M2-branes and Supersymmetric Chern-Simons Theories

1 Exercises

- **Non-abelian $\mathcal{N} = 2$ Chern-Simons**

  In the first lecture, I focused on the abelian theory for simplicity. Check that the $\mathcal{N} = 2$ superspace action,

  $$\frac{k}{2\pi} \int d^4\theta \int dt \ \mathrm{Tr} \left[V \bar{D}^{\alpha} (e^{-tV} D_{\alpha} e^{tV}) \right] ,$$

  reproduces the $\mathcal{N} = 2$ $U(N)$ Chern-Simons action in components, proving that the latter is indeed supersymmetric.

- **$\mathcal{N} = 3$ Chern-Simons-matter**

  Check that the $\mathcal{N} = 3$ Chern-Simons term preserves exactly $\mathcal{N} = 3$ supersymmetry when added to the $\mathcal{N} = 4$ Yang-Mills-matter theory in 3d. You may find it useful to work in components or 3d $\mathcal{N} = 1$ notation, and show that the Spin(4) R-symmetry is broken exactly to $\mathrm{Spin}(3)_R$.

  In the IR, the whole $\mathcal{N} = 4$ Yang-Mills term is an irrelevant operator. What is the bosonic potential, after integrating out massive fields, of the resulting pure $\mathcal{N} = 3$ Chern-Simons-matter theory?

- **Generation of YM terms on the moduli space**

  Consider the $\mathcal{N} = 3$ $U(1) \times U(1)$ Chern-Simons-matter theory with levels k and $-k$ and a single bifundamental hyper multiplet (the supersymmetry is in fact enhanced to $\mathcal{N} = 4$ for this choice of gauge group and matter content, but this will not be relevant for this exercise). The moduli space is parametrized by the VEVs of the scalars in the hypermultiplet. Find the effective action on the moduli space after integrating out the massive fields.

You should find that a YM kinetic term is generated for the diagonal combination of the CS gauge fields, which is unbroken on the moduli space. Dualize this gauge field, under which no matter is charged, to a scalar (ie. such that $\frac{g_{YM}^2}{4\pi}\, d\tau = \star F$).

What is the periodicity of the scalar, τ? How does it transform under gauge transformations? Set $\tau = 0$ to fix the gauge symmetry that is broken on the moduli space. Does this completely fix the gauge symmetry? What is the moduli space after quotienting by any remaining gauge symmetry?

1.1 Optional exercise

- **Chern-Simons terms from $(1,k)$ fivebranes**

  A $(1,k)$ fivebrane can be thought of as a bound state of k D5 branes and one NS5 brane. When the IIB coupling is weak, the NS5 brane is parametrically heavier. The k D5 branes can break on an NS5, causing it to bend in between the two ends of the D5 branes, where the D5 bind with it to form a $(1,k)$ 5-brane. The field theory on a D3 brane stretched between an NS5 brane and such a fivebrane web preserves $\mathcal{N}=2$ supersymmetry in the three noncompact dimensions.

  The resulting effective field theory in 3d is a $U(1)$ $\mathcal{N}=2$ Yang-Mills theory with k conjugate flavor pairs, Φ_i of charge 1, and $\tilde{\Phi}_i$ of charge -1, $i = 1, ..., k$ (there is also an adjoint chiral matter field, but in this abelian theory it is completely decoupled and will not affect us). The breaking of the D5 branes corresponds to turning on real masses

$$\int d^4\theta \ \left(\Phi_i^\dagger e^{m\theta\bar{\theta}}\Phi_i + \tilde{\Phi}_i^\dagger e^{m\theta\bar{\theta}}\tilde{\Phi}_i\right).$$

  Note that conjugate fields are given the *same* sign real mass - this is the opposite of the masses they are given on the Coulomb branch of the $U(1)$ theory.

  Show that integrating out the massive flavors generates a Chern-Simons term at level k.