

## PiTP 2009: Computational Astrophysics

### Computational Methods for Numerical Relativity

#### Lecture 4: Generalized Harmonic Evolution

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## Outline

- Generalized Harmonic Decomposition of the Einstein Equations
  - generalized harmonic coordinates and how they can be used to solve the field equations
  - black hole merger example
  - brick-a-brack
    - compactification, nature of the solution, damped harmonic coordinates
- Concluding Remarks

## Generalized Harmonic Decomposition

- Why more formalism?
  - It is largely through understanding the structure of the field equations, and not development of new numerical methods, that recent advances in the field have been made
    - Except for “linking” with the algorithms here and there, the methods used by all numerical groups have been around for a while: finite difference with AMR, multigrid, multipatch, pseudo-spectral, domain decomposition, ...
    - Contemporary computer systems are powerful enough that we do not need to develop clever techniques to approximate the physics of gravity in many situations; we can solve the full field equations
    - The difficulty then in a sense is all formalism – controlling constraints, choosing the gauge, imposing boundary conditions, etc., and in this regard the ADM formalism by itself is not adequate in all situations
- Why not just do generalized harmonic (GH) to begin with?
  - certain aspects of GH are a bit obscure, in particular how the constraints enter, and how to choose the gauge; describing these things in the language of ADM allows a better intuition about what’s going on
  - though will not cover it here in detail, BSSN is based on ADM

## Harmonic Coordinates

- Harmonic coordinates

$$\nabla^\alpha \nabla_\alpha x^\mu \equiv \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} g^{\alpha\mu}) = 0$$

does to the Einstein equations what the Lorenz gauge does to Maxwell’s equations ... the *principle part* of each component of the Einstein tensor becomes a wave equation for the corresponding metric element

$$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} + 2g^{\gamma\delta}{}_{,\alpha} g_{\beta\delta,\gamma} + 2\Gamma_{\beta\gamma}^\gamma \Gamma_{\alpha\delta}^\delta + 8\pi(2T_{\alpha\beta} - g_{\alpha\beta}T) = 0$$

where

$$\Gamma_{\alpha\beta}^\gamma \equiv \frac{1}{2} g^{\delta\epsilon} (g_{\alpha\epsilon,\beta} + g_{\beta\epsilon,\alpha} - g_{\alpha\beta,\epsilon})$$

- the character of each field equation is now manifestly hyperbolic, though at the expense of having used up (essentially) all our coordinate degrees of freedom

### Harmonic Coordinates

- Harmonic coordinates are in a sense older than the field equations themselves, as they were used by Einstein as early as 1912 while searching for a relativistic theory of gravity (then they were called isothermal coordinates)
- over the years they have played an instrumental role in the formal analysis of the field equations, and the study of gravitational radiation
  - study of the characteristic structure of the field equations (deDonder 1921), first proofs of existence and uniqueness of solutions to the field equations (Choquet-Bruhat 1952), analysis of gravitational waves (Fock 1955), the stability of Minkowski spacetime (Lindblad & Rodnianski 2004), ...
- avoided in numerical relativity because of the largely misguided belief that they were prone to developing coordinate pathologies in generic scenarios
  - loosely speaking, if the coordinates are "wavelike", superposition of coordinate waves could result in large amplitude variations, which could result in the coordinates ceasing to be linearly independent; e.g., if the amplitude of waves in the time coordinate become too large, time could start flowing backwards in the troughs of the waves
  - Garfinkle [PRD 65, 044029 (2002)] however noted a possible resolution to this problem

### Generalized Harmonic Coordinates

- Generalized harmonic coordinates [Friedrich, *Comm. Math. Phys.* 110, 525 (1985)] introduce a set of arbitrary *source functions*  $H^\mu$  into the usual definition of harmonic coordinates

$$\nabla^\alpha \nabla_\alpha x^\mu = \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} g^{\alpha\mu}) = H^\mu$$

- note that *any* metric in *any* coordinate system can be viewed as a generalized harmonic metric
- If we now treat the  $H^\mu$  as *independent functions*, we can still write the field equations in the desirable wave-like form of harmonic coordinate

$$g^{\gamma\delta} g_{\alpha\beta;\gamma\delta} + 2g^{\gamma\delta}{}_{;\alpha} g_{\beta\gamma;\delta} + 2\Gamma_{\delta\beta}^\gamma \Gamma_{\gamma\alpha}^\delta + 2H_{(\alpha,\beta)} - 2H_\delta \Gamma_{\alpha\beta}^\delta + 8\pi(2T_{\alpha\beta} - g_{\alpha\beta} T) = 0$$

- The source functions now encode the coordinate freedom in general relativity, and to close the system we must specify some additional equations for the  $H^\mu$

### Generalized Harmonic Evolution

- The claim then is that a solution to the coupled Einstein-harmonic equations

$$g^{\gamma\delta} g_{\alpha\beta;\gamma\delta} + 2g^{\gamma\delta}{}_{;\alpha} g_{\beta\gamma;\delta} + 2\Gamma_{\delta\beta}^\gamma \Gamma_{\gamma\alpha}^\delta + 2H_{(\alpha,\beta)} - 2H_\delta \Gamma_{\alpha\beta}^\delta + 8\pi(2T_{\alpha\beta} - g_{\alpha\beta} T) = 0$$

$$\mathcal{L}^\alpha(H^\alpha) = 0$$

where  $\mathcal{L}^\alpha H^\alpha$  (no summation) are some (arbitrary) set of evolution equations for the source functions, plus additional matter evolution equations, will also be a solution to the Einstein equations *provided* the harmonic constraints

$$C^\mu \equiv H^\mu - \nabla^\alpha \nabla_\alpha x^\mu$$

and their first time derivatives are satisfied at the initial time.

- "Proof": one can show that the constraints satisfy the following equation

$$\nabla^\alpha \nabla_\alpha C^\mu = -R^\mu_\nu C^\nu$$

### Generalized Harmonic Evolution

- At a first glance this seems a bit like magic ... we've been able to put the equations into strongly hyperbolic form, and keep all our coordinate freedom, by "ignoring" certain relationships that must exist between variables of the equations

- What we've really done is *expanded* the system of equations in such a manner that the expanded system has the desired mathematical properties

- The solutions to the Einstein equations now form a subset of the solutions to this expanded system, selected through appropriate choice of initial conditions (this is the case for *all* free, Cauchy evolution schemes)

- The important question that still remains, is how does the system evolve under perturbations; in particular for numerical evolution, under perturbations sourced by truncation error?

- For PDEs without constraints, unless you're in a regime where the continuum solution exhibits exponential sensitivity to initial conditions, it would rare for global truncation to grow faster than linearly at a given resolution, since after each time step, even if truncation error has changed the evolution, we now have perfectly consistent initial data for the next step

- If there are constraints, it is not unreasonable to expect that the amount by which the constraint-violation grows in one time step is proportional to the amount present at the beginning of the time; this is the recipe for exponential growth

- do the GH form of the EFE admit exponential growth of the constraints?

### Constraint damping

- It turns out that free evolution of spacetimes containing black holes with "plain" harmonic evolution does not have desirable evolution properties if the numerical data contains small violations of the constraints
- The (apparent) cure, as suggestion by C. Gundlach et al (*J.C. Gundlach, J.M. Martin-Garcia, G. Calabrese, I. Hinder, gr-qc/0504114*) based on earlier work by Brodbeck et al (*J. Math. Phys. 40, 909 (1999)*) is to modify the Einstein equations in harmonic form as follows:

$$g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \dots + k(n_\mu C_\nu + n_\nu C_\mu - g_{\mu\nu} n^\alpha C_\alpha) = 0$$

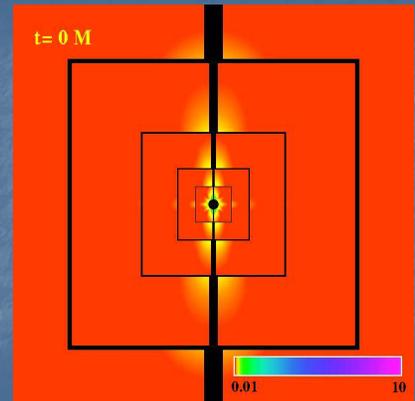
$n = \alpha \partial_t$  is a unit timelike vector normal to  $t = \text{const}$  hypersurfaces, with proper time measured by an observer moving along  $n_\mu$  given by the lapse function  $\alpha$ , and  $k$  is a constant parameter

- note that any solution to the field equations must have  $C=0$ , so we are adding "nothing" to them! (i.e., we are modifying the system of equations, but not the sub-system satisfying the Einstein equations)
- however, if  $C$  happens to be non-zero but small, and the spacetime being evolved is a small perturbation of Minkowski space, Gundlach et al showed that this modification causes all finite wavelength components of  $C$  to be exponentially damped with time
  - no proof that this damping property extends to more general scenarios, but at least empirical evidence that in works in binary black hole spacetimes
- The equations governing the dynamics of the constraints are still homogenous in the constraints, hence the arguments for obtaining valid solutions with the GH system also apply to the GH system with constraint damping

$$\nabla^\alpha \nabla_\alpha C^\mu = -R^\mu_\alpha C^\alpha + 2k \nabla_\alpha (n^\mu C^\alpha)$$

### Constraint damping example

- An animation of the evolution of the constraints in an axisymmetric simulation of a Schwarzschild black hole using GH coordinate with (right) and without (left) constraint damping



(for simplicity the constraints were not evaluated next to refinement boundaries, hence the black borders)

### Coordinate Issues

- The source functions encode the coordinate degrees of freedom of the spacetime

- how does one specify  $H^\nu$  to achieve a particular slicing/spatial gauge?
- what class of evolutions equations for  $H^\nu$  can be used that will not adversely affect the well-posedness of the system of equations?

### Specifying the spacetime coordinates

- A way to gain insight into how a given  $H^\nu$  could affect the coordinates is to appeal to the ADM decomposition of the spacetime

$$ds^2 = -\alpha^2 dt^2 + h_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- In terms of the ADM variables, the source functions are

$$H \cdot n \equiv H_\mu n^\mu = -n^\mu \partial_\mu \ln \alpha - K$$

$$\perp H^i \equiv H_\mu h^{\mu i} = \frac{1}{\alpha} n^\mu \partial_\mu \beta^i + h^{ij} \partial_j \ln \alpha - \Gamma^i_{jk} h^k$$

or

$$\partial_t \alpha = -\alpha^2 H \cdot n + \dots$$

$$\partial_t \beta^i = \alpha^2 \perp H^i + \dots$$

( $\Gamma$ -bar is the spatial connection)

### Specifying the spacetime coordinates

- Therefore, if you know what kind of coordinates you want in terms of the lapse and shift, you can design equations for  $H^i$  ( $H^i$ ) to drive  $\alpha$  ( $\beta^i$ ) to desired values
- For example, I have found it useful when evolving black holes to not have the lapse function evolve to close to 0; the following equation is thus designed to drive the lapse to 1

$$\nabla^\mu \nabla_\mu H_i = -\xi_1 \frac{\alpha - 1}{\alpha^n} + \xi_2 \partial_\mu H_i \cdot n^\mu$$

- implemented as a driven wave equation to not adversely affect the hyperbolicity of the coupled GH-gauge system, and with an added damping term to help stabilize the gauge

### "Damped" harmonic coordinates

- As an another example of gauge, suppose the worries about harmonic gauge becoming singular during the interaction of large amplitude gauge waves occurred in some scenario, perhaps it could be avoided by demanding that the coordinates instead satisfied a *damped wave equation*
- let unbarred coordinates  $x^\nu$  represent the actual (code) coordinates, and assume there exists a set of "well-behaved" harmonic coordinates

$$\nabla_\mu \nabla^\mu \bar{x}^\nu = 0 \quad \text{such that} \quad x^\nu = \bar{x}^\nu + \bar{f}^\nu(x^\nu)$$

where gradients are with respect to code coordinates, and  $f^\nu$  are some set of *scalar* functions; i.e. the above define a set of 4 scalar relations

- The functions  $f^\nu$  are going to represent the wave-like perturbations to well-behaved coordinates that we want to damp, and so we will require that each  $f^\nu$  satisfies a damped wave equation relative to the timelike normal vector field  $n^\mu$ :

$$\nabla_\mu \nabla^\mu f^\nu = \kappa \partial_\mu f^\nu \cdot n^\mu$$

where  $\kappa$  is a constant

### Damped Harmonic Coordinates

- Then, the source functions  $H^\nu$  in a generalized harmonic formalism are

$$\nabla_\mu \nabla^\mu x^\nu \equiv H^\nu = \kappa [n^\nu - \bar{n}^\nu] \quad \text{where} \quad \bar{n}^\nu = n^\mu \frac{\partial \bar{x}^\nu}{\partial x^\mu}$$

- Of course, we don't know what the transformation between the coordinates is, so let's just *choose* a form for the barred-normal in code coordinates; e.g. to "drive" the normal to the form it would have in geodesic coordinates, giving *algebraic conditions* on the source functions

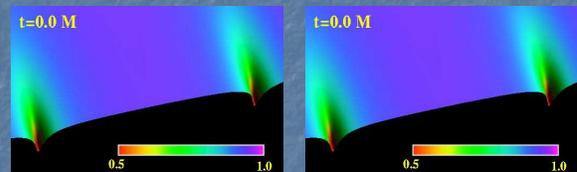
$$\bar{n}^\nu \equiv \left(\frac{\partial}{\partial t}\right)^\nu \Rightarrow H_i = -\kappa[\alpha + g_{ii}]; \quad H_i = -\kappa_i g_{ii}$$

- Note: this condition is quite similar to that recently proposed by Lindblom & Szilagyi [arXiv:0904.4873]; in fact their condition can be written in the above form with

$$\bar{n}^\nu \equiv \frac{1}{\alpha} \left(\frac{\partial}{\partial t}\right)^\nu + \log(\sigma/\sqrt{|h|}) n^\nu$$

### Damped Harmonic Coordinates

- Damped harmonic coordinates have so far not been needed in binary merger spacetimes, though have been of use in high-speed soliton collision simulations. The following shows the lapse function from identical sets of initial data; one with harmonic gauge (left), the other (left) beginning with harmonic, but then transitioning to damped harmonic



With harmonic coordinates when the waves in the lapse collide, the slice goes singular; with damped harmonic coordinates evolution continues through this point, and here the collision results in black hole formation (after that we switch to Harmonic again, as geodesic coordinates are not good in a black hole spacetime)

### Finding consistent initial data

- For the metric, the GH equations are second order PDEs, hence for the IBVP we need to specify the metric and its first time derivative at the initial time. How do we do so in a manner such that  $C^i=0$  and  $\partial C^i/\partial t=0$ ?
  - Many conceivable ways; the following is perhaps the most straight-forward, utilizing the many years of research gone into solving the ID problem in the ADM formalism:
- 1. solve the ID problem in the ADM formalism, giving  $\alpha, \beta, h_{ij}$  and  $\partial h_{ij}/\partial t$  at  $t=0$ .
- 2. choose  $H^i$  at  $t=0$ ; from the definition  $C^i=0$  this then gives  $d\alpha/dt$  and  $d\beta/dt$  at  $t=0$ .
- 3. given 1 & 2, we can construct  $g_{ij}$  and  $\partial g_{ij}/\partial t$

### Finding consistent initial data

- Step 2 on the previous slide directly gives  $C^i=0$ ; to see that  $\partial C^i/\partial t=0$  is also satisfied, one can show that the following is a geometric identity
 
$$\mathcal{M}_u = \nabla_u \nabla_v C^v - \frac{1}{2} n_u \nabla_v C^v$$
- where
 
$$\mathcal{M}_u \equiv (R_{uv} - \frac{1}{2} g_{uv} R - 8\pi T_{uv}) n^v$$
- i.e.,  $\mathcal{M}_i n^i$  is just the Hamiltonian constraint, and  $\perp \mathcal{M}_i$  are the momentum constraints.
- Thus, step 1 gives  $\mathcal{M}_i=0$ , and since  $C^i=0$ , all its spatial gradients will be zero, and then from the above identity we get that  $\partial C^i/\partial t$  must be zero at  $t=0$

### Example : Binary Black hole merger

- Summary of equations solved : Einstein equations in generalized harmonic form with constraint damping,

$$g^{\gamma\delta} g_{\alpha\beta;\gamma\delta} + 2g^{\gamma\delta} (\alpha g_{\beta\gamma})_{;\delta} + 2H_{(\alpha\beta)} - 2H_{\delta} \Gamma_{\alpha\beta}^{\delta} + 2\Gamma_{\beta\gamma}^{\gamma} \Gamma_{\alpha}^{\delta} + 8\pi(2T_{\alpha\beta} - g_{\alpha\beta} T) + \kappa(n_{\mu} C_{\nu} + n_{\nu} C_{\mu} - g_{\mu\nu} n^{\alpha} C_{\alpha}) = 0$$

with the following gauge equations

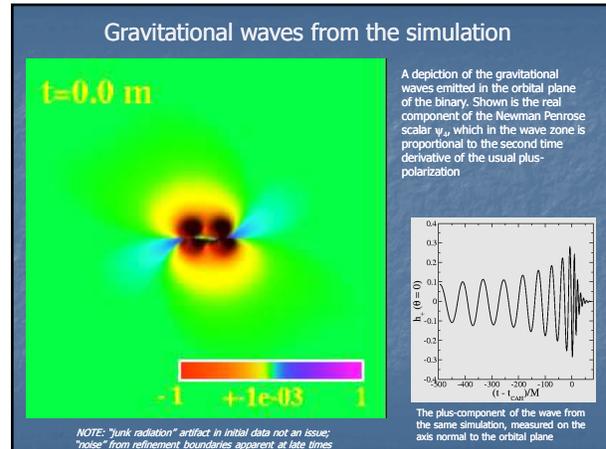
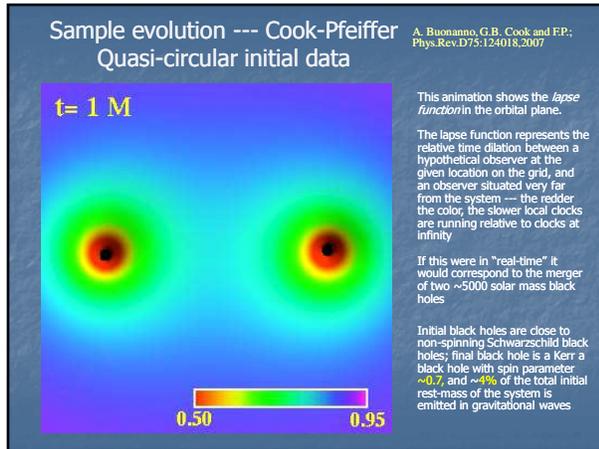
$$\nabla^{\mu} \nabla_{\mu} H_i = -\xi_i \frac{\alpha - 1}{\alpha^n} + \xi_i^{\alpha} \partial_{\alpha} H_i - n^{\mu}; \quad H_x = H_y = H_z = 0$$

- Will show a simulation of the merger of two equal-mass initially non-spinning black holes
- At the time solved the equations with 1-step NGS relaxation, though used maple to generate the residuals and Jacobians (would have been too "painful" to do it by hand in RNPL, in particular with excision)
  - highest resolution simulation took ~ 1 month CPU time on 128 nodes of the Lonestar cluster at UT Austin, in 2006 (took about 2 months wall time, because of 1-day run-time limits they had in place at the time)
  - Just in case you're skeptical that the above "innocent" looking equations couldn't possibly have taken so much compute power ...

the inner loop of the GH 3D code that is evaluated every iteration of every time step at every grid point in the domain; ~ 5000 lines of code (4,500 of which are maple 'optimized' fortran).

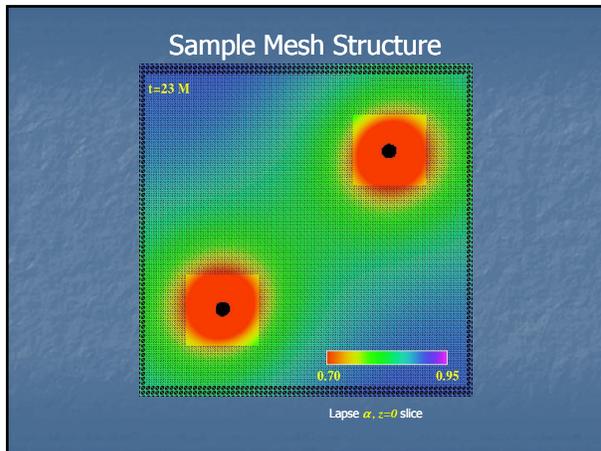
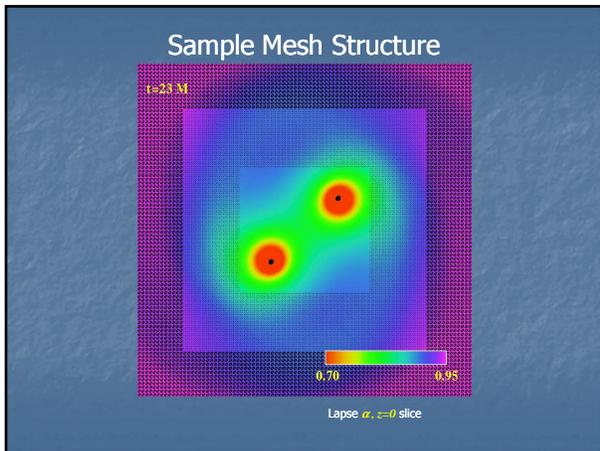
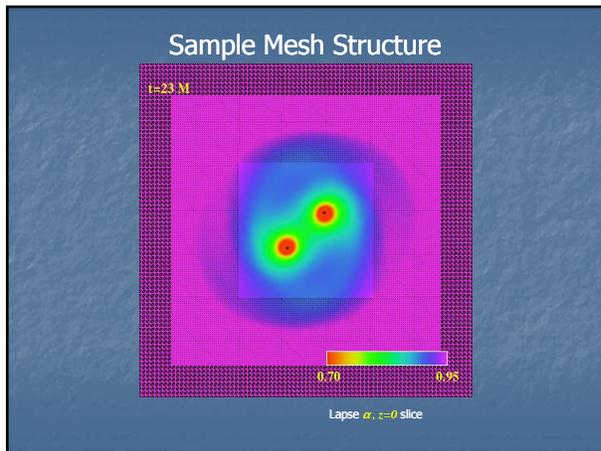
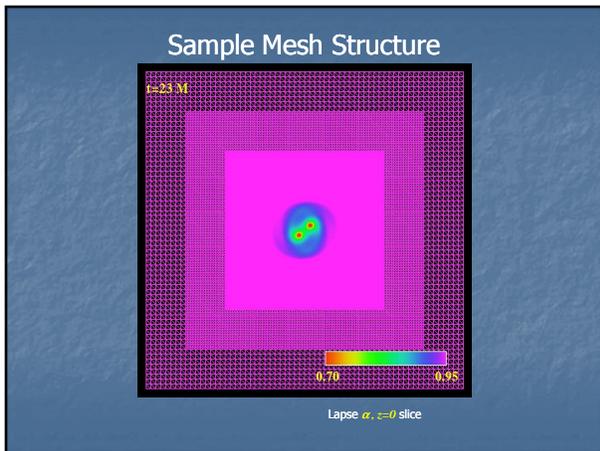
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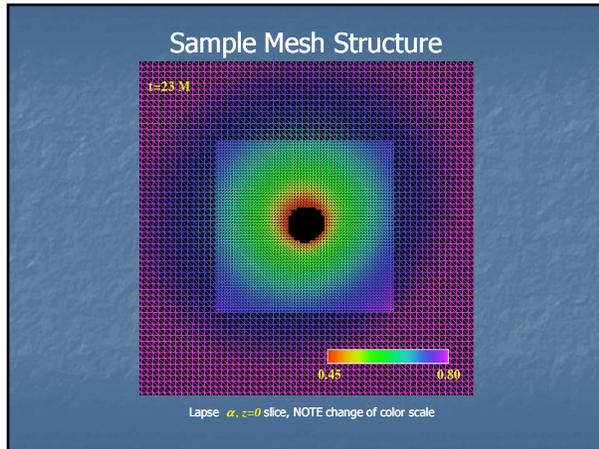
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- ### What does the merger wave represent?
- Scale the system to two **10 solar mass** ( $\sim 2 \times 10^{31}$  kg) BHs
    - radius of each black hole in the binary is  $\sim 30$  km
    - radius of final black hole is  $\sim 60$  km
    - distance from the final black hole where the wave was measured  $\sim 1500$  km
    - frequency of the wave  $\sim 200$  Hz (early inspiral) - 800 Hz (ring-down)

- ### What does the merger wave represent?
- fractional oscillatory "distortion" in space induced by the wave transverse to the direction of propagation has a *maximum* amplitude  $\Delta L/L \sim 3 \times 10^{-3}$ 
    - a 2m tall person will get stretched/squeezed by  $\sim 6$  mm as the wave passes
    - LIGO's arm length would change by  $\sim 12$  m. Wave amplitude decays like  $1/\text{distance}$  from source; e.g. at 10Mpc the change in arms  $\sim 5 \times 10^{-17}$  m (1/20 the radius of a proton, which is well within the ballpark of what LIGO is trying to measure!!)
  - despite the seemingly small amplitude for the wave, the energy it carries is enormous --- around  $10^{30}$  kg  $c^2 \sim 10^{47}$  J  $\sim 10^{54}$  ergs
    - peak luminosity is about  $1/100^{\text{th}}$  the Planck luminosity of  $10^{39}$  ergs/s !!
    - luminosity of the sun  $\sim 10^{33}$  ergs/s, a bright supernova or milky-way type galaxy  $\sim 10^{42}$  ergs/s
    - if all the energy reaching LIGO from the 10Mpc event could directly be converted to sound waves, it would have an intensity level of  $\sim 80$  dB





### Compactification

- Mentioned before that it is usually difficult to impose self-consistent boundary conditions. However, there is one simple solution to the problem : *compactification*
- Since in GR we know from the get-go that we are not going to have to deal with non-trivial coordinates in interesting situations, once might as well take the covariance of the theory to full advantage.
- Compactification here means performing an additional coordinate transformation on the metric to bring spatial infinity to a finite coordinate location on the computational domain (in principle could also do this with null infinity).
- We know exactly what the boundary conditions are here ... for an asymptotically flat spacetime, the metric is Minkowski.
- In the harmonic code of the previous example, the Cartesian-like coordinate are independently compactified via

$$\bar{x} = \tan\left(\frac{\pi x}{2}\right); \quad \bar{y} = \tan\left(\frac{\pi y}{2}\right); \quad \bar{z} = \tan\left(\frac{\pi z}{2}\right)$$

- This maps the barred physical coordinates from  $[-\infty, \infty]$  to the unbarred code coordinates spanning  $[-1, 1]$

### Compactification

- The one possible "problem" with this form of compactification is that the resolution gets very poor approaching the outer boundary
  - outward propagating waves are "blue-shifted" and slow down as they approach the outer boundary
  - eventually, for any finite resolution, the waves will become poorly resolved
  - with dissipation (which is needed with excision and AMR in any case), at this stage the waves then get dissipated away, so not a problem unless you want to keep tracking the waves
- Note however that the goal of this compactification was not to extract gravitational waves at infinity, rather, to it was to be able to place the physically correct boundary conditions for the evolution; thus gravitational waves still need to be extracted at a finite radius (null compactification could solve these issues)

t= 0 m

-1e-03      1e-03

t= 0 m

-1e-03      1e-03

"Scalar field collapse" binary merger example; left, code coordinates, right, same data but transformed to barred-coordinates

### Comment about the solution

- In a GR code, the end product of evolution is the metric; the things we often use to describe or understand the geometry, e.g. black holes, gravitational waves, geodesics, energies, etc. need to be extracted from the metric.
- Of particular importance, in particular if we are using excision, is the black hole --- if a "black hole" is not a distinct or separate entity in the simulation that we explicitly model, how do we know where it is, if it's even there, and what then are we excising?
- Answer : we search for an apparent horizon---a marginally outer-trapped 2-sphere
- A trapped surface is one in which the expansion of null wavefronts moving "away" from the surface is everywhere non-positive
- The apparent horizon (AH) is the outer-most trapped surface in a volume of spacetime that has them, and on the AH the outward null expansion is zero
- If cosmic censorship holds, an AH will always be inside the event horizon (EH) of a black hole; in situations where both these entities have been studied, the AH is for most of the evolution almost exactly coincident with the EH
- One can find the apparent horizon locally (i.e. at each time), and this then guides excision. Properties of the AH also give valuable information about the black hole, e.g. its mass and spin.

t= 3 m

Sample AH shapes from BH merger

### Concluding remarks

- Future of numerical relativity? In my biased and humble opinion:
  - there is much still to uncover about the general relativity using numerical methods, however, for better or worse, the future of NR as a vibrant, exciting and *fundamental* branch of science is intimately tied to the gravitational wave astronomy endeavor
  - In that regard, as of now, the future is bright! LIGO has reached initial design sensitivity, a remarkable feat, and Advanced LIGO is funded. Now it's up to the universe to be kind, and send some gravitational waves this way! (an order of EM counter-part on the side would be nice too).
  - What role will NR specifically play here?
    - The "in principle" vacuum problem may be "solved", at least those aspects that are amenable to full numerics as of today, namely a modest number of orbits, non-extreme mass ratios
      - though near-extremal mergers, and orbital eccentricity, may still be harboring some surprises

### Concluding remarks

- What role will NR specifically play here?
  - Still on the vacuum problem, there is of course much work that needs to be done to produce useful templates of all the relevant parameter space for GW detection --- will be a several year endeavor at least
  - The frontier for novel and interesting research is to understand what happens with strong-field GR/matter interactions [for an excellent introduction to the cutting-edge computational methods and open questions, see this summer school]; e.g. binary neutron star mergers, neutron star/BH mergers, pulsars, accretion disks

### Concluding remarks

- What role will NR specifically play here?
  - GW detection is sometimes "sold" as opening up a new window to the universe, and as history tells, we should thus expect the unexpected.
  - This cannot happen off-the-bat with (Advanced) LIGO, as, for good reason, the detector is too biased with predictions of what it is expected to see
  - The "best" (most exciting) scenario, again IMHO, would be detections that are unexplained (excess power detections, in particular if there are EM counterparts) or signals that don't quite fit expectations (e.g. separate detections with inspiral and burst templates, though the best fit parameters don't match a single binary inspiral and merger)
  - If such events are detected, then GW source modeling will enter a very exciting era, as historically the most significant advances in science have come from a vibrant interplay between theory and new observational/experimental data
  - In all then, as a research endeavor, which by definition is uncertain and flourishes off unanswered questions, the future couldn't be brighter!