GROUP 12

Maldacena 3

(a) $S = \int d^4x \sqrt{g}\; \partial_\mu\phi\, \partial_\nu\phi\; g^{\mu\nu}$

$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2}$$

$$\Rightarrow \quad S = \int d^4x \; \frac{\eta^{-2}}{2} (\partial\phi)^2$$

$$\Rightarrow \quad \phi_k'' - \frac{2}{\eta}\phi_k' + k^2\phi_k = 0$$

$\phi_k \sim e^{ik\eta}$ @ $\eta \to -\infty$

$\phi_k \sim \phi_0$ @ η_c : cuts off the integral for S = finite.

(b) $\cancel{\phi_k(\eta) = \phi_0 (1 - ik(\eta - \eta_c)) e^{ik(\eta-\eta_c)}}$

Minkowski now

$$S_{classical} = \int d\eta\, d^3x \; \frac{\eta^{-2}}{2} (\partial\phi)^2$$

$$\cancel{\Rightarrow S_{cl}[\phi] = \int d^3k \frac{k^2}{2\eta_c} \phi_0(k)\phi_0(-k)}$$

From F.T.

$$\phi(x,\eta) = \int d^3k \; e^{i\underline{k}\cdot x} \phi_{\underline{k}}$$

$\Rightarrow \int d^3k \, d^3k' \, d^3x$ in action gives

$$\int d^3x \Rightarrow \delta(\underline{k} + \underline{k}')$$

$$\Rightarrow \int d^3k' \rightarrow \underline{k}' \rightarrow -\underline{k}$$

$$\phi_k^{cl}(\eta) = \phi_0(k) \left[\frac{(1 - ik\eta)}{(1 - ik\eta_c)} \right] e^{ik(\eta - \eta_c)}$$

correct sol^n with b.c.'s

$$(\partial\phi)^2 = \phi'^2 - k^2\phi$$

$$\phi' = \phi_0 \frac{k^2\eta^2}{(1 - ik\eta_c)} e^{ik(\eta - \eta_c)}$$

$$\Rightarrow S = \int_{-\infty}^{\eta_c} d\eta \, \frac{\eta^{-2}}{2} \int d^3k \; \phi_0(\underline{k})\,\phi_0(-\underline{k})$$

$$\left[\frac{k^4\eta^2}{1+k^2\eta_c^2} - k^2\left(\frac{1+k^2\eta^2}{1+k^2\eta_c^2}\right) \right]$$

$$\Rightarrow S = \int_{-\infty}^{\eta_c} d\eta\, d^3k \, \frac{(-k^2\eta^{-2})}{(1+k^2\eta_c^2)} \; \phi_0(\underline{k})\,\phi_0(-\underline{k})$$

$$\boxed{S = \int d^3k \; \frac{k^2}{\eta_c(1+k^2\eta_c)} \; \phi_0(\underline{k})\,\phi_0(-\underline{k})}$$

(c) $$e^{\,i\int d^3k \, \frac{k^2}{\eta_c(1+k^2\eta_c)} \phi_0(\underline{k})\phi_0(-\underline{k})}$$

(d) Euclidean AdS

$$ds^2 = \frac{1}{z^2}\left(dz^2 + dx^2\right) \; ; \; z \in [-\infty, 0]$$

c.f. Euclidean dS

$$ds^2 = -\frac{1}{z^2}\left(dz^2 + dx^2\right)$$

$$\phi_{\underline{k}}'' - \frac{2}{z}\phi_{\underline{k}}' - k^2\phi = 0$$

but also same as above $\boxed{k \to -ik}$

$$\Rightarrow \phi_k \simeq \phi_b(\underline{k}) \left[\frac{1-kz}{1-kz_c}\right] e^{k(z-z_c)}$$

decays as $z \to -\infty$

Check in action

$$S_{cl} = (-i)^5 \int d^3k \, \frac{k^2}{z_c(1-k^2z_c)} \phi_b(\underline{k})\phi_b(-\underline{k})$$

$$\boxed{S = -i \int d^3k \, \frac{k^2}{z_c(1-k^2z_c)} \phi_b(\underline{k})\phi_b(-\underline{k})}$$

Euclidean $Z = e^{-S_{cl}}$

$$= e^{i\int d^3k \, \frac{k^2}{z_c(1-k^2z_c)} \phi_b(\underline{k})\phi_b(-\underline{k})}$$

- Looks like Minkowski dS with Wick rotation on <u>space</u>

makes sense since

AdS warps on space co-ord
dS "warps" in time co-ord

Euclidean makes space = time.

So rotating <u>space</u> (and overall minus) sends dS $\rightarrow$ E AdS

- Main difference in action is denominator

$$\frac{1}{(1-k^2 z_c)} \quad \text{not} \quad \frac{1}{(1+k^2 \eta_c)}$$

Which has change in sign, but for small, -ve z_c η_c or taking cut-off to zero this difference vanishes to first order.

(d)

Wave fn

$$\Psi(x) = e^{iS}$$

$$\approx e^{iS_{cl}}$$

by saddle point

$$S = \int d^4x \, \mathcal{L}$$

$\delta S = 0$ satisfies e.o.m.

$\Rightarrow$ S_{cl} has $\delta S = 0$

• Taylor expanding around this

$$\Psi(\phi) = \Psi(\phi_{cl}) + \left.\frac{\delta \Psi}{\delta \phi}\right|_{\phi_{cl}} (\phi - \phi_{cl})$$

$$+ \frac{1}{2} \frac{\delta^2 \Psi}{\delta \phi^2} (\phi - \phi_{cl})^2$$

$$\frac{\delta \Psi}{\delta \phi} = \frac{\delta \Psi}{\delta S} \frac{\delta S}{\delta \phi}$$ but $\delta S = 0$ @ classical

$$\cancel{\frac{\delta^2\Phi}{\delta\phi^2} = \frac{\delta^2\Phi}{\delta\phi^2}} \quad \frac{\delta^2\Phi}{\delta\phi^2}\bigg|_{cl} \overset{*}{=} \frac{\delta}{\delta\phi}\left[\frac{\delta\Phi}{\delta S}\frac{\delta S}{\delta\phi}\right]\bigg|_{cl}$$

$$\Rightarrow \left[\left(\frac{\delta S}{\delta\phi}\right)^2 \frac{\delta^2\Phi}{\delta S} + \frac{\delta\Phi}{\delta S}\frac{\delta^2 S}{\delta\phi^2}\right]\bigg|_{cl} \qquad \cancel{= \frac{\delta}{\delta S}\frac{\delta S}{\delta\phi}\left[\frac{\delta\Phi}{\delta S}\frac{\delta S}{\delta\phi}\right]} \quad \cancel{= \left(\frac{\delta S}{\delta\phi}\right)^2\frac{\delta^2\Phi}{\delta S^2}\bigg|_{S=S_{cl}}} \quad \cancel{= -S_{cl}^2 e^{iS_{cl}}}$$

$$\Rightarrow \Phi = \Phi_{cl} + iS_{cl}\Phi_{cl}\frac{\delta^2 S}{\delta\phi}\bigg|_{\phi=\phi_{cl}}(\phi-\phi_{cl})^2 + \cdots$$

$$= \Phi_{cl}\left(1 + iS_{cl}\frac{\delta^2 S}{\delta\phi}\bigg|_{\phi=\phi_{cl}}(\phi-\phi_{cl})^2\right) + \cdots$$

* gives us a 2-pt. f^n as the correction to the wave f^n. (by part (e))
Also makes sense since picked up an explicit imaginary piece.
Quantum wave f^n has non-zero imaginary piece

Creminelli Question One

We seek to calculate the two-point correlation function $\langle \phi_{\vec{k}} \phi_{\vec{k}'} \rangle$ on a de Sitter background with metric

$$\mathrm{d}s^2 = \frac{1}{H^2\eta^2}\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right), \tag{1}$$

where

$$\phi(\eta, \vec{x}) = \int \mathrm{d}^3\vec{x}\, \phi_{\vec{k}}(\eta) e^{i\vec{k}\dot{x}}, \tag{2}$$

and η represents conformal time. Both the action for the field,

$$S = \int \mathrm{d}^4\vec{x}\sqrt{-g}\left[-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m^2\phi^2\right], \tag{3}$$

and the metric 1 are invariant under dilations in both space and conformal time, $(\eta, \vec{x}) \to (\lambda\eta, \lambda\vec{x})$. It then follows from 2 that $\phi_{\vec{k}} \to \lambda^3\phi_{\vec{k}}$ and $\vec{k} \to \vec{k}/\lambda$. The metric is also invariant under spatial translations and so momentum $(\vec{k})$ is conserved. In this case the most general form of the two-point correlation function that respects both of these symmetries is

$$\langle \phi_{\vec{k}} \phi_{\vec{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}') \frac{F(k\eta)}{k^3}, \tag{4}$$

where the right hand side depends only on the magnitude of $\vec{k}$ in accordance with the rotational symmetry of the metric.

In order to calculate the tilt $(\langle \phi_{\vec{k}} \phi_{\vec{k}'} \rangle \sim k^{-3+(n_s-1)})$ we need to compute the k dependence of $\phi_{\vec{k}}$ explicitly. Our starting point is the Klein-Gordon equation in conformal time

$$\ddot{\phi}_{\vec{k}} + 2\frac{\dot{a}}{a}\dot{\phi}_{\vec{k}} + \left(k^2 + m^2a^2\right)\phi_{\vec{k}} = 0, \tag{5}$$

where $a = -1/H\eta$ is the scale factor and a dot denotes a derivative with respect to conformal time. We are interested in the solution for modes well outside the horizon at late times $(\eta \to 0)$ in which case we may neglect the term k^2. Inserting an ansatz solution of the form $\phi_{\vec{k}} \propto A(k)\eta^\alpha$ we find that

$$\alpha = \frac{3}{2}\left(1 \pm \sqrt{1 - \frac{4m^2}{9H^2}}\right), \tag{6}$$

where, since *eta* approaches zero from below, we take the negative sign in order to focus on the fastest growing mode (which tends to the constant mode in the case $m \to 0$). In this case the correlation function will scale like $\delta^{(3)}(\vec{k} + \vec{k}')A(k)^2\eta^2\alpha$ from which $A(k)$ may be determined by demanding consistency with the general form 4:

$$\langle \phi_{\vec{k}} \phi_{\vec{k}'} \rangle \propto \frac{\delta^{(3)}(\vec{k} + \vec{k}')}{k^3}(k\eta)^{3(1-\sqrt{1-\frac{4m^2}{9H^2}})}. \tag{7}$$

The tilt is then

$$n_s - 1 = 3\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}} \approx \frac{2m^2}{3H^2}\right). \tag{8}$$

GROUP 12

Silverstein Q4

Consider the $\bar{D}3$-$D3$ system in a warped throat, with metric given by

$$ds^2 = \left(\frac{r^2}{R^2}\right) dx_4^2 + \left(\frac{R^2}{r^2}\right) dr^2 + dS_5^2$$

where R is the AdS curvature radius. We work in units $M_p = 1$. The $\bar{D}3$ is at fixed coordinate $r = r_0$, while the $D3$ position is described by $r = r_1$, which will serve as our scalar inflaton field in 4 dimensions. The potential between the $\bar{D}3$-$D3$ is given by:

$$V_{\bar{3}3} = 2T_3 \left(\frac{r_0}{R^4}\right) \left(1 - \frac{1}{N}\left(\frac{r_0}{r_1}\right)^4\right)$$

N is the number of additional $D3$ branes at the bottom of the throat, which give rise to the warping according to:

$$N = \frac{R^4}{g_s \alpha'}$$

and T_3 is the $D3$ tension, given by:

$$T_3 = \frac{g_s}{\alpha'^4}$$

In order to be in control of the calculation, we will work in the large N limit, which can be understood in terms of the AdS/CFT correspondence. In the large N limit, the correspondence is exact, so that gravitational physics in the throat is dual to the CFT $U(N)$ gauge theory on the branes. If, in addition, we make this gauge theory strongly coupled, then strong-weak duality implies that the gravitational physics will be weakly coupled. Therefore we can calculate in classical gravity/perturbative string theory and trust our results.

Inflation occurs as the $D3$ slowly rolls in its potential, moving towards the $\bar{D}3$. When the branes collide at $r_1 = r_0$ they annihilate, causing reheating by production of radiation, and also production of what will become cosmic strings. In order that these string defects be produced in the correct manner, we require that inflation end as the $D3$ exits slow roll, falling off the steep end of its potential, with $\eta \approx 1$.

Fundamental strings in the UV have tension $T_s^{UV} = 1/\alpha'$. However, the strings are produced in the IR, where this energy scale is warped à la Randall-Sundrum to $T_s^{IR} = g^{00} T_s^{UV}$. This should be evaluated also at $r = r_0$, giving:

$$T_s^{IR} = \left(\frac{r_0}{R}\right)^2 \frac{1}{\alpha'} \tag{1}$$

We would like to fix r_0 and R from our general conditions for successful inflation, and thus fix the cosmic string tension in terms of the fundamental parameters g_s and α'. In the calculation that follows we will typically drop $\mathcal{O}(1)$ coefficients to get order of magnitude estimates and parameter scalings only.

Firstly, we will use the 2-point scalar perturbation power spectrum:

$$P_{\zeta\zeta} \sim \frac{H^4}{\dot{r}_1^2} \sim \frac{H^2}{\epsilon}$$

where the slow roll parameter $\epsilon \sim \left(\frac{V'}{V}\right)^2$. This should also be evaluated at $\bar{D}3$-$D3$ annihilation $r_1 = r_0$. The amplitude of the scalar power spectrum is fixed by the CMB to be of order 10^{-10}, so we have:

$$\frac{H^2}{\epsilon}|_{r_1=r_0} \sim 10^{-10}$$

The inflaton potential dominates the expansion, so that $H^2 \approx \frac{1}{3}V$. Working to leading order in N, some simple algebra shows that:

$$\frac{1}{g_s\alpha'^6}R^4 r_0^2 \sim 10^{-10} \tag{2}$$

Now we use our earlier condition that $\eta \sim \mathcal{O}(1)$ at the end of inflation, giving:

$$|\frac{V''}{V^2}|_{r_1=r_0} \sim 1$$

To leading order in N this gives:

$$\frac{r_0^6}{\alpha'^5} \sim 1 \tag{3}$$

Finally, we use Eqns. 2 and 3 to solve for r_0 and R, and substitute into Eqn. 1, giving the final result for the cosmic string tension:

$$T_s^{IR} = \frac{10^5}{\sqrt{g_s\alpha'}}\frac{1}{\alpha'}$$