MALDACENA 1.5 (group 11)

Make the wick rotation : $\tau \to it$

$$dS^2 = dt^2 + \cos^2 t \, d\Omega_3^2$$

$\Rightarrow R = 12$ (Ricci scalar)

$$d\Omega_3^2 = d\psi^2 + \sin^2\psi \left[d\theta^2 + \sin^2\theta \, d\varphi^2 \right]$$

$$\sqrt{g} = \cos^3 t \sin^2\psi \sin\theta$$

So, the first integral in the action is:

$$\int_{\Sigma_4} \sqrt{g}\,(R+6) = 18 \int_0^{\pi} d\psi \int_0^{\pi} d\theta \int_0^{2\pi} d\varphi \sin^2\psi \sin\theta \int_0^{t_c} \cos^3 t \, dt.$$

where $t_c = \text{const.}$ (cutoff)

$$= +36\pi^2 \left[\frac{3}{4} \sin t_c + \frac{1}{12} \sin(3t_c) \right]$$

now use : $t_c = -i\tau_c$

① $$\int_{\Sigma_4} \sqrt{g}\,(R+6) = 36\pi^2 \, i \left[\frac{3}{4} \sinh \tau_c + \frac{1}{12} \sinh(3\tau_c) \right]$$

h_{ab} is given by:

$$ds_h^2 = d\Omega_3^2 \cdot \cos^2 t \quad , \sqrt{h} = \cos^3 t_c \sin^2\psi \sin\theta$$

$$K = \frac{-1}{2} h^{ab} \partial_t h_{ab} = + \cancel{\frac{1}{2}} \cdot \cancel{2} \cdot \frac{\cos t \sin t}{\cos^2 t} \cdot 3 \Big|_{t=t_c} =$$

$$= 3 \frac{\sin t_c}{\cos t_c}$$

Now,

$$2\int_{\partial\Sigma_4} \sqrt{h} \cdot K = 2\cos^{\cancel{3}} t_c \int_0^{\pi} d\psi \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \, \sin^2\psi \sin\theta \cdot 3 \frac{\sin t_c}{\cancel{\cos t_c}}$$

$$= 12\pi^2 \sin t_c \cos^2 t_c$$

now rotating back:

$$2\int_{\partial\Sigma_4} \sqrt{h} K = 12\pi^2 i \cosh^2 \tau_c \sinh \tau_c \quad (2)$$

$A(\tau_c)$

$$\Rightarrow S = -\frac{R_{ds}^2}{\cancel{16}_4 \pi G_N} \cancel{12}^3 i\pi^2 \left[\cosh^2\tau_c \sinh\tau_c + \frac{9}{4}\sinh\tau_c + \frac{1}{4}\sinh(3\tau_c)\right]$$

$$\Rightarrow e^{iS} = e^{-S_E}$$

Note that there is no convergent part of S unlike the AdS case from problem 1.4.

The reason why S diverges is that in dS, taking the limit of time-like infinity means integrating over exponentially growing space-time volumes; and there is ~~[illegible]~~ infinite proper distance to time-like infinity. ~~[illegible]~~

(There is no geodesic that goes from the bulk to the boundary in finite proper time).

CREMINELLI 2. (group 11)

$$\langle \zeta_{\vec{k}_1} \dots \zeta_{\vec{k}_n} \rangle = (2\pi)^3 \delta(\sum_i \vec{k}_i) F(k_i)$$

Due to homogeneity & isotropy of de Sitter space, translations should be a symmetry of this theory. In other words, adding: $\vec{x}_i \to \vec{x}_i + \vec{a}_i$ where $\vec{a}_i = \text{const.}$ should not change the RHS. of the above equation. On the LHS, this induces a phase $\sim e^{-i\vec{a}_i \cdot \sum_i (\vec{k}_i)}$ which should be 0; this gives $\delta(\sum_i (\vec{k}))$ on the RHS.

By the same symmetries, F can only be a function of the magnitudes of $|\vec{k}_i|$, not on their directions.

Finally, we can apply dimensional analysis to get the power of k in F.

Since $\zeta_{\vec{k}_i}$ has mass dim. -3, the LHS has dimension $-3n$. On the RHS, the δ-function has dim. -3, so F has to be $+3-3n =$
$= -3(n-1)$.

This works under the assumption that F is a homogeneous function of k_i and that there are no other dimensionful parameters on the RHS.

Group 11.

1 Problem 3.5 (Susskind)

In general, the transition matrix for three vacua is

$$\Gamma = \begin{pmatrix} -\gamma_{10} - \gamma_{20} & \gamma_{01} & \gamma_{02} \\ \gamma_{10} & -\gamma_{01} - \gamma_{21} & \gamma_{12} \\ \gamma_{20} & \gamma_{21} & -\gamma_{02} - \gamma_{12} \end{pmatrix}.$$

Since the vacuum 0 is terminal, $\gamma_{10} = \gamma_{20} = 0$ and the matrix takes form

$$\Gamma = \begin{pmatrix} 0 & \gamma_{01} & \gamma_{02} \\ 0 & -\gamma_{01} - \gamma_{21} & \gamma_{12} \\ 0 & \gamma_{21} & -\gamma_{02} - \gamma_{12} \end{pmatrix}. \tag{1}$$

Clearly, the matrix is singular and the vector

$$v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

satisfies $\Gamma v = 0$, and therefore v is the normalized eigenvector with eigenvalue $\lambda = 0$.

Now we will show that two remaining eigenvalues are real and negative, assuming all γ factors in (1) are strictly positive. The characteristic polynomial is

$$\det(\Gamma - \lambda I) = -\lambda\left[\lambda^2 + \lambda(\gamma_{01} + \gamma_{02} + \gamma_{12} + \gamma_{21}) + (\gamma_{01}\gamma_{12} + \gamma_{01}\gamma_{02} + \gamma_{02}\gamma_{21})\right].$$

Let λ_1, λ_2 be two zeroes of the polynomial in square brackets. First we show that they are real. In order to do so, we want to show that

$$(\gamma_{01} + \gamma_{02} + \gamma_{12} + \gamma_{21})^2 - 4(\gamma_{01}\gamma_{12} + \gamma_{01}\gamma_{02} + \gamma_{02}\gamma_{21}) > 0.$$

We have

$$\begin{aligned} 0 &< (\gamma_{01} + \gamma_{12} - \gamma_{02} - \gamma_{21})^2 + 4\gamma_{12}\gamma_{21} \\ &= \gamma_{01}^2 + \gamma_{12}^2 - \gamma_{02}^2 - \gamma_{21}^2 + 2(\gamma_{01}\gamma_{12} + \gamma_{02}\gamma_{21} + \gamma_{12}\gamma_{21} - \gamma_{01}\gamma_{02} - \gamma_{01}\gamma_{21} - \gamma_{12}\gamma_{02}) \\ &= (\gamma_{01} + \gamma_{02} + \gamma_{12} + \gamma_{21})^2 - 4(\gamma_{01}\gamma_{12} + \gamma_{01}\gamma_{02} + \gamma_{02}\gamma_{21}). \end{aligned}$$

Now we can apply Vieté's formulas to find

$$\begin{aligned} \lambda_1 + \lambda_2 &= -(\gamma_{01} + \gamma_{02} + \gamma_{12} + \gamma_{21}) < 0, \\ \lambda_1\lambda_2 &= \gamma_{01}\gamma_{12} + \gamma_{01}\gamma_{02} + \gamma_{02}\gamma_{21} > 0, \end{aligned}$$

from which follows that $\lambda_1, \lambda_2 < 0$.

Finally we show that the probability of finding the vacua 1 and 2 decreases to zero, but the total number of the vacua grows, as long as all transition amplitudes are small. To do it, we add the number of vacua 1 and 2 together and we denote it by N_{12}. We arrive at the problem of decay $1, 2 \mapsto 0$, with some cumulative rate of decay $\gamma_{0,12}(t)$. However, this rate depends on time now, since we can write

$$\gamma_{0,12}(n) = \gamma_{01}P(\text{we are at 1 at step } n) + \gamma_{02}P(\text{we are at 2 at step } n)$$

and estimate

$$\min(\gamma_{01}, \gamma_{02}) \le \gamma_{0,12}(n) \le \max(\gamma_{01}, \gamma_{02})$$

for each $n = 0, 1, 2, \ldots$. We arrived at the problem discussed during the lecture so we can directly write the probabilities of finding vacua 0 and 1 or 2,

$$\begin{aligned} P_0(n+1) &= P_0(n) \cdot (1 + \gamma_{0,12}(n)), \\ P_{12}(n+1) &= P_{12}(n) \cdot (1 - \gamma_{0,12}(n)). \end{aligned}$$

The probability of finding the vacuum 1 or 2 decreases exponentially,

$$P_{12}(n) \le (1 - \min(\gamma_{01}, \gamma_{02}))^n, \qquad n = 0, 1, 2, \ldots,$$

but the total number of the vacua is

$$N_{12}(n) = P_{12}(n) \cdot 2^{3n} \ge [8\,(1 - \max(\gamma_{01}, \gamma_{02}))]^n,$$

and grows infinitely, as long as $\gamma_{0,12}(n) < 7/8$, which is satisfied since $\gamma_{0,12}(n) < \gamma_{01}, \gamma_{02}$ are small.

SILVERSTEIN 3 (group 11)

$$S = -\int d^4x \sqrt{-g}\left[\underbrace{\frac{\phi^4}{\lambda}\sqrt{1-\frac{2}{\phi^4}(\partial_\mu\phi\,\partial_\nu\phi)g^{\mu\nu}} - V(\phi)}_{\mathcal{L}} \right]$$

Euler-Lagrange eqn:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \nabla_\alpha \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{\delta V}{\delta \phi}$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha\phi)} = \cancel{2}\cdot\frac{1}{\cancel{2}}\;\frac{-\frac{\lambda}{\phi^4}\,\delta^\alpha_\mu\,\partial_\nu\phi\, g^{\mu\nu}}{\sqrt{1-\frac{2}{\phi^4}\partial_\mu\phi\,\partial_\nu\phi\, g^{\mu\nu}}}$$

$$\Rightarrow \nabla_\alpha \frac{\partial \mathcal{L}}{\partial(\partial_\alpha\phi)} = \nabla_\mu\left[\frac{-\partial_\nu\phi\, g^{\mu\nu}}{\sqrt{1-\frac{2}{\phi^4}\partial_\mu\phi\,\partial_\nu\phi\, g^{\mu\nu}}} \right]$$

$\Rightarrow$ The equation of motion for the field ϕ is:

$$\boxed{\nabla_\mu\left(\frac{\nabla^\mu\phi}{\sqrt{1-\frac{2}{\phi^4}\nabla_\mu\phi\nabla^\mu\phi}}\right) + \frac{4\phi^3}{\lambda}\sqrt{1-\frac{2}{\phi^4}\nabla_\mu\phi\nabla^\mu\phi} + \frac{2}{\phi}\,\frac{\nabla_\mu\phi\nabla^\mu\phi}{\sqrt{1-\frac{2}{\phi^4}\nabla_\mu\phi\nabla^\mu\phi}} - \frac{\delta V}{\delta\phi} = 0}$$

~~Let~~ Let us now assume that ϕ consists of a homogeneous part $\phi_0(t)$, which has no gradient, and a ~~small~~ perturbation $\delta\phi(t, \bar{x})$, which depends both on space and time: $\phi(t, \bar{x}) = \phi_0(t) + \delta\phi(t, \bar{x})$.

We can get the EOM for $\delta\phi$ by perturbing the EOM for ϕ, and taking gradients $\nabla_i \phi_0 = 0$, and keeping only the first-order terms in this expansion.

TO BE CONTINUED

Problem Solution: Zaldarriaga 5 (by Group 11)

(Dated: July 29, 2011)

The aim of this problem is to investigate how the CMB–power-spectrum acoustic peaks depend on the contents of the universe. For this purpose, we use CAMB online tool (http://lambda.gsfc.nasa.gov/), to generate three figures, each of which illustrates how variations in $\Omega_b h^2$, $\Omega_{dm} h^2$, and Ω_k, respectively, affect the temperature power spectrum.

The plots show WMAP-7 measurements of the TT power spectrum with corresponding error bars, and the CAMB-generated TT power spectra, for three different cosmologies. In each figure, we vary only one of the above mentioned Ω parameters, while holding all other parameters fixed to the best-fit WMAP-7 values. The black curves represent the best-fit, the green curves are produced using a value for Ω that is smaller than the best-fit value, and the red ones using a larger value (see Table I for numbers).

varied parameter	$\Omega_b h^2$ (Fig. 1)	$\Omega_{dm} h^2$ (Fig. 2)	Ω_k (Fig. 3)
black (best fit)	0.0226	0.114	0
green	0.0126	0.08	-0.1
red	0.0326	0.214	0.1

TABLE I: Parameters used to generate the theoretical TT power spectra for Figures in this solution.

We see that adding baryons increases the amplitude of the odd peaks (relative to the even ones); this is "baryon loading": adding baryons deepens the potential wells, and enhanses the amplitude of the "compressed" modes (odd peaks), relative to those that were most diluted at the time of last scattering (even peaks). Reducing dark matter content reduces the driving force for the acoustic oscillations, and so the overall amplitude of the acoustic peaks falls down sharply. Finally, changing the curvature changes the angular diameter distance to the last scattering surface, and thus shifts the position of the first acoustic peak (adding positive curvature, for a fixed physical distance, leads to a smaller angular scale on the sky, and the first peak shifts to larger L multipoles; this is equivalent to reducing the amount of $\Omega_\Lambda = 1 - (\Omega_k + \Omega_{dm} + \Omega_b)$).

From these plots, we can guessestimate the precision with which each of these three cosmological parameters is measured using just WMAP-7, by seeing how much wiggle-room we have before these curves no longer seem to fit the data. We see that the level at which the best-fit values can vary, and still be acceptable at 1-σ confidence level is, very roughly,

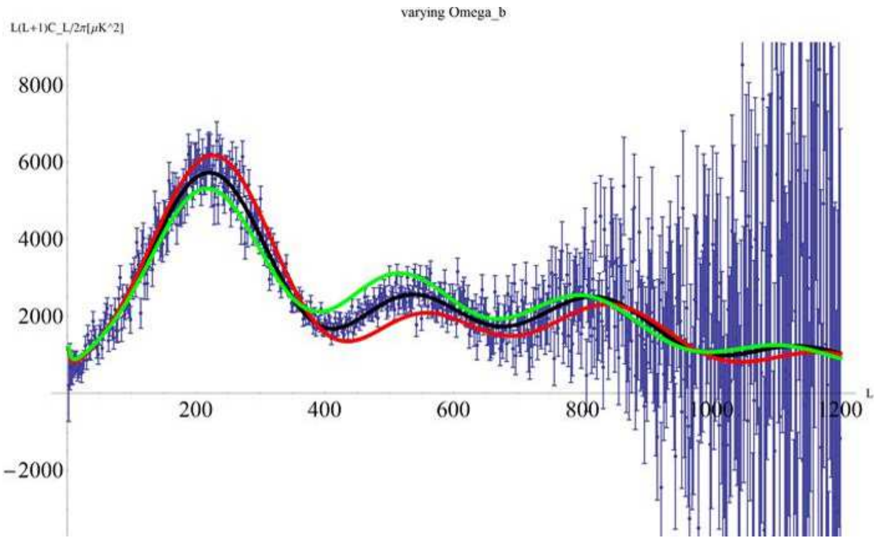

FIG. 1: Varying Ω_b.

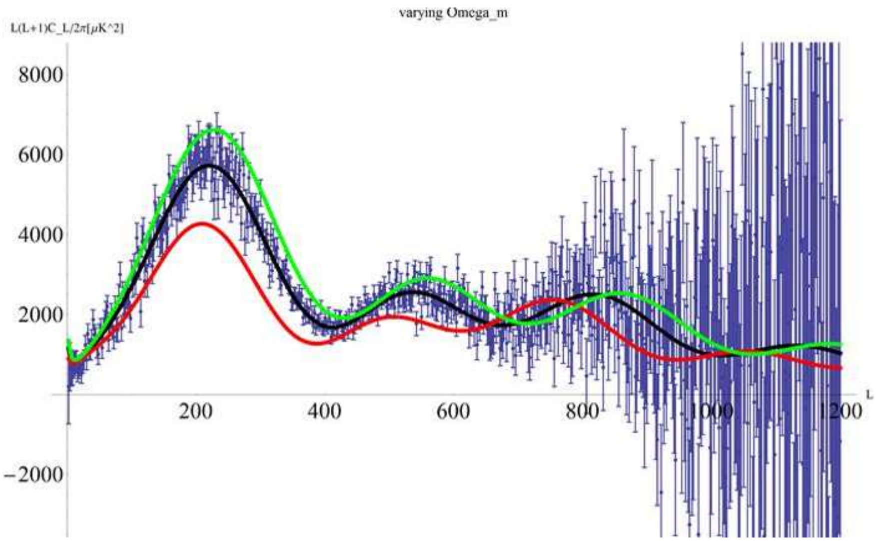

FIG. 2: Varying Ω_{dm}.

5%. For a more precise analysis, see Komatsu et al., 2010.

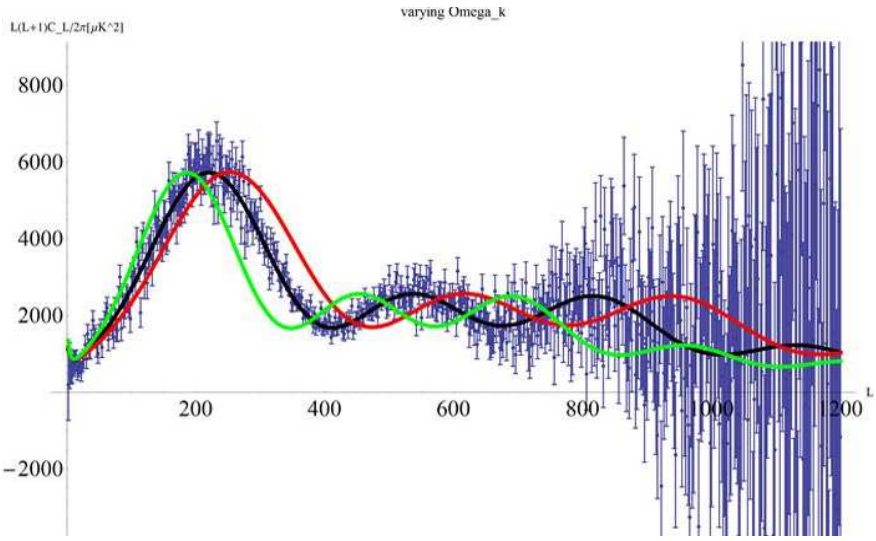

FIG. 3: Varying Ω_k.

References:

CMB tutorial: [http://background.uchicago.edu/ whu/intermediate/intermediate.html]

Komatsu et al., 2010 [arXiv:1001.4538]