Problem 1.2: Interactions

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A scalar field on an expanding universe evolves according to:

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}\nabla^2\phi + \frac{dV}{d\phi} = 0 \tag{1}$$

The correlation functions of any operator can be calculated in the IN-IN formalism:

$$\langle\Omega|O_H(t)|\Omega\rangle = \langle 0|\bar{T}\exp\left[+i\int_{-\infty}^{t} dt' H_I(t')\right] O_I(t)\; T\exp\left[-i\int_{-\infty}^{t} dt' H_I(t')\right]|0\rangle \tag{2}$$

where the interaction picture fields obey linear equations. In the present case, we have to evaluate the three point function for a massless scalar field with cubic self-interactions. The Hamiltonian density is:

$$H = a^3\left[\frac{\dot{\phi}^2}{2} + \frac{(\nabla\vec{\phi})^2}{2a^2} + V(\phi)\right] \tag{3}$$

So,

$$H_I(t') = \int d^3\vec{x}' V[\phi_I(t',\vec{x}')]a^3(t') \tag{4}$$

We this get

$$\langle\Omega|\phi_H(t,\vec{x_1})\phi_H(t,\vec{x_2})\phi_H(t,\vec{x_3})|\Omega\rangle = -i\int_{-\infty}^{t} dt'\langle 0|[\phi_I(t,\vec{x_1})\phi_I(t,\vec{x_2})\phi_I(t,\vec{x_3}), H_I(t')]|0\rangle \tag{5}$$

which becomes (since $H_I(t') = \int d^3\vec{x}' V[\phi_I(t',\vec{x}')]a^3(t')$):

$$\langle\Omega|\phi_H(t,\vec{x}_1)\phi_H(t,\vec{x}_2)\phi_H(t,\vec{x}_3)|\Omega\rangle = -i\int d^3\vec{x}'\int_{-\infty}^{t} dt' a^3(t')\langle 0|[\phi_I(t,\vec{x}_1)\phi_I(t,\vec{x}_2)\phi_I(t,\vec{x}_3), V(\phi_I(t',\vec{x}'))]|0\rangle \tag{6}$$

Taking Fourier transforms:

$$\langle\Omega|\phi_H(t,\vec{x}_1)\phi_H(t,\vec{x}_2)\phi_H(t,\vec{x}_3)|\Omega\rangle = \int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{d^3\vec{p}_2}{(2\pi)^3}\frac{d^3\vec{p}_3}{(2\pi)^3}\langle\Omega|\phi_H(t,\vec{p}_1)\phi_H(t,\vec{p}_2)\phi_H(t,\vec{p}_3)|\Omega\rangle e^{i(\vec{p}_1\vec{x}_1+\cdots+\vec{p}_3\vec{x}_3)} \tag{7}$$

Since ϕ_Is are interaction picture fields, they satisfy linear equations, so, they can be expanded as:

$$\hat{\phi}_I(t,\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^{3/2}}\left[\, e^{i\vec{p}\cdot\vec{x}}\;\hat{a}(\vec{p})\;\phi(\vec{p},t) \;+\; e^{-i\vec{p}\cdot\vec{x}}\;\hat{a}^\dagger(\vec{p})\;\phi^*(\vec{p},t)\right] \tag{8}$$

using this mode decomposition (assuming that $V = g\phi^3/3$), we get:

$$\langle\Omega|\phi_H(t,\vec{p}_1)\phi_H(t,\vec{p}_2)\phi_H(t,\vec{p}_3)|\Omega\rangle = (2\pi)^3\;\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)\;B_\phi(\vec{p}_1,\vec{p}_2,\vec{p}_3,t) \tag{9}$$

where B_ϕ is called the bispectrum and its connected part is given by:

$$B_\phi(\vec{p}_1,\vec{p}_2,\vec{p}_3,t)|_c = \frac{-ig}{3}\cdot 16\cdot\int_{-\infty}^{t} dt' a^3(t')\left[\phi_{\vec{p}_1}(t)\phi^*_{\vec{p}_1}(t')\cdot\phi_{\vec{p}_2}(t)\phi^*_{\vec{p}_2}(t')\cdot\phi_{\vec{p}_3}(t)\phi^*_{\vec{p}_3}(t') - C.C.\right] \tag{10}$$

If $c = \phi_{\vec{p}_1}(t)\phi^*_{\vec{p}_1}(t')\cdot\phi_{\vec{p}_2}(t)\phi^*_{\vec{p}_2}(t')\cdot\phi_{\vec{p}_3}(t)\phi^*_{\vec{p}_3}(t')$, then, the expression for connected contribution to the bispectrum is-

$$B_\phi(\vec{p}_1,\vec{p}_2,\vec{p}_3,t)|_c = \frac{-ig}{3}\cdot 16\cdot\int_{-\infty}^{t} dt' a^3(t')\,[c - c*] \tag{11}$$

now $(-i)(c - c*) = 2\mathrm{Im}(c)$, so we have:

$$B_\phi(\vec{p}_1,\vec{p}_2,\vec{p}_3,t)|_c = \frac{g}{3}\cdot 16\cdot\int_{-\infty}^{t} dt' a^3(t')\,[2\mathrm{Im}(c)] = \frac{g}{3}\cdot 16\cdot\int_{-\infty}^{\eta} d\eta' a^4(\eta')\,[2\mathrm{Im}(c)] \tag{12}$$

Since $c = \phi_{\vec{p}_1}(t)\phi^*_{\vec{p}_1}(t')\cdot\phi_{\vec{p}_2}(t)\phi^*_{\vec{p}_2}(t')\cdot\phi_{\vec{p}_3}(t)\phi^*_{\vec{p}_3}(t')$, at late times, if $\phi_{\vec{p}}(t)$ freezes (i.e. $\phi_{\vec{p}_1}(t') = \phi_{\vec{p}_1}(t)$ etc), then, $c(t')$, at late times will be $|\phi_{\vec{p}_1}(t')|^2\cdot|\phi_{\vec{p}_2}(t')|^2\cdot|\phi_{\vec{p}_3}(t')|^2$ which does not have an imaginary part, and so the integrand in the above expression vanishes when the mode functions freeze. This ensures that the bispectrum also freezes when the power spectrum does.

But this expression is finite only if we do not integrate till $\eta = 0$ due to logarithmic dependence on the cut-off.

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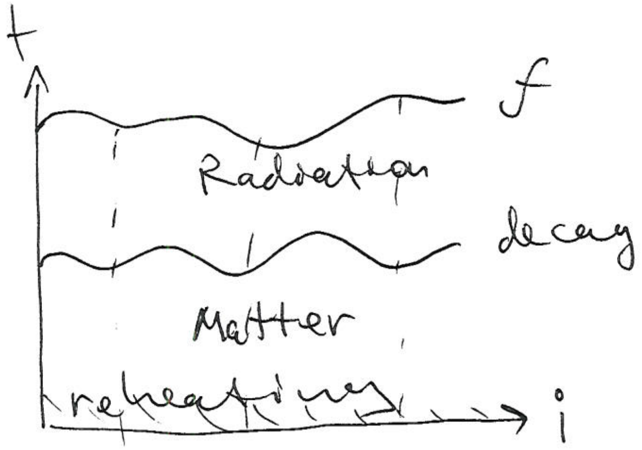


This is how we understood this problem: Some field has no influence on the metric on it's own, but it influences the moment of time when some other matter (say, inflaton) decays into radiation. In this case

$i \to \text{decay} \quad a \sim H^{-2/3}$

$\text{decay} \to f \quad a \sim H^{-1/2}$ this gives for the amplitude of perturbations

$\Gamma \sim H$

$$e^{\zeta(x)} = \frac{a(f)}{a(\text{decay})} \cdot \frac{a(\text{decay})}{a(i)} \sim H^{-\frac{1}{6}} \sim \Gamma^{-\frac{1}{6}}$$

$$\text{so } \zeta = \cancel{-\frac{1}{6}\frac{\delta\Gamma}{\Gamma}} = -\frac{1}{6}\left[\frac{\delta\Gamma}{\Gamma} - \frac{1}{2}\left(\frac{\delta\Gamma}{\Gamma}\right)^2 + \dots\right] =$$

$$= -\frac{1}{6}\left[\frac{r'}{r}\delta\varphi + \frac{1}{2}\frac{r''}{r}\delta\varphi^2 - \frac{1}{2}\left(\frac{r'}{r}\delta\varphi\right)^2 + \dots\right]$$

~~since~~ where $\delta r = r'\delta\varphi$, $\delta r^2 = r''\delta\varphi^2$

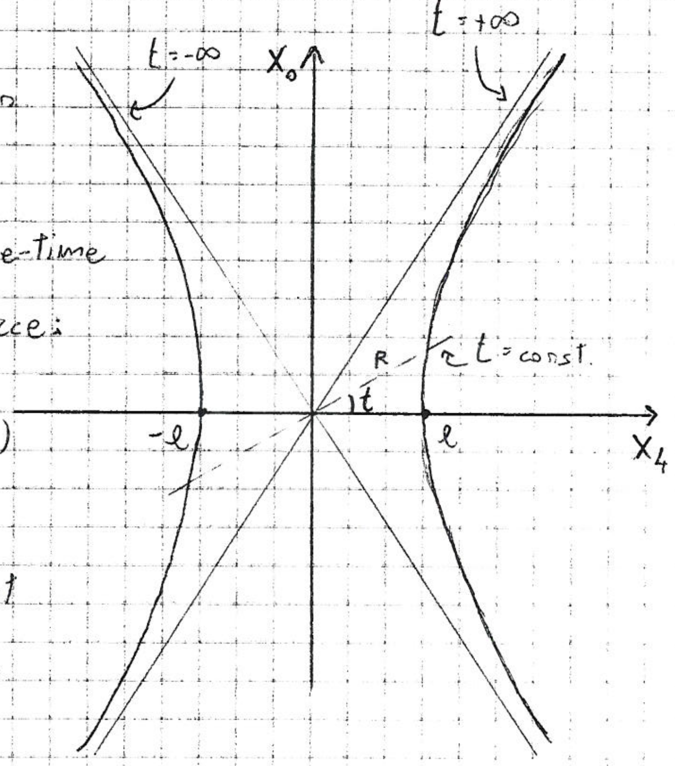
Now $\zeta_g = -\frac{1}{6}\frac{r'}{r}\delta\varphi$, and consequently

$$\zeta = \zeta_g - 5\cdot\frac{3}{5}\left[\zeta_g^2 - \cancelto{0}{\langle\zeta_g\rangle^2}\right]\cdot\left[\frac{r''r}{r'^2} - 1\right]$$

$$f_{NL} = -5\left[\frac{r''r}{r'^2} - 1\right]$$

Group 14 Problem 2 - Susskind

Let's start by writing the equation for an hyperboloid in 5 dimensions, which represent a 4-D de Sitter space-time embedded in a 5-D Minkowski space:



$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2 \quad (*)$$

The coordinates X_1, X_2, X_3 represent a 3-D euclidean space. Thus we can use spherical coordinates:

$$\begin{cases} X_1 = r\cos\theta_1 \\ X_2 = r\sin\theta_1\cos\theta_2 \\ X_3 = r\sin\theta_1\sin\theta_2 \end{cases}$$

In figure we represent the projection of the hyperboloid in the plain (X_4, X_0). We can use (t, R) like in figure to cover the entire hyperboloid, so:

$$\begin{cases} X_0 = R\sinh t \\ X_4 = R\cosh t \end{cases}$$

where $R = \sqrt{\ell^2 - r^2}$ follow from equation $(*)$.

It's now straightforward to find equal-time surfaces, precisely:

$$X_0 = tgh\, t \; X_4$$

The coordinate parameterization we have chosen is:

$$\begin{cases} X_0 = \sqrt{\ell^2 - r^2}\ \sinh t \\ X_1 = r\cos\theta_1 \\ X_2 = r\sin\theta_1 \cos\theta_2 \\ X_3 = r\sin\theta_1 \sin\theta_2 \\ X_4 = \sqrt{\ell^2 - r^2}\ \cosh t \end{cases}$$

Differentiating these we obtain:

$$\begin{cases} dX_0 = \sqrt{\ell^2 - r^2}\ \cosh t\ dt - \dfrac{r\sinh t}{\sqrt{\ell^2 - r^2}}\ dr \\ dX_4 = \sqrt{\ell^2 - r^2}\ \sinh t\ dt - \dfrac{r\cosh t}{\sqrt{\ell^2 - r^2}}\ dr \end{cases}$$

The missing equations give us the usual result:

$$dX_1^2 + dX_2^2 + dX_3^2 = r^2 d\Omega_2^2 + dr^2$$

So:

$$ds^2 = -dX_0^2 + dX_4^2 + r^2 d\Omega_2^2 =$$

$$= -(\ell^2 - r^2)\cosh^2 t\ dt^2 - \frac{r^2 \sinh^2 t}{(\ell^2 - r^2)}\ dr^2 + 2r\sinh t\cosh t\ dt\,dr +$$

$$+ (\ell^2 - r^2)\sinh^2 t\ dt^2 + \frac{r^2\cosh^2 t}{(\ell^2 - r^2)}\ dr^2 - 2r\sinh t\cosh t\ dt\,dr + dr^2 + r^2 d\Omega_2^2$$

$$\Rightarrow ds^2 = -(\ell^2 - r^2)\,dt^2 + \frac{\ell^2}{\ell^2 - r^2}\ dr^2 + r^2 d\Omega_2^2 ,$$

which is a static metric.

Group 14 Problem 3 - Silverstein

Derivation of the perturbed field equation for the scalar field:

$$\phi \to \phi(t) + \delta\phi(t, \vec{x})$$

Let's start with the field equation:

$$-V'(\phi) + \frac{4\phi^3}{\lambda\gamma} + 4\gamma\frac{X}{\phi} + \gamma\Box\phi - 8\lambda\gamma^3\frac{X^2}{\phi^5} + \frac{\lambda}{\phi^4}\gamma^3\phi_{;\mu\nu}\phi^{;\mu}\phi^{;\nu} = 0$$

with: $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$, $\gamma = \left(1 - \frac{2\lambda X}{\phi^4}\right)^{-1/2}$

$$\Rightarrow \bullet\ X = \frac{1}{2}\partial_\mu(\phi+\delta\phi)\partial^\mu(\phi+\delta\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \partial_\mu\phi\partial^\mu\delta\phi + O(\delta\phi^2)$$

$$= X^{(0)} + \partial_\mu\phi\partial^\mu\delta\phi + O(\delta\phi^2)$$

$$\bullet\ \gamma = \left[1 - \frac{2\lambda\left(X^{(0)} + \partial_\mu\phi\partial^\mu\delta\phi\right)}{(\phi+\delta\phi)^4}\right]^{-1/2} = \left[1 - \frac{2\lambda\left(X^{0} + \partial_\mu\phi\partial^\mu\delta\phi\right)}{\phi^4\left(1 + 4\delta\phi/\phi\right)}\right]^{-1/2} =$$

$$= \left[1 - \frac{2\lambda}{\phi^4}\left(X^{(0)} + \partial_\mu\phi\partial^\mu\delta\phi\right)\left(1 - \frac{4\delta\phi}{\phi}\right)\right]^{-1/2} =$$

$$= \left[1 - \frac{2\lambda X^{(0)}}{\phi^4}\left(1 + \frac{\partial_\mu\phi\partial^\mu\delta\phi}{X^{(0)}} - \frac{4\delta\phi}{\phi}\right)\right]^{-1/2} =$$

$$= \gamma^{(0)}\left[1 - \frac{\partial_\mu\phi\partial^\mu\delta\phi}{2X^{(0)}} + \frac{2\delta\phi}{\phi}\right]$$

$\Rightarrow$ The equation of motion, without background terms, is:

$$\begin{aligned}
&-V''(\phi)\delta\phi + \frac{4}{\lambda}\left(1 + 3\frac{\delta\phi}{\phi}\right)\phi^3 \cdot \frac{1}{\gamma^{(0)}}\left(1 + \frac{\partial_\mu\phi\partial^\mu\delta\phi}{2X^{(0)}} - \frac{2\delta\phi}{\phi}\right) + \\
&+ \frac{4\gamma^{(0)}}{\phi}\left(1 - \frac{\partial_\mu\phi\partial^\mu\delta\phi}{2X^{(0)}} + \frac{2\delta\phi}{\phi}\right)\left(1 - \frac{\delta\phi}{\phi}\right)X^{(0)}\left(1 + \frac{\partial_\mu\phi\partial^\mu\delta\phi}{X^{(0)}}\right) + \\
&+ \gamma^{(0)}\left(1 - \frac{\partial_\mu\phi\partial^\mu\delta\phi}{2X^{(0)}} + \frac{2\delta\phi}{\phi}\right)\Box(\phi+\delta\phi) - 8\lambda\frac{\gamma^{(0)3}X^{(0)2}}{\phi^5}\cdot\left(1 - \frac{3\partial_\mu\phi\partial^\mu\delta\phi}{2X^{(0)}} + \right. \\
&\left. + \frac{6\delta\phi}{\phi}\right)\left(1 + \frac{2\partial_\mu\phi\partial^\mu\delta\phi}{X^{(0)}}\right)\left(1 - \frac{5\delta\phi}{\phi}\right) + \frac{\lambda\gamma^{(0)3}}{\phi^4}\left(1 - \frac{3\partial_\mu\phi\partial^\mu\delta\phi}{2X^{(0)}} + \right. \\
&\left. + \frac{6\delta\phi}{\phi}\right)\left(1 - 4\frac{\delta\phi}{\phi}\right)(\phi+\delta\phi)_{;\mu\nu}(\phi+\delta\phi)^{;\mu}(\phi+\delta\phi)^{;\nu} =
\end{aligned}$$

$$-V''(\phi)\,\delta\phi + \frac{4\phi^3}{\lambda\gamma^{(0)}}\left(\frac{\delta\phi}{\phi} + \frac{\partial_\mu\phi\,\partial^\mu\delta\phi}{2X^{(0)}}\right) + \frac{4X^{(0)}\gamma^{(0)}}{\phi}\left(\frac{\delta\phi}{\phi} + \frac{\partial_\mu\phi\,\partial^\mu\delta\phi}{2X^{(0)}}\right) +$$

$$+\gamma^{(0)}\Box\delta\phi + \gamma^{(0)}\left(\frac{2\delta\phi}{\phi} - \frac{\partial_\mu\phi\,\partial^\mu\delta\phi}{2X^{(0)}}\right)\Box\phi - \frac{8\lambda X^{(0)2}\gamma^{(0)3}}{\phi^5}\left(\frac{\delta\phi}{\phi} +\right.$$

$$\left.+\frac{\partial_\mu\phi\,\partial^\mu\delta\phi}{2X^{(0)}}\right) + \frac{\lambda\gamma^{(0)3}}{\phi^4}\left(\frac{2\delta\phi}{\phi} - 3\frac{\partial_\mu\phi\,\partial^\mu\delta\phi}{2X^{(0)}}\right)\phi_{;\mu\nu}\phi^{;\mu}\phi^{;\nu} +$$

$$+\frac{\lambda\gamma^{(0)3}}{\phi^4}\left(\phi_{;\mu\nu}\phi^{;\mu}\delta\phi^{;\nu} + \phi_{;\mu\nu}\phi^{;\nu}\delta\phi^{;\mu} + \delta\phi_{;\mu\nu}\phi^{;\mu}\phi^{;\nu}\right) = 0$$

Dropping the $^{(0)}$'s, in a FRW universe with a metric of the form:

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2 d\Omega_2^2\right],$$

the above equation becomes:

$$-V''(\phi)\,\delta\phi + \left(\frac{4\phi^3}{\lambda\gamma} + \frac{4X\gamma}{\phi} - \frac{8\lambda X^2\gamma^3}{\phi^5}\right)\left(\frac{\delta\phi}{\phi} - \frac{\delta\dot\phi}{\dot\phi}\right) -$$

$$-\gamma\left(\delta\ddot\phi + 3H\delta\dot\phi - \frac{\nabla^2\delta\phi}{a^2}\right) - \gamma\left(\ddot\phi + 3H\dot\phi\right)\left(2\frac{\delta\phi}{\phi} - \frac{\delta\dot\phi}{\dot\phi}\right) +$$

$$+\frac{\lambda\gamma^3}{\phi^4}\dot\phi^2\ddot\phi\left(2\frac{\delta\phi}{\phi} - \frac{3\delta\dot\phi}{\dot\phi}\right) + \frac{\lambda\gamma^3}{\phi^4}\dot\phi\left(2\ddot\phi\,\delta\dot\phi + \dot\phi\,\delta\ddot\phi\right) = 0$$

with $X = -\frac{1}{2}\dot\phi^2$, $\gamma = \left(1 + \frac{\lambda\dot\phi^2}{\phi^4}\right)^{-1/2}$

$$\Rightarrow \left(\frac{\lambda\gamma^3}{\phi^4}\dot\phi^2 - \gamma\right)\delta\ddot\phi + \left(\cancel{\frac{2\lambda\dot\phi\ddot\phi}{\phi^4}\gamma^3} - \frac{3\lambda\dot\phi\ddot\phi}{\phi^4}\gamma^3 + \frac{\ddot\phi}{\dot\phi}\gamma + 3\cancel{H}\gamma - 3\cancel{H}\gamma +\right.$$

$$\left.+\frac{2\lambda\dot\phi^3}{\phi^5}\gamma^3 + \frac{2\dot\phi}{\phi}\gamma - \frac{4\phi^3}{\lambda\gamma\dot\phi}\right)\delta\dot\phi + \left(\frac{2\lambda\dot\phi^2\ddot\phi}{\phi^5}\gamma^3 - 2\frac{\ddot\phi}{\phi}\gamma - 6\frac{H\dot\phi}{\phi}\gamma +\right.$$

$$\left.+\frac{4\phi^2}{\lambda\gamma} - \frac{2\dot\phi^2}{\phi^2}\gamma - 2\lambda\frac{\dot\phi^4}{\phi^6}\gamma^3 - V''(\phi)\right)\delta\phi + \gamma\frac{\nabla^2\delta\phi}{a^2} = 0$$

$$\Rightarrow \left(\frac{\lambda\dot\phi^2}{\phi^4}\gamma^3 - \gamma\right)\delta\ddot\phi + \left(\gamma\frac{\ddot\phi}{\dot\phi} - \frac{\lambda\dot\phi\ddot\phi}{\phi^4}\gamma^3 + \frac{2\lambda\dot\phi^3}{\phi^5}\gamma^3 + 2\frac{\dot\phi}{\phi}\gamma - \frac{4\phi^3}{\lambda\gamma\dot\phi}\right)\delta\dot\phi +$$

$$+\left(\frac{2\lambda\dot\phi^2\ddot\phi}{\phi^5}\gamma^3 - \frac{2\ddot\phi}{\phi}\gamma - 6\frac{H\dot\phi}{\phi}\gamma + \frac{4\phi^2}{\lambda\gamma} - \frac{2\dot\phi^2}{\phi^2}\gamma - \frac{2\lambda\dot\phi^2}{\phi^6}\gamma^3 - V''(\phi)\right)\delta\phi +$$

$$+\gamma\frac{\nabla^2\delta\phi}{a^2} = 0$$

Silverstein 3 | Group 14

$$S = -\int d^4x \sqrt{-g}\left[\frac{\varphi^4}{\lambda}\sqrt{1+\lambda(\partial_\mu\varphi)^2\varphi^{-4}} - V(\varphi)\right] \qquad (1)$$

we first consider homogenious solutions:

$$ds^2 = -dt^2 + a(t)^2 dx^2, \quad \varphi = \varphi(t)$$

in this case the equation of motion for φ reads

$$\ddot{\varphi} - 6\frac{\dot{\varphi}^2}{\varphi} + \frac{4\varphi^3}{\lambda} + \frac{3H}{\gamma^2}\dot{\varphi} - V'\frac{1}{\gamma^3} = 0 \qquad (2)$$

and the Friedman equations are

$$\rho = \frac{\varphi^4}{\lambda}\cdot\gamma - V = 3H^2 \qquad (3,4)$$

$$-p = \frac{\varphi^4}{\lambda}\gamma^{-1} - V = 3H^2 + 2\dot{H}$$

, where

$$\gamma = \sqrt{1-\lambda\dot{\varphi}\varphi^{-4}}^{\,-1}, \quad H = \frac{\dot{a}}{a} \text{ and } M_{pl} \equiv 1$$

we will now make a conjecture, that inflation happens when γ is large.

To check it, let us denote by $\zeta(t)$

$\zeta = \varphi - \frac{\sqrt{\lambda}}{t}$, so that when $\zeta \to 0 \Rightarrow \gamma^{-1} \to \infty$

Then for the slow roll parameter ε we get

$$\frac{|\dot{H}|}{H^2} = + \frac{\rho + p}{\rho} = \frac{-\gamma^{-2} + 1}{1 + V\lambda\varphi^{-4}\gamma^{-1}} = \varepsilon \qquad (5)$$

Because among 3 eq:s (2, 3, 4) only two are independent it is enough to check that the assumption $\zeta \ll \varphi$ is consistent with (5) and (2).

$$\gamma^{-1} = \sqrt{\frac{2}{\sqrt{\lambda}} t^2 \dot{\zeta} + \frac{4}{\sqrt{\lambda}} + \zeta} + O(\zeta^2) \qquad (6)$$

now neglecting terms $\sim \zeta^2$ in (5) we get $\gamma^{-1} = \dfrac{1}{V\lambda\varphi^{-4}}\left(\dfrac{1}{\varepsilon} - 1\right)$.

To proceed, we assume a simple form of potential $V = V_2\varphi^2$; ~~and also that~~

now for ζ we get the equation

$$\frac{2}{\sqrt{\lambda}}t^2\dot{\zeta} + \frac{4}{\sqrt{\lambda}}t\zeta = \left(\frac{1}{\varepsilon}-1\right)^2\frac{1}{V_2^2\lambda^2t^4} \qquad (7)$$

where we neglected terms $\sim\zeta$ in r.h.s. because there're already enough powers of t in denominator. (7) has a solution:

$$\zeta = \frac{c}{t^2} - \frac{1}{6}\left(\frac{1}{\varepsilon}-1\right)^2\frac{1}{V_2^2\lambda^{3/2}}\cdot\frac{1}{t^5}$$

Substituting this in (2) we get

$$\cancel{\ddot{\zeta} + \frac{12}{t}\dot{\zeta} + \frac{18}{t^2}\zeta +}$$

$$\ddot{\zeta} + \frac{12}{t}\dot{\zeta} + \frac{18}{t^2}\zeta - 3\left(\frac{1}{\varepsilon}-1\right)^2 \frac{\sqrt{\lambda}}{V_2^2\lambda^2 t^6} H - 2V_2 \frac{\sqrt{\lambda}\left(\frac{1}{\varepsilon}-1\right)^3}{V_2^3\lambda^3 t^7}$$

for H in the first order we have

$H = \frac{1}{\varepsilon t}$, and for the case $\frac{1}{\varepsilon} \gg 1$

we arrive at the following equation:

$$36\varepsilon - 3 - \frac{2}{\lambda} = 0 \ .$$

This is a pretty weird result!

ε can really be made small, but λ needs to be tuned to achieve this.

Probably I've screwed up smth. in algebra, but anyway, if V_2 is large enough ζ is in fact $\ll \varphi$, and it behaves as t^{-5} (c ~~never~~ always cancells out, so we can set it to 0), so

$\frac{\sqrt{\lambda}}{t}$ and $\frac{1}{\varepsilon t}$ really become asymptotics

for φ and H at late times meaning

that we get a power-law inflation.