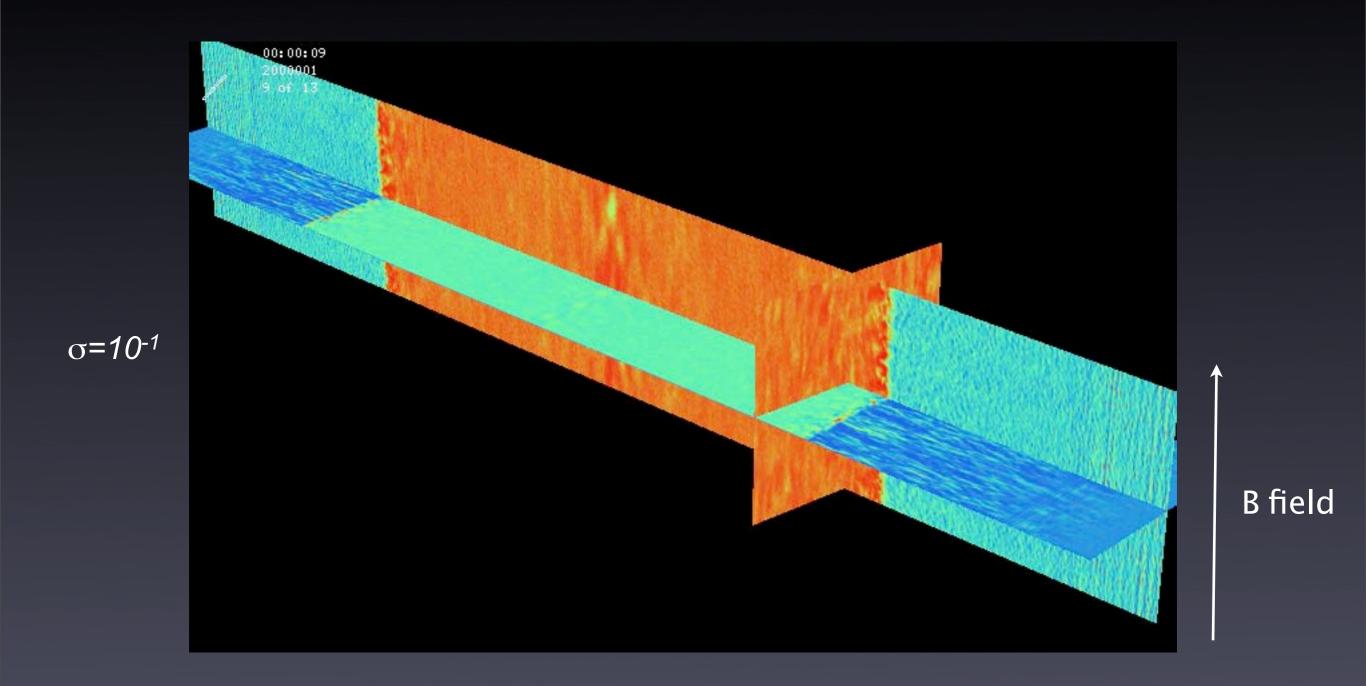
Transition between magnetized and unmagnetized shocks:



Transition between magnetized and unmagnetized shocks:



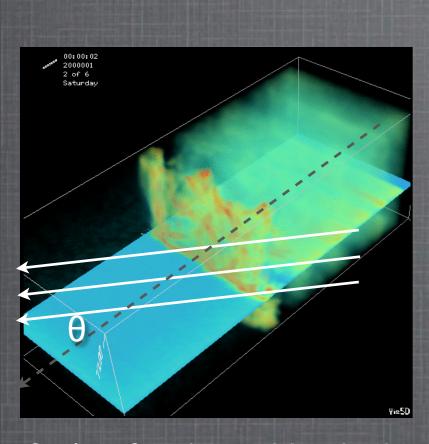
Transition between magnetized and unmagnetized shocks:



Acceleration: σ <10⁻³ produce power laws, σ >10⁻³ just thermalize

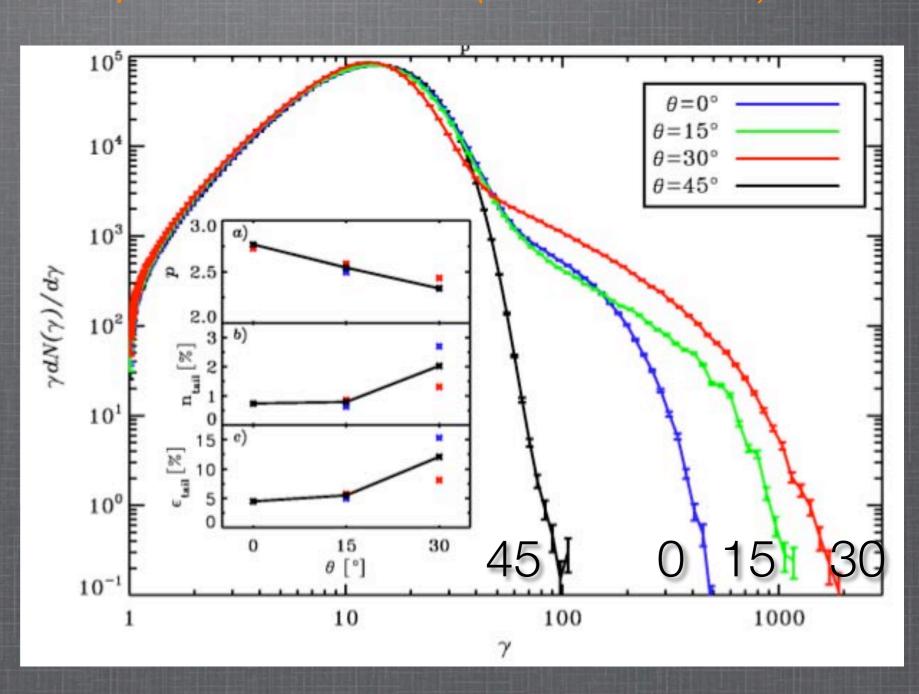
Magnetized pair shocks: acceleration

Pair shocks: σ =0.1, γ =15; Find p-law index near -2.3 (Sironi & AS 2009)



 $\beta_{sh}/cos\theta < 1$ -- subluminal

Self-turbulence is not enough to exceed superluminal constraint



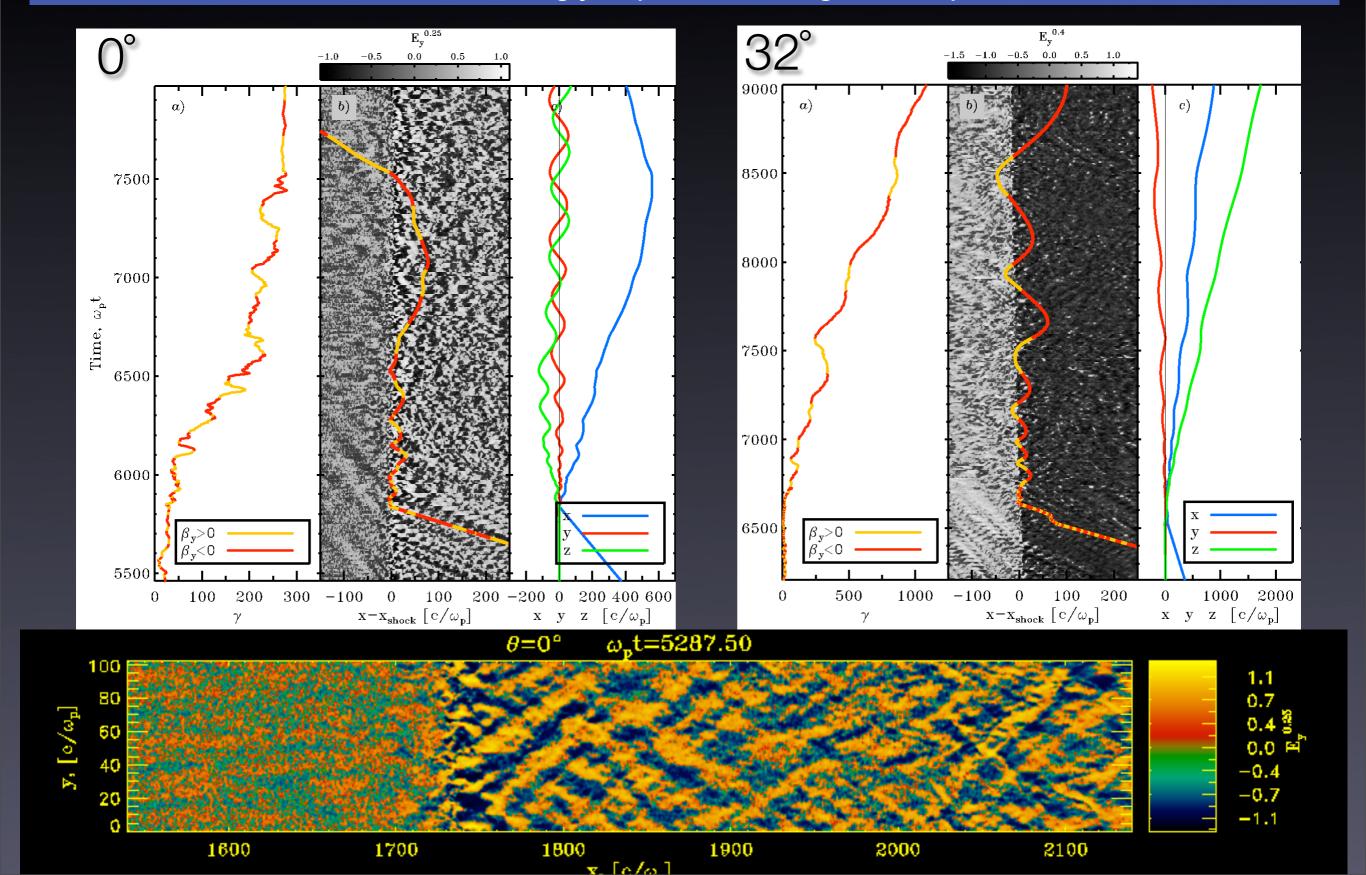
Observe transition between subluminal and superluminal shocks. Shock drift acceleration is important near transition.

Perpendicular shocks are poor accelerators.

In upstream frame need: $\theta_{upstream} < 32^{\circ}/\gamma$ for acceleration

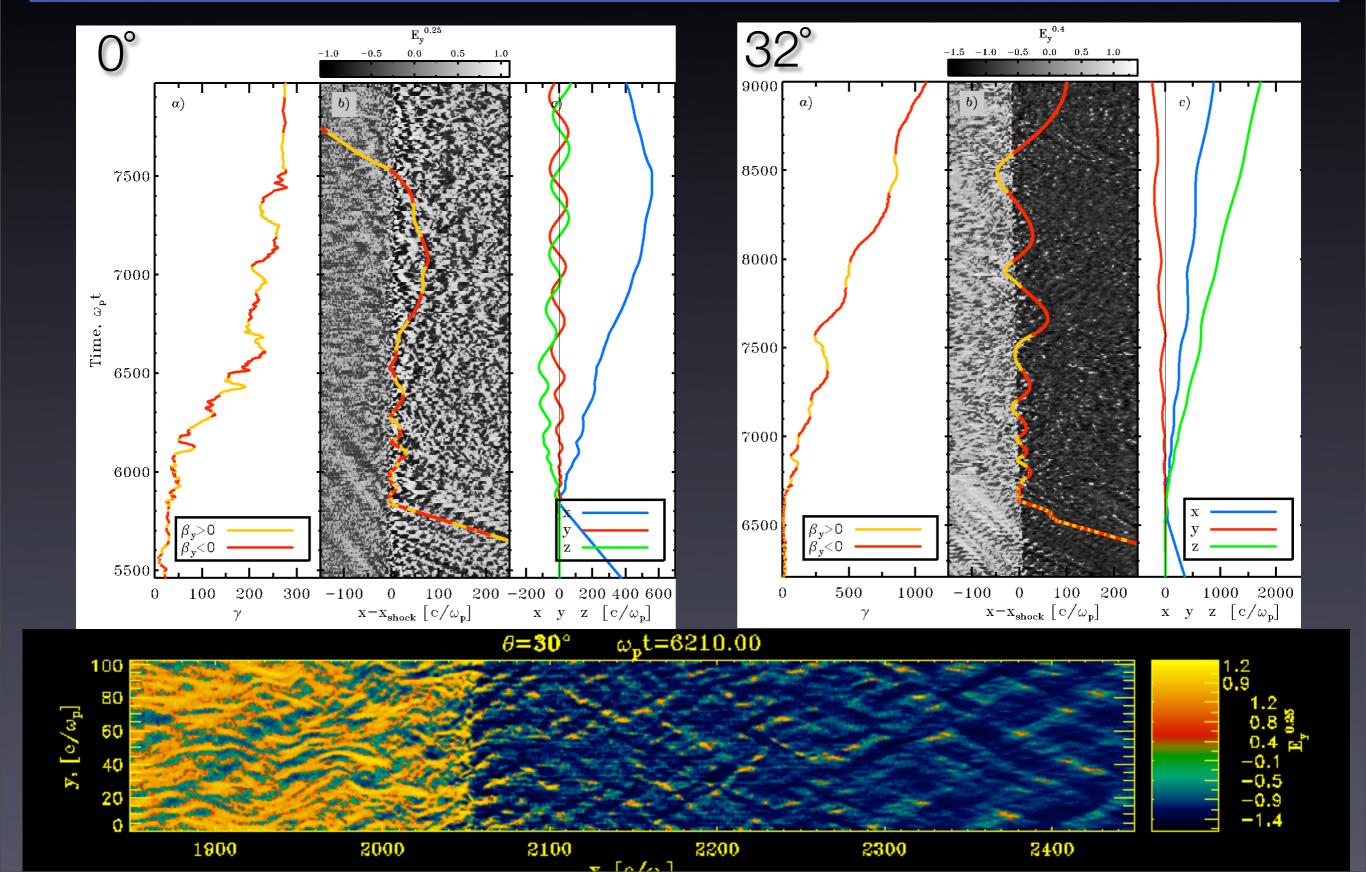
Acceleration mechanisms: "Fermi" vs shock-drift

Drift acceleration becomes increasingly important for higher obliquities.



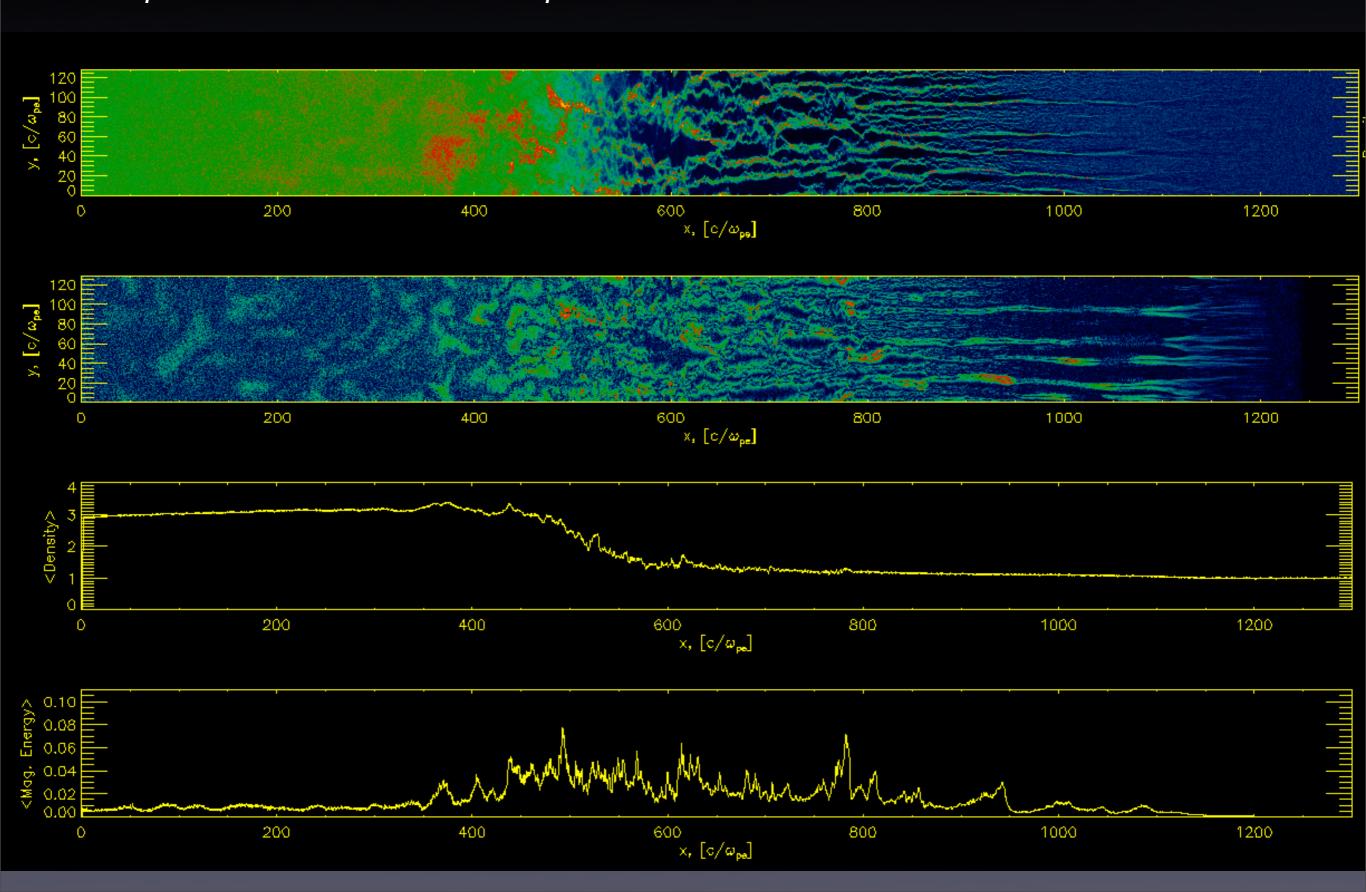
Acceleration mechanisms: "Fermi" vs shock-drift

Drift acceleration becomes increasingly important for higher obliquities.



Relativistic Electron-ion shocks

We explored electron-ion shocks up to mass ratio of 1000.



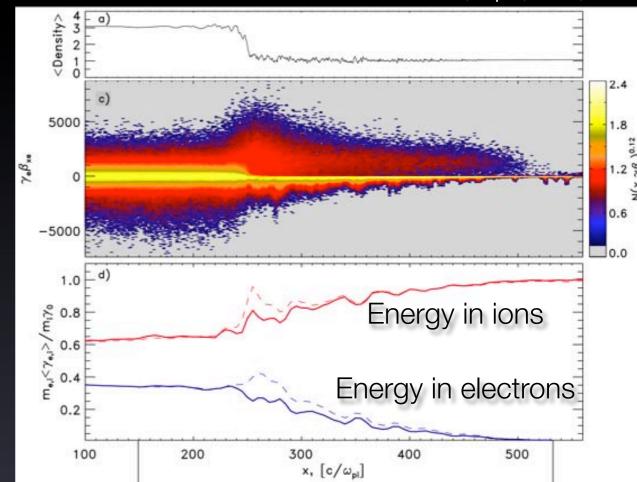
Medvedev 06

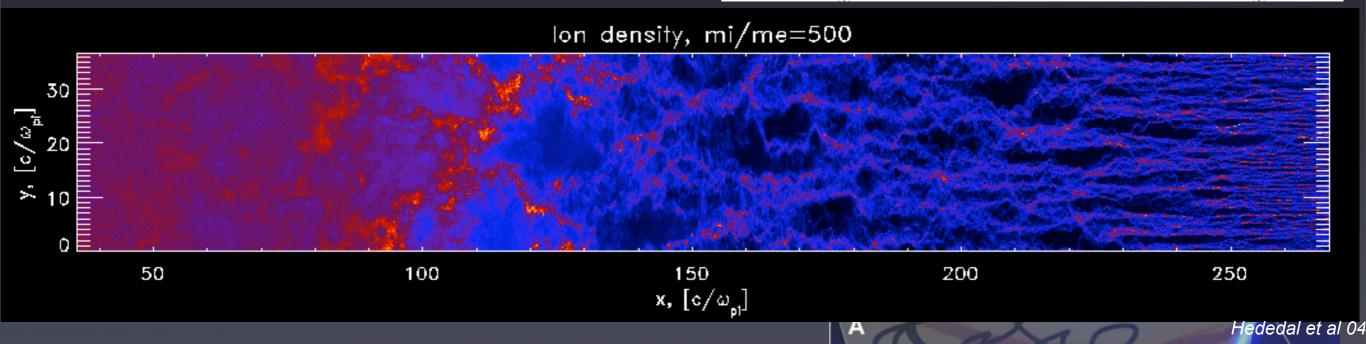
Relativistic Electron-ion shocks

We observe electron-ion energy exchange in the shock. Electrons come close to equipartition with the ions. *Behaves like pair shock!* This helps to explain the high electron energy fraction inferred in GRB afterglows.

Fermi acceleration proceeds very similarly in unmagnetized e-ion shocks

Perpendicular e-ion shocks do heating, but not significant acceleration.





Electron heating is related to electron oscillation in ion filaments, and the longitudinal instability of the filaments.

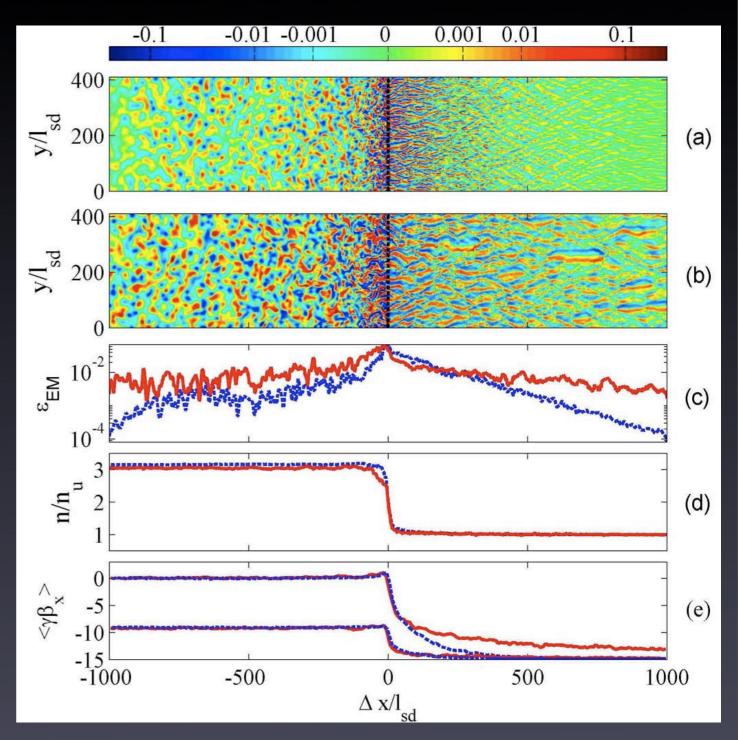
Pair shocks: magnetic field evolution

Can Weibel shocks generate enough field for downstream synchrotron emission?

Returning particles cause filamentation far in the upstream region and cause growth of the scale and amplitude of the upstream field.

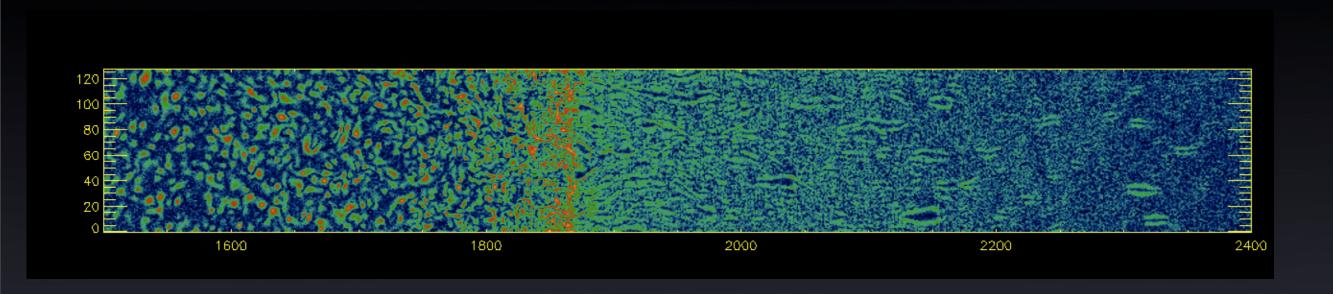
This affects the rate of decay of the field in the downsream (longer wavelengths decay slower).

1% magnetization is not unreasonable (Keshet, Katz, A.S., Waxman 2008).



we see growth of field energy and scale with time near shock, and slower decay downstream at 10⁴ skindepths

Pair shocks: magnetic field evolution

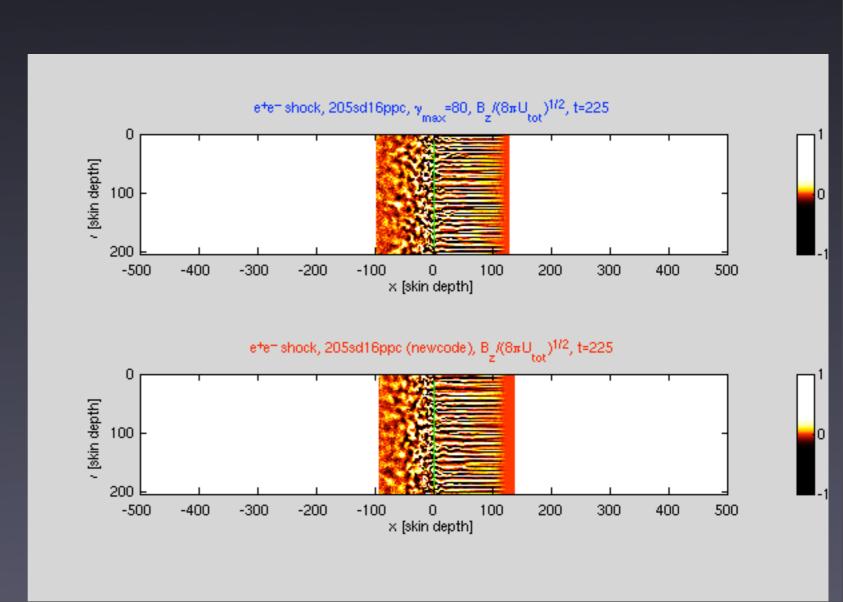


Field evolution:

Without high energy particles:

With high energy particles:

Scale growth is caused by accelerated particles. Larger field accelerates more particles -- bootstrapping!



B field amplification

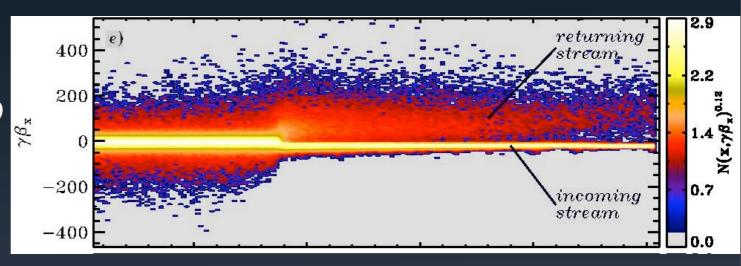
CR accelerating shocks can cause a current of protons to propagate through the upstream. Bell (04, 05) found an MHD instability of CRs flying through magnetized plasma.

The interaction is nonresonant at wavelength << Larmor radius of CRs.

We simulated this instability with PIC in 2D and 3D (Riquelme and A.S. 08)

Saturation is due to plasma motion (V_A~ V_{d,CR}), or CR deflection; for SNR conditions expect ~10 field increase.

Bell's nonresonant CR instability





Cosmic ray current J_{cr}=en_{cr}v_{sh}

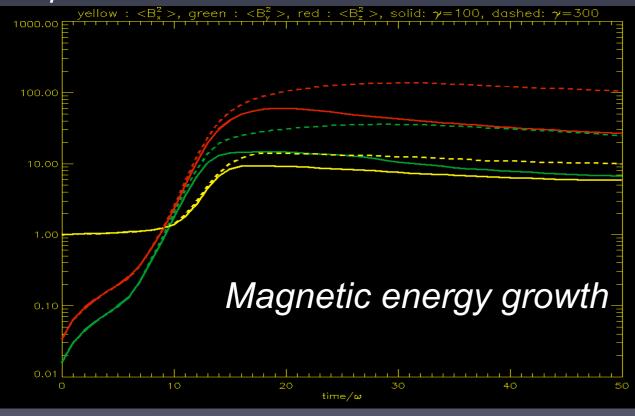
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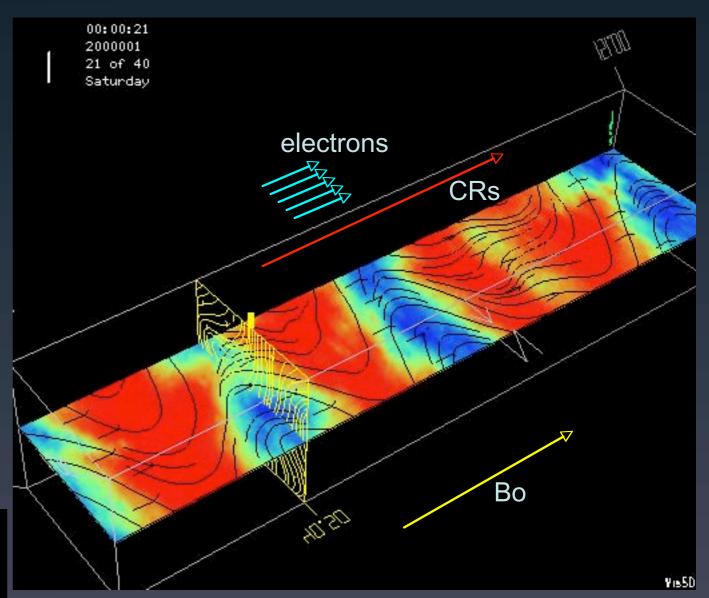
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Bell's nonresonant CR instability



$$k_{\text{max}} c = 2\pi J_{\text{cr}}/B_0$$

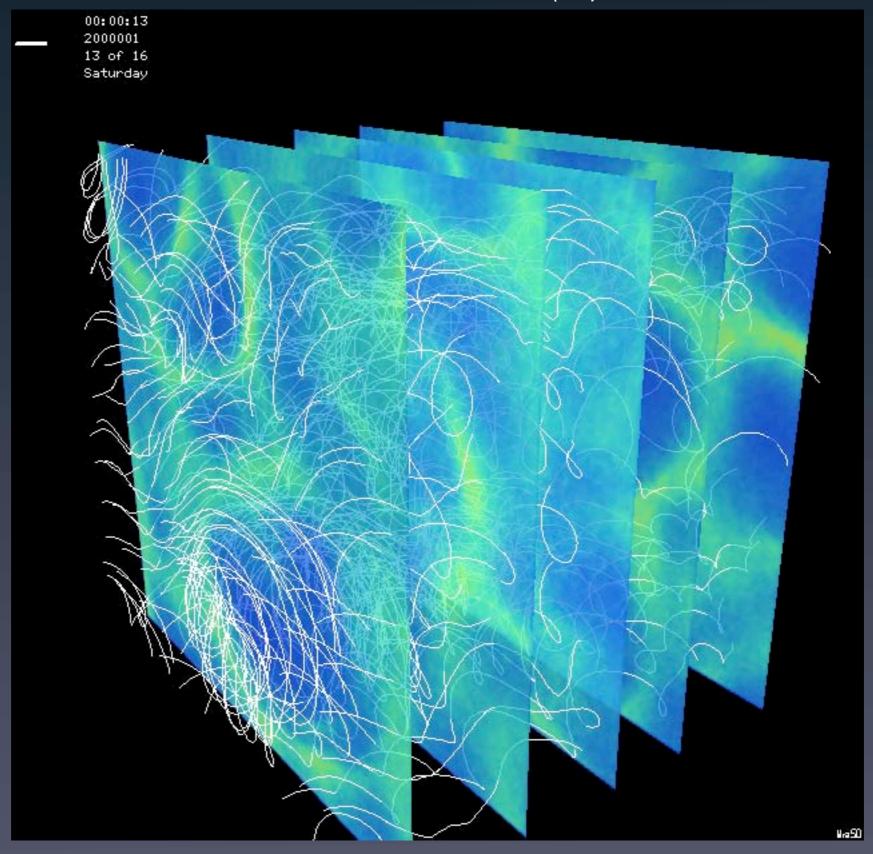
 $\gamma_{\text{max}} = k_{\text{max}} V_{\text{Alfven,0}}$

Need magnetized plasma: $\omega_{ci} >> \gamma_{max}$

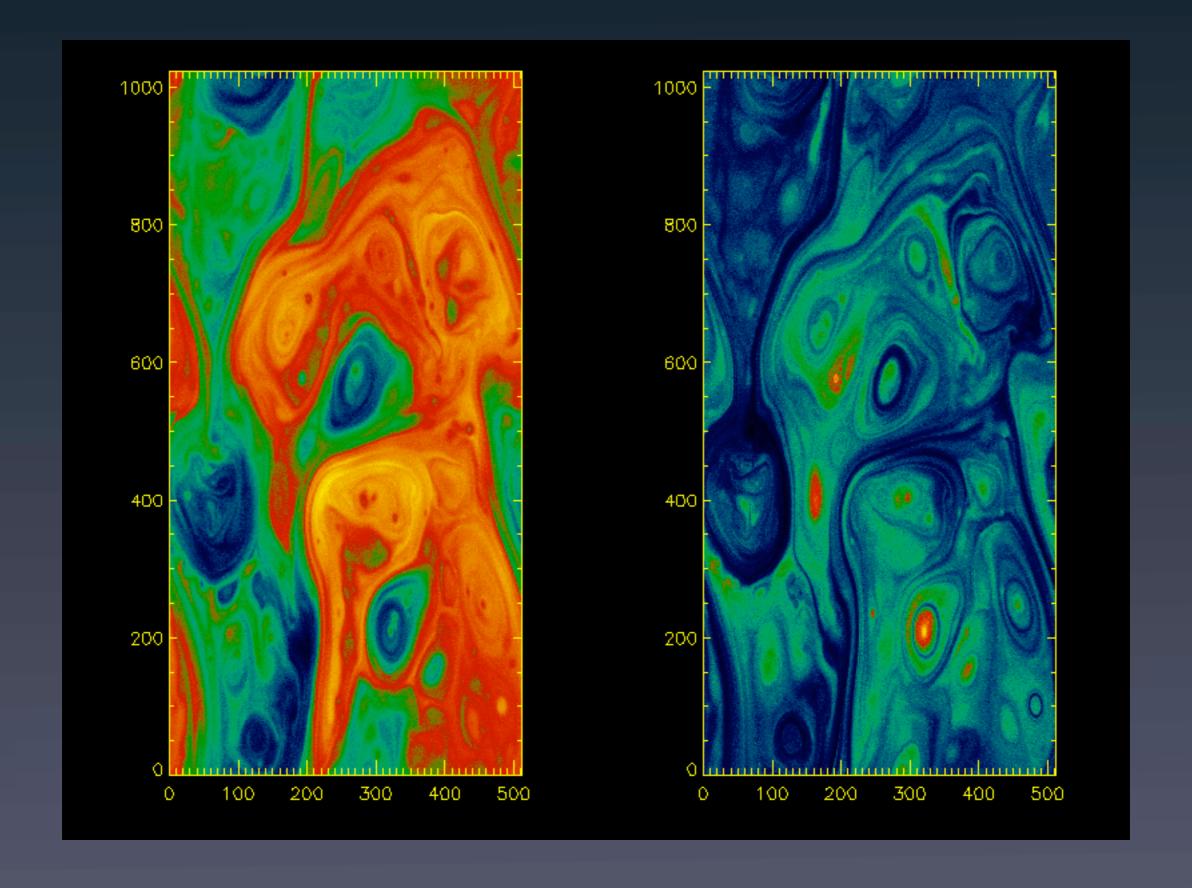
B field amplification: 3D runs

Bell's nonresonant CR instability

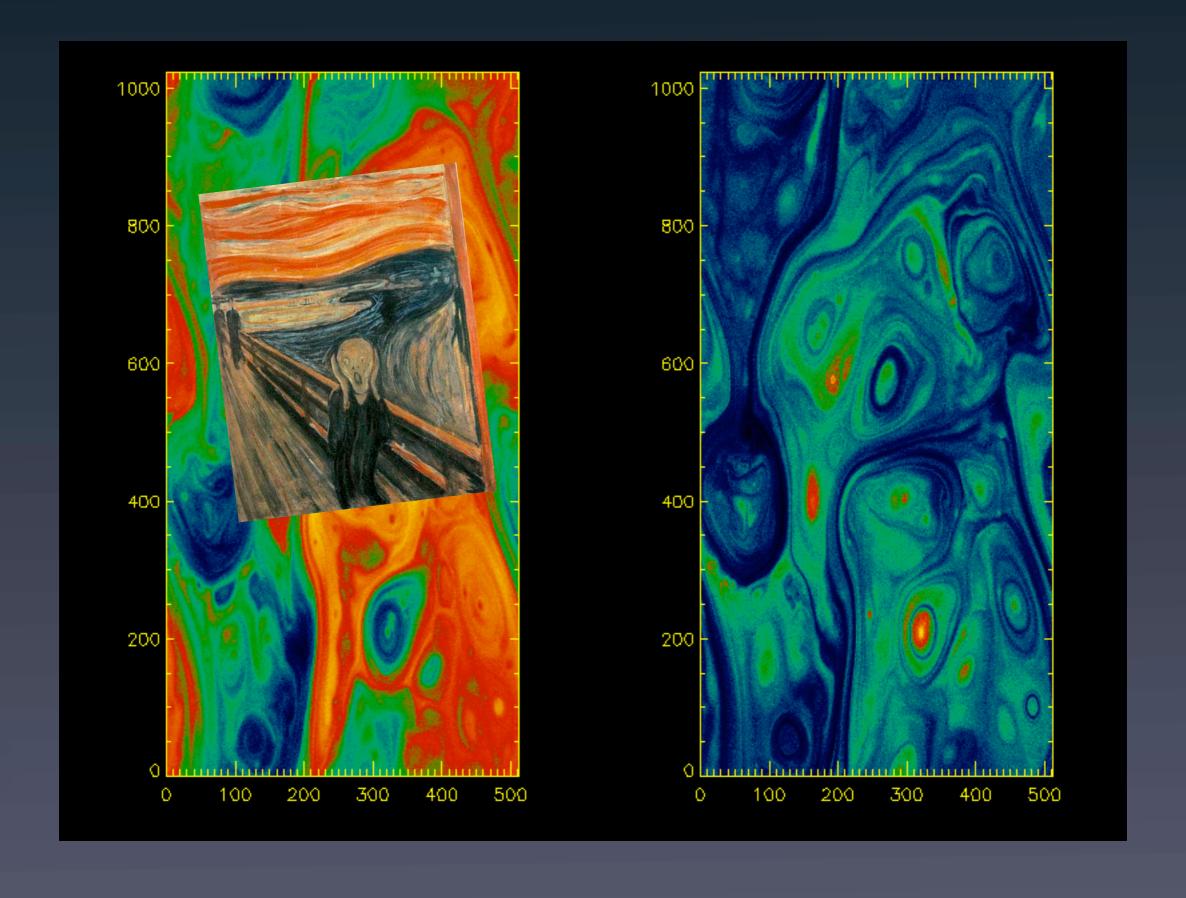
(Riquelme and A.S. arXiv:0810.4565)



Field amplification of ~10 in SNRs can be due to Bell's instability



PIC simulations of shocks



Shocking astrophysics

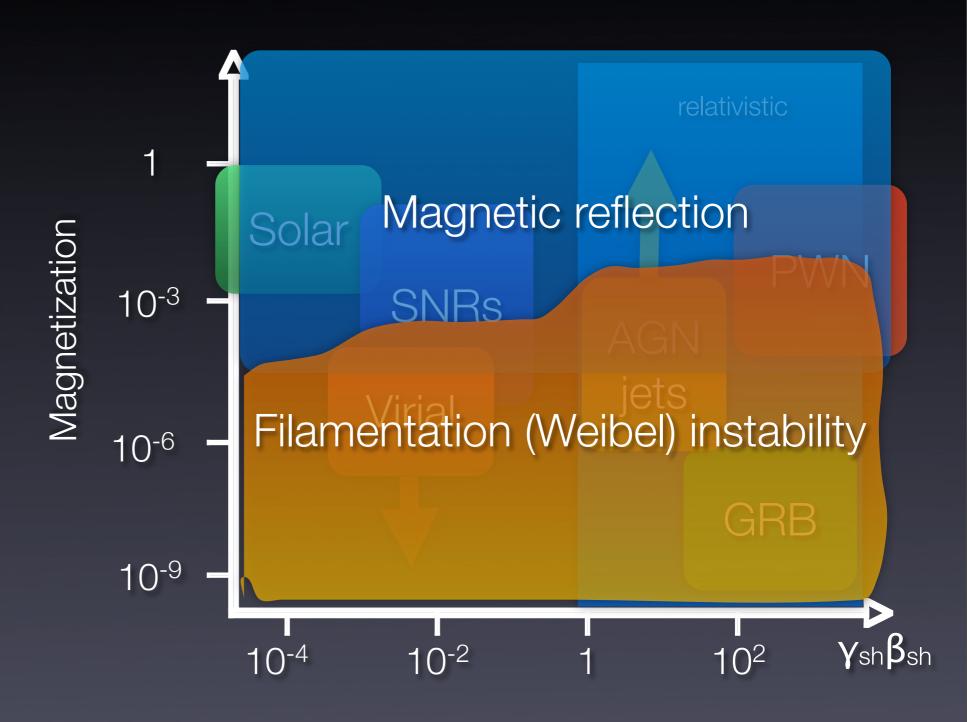
Open issues:

What is the structure of collisionless shocks? Do they exist? How do you collide without collisions?

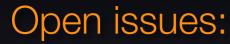
Particle acceleration -- Fermi mechanism? Other? Efficiency?

Equilibration between ions and electrons?

Generation of magnetic fields? GRB/SNR shocks, primordial fields?



Shocking astrophysics

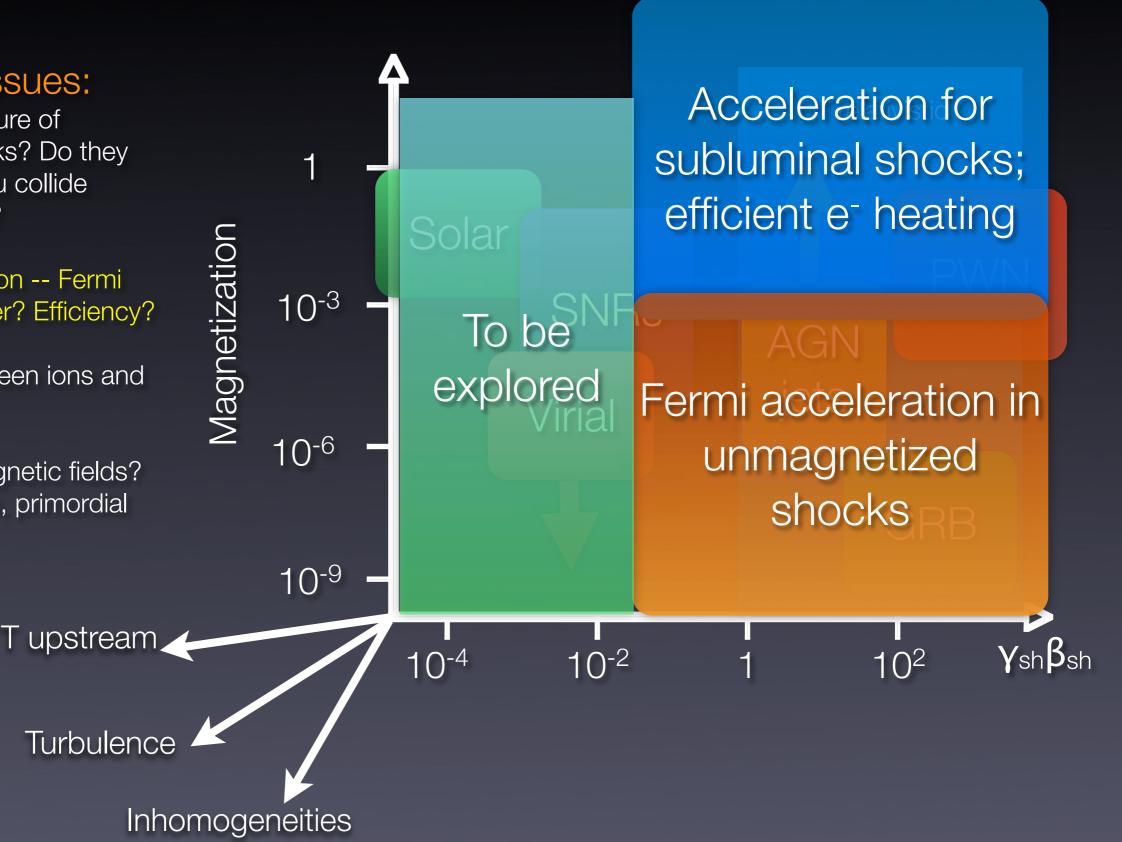


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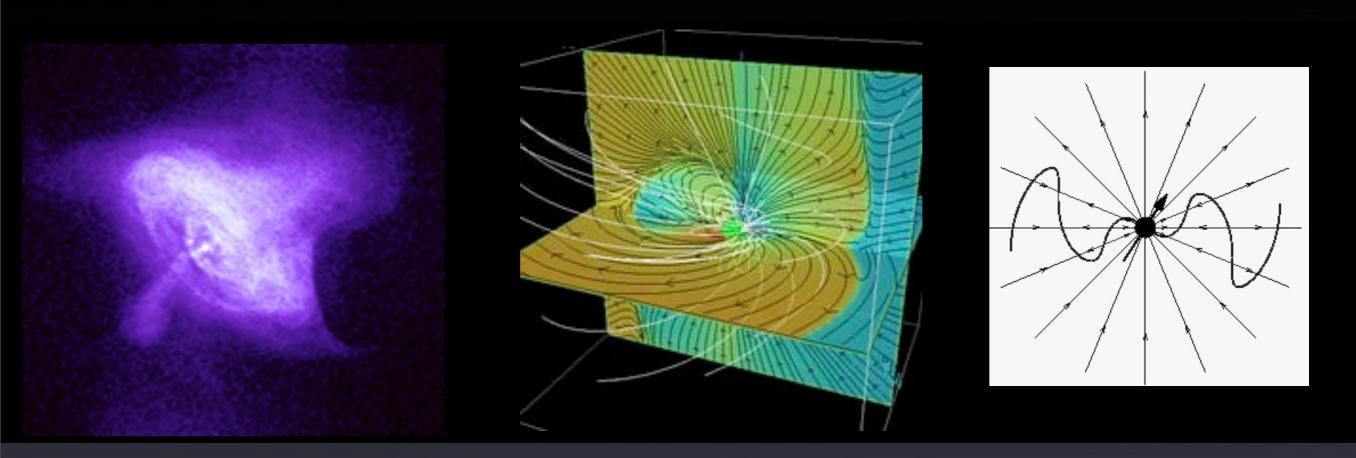
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Equilibration between ions and electrons?

Generation of magnetic fields? GRB/SNR shocks, primordial fields?



Astrophysical implications: Pulsar Wind Nebulae (PWNe)



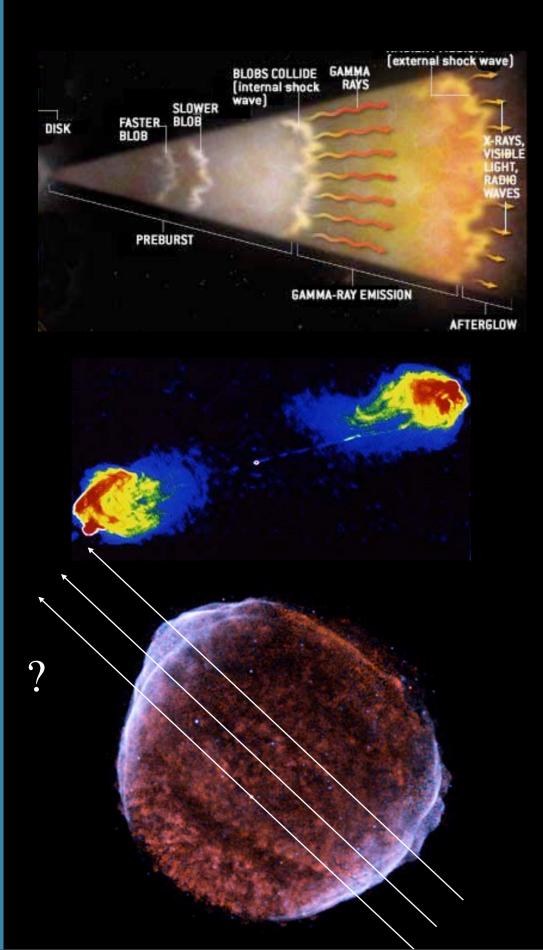
Shock acceleration in PWN implies low magnetization shock. σ =0.001-0.01 is inferred from modeling of the nebulae. This is a "transition" regime between magnetized and unmagnetized shocks -- expect Weibel instability to dominate the shock.

Equatorial shock occurs where the current sheet lies -- hence expect a weakly magnetized "equatorial wedge" -- consistent with shock physics.

At the moment pair composition could be ok, although other arguments suggest the presence of pair-ion plasma (A.S. & Arons 04).

Alternative -- reconnecting flow at the termination shock (Lyubarsky & Petri 07)

Astrophysical Implications



Gamma Ray Bursts

Very low magnetization σ =10⁻⁸ shocks can operate even in electron-ion plasma.

Electron heating to near equipartition with the ions implies that high electron energy fraction (ε_e =0.1) is not unreasonable. Magnetic fields near (ε_B =0.01) could also be generated. Can we see thermal component?

AGN and other jets

High magnetization perpendicular pair flows are unlikely to generate nonthermal particles through Fermi acceleration. Other physics needed? Not pure pair flows? Sheath flow?

Supernova Remnants

We see field amplification due to streaming CRs: Bell's instability is part of the amplification puzzle.

Parallel shocks are more likely to accelerate particles than perpendicular shocks (e.g. SN1006?).

Nonrelativistic shocks

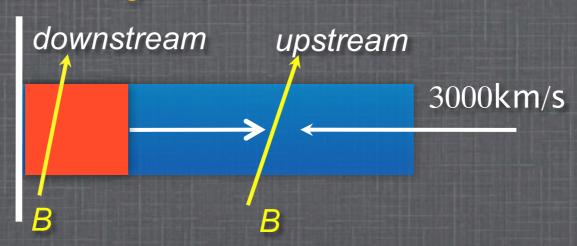
New scales: speed is no longer c, so Debye and skin depth are different, thermal velocity no longer c.

Difficulties: longer runtimes (still resolve speed of light) Acceleration is intrinsically slower (vshock/c)^2

Injection problem -- how to pre-accelerate particles so their larmor radius exceeds the scale of the shock?

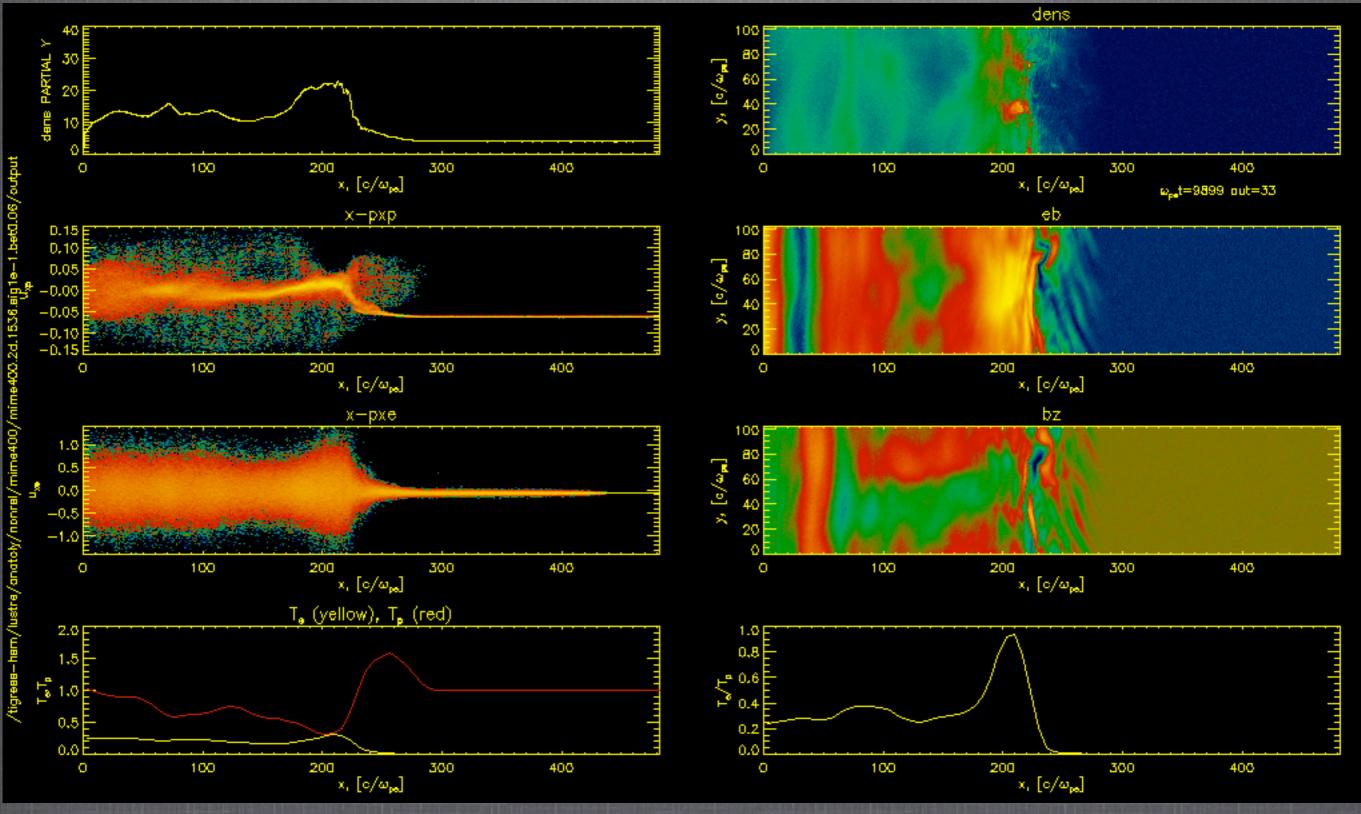
Two types of shocks -- quasi-parallel and quasi-perpendicular

We investigated quasi-perpendicular shocks (inclination angle 15 degrees), with mass ratios from 100 to 1000, and speeds from 3000 to 30000 km/s, Alfvenic Mach number from 3 to 100. For 1000km/s, B=25uG: Ma=18; 3000km/s->Ma=54 We are essentially in a realistic regime, albeit in 2.5D.



Nonrelativistic shocks: shock structure

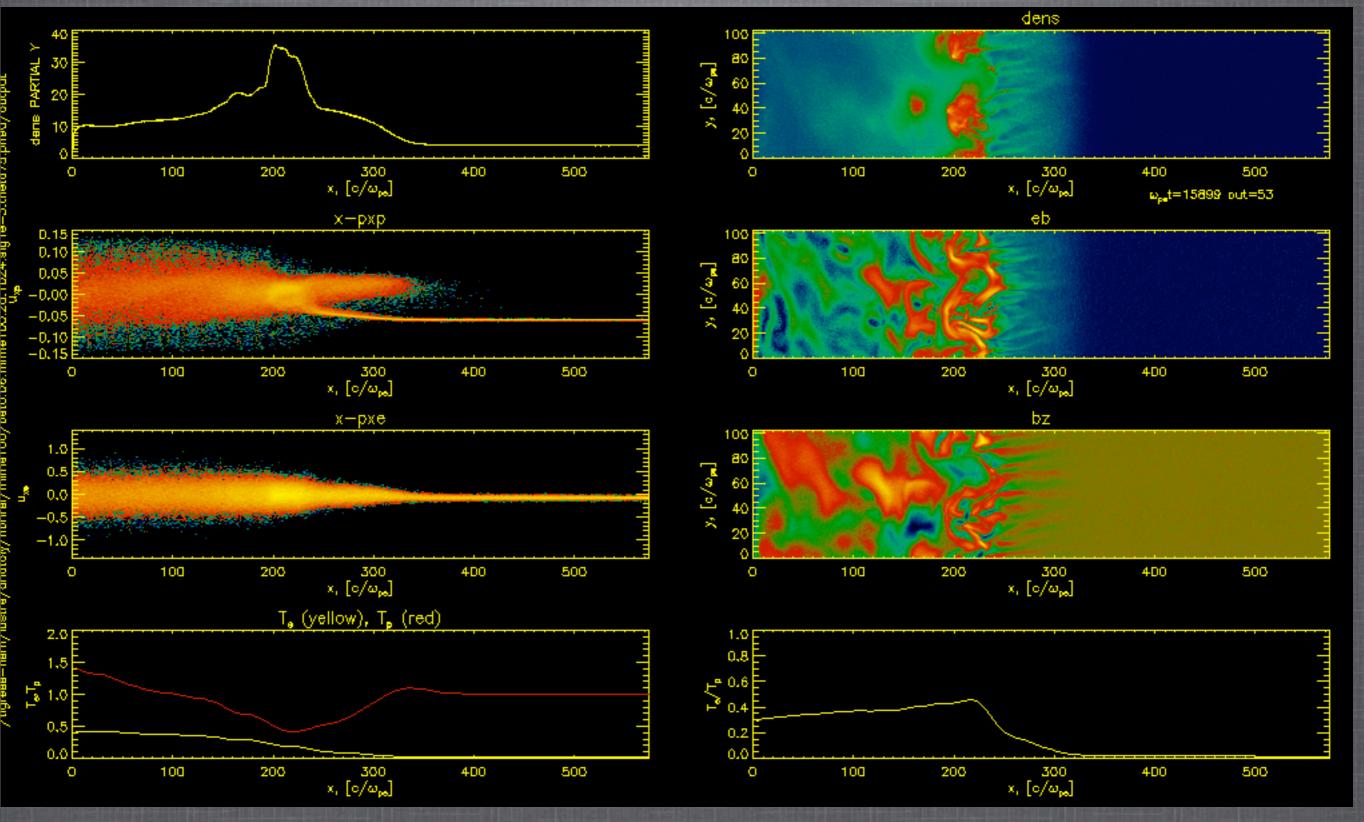
mi/me=400, v=18,000km/s, Ma=5



Shock foot, ramp, overshoot, returning ions, electron heating, whistler(?) waves.

Nonrelativistic shocks: shock structure

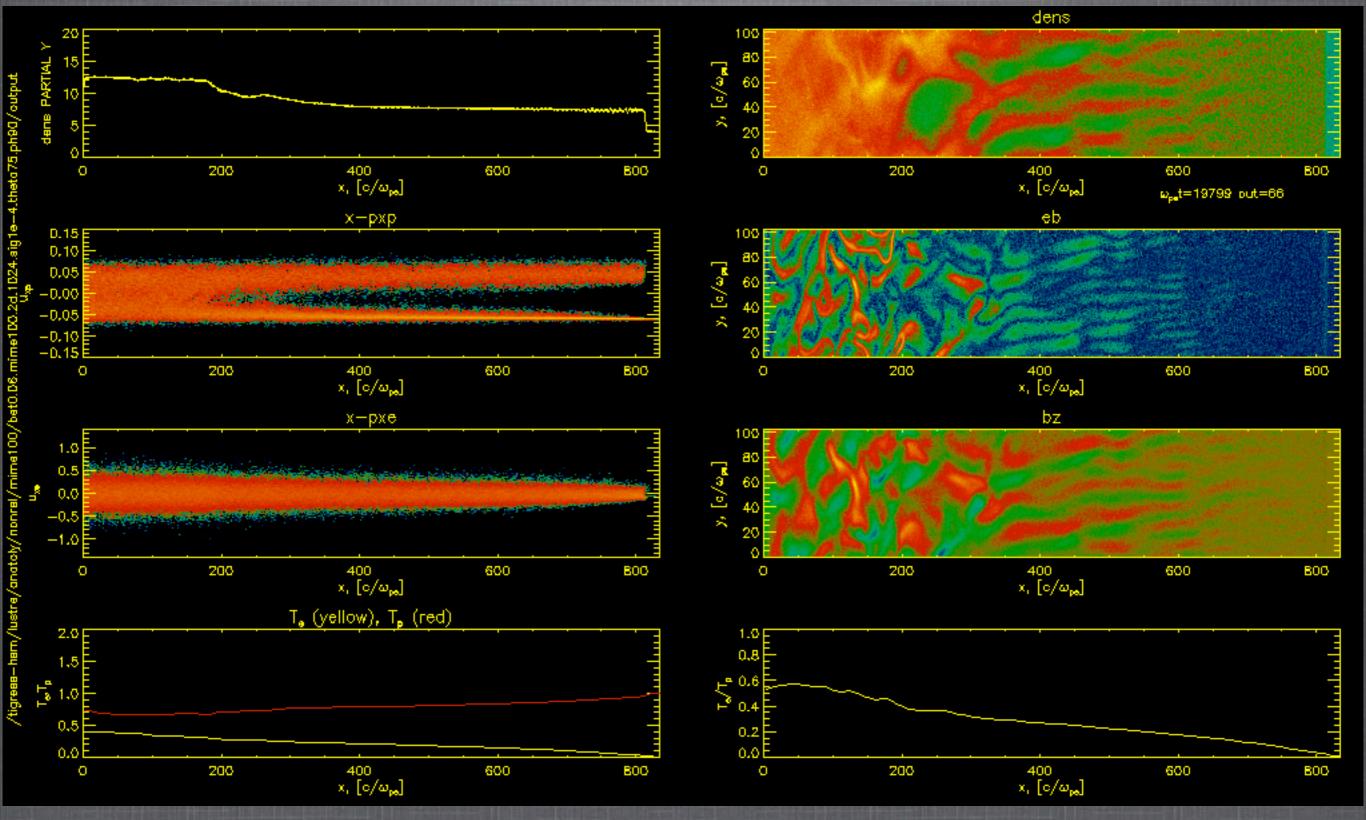
mi/me=100, v=18,000km/s, Ma=45



Shock foot, ramp, overshoot, returning ions, electron heating, whistler(?) waves, spectra.

Nonrelativistic shocks: shock structure

mi/me=100, v=18,000km/s, Ma=140



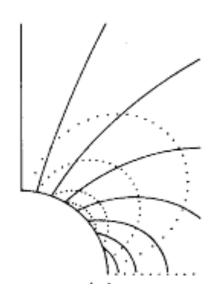
Shock foot, ramp, overshoot, returning ions, electron heating, whistler(?) waves, spectra.

Application: magnetospheres

Applications

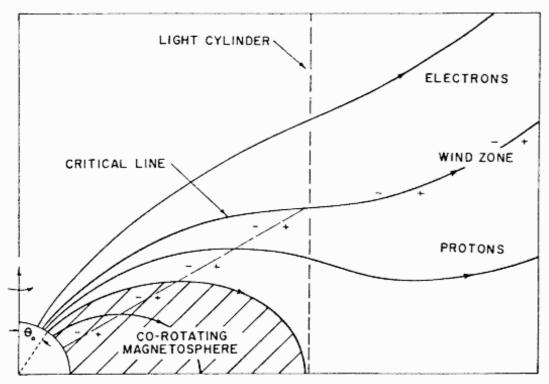
Astrophysics:

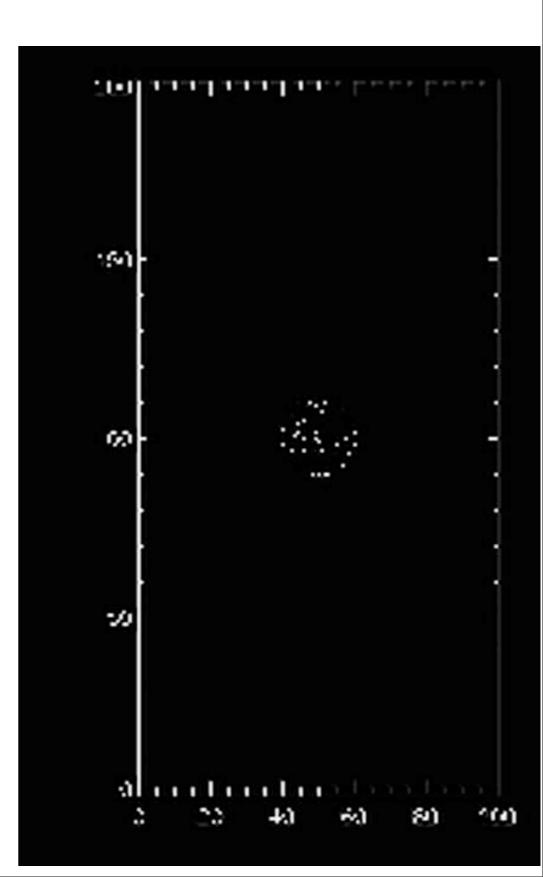
Nonneutral plasma physics in pulsar magnetospheres



Electric field on the surface extracts charges. Does magnetosphere form?

Expect to see this:

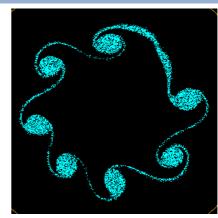


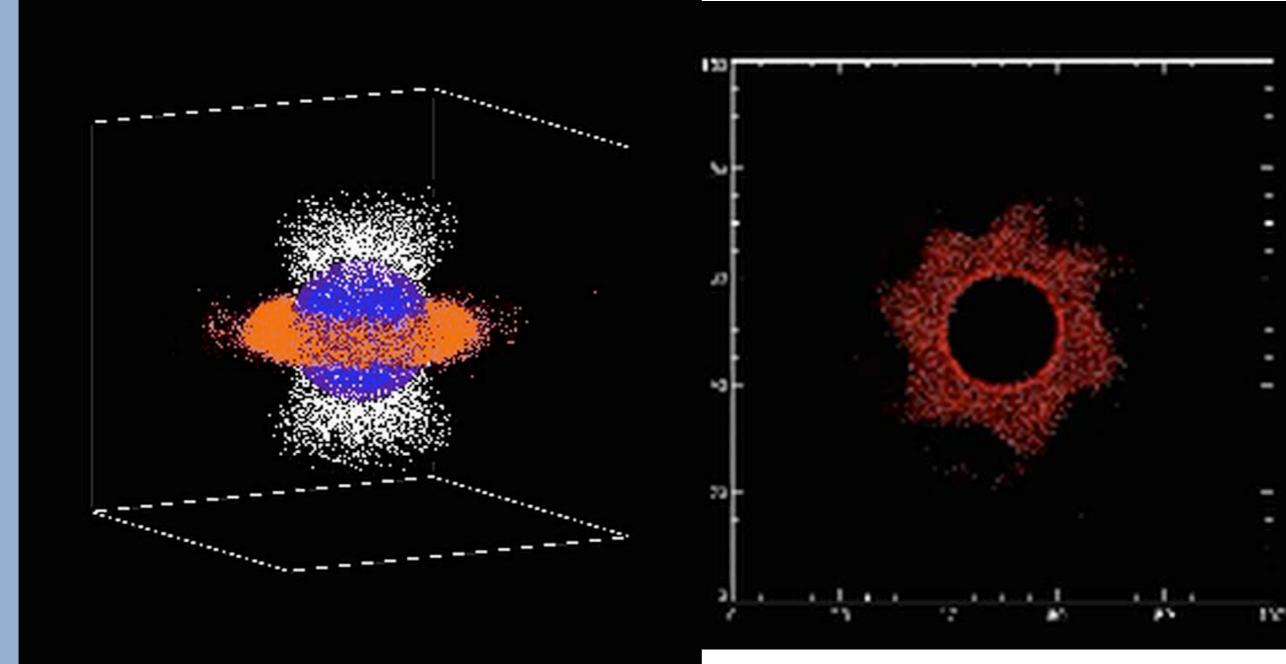


Applications

Astrophysics:

Nonneutral plasma physics in pulsar magnetospheres. Diocotron instability



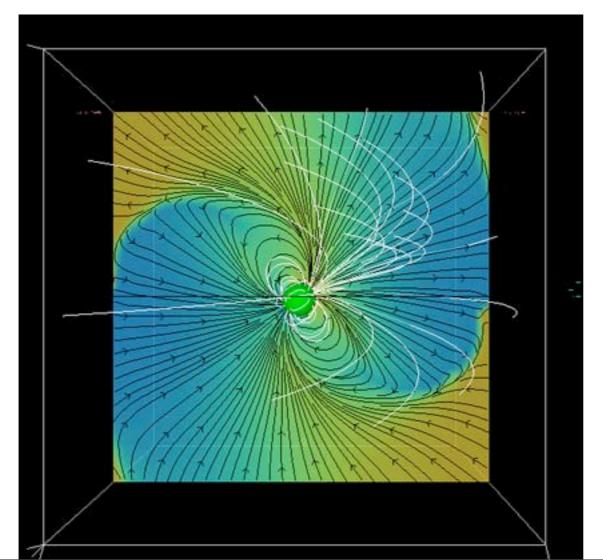


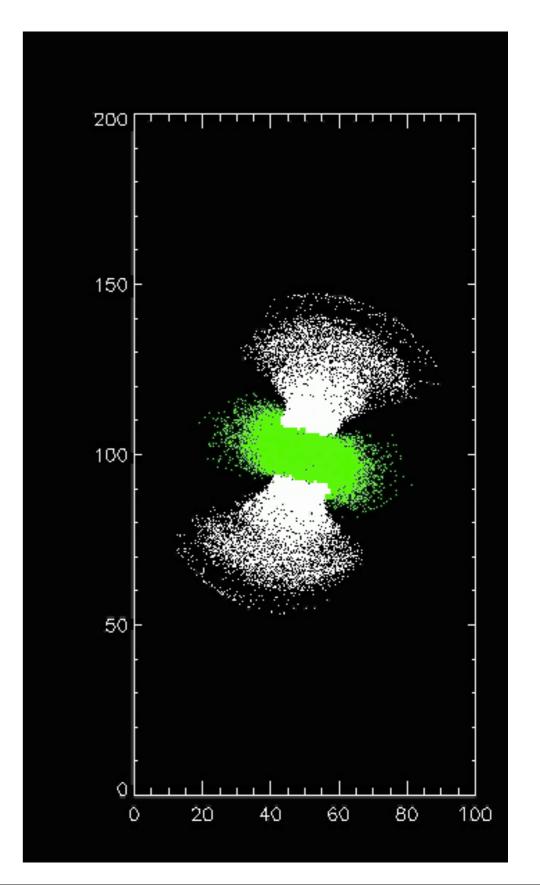
Applications

Astrophysics:

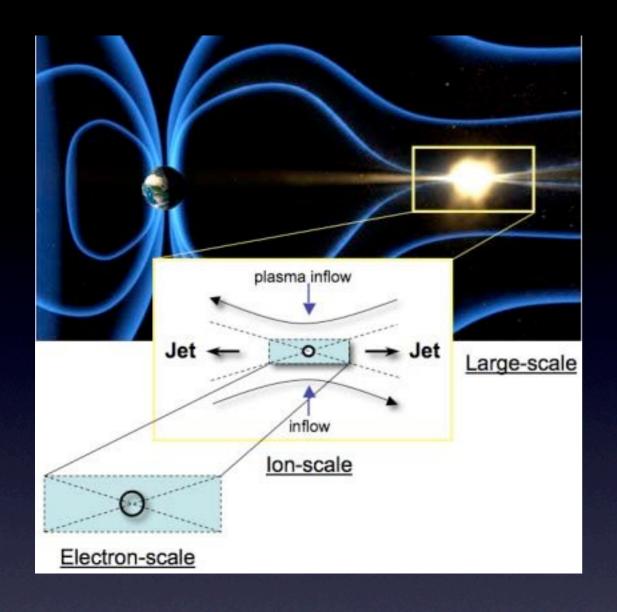
Nonneutral plasma physics in pulsar magnetospheres. Diocotron instability

Space-charge limited flow dynamics in presence of pair formation needs to be addressed.



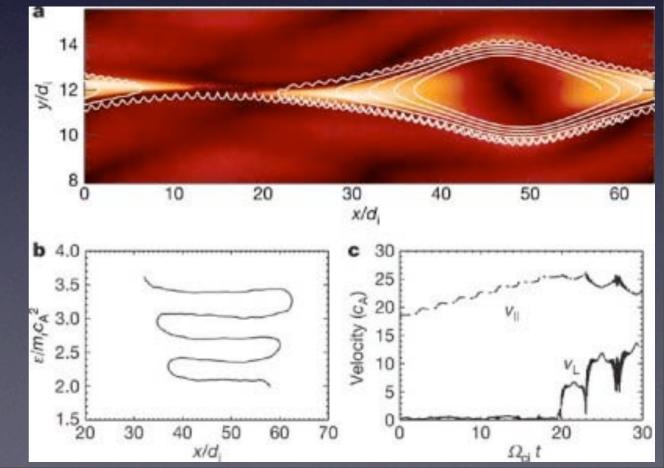


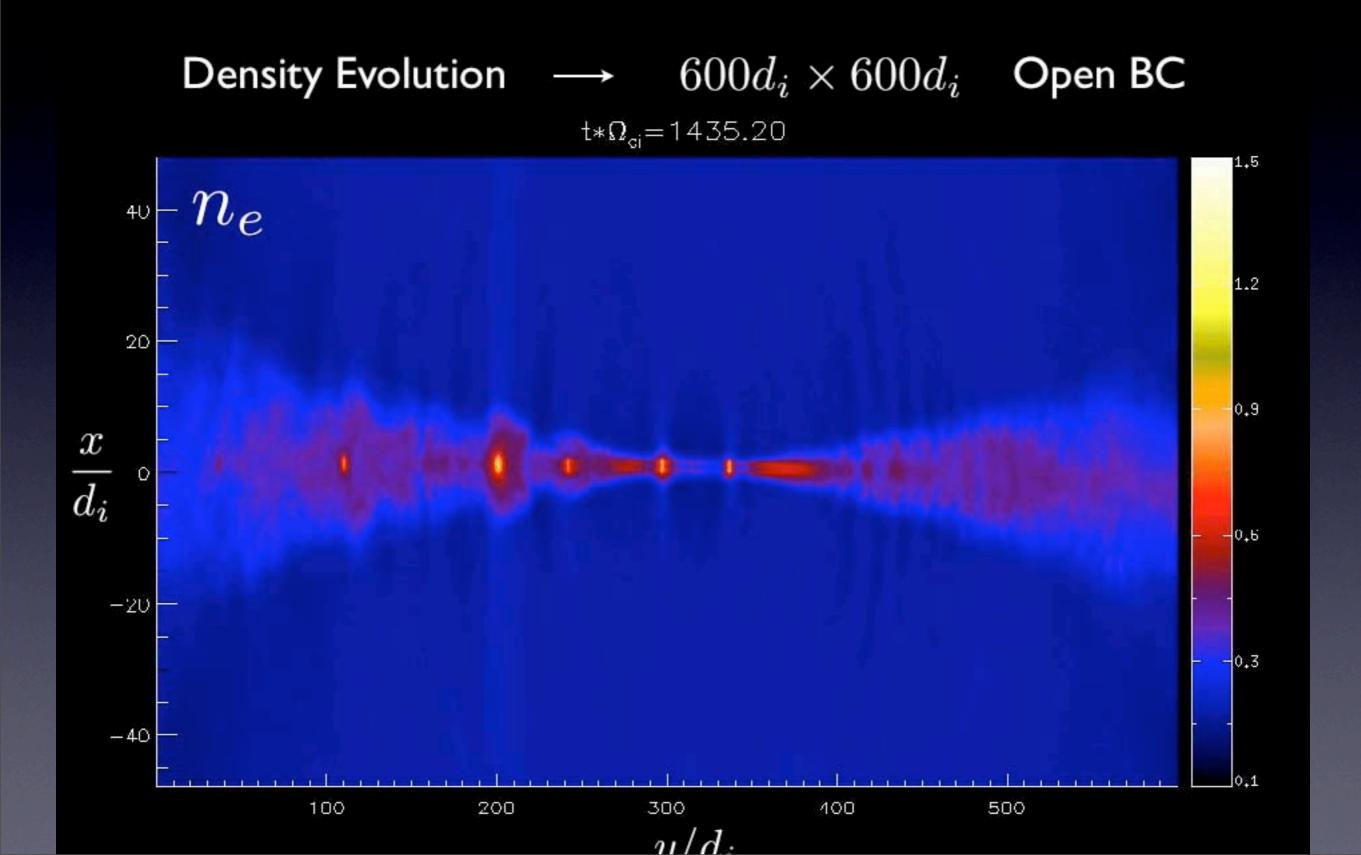
Application: reconnection



Reconnection questions:
Rate of reconection
Partilce energization
e-ion vs e-positron

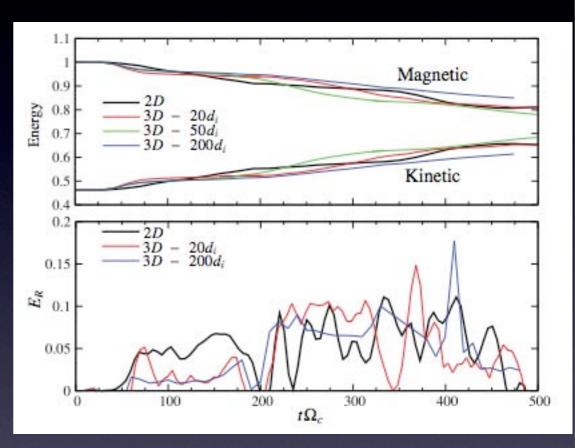
BC arguments: periodic vs open





$\frac{|\mathbf{J}|/J_o}{0.4}$ B_y/B_o See cut below 0.0 0.220 Current Tube Diffusion Region -20 50 100 150 ${0\atop |\mathbf{J}|/J_o}^1$ z/d_i

Yin et al 08



Outlook

PIC is a versatile robust tool for self-consistent solution of plasma physics.

- •Electrostatic method is well understood, and analytical theory of numerical plasma exists.
- •Electromagnetic model is more diverse, and many alternative formulations exist. Multidimensional theory of the simulation is not as well developed.
- Implicit methods are now common for large timestep solutions.
- Long term stability is an issue for largest runs.
- •In astrophysics PIC has the potential to answer the most fundamental theoretical questions: particle acceleration, viability of two-temperature plasmas, dissipation of turbulence.

Outlook

Current results:

- ab-initio evidence for particle acceleration in shocks
- •conditions for particle acceleration -- constraints on models!
- measurements of ion-electron energy exchange in shocks
- CR feedback and field amplification
- Pulsars: instabilities that lead to charge transfer in the magnetosphere.
- •Reconnection: rate of reconnection, physics of the reconnection layer.

Outlook

- Computational issues:
- optimization
- load balancing
- visualization -- what to do with 100 billion particles?!!!
- treating simulations as experiments?
- How to figure what is going on? Dispersion relations, test particle trajectories, reproducibility.